Name: ..... **Maths Class:** .....

SYDNEY TECHNICAL HIGH SCHOOL

Year 10
Mathematics

2019

Yearly Exam

Time allowed: 70 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- Write using black or blue pen
- Write your answers in the space provided

Multiple choice Questions 1-5	**/5**
Question 6	**/11**
Question 7	**/12**
Question 8	**/12**
Question 9	**/12**
Question 10	**/12**
TOTAL	**/64**

Section 1 Multiple Choice

Colour the bubble of the correct answer on your multiple choice answer sheet in your answer booklet

1. Given $a = 3, b = -2; \quad ab^2 =$

A. 16 B. 22 C. 12 D. -26

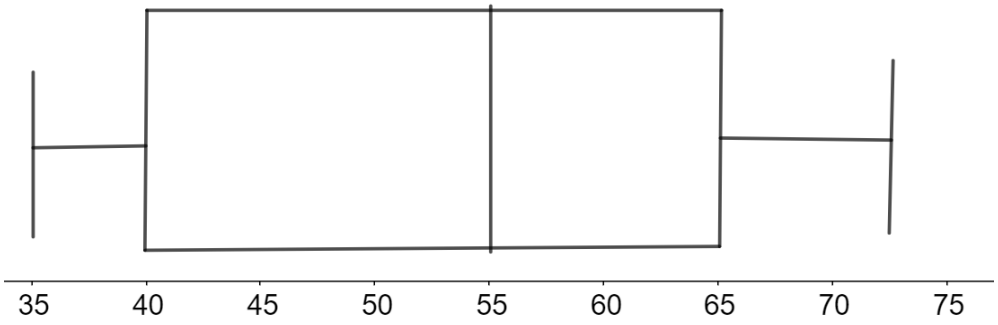
2. After 3 Maths tests Andy's mean score was 73%. In his next test Andy scored 81%. Andy's new mean is calculated to be

A. 48.5% B. 75% C. 76% D. 74%

3. At a sale, Kay buys shoes for $95. The price before the sale was $120. The sale price represents a saving of approximately

A. 21% B. 15% C. 28% D. 70%

4. The results of a test are displayed in the box-and-whisker plot.



Which of the following statements is **false**?

A. 50% of the scores lie between 40 and 50. B. 50% of the scores are above 55.

C. 25% of the scores are below 40. D. 75% of the scores are below 65.

5. The solutions to $(3x + 1)(5x - 2) = 0$ are:

A. $x = -2$ or $x = 1$ B. $x = -\frac{2}{3}$ or $x = \frac{1}{5}$

C. $x = -\frac{3}{2}$ or $x = 5$ D. $x = \frac{2}{5}$ or $x = -\frac{1}{3}$

Section 2

All answers are to be completed in your answer booklet. Show all necessary working

Question 6 (Start a New Page) **(11 marks)**

a) For the points $A\ (2, 2)$ and $B\ (1, -3)$, find:

i) The distance of the line AB (to two decimal places). **1**

ii) The midpoint of AB. **1**

iii) The gradient of AB. **1**

b) Fully factorise $2x^2 - 9x + 9$ **1**

c) For the scores 1, 3, 4, 6, 8, 3, find

i) the mean, correct to 2 decimal places **1**

ii) the standard deviation, correct to 1 decimal place **1**

d) The two way table below represents how many students like Barcelona and Real Madrid.

	Like Barcelona	Do not like Barcelona	Total
Like Real Madrid	10	12	22
Do not like Real Madrid	15	5	20
Total	25	17	42

For a randomly selected student, what is the probability that:

i) The student likes Barcelona? **1**

ii) The student likes both Barcelona and Real Madrid? **1**

iii) The student likes only 1 team out of Barcelona and Real Madrid? **1**

e)

stem	leaf			
0	2	4	4	5
1	0	?	7	
2	1	2		
3	0			

2 | 6 represents 26

One digit is missing from the above stem and leaf plot. The median of all scores is 13. **2**

Find the missing digit.

