Mrs Cooper	Mr Feng	**Name:** ……..
Mrs Greenberg	Mrs Kerr	
Ms Lau	Mrs Squires	**Teacher:**……

MATHEMATICS EXTENSION 1

Year 11 ASSESSMENT TASK 1

Term 1 2023

General Instructions

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using black or blue pen
- Erasable pens are not to be used
- Calculators approved by NESA may be used
- Reference sheets are provided

Total marks
30

SECTION 1 – 5 marks

- Attempt Questions 1-5

SECTION II – 25 marks

- Attempt all Questions
- Answer each question in the appropriate space in the Answer Booklet.

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SECTION I

Multiple Choice (5 marks)

Answer using the Multiple-Choice Answer Sheet.

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1 What values of k satisfy $(2k+1)^2 - 4(2k+1) \geq 12$?

(A) $k \leq -2$ or $k \geq 6$

(B) $-2 \leq k \leq 6$

(C) $\frac{-3}{2} \leq k \leq \frac{5}{2}$

(D) $k \leq \frac{-3}{2}$ or $k \geq \frac{5}{2}$

2 What is the domain and range of $y = \frac{-2}{\sqrt{x+1}}$ respectively?

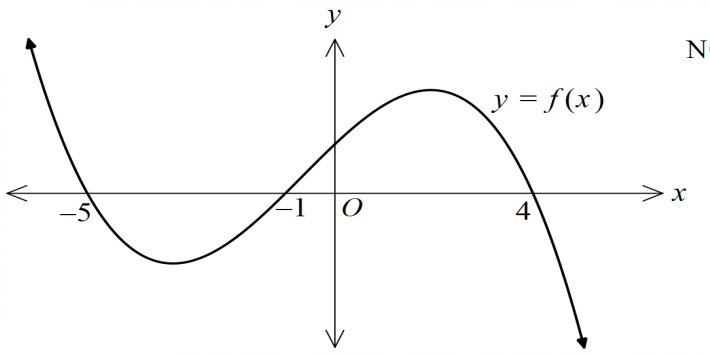
(A) $x \in (-\infty, -1]$ and $y \in (0, \infty)$

(B) $x \in (-1, \infty)$ and $y \in (-\infty, 0)$

(C) $x \in [-1, \infty)$ and $y \in (-\infty, 0]$

(D) $x \in (-\infty, -1)$ and $y \in [0, \infty)$

3 The graph of $y = f(x)$ is given below.



NOT TO SCALE

What are the values of x, such that $f(-x) < 0$?

(A) $-5 < x < -1$ or $x > 4$

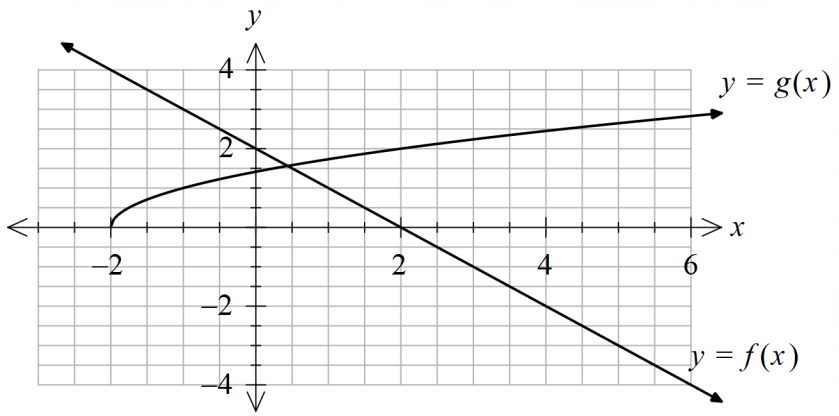
(B) $x < -4$ or $1 < x < 5$

(C) $x < -5$ or $-1 < x < 4$

(D) $-4 < x < 1$ or $x > 5$

4 Given below are the graphs of $y = f(x)$ and $y = g(x)$.

What is the domain and range of $y = f(x) - g(x)$?



