Girraween High School

2018 Year 11 Mathematics Extension 1

Task 2

General Instructions

- Working Time - 1 hour & 30 minutes
- Calculators and ruler may be used
- All necessary working out must be shown
- Write on one side of the paper only

Total Marks - 84

- Attempt all questions
- Marks may be deducted for careless or badly arranged work
- Start each question on a new sheet of paper

Question 1 (1 mark)

How many ways can n people be seated at a round table?

A. $n-1$ B. $(n-1)!$ C. $n!$ D. $(n+1)!$

Question 2 (1 mark)

If θ is the acute angle between the lines $x=1$ and $y=2x+1$ then the value of $\tan\theta$ is:

A. 2 B. $\frac{1}{2}$ C. -2 D. $-\frac{1}{2}$

Question 3 (1 mark)

The letters of the word KITTEN are shuffled, what is the probability that the two T's are together?

A. $\frac{1}{3}$ B. $\frac{1}{6}$ C. $\frac{2}{3}$ D. $\frac{1}{2}$

Question 4 (1 mark)

The exact value of $\cos 75^\circ$ is

A. $\frac{1-\sqrt{3}}{\sqrt{2}}$ B. $\frac{\sqrt{3}-1}{\sqrt{2}}$ C. $\frac{1-\sqrt{3}}{2\sqrt{2}}$ D. $\frac{\sqrt{3}-1}{2\sqrt{2}}$

Question 5 (1 mark)

Which of the following would imply that A is the midpoint of PB?

A. P divides AB internally in the ratio $1:1$

B. P divides AB externally in the ratio $1:1$

C. P divides AB externally in the ratio $1:2$

D. P divides BA externally in the ratio $1:2$

Question 6 on the next page

Question 6 (13 marks)

(a) Find the exact value of:

i. $\sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ$ [1]

ii. $2\cos^2 15^\circ - 1$ [1]

iii. $\dfrac{\tan 45^\circ - \tan 15^\circ}{1 + \tan 45^\circ \tan 15^\circ}$ [1]

iv. $\cos 75^\circ$ [2]

(b) Let A be the point $(3, -1)$ and B be the point $(9, 2)$. Find the coordinates of the point P which divides the interval AB externally in the ratio $5 : 2$. [2]

(c) Find the acute angle between the lines $2x + 2y = 5$ and $y = 3x + 1$ to the nearest minute. [3]

(d) The line $y = mx$ is inclined at 45° to the line $y = 2x + 1$. Find all possible values of m. [3]

Question 7 (22 marks)

(a) If $t = \tan \dfrac{\theta}{2}$ find $\cot\theta + \operatorname{cosec}\theta$ in terms of t. [2]

(b) If $\sin\alpha = \dfrac{3}{5}$ and α is acute, find the exact values of:

i. $\cos 2\alpha$ [1]

ii. $\sin 2\alpha$ [2]

iii. $\tan 4\alpha$ [2]

(c) The letters A, B, C, D, E, F are to be arranged in a circle. How many ways can this be done if:

i. there are no restrictions [1]

ii. A and B are together [2]

iii. A and B are not together [1]

iv. A is directly opposite B [1]

v. none of the three letters A, B and C are together [2]

vi. only four letters are to be used [2]

(d) A hand of five cards is drawn from a standard deck of 52 cards. Find the probability of obtaining a hand that:

i. contains four aces [2]

ii. consists of only one suit [2]

iii. contains exactly two cards of the same suit [2]

Question 8 on the next page

Question 8 (11 marks)

Solve for x where $0° \leq x \leq 360°$

(a) $\sin(x - 300°) = \dfrac{1}{2}$ [2]

(b) $\sin 2x = \cos x$ [3]

(c) $\cos 2x + \sin x = 0$ [3]

(d) $-2\sin^2 2x + \cos 2x + 1 = 0$ [3]

Question 9 (20 marks)

(a) i. Write $\cos x - \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° \leq \alpha \leq 360°$. [3]

ii. Hence solve $\cos - \sin x = \dfrac{1}{\sqrt{2}}$ for $0° \leq x \leq 360°$ [3]

(b) Solve $2\sin x - \cos x = 1$ for $0° \leq x \leq 360°$ using the t-method [4]

(c) Prove the following identities:

i. $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$ [3]

ii. $\dfrac{1 - \tan^2 x}{1 + \tan^2 x} = \cos 2x$ [3]

iii. $\sin 2x - \cos 2x \tan x = \tan x$ [4]

Question 10 (13 marks)

(a) Find the ratio in which the point $(1, 2)$ divides the line segment connecting the points $(-2, -1)$ and $(2, 3)$. [3]

(b) Three people are randomly seated in a row of six seats. What is the probability that no two of them are seated next to each other? [3]

(c) Prove that $(\sin^3\theta + \cos^3\theta)(\cos\theta - \sin\theta) = \cos 2\theta - \dfrac{1}{4}\sin 4\theta$ [4]

(d) Show that $\sin\theta\left(\cot\theta - \cot\dfrac{\theta}{2}\right)$ is independent of θ [3]

End of exam