End of Question 6

Question 7 (Start a New Page) **(12 marks)**

a) Find the simple interest (to the nearest cent) earned on \$788 invested at 1.5% p.a. for 200 days. **1**

b) A new computer costs \$2000. If it depreciates by 15% per year, what will the computer be worth (to the nearest dollar) in 5 years? **2**

c) i) Given $xy + 3y - x = 0$, make y the subject. **2**

ii) What value(s) can m not take in the equation $g = \dfrac{m}{m+1}$? **1**

d) Solve these simultaneous equations using the elimination method. **2**

$3x - 7y = 13$

$4x + 5y = 3$

e) Solve $3x^2 - 5x - 1 = 0$, leave your answer in an exact form. **2**

f) Given that $\sin\theta = -\dfrac{2}{5}$ and $\cos\theta$ is positive, find the exact value of $\tan\theta$. **2**

End of Question 7

Question 8 (Start a New Page) **(12 marks)**

a) Consider $y = \dfrac{1}{x - 1}$,

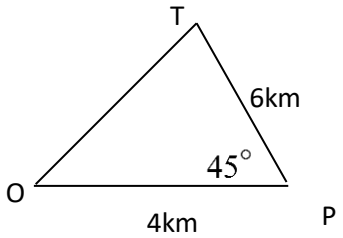
i) State the equation(s) of the asymptote(s). **2**

ii) Find the y-intercept. **1**

iii) Sketch the graph of $y = \dfrac{1}{x - 1}$, showing all the above features. **2**

b) Find the equation of the line perpendicular to $y = -2x + 1$, which passes the point $(-2, 5)$. **2**
Express your answer in general form.

c) A triangular piece of property is pegged out for cattle to graze, as shown. **2**
Find the exact area of this grazing land.



d) Find the exact value of

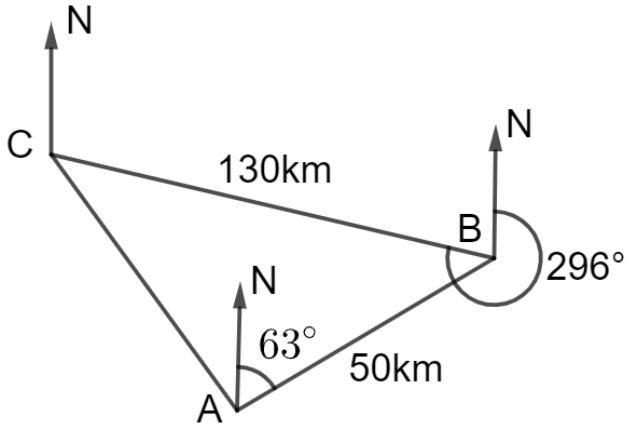
(i) $\cos 210°$. **1**

(ii) $\tan^2 210° - \sin 135°$ **2**

End of Question 8

Question 9 (Start a New Page) **(12 marks)**

a) A ship sails 50 km from Port A to Port B on a bearing of $063°$, then sails 130 km from Port B to Port C on a bearing of $296°$.



i) Show that $\angle ABC = 53^\circ$ **1**

ii) Find, to the nearest km, the distance of Port A to Port C. **2**

iii) Find the bearing of Port A from Port C, correct to the nearest degree. **3**

b) Solve the simultaneous equations below. **3**

$y = x^2 + 1$

$6x - y = 8$

c) (i) Consider the equation $(x - 2)^2 + (y + 1)^2 = 9$, state the centre and radius. **2**