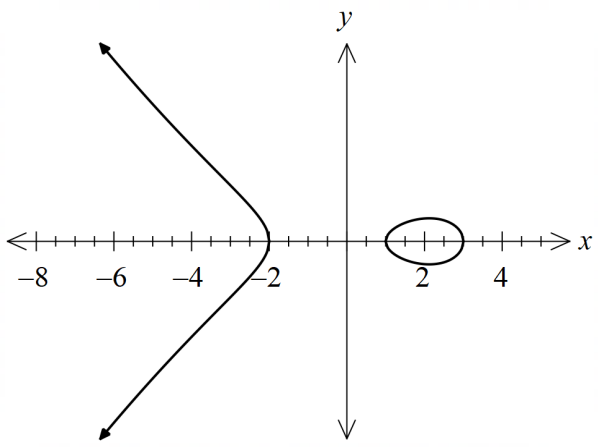
(A) Domain $x \geq -2$, Range $y \in \mathbb{R}$

(B) Domain $x \in \mathbb{R}$, Range $y \leq -2$

(C) Domain $x \leq -2$, Range $y \leq 4$

(D) Domain $x \geq -2$, Range $y \leq 4$

5 Given the graph of $y^2 = f(x)$, which of the following is the possible equation of $y = f(x)$?



(A) $y = -(x+2)(x-1)(x-3)$

(B) $y = (x+2)(x-1)(x-3)$

(C) $y = -(x-3)(x-1)(x+2)^2$

(D) $y = (3-x)(x+1)(x+2)$

SECTION II
25 Marks

Answer these questions in the Answer Book provided

Your responses should include relevant mathematical reasoning and/or calculations.

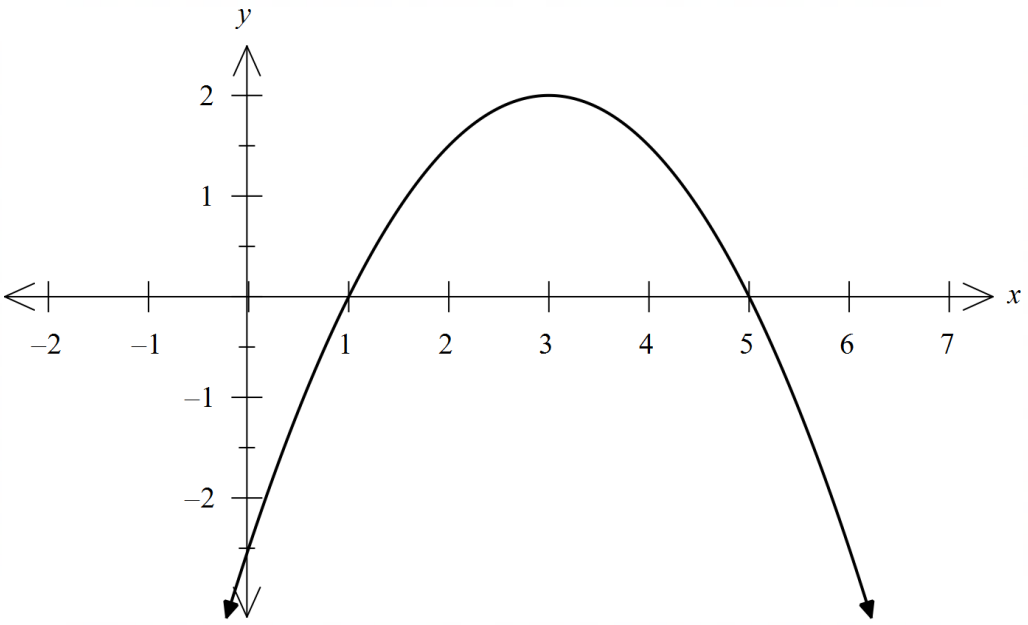
Question 6 (9 marks) **Marks**

(a) Given $f(x)=\dfrac{4x}{(x-1)^2}$, show that $f(x)=f\left(\dfrac{1}{x}\right)$. **2**

