GIRRAWEEN HIGH SCHOOL

Year 11 Mathematics Extension Task 2 June 2024

Time Allowed: 90 minutes

Instructions:

- There are 11 questions in this paper. All questions are compulsory.
- Use black or blue pen.
- For Questions 1-5, colour in the bubble next to the letter corresponding to the correct response on your answer paper.
- Start each question (6 -11) on a new page.
- Show all necessary working.
- NESA approved calculators may be used.
- Mathematics Reference sheets are provided.
- Diagrams are NOT to scale.
- Marks may be deducted for careless or badly arranged work.

Section 1: Multiple Choice questions(5 marks)

Question 1

Given $f(x) = 1 + \sqrt{x}$, what are the domain and range of $f^{-1}(x)$

(A) $x \geq 0, y \geq 0$
(B) $x \geq 0, y \geq 1$
(C) $x \geq 1, y \geq 0$
(D) $x \geq 1, y \geq 1$

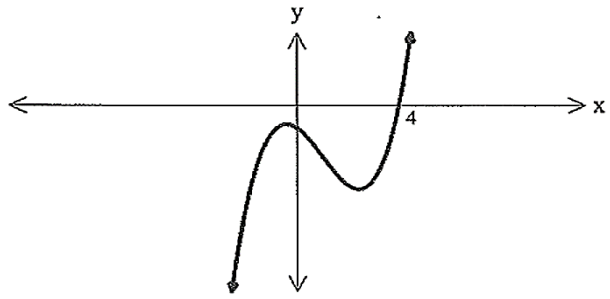
Question 2

A curve is defined parametrically by the equations $x = \sin\theta,\ y = 2\cos\theta$

What is the Cartesian equation of the curve?

(A) $4x^2 + y^2 = 4$
(B) $4x^2 + y^2 = 1$
(C) $x^2 + y^2 = 1$
(D) $x^2 + y^2 = 4$

Question 3



The graph of the polynomial shown above, could be:

(A) $y = (x-4)(x^2+x+1)$
(B) $y = (x+4)(x^2+x+1)$
(C) $y = (4-x)(x^2+x+1)$
(D) $y = (x-4)^3$

Examination continues on the following page.

Question 4

A polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has real roots at x_1, x_2 and x_3, which are all distinct values of x.

Given that $P'(x_1) = 0, P'(x_2) = P''(x_2) = 0$ and $P'(x_3) = P''(x_3) = P'''(x_3) = 0$,

What is the lowest possible degree of $P(x)$?

(A) $n = 9$
(B) $n = 7$
(C) $n = 6$
(D) $n = 3$

Question 5

When a polynomial $(3x^2 + 8x - 3)$ is multiplied by $(px - 1)$and the resulting product is divided by $(x + 1)$ the remainder is 24. What is the value of p ?

(A) -4
(B) 2
(C) 4
(D) $\frac{11}{4}$

Examination continues on the following page.

Question 6 (10 marks) (Start solutions on a new page)

a. The parametric equations of a curve are $x = 4 - t, y = (1 - t)^2$.
Find the Cartesian equation of the curve, simplifying your answer. 2 marks

b. Given the function $f(x) = x^2 - 2x - 1, x \leq 1, y \geq -2$

i. State the domain and range of $f^{-1}(x)$. 2 marks

ii. Find $f^{-1}(x)$ 3 marks

iii. Show that $f(x)$ and $f^{-1}(x)$ intersect where $x = \frac{3-\sqrt{13}}{2}$ 3 marks

Examination continues on the following page.