(ii) Shade the region $(x - 2)^2 + (y + 1)^2 \leq 9$ on a number plane. **1**

End of Question 9

Question 10 (Start a New Page) **(12 marks)**

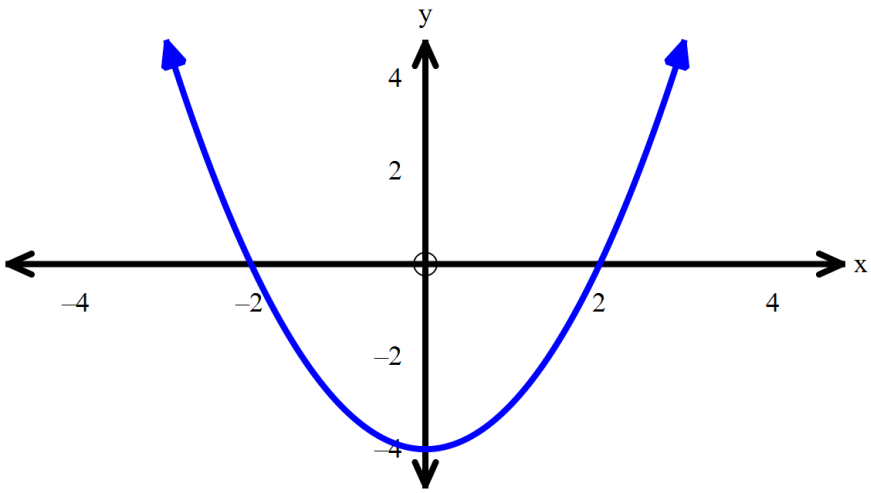
a) Solve $2\cos\theta = -\sqrt{3}$ for $0° \leq \theta \leq 360°$ **2**

b) Consider the function $y = -2x(x-3)(2x+1)$,

i) Find the x-intercepts. **1**

ii) Sketch the graph. **2**

c) For the graph $y = f(x)$ shown below, sketch the transformations indicated (use a new number plane for each one). Show all key features.



i) $y = -f(x)$ **1**

ii) $y = -f(x-2)$ **2**

d) Solve $3x^4 - 10x^2 + 8 = 0$ **2**

e) Show $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$ **2**

End of examination

udent Name: _____ Teacher Name: _____

Solutions

Q1

$$ab^2$$
$$= 3(-2)^2$$
$$= 12$$

C

Q2

$$\frac{0.73 \times 3 + 0.81}{4}$$
$$= 0.75$$

B

Q3

$$\frac{\$120 - \$95}{\$120} \times 100\%$$
$$= 20.8\dot{3}\%$$

A

Student Name: _____ Teacher Name: _____

Q4

A

Q5

$x = -\frac{1}{3}$ or $x = \frac{2}{5}$

D

udent Name: _____ Teacher Name: _____

Q 6 (a)(i)

$$AB = \sqrt{(-3-2)^2 + (1-2)^2}$$

$$= \sqrt{5^2 + 1^2}$$

$$= 5.10 \text{ units} \quad (2 \text{ d.p.})$$

(a)(ii)

Midpoint of AB : $\left(\frac{2+1}{2}, \frac{2-3}{2}\right)$

$\left(\frac{3}{2}, -\frac{1}{2}\right)$

(a)(iii)

$$m_{AB} = \frac{-3-2}{1-2} = 5$$

(b) $2x^2 - 9x + 9$

$= (2x-3)(x-3)$

Student Name: _____ Teacher Name: _____

(c)(i)

$$\text{Mean} = \frac{1+3+4+6+8+3}{6}$$

$$= \frac{25}{6} = 4.17 \text{ (2 d.p.)}$$

(c)(ii)

Standard deviation = 2.3 (1 decimal place)