(b) Solve $\dfrac{2}{2x-1}\geq 1$. **2**

(c) The graph of $y=f(x)$ is drawn below. **2**

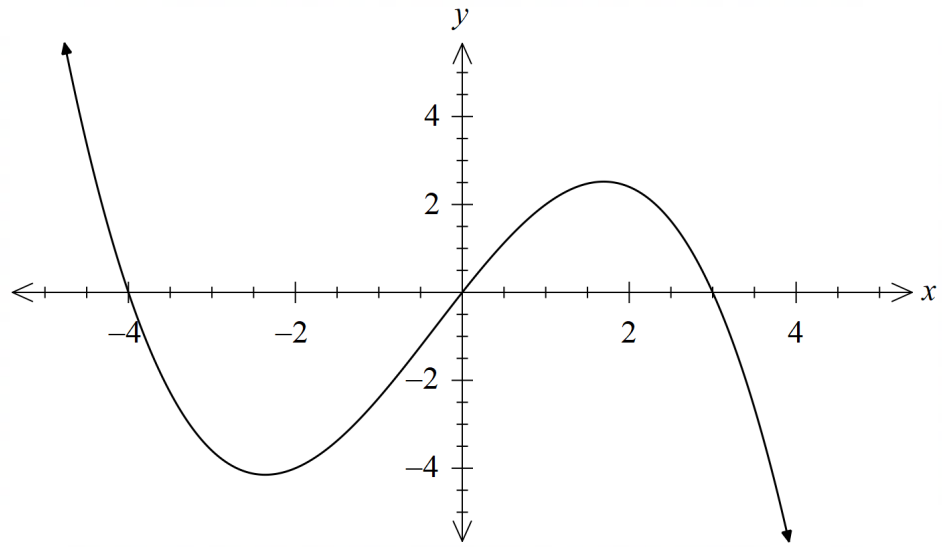
On the number plane provided in your answer booklet, draw the graph of $y=\dfrac{1}{f(x)}$, showing all important features.



(d) Find the values of p and q such that $\dfrac{(0.5)^4\times 9^{\frac{-3}{2}}}{12^{-\frac{1}{2}}}=2^p3^q$. **3**

Question 7 (8 marks) **Marks**

(a) The graph of $y = f(x)$ is drawn below.



On the number planes provided in your answer booklet, draw the graphs of the following relations showing all important features.

(i) $y = f(|x|)$ **2**

(ii) $y = |f(x)|$ **1**

(iii) $y = \sqrt{f(x)}$ **2**

(b) Solve $3^{2x-1} = 10\left(3^{x-1}\right) - 3$. **3**

Question 8 (8 marks) **Marks**

(a) (i) Sketch $y = |2x - 5|$. **2**

(ii) Find the value of *k,* such that $|2x - 5| - 4 = k$ has exactly one solution. **1**

(iii) Hence, or otherwise, solve $|2x - 5| > 5$. **2**

(b) A function is defined by $h(x) = \dfrac{x}{x-1}$; $x \neq 1$.

(i) If $h(h(x))$ and $h(h(h(x)))$ are denoted by $h^2(x)$ and $h^3(x)$ respectively, find the expressions for $h^2(x)$ and $h^3(x)$. **2**

(ii) Evaluate $h^{2023}(-1)$. **1**

END OF ASSESSMENT TASK

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NSW Education Standards Authority

2022 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

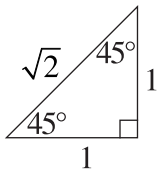
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

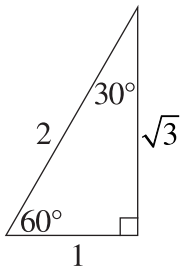
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

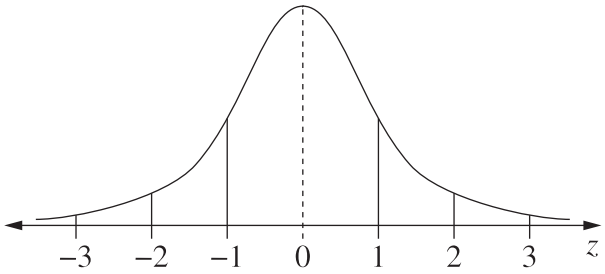
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow\ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

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