Question 7 (12 marks) (Start solutions on a new page)

a.

i. Show that $x = 4$ is a root of the equation $2x^3 - x^2 - 32x + 16 = 0$. 1 mark

ii. Hence, solve the equation $2x^3 - x^2 - 32x + 16 = 0$. 3 marks

b. If α, β and γ are the roots of $3x^3 + 8x^2 + 1 = 0$, find the values of

i. $\alpha + \beta + \gamma$ 1 mark

ii. $\alpha\beta + \beta\gamma + \alpha\gamma$ 1 mark

iii. $\alpha\beta\gamma$ 1 mark

iv. $(\alpha - 1)(\beta - 1)(\gamma - 1)$ 2 marks

v. $(\alpha\beta)^{-2} + (\alpha\gamma)^{-2} + (\beta\gamma)^{-2}$ 3 marks

Examination continues on the following page.

Question 8 (11 marks) (Start solutions on a new page)

a. A committee of 3 students is to be chosen from a group consists of 5 girls and 4 boys. What is the probability that 2 boys will be selected?

2 marks

b. Consider the digits 7,6,5,4,3,2,1 and 0. Find how many four-digit numbers are possible if the digits are in:

i. descending order. 1 mark

ii. ascending order. 2 marks

c. Students completing Legal Studies for the NSW HSC are awarded an integer mark between zero and 100, inclusive. In the year 2021, a total of 10935 students completed Legal Studies. What is the largest number of students that were guaranteed to receive the same mark? 2 marks

d. Eight students, including Ari, Bobbi and Cali, form a single line and walk into their classroom.

i. How many ways can the students walk into the classroom if Ari and Bobbi are next to each other, with no students in between? 2 marks

ii. When the students enter the classroom, they are seated around a circular table. How many ways can the students be seated around the table if Bobbi and Cali cannot sit next to each other? 2 marks

Examination continues on the following page.

Question 9 (12 marks) (Start solutions on a new page)

a. Write $\frac{1}{(n+1)!} - \frac{1}{(n-1)!}$ as a single fraction. 2 marks

b. All the letters of the word DEFINITION are arranged in a row.

i. How many distinct arrangements are possible? 1 mark

ii. In how many of these arrangements are all the vowels together? 2 marks

c. A netball team's record for the 2022 season was 16 wins and 4 losses. None of their games were drawn. Prove that the team must have won at least 4 games in a row somewhere during the season. 2 marks

d. At a particular dinner, each rectangular table has nine seats, five facing the stage and four with their backs to the stage.

i. In how many ways can 9 people be seated if John and Mary sit on the same side? 2 marks

ii. What is the probability of John and Mary sitting on the opposite sides of the table? 3 marks

Examination continues on the following page.

Question 10 (10 marks) (Start solutions on a new page)

a. Solve for x.

$\frac{4x-1}{x-1} \geq 3,\ x \neq 1$ 3 marks

b. The polynomial $P(x) = x^3 - 4x^2 - 3x + 18$ has a double zero.

i. Find $P'(x)$. 1 mark

ii. Hence, find the double zero. 2 marks

iii. Find the remaining simple zero. 2 marks

c. A polynomial $P(x)$ has a remainder of $2x + 1$ when divided by $x^2 - 5x + 6$.

What is the remainder when $P(x)$ is divided by $x - 3$? 2 marks

Examination continues on the following page.

Question 11 (10 marks) (Start solutions on a new page)

a. A monic polynomial $P(x)$ of degree 4 has a single root at $x = 2$ and a double root at $x = -1$. If the polynomial passes through the point (0,6) find the equation of $P(x)$.

2 marks

b.

i. Sketch the graph of $P(x) = -x(x-3)^3$ 2 marks

ii. Hence, solve $P(x) > 0$ 1 mark

c. Given α and β are the roots of the equation $x^2 - bx + c = 0$.

i. Show that $(\alpha^2 + 1)(\beta^2 + 1) = (c-1)^2 + b^2$ 2 marks

ii. Show, in terms of b and c, the quadratic equation whose roots are $\frac{\alpha}{\alpha^2+1}$ and $\frac{\beta}{\beta^2+1}$ is given by:

$[(c-1)^2 + b^2]x^2 - b(c+1)x + c = 0$ 3 marks

END OF EXAMINATION.