(d)(i) $\frac{25}{5+15+10+12} = \frac{25}{42}$

(d)(ii) $\frac{10}{42} = \frac{5}{21}$

(d)(iii) $\frac{27}{42} = \frac{9}{14}$

(e)

$$\frac{10 + 10 + ?}{2} = 13$$

$$? = 6$$

Student Name: _____ Teacher Name: _____

Q7

(a) $\$788 \times 0.015 \times \frac{200}{365}$

$= \$6.48$

(b) $\$2000 \times (1-0.15)^5$

$= \$887$

(c)(i) $xy + 3y - x = 0$

$y(x+3) = x$

$y = \frac{x}{x+3}$

(c)(ii) -1

(d) $3x - 7y = 13 \quad \cdots ①$

$4x + 5y = 3 \quad \cdots ②$

$4 \times ① - 3 \times ②$: $-43y = 43$

$y = -1$

Substitute $y = -1$ into ①

$3x + 7 = 13$

$x = 2$

Student Name: _____ Teacher Name: _____

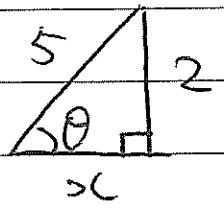
7(e) $x = \frac{+5 \pm \sqrt{25 + 4 \times 3}}{6}$

$= \frac{5 \pm \sqrt{37}}{6}$

7(f)

