Girraween High School

2021 Year 11 Mathematics Extension 1 Task 2

General Instructions

- Working Time - 90 minutes
- There are 10 questions in this paper. All questions are compulsory
- Write using black or blue pen only
- For Questions 1-5, color in the bubble next to the letter corresponding to the correct response on your paper.
- Start each question (6-10) on a new page, showing all the necessary working
- Calculators approved by NESA may be used
- Mathematics reference sheets are provided
- Marks may be deducted for careless or badly arranged work

Section I

Use the multiple-choice answer sheet for question 1-5. [5 marks]

Question 1 (1 mark)

A university offers 6 courses in the morning and 4 in the evening. The possible number of choices that a student can make if he wants to study one course in the morning and one in the evening is:

A. 24 B. 2 C. 12 D. 10

Question 2 (1 mark)

The number of ways of arranging 8 people in a straight line:

A. 8! B. ${}^8C_8$ C. $\dfrac{8!}{4 \times 2!}$ D. ${}^8P_4$

Question 3 (1 mark)

The number of combinations of 10 different things taking 5 at a time when 2 particular things are always to be included:

A. ${}^{10}C_2$ B. ${}^8C_2$ C. ${}^8C_3$ D. ${}^{10}C_5 - 2$

Question 4 (1 mark)

If $(x + 2)$ is a factor of $P(x) = x(x + 6)(x - 1) - k$, find the value of k:

A. 24 B. 16 C. -16 D. 0

Question 5 (1 mark)

The remainder when $P(x) = 2x^3 - 6x^2 + 4x + 3$ is divided by $(2x - 1)$?

A. 3 B. 117 C. $2\dfrac{25}{27}$ D. $3\dfrac{3}{4}$

Y11 Mathematics Extension

Section II - 75 marks

Attempt Questions 6-10

Allow about 1 hours and 20 minutes for this section

Question 6 (17 marks)

(a) State if the following are polynomials are not, state why.

i. $y = x\sqrt{x} + 1$ [2 marks]

ii. $y = 2x^4 + x + 1$ [2 marks]

iii. $y = 5^x - 5x - 1$ [2 marks]

(b) Consider the polynomial $P(x) = 2x^3 - 6x^2 - 18x - 10$.

i. Show that $x = -1$ is a root of equation $P(x) = 0$ [1 mark]

ii. Hence, use method of long division to factorise $P(x)$ [3 marks]

iii. Sketch $P(x)$ clearly showing all the intercepts [2 marks]

iv. Hence, solve $P(x) > 0$ for all x. [1 mark]

(c) The polynomial $P(x) = x^3 - 3ax^2 + 3a^2x - a^3 + b$ has a zero at $x = 1$, and when divided by x, the remainder is -7. Find all the possible values of the pair a, b. [4 marks]

Question 7 (13 marks)

(a) If α, β, γ are the roots of the equations $2x^3 + 3x^2 - 2x + 1 = 0$. Find the value of:

i. $\alpha + \beta + \gamma$ [1 mark]

ii. $\alpha\beta\gamma$ [1 mark]

iii. $\alpha\beta + \beta\gamma + \alpha\gamma$ [1 mark]

iv. $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ [2 marks]

v. $\alpha^2 + \beta^2 + \gamma^2$ [2 marks]

vi. $(\alpha - 1)(\beta - 1)(\gamma - 1)$ [2 marks]

(b) Solve the equation $5x^3 - 21x^2 - 21x + 5 = 0$ if one root is a reciprocal of the other. [4 marks]

Question 8 (16 marks)

(a) The polynomial $P(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$ has a triple zero.

i. Find $P'(x)$, $P''(x)$ and $P'''(x)$. [3 marks]

ii. Hence, find the triple zero of $P(x)$ [4 marks]

iii. Hence, factor $P(x)$ fully. [1 mark]

(b) Simplify $\frac{1}{n!} + \frac{1}{(n-2)!}$. [2 marks]

(c) How many 6 digit odd numbers greater than 600 000 can be formed from the digits 5, 6, 7, 8, 9, 0 if

i. repetitions are not allowed? [3 marks]

ii. repetitions are allowed? [3 marks]

Question 9 (14 marks)

(a) In how many ways can the letters of the word "PARALLELOPIPED" can be arranged if:

i. there are no restrictions. [1 mark]

ii. all the P's are together. [2 marks]

iii. the two A's have to be separate. [2 marks]

iv. All the vowels are together. [2 marks]

(b) A committee of four students is to be formed from a total of 12 students. In how many ways can the committee be selected if

i. there are no restrictions. [1 mark]

ii. two particular students have to be included in the committee? [2 marks]

iii. two particular students do not wish to be together in the committee? [1 mark]

iv. two particular students wish to be included together only? [3 marks]

Question 10 (15 marks)

(a) How many ways can 10 people be seated around a round table if:

i. there are no restrictions. [1 mark]

ii. Sri and Ash must be next to each other [2 marks]

iii. Sri and Ash must not be next to each other [1 mark]

iv. Sri sits directly opposite Ash [1 mark]

v. none of Sri, Ash and Nate are next to each other [2 marks]

(b) Out of 30 consecutive integers, two integers are drawn at random. Find the probability that their sum is odd. [2 marks]

(c) i. The are 7 pairs of socks in a drawer, one of each color of the rainbow. How many socks have to be drawn out in order to guarantee that there is at least one matching pair? [2 marks]

ii. If there are 7 pairs of socks and 7 pairs of gloves, one of each colour of the rainbow and one cannot tell the difference between gloves and socks. How many socks and/or gloves have to drawn to guarantee that there is at least one matching set of both socks and gloves? [2 marks]

(d) Find the number of ways of dividing 52 cards among 4 players equally such that each gets exactly one Ace. [2 marks]

End of exam