S	A
T	C

$\tan \theta$ is negative



$x^2 + 2^2 = 5^2$

$x = \sqrt{21}$

$\tan \theta = -\frac{2}{\sqrt{21}}$

Student Name: _____ Teacher Name: _____

Q8

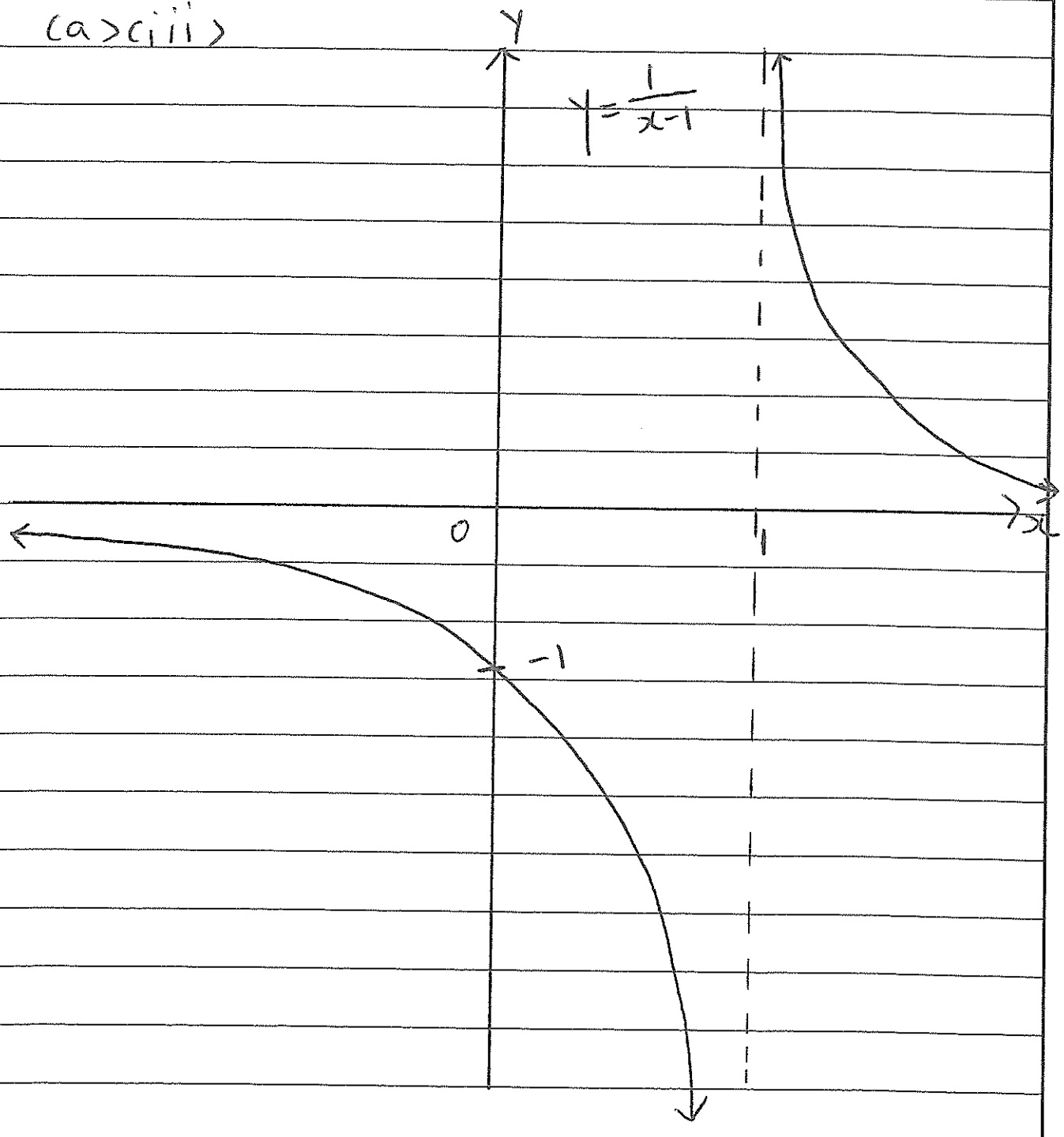
(a)(i) $x=1$ and $y=0$

(a)(ii)

When $x=0$, $y=\frac{1}{-1}$

$y=-1$

(a)(iii)



Student Name: _____ Teacher Name: _____

(b) $y = -2x + 1$

$m = -2$

So the gradient of the line perpendicular to $y = -2x + 1$ is $\frac{1}{2}$.

$$\frac{y-5}{x+2} = \frac{1}{2}$$

$$x + 2 = 2y - 10$$

$$x + 12 = 2y$$

$$x - 2y + 12 = 0$$

Equation of the line perpendicular to $y = -2x + 1$ which passes the point $(-2, 5)$ is : $x - 2y + 12 = 0$

(c) Area $= \frac{1}{2}(6)(4)\sin 45°$

$= \frac{1}{2}(24)\left(\frac{\sqrt{2}}{2}\right) = 6\sqrt{2}$ km^2

(d)(i)

S	A
T	C

$\cos 210°$

$= -\cos 30°$

$= -\frac{\sqrt{3}}{2}$

(d)(ii) $\tan^2 210° - \sin 135°$

$= \left(\frac{1}{\sqrt{3}}\right)^2 - \frac{\sqrt{2}}{2}$

$= \frac{1}{3} - \frac{\sqrt{2}}{2}$

$= \frac{2 - 3\sqrt{2}}{6}$

Student Name: _____ Teacher Name: _____

Q9

(a)(i) $\angle ABC$

$= 296^\circ - 180^\circ - 63^\circ$

$= 53^\circ$

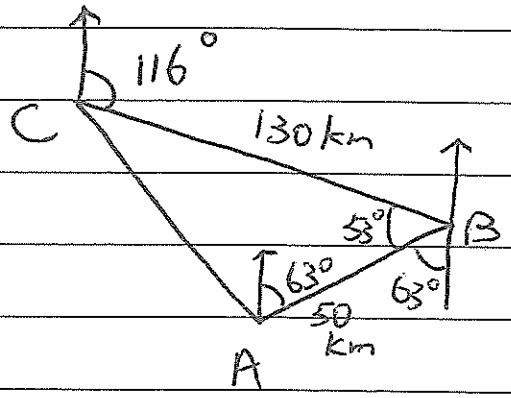
(a)(ii)

$AC^2 = 130^2 + 50^2 - 2(50)(130)\cos 53^\circ$

$AC^2 = 11576.4047$

$AC = 108$ km

(a)(iii)



$$\frac{50}{\sin \angle ACB} = \frac{AC}{\sin 53^\circ}$$

$$\sin \angle ACB = \frac{50 \sin 53^\circ}{107.5937019}$$

$\angle ACB = 21.78562521^\circ$

$\angle ACB \doteq 22^\circ$ (to the nearest degree)

The Bearing of A from C is $116^\circ + 22^\circ$

$= 138^\circ T$

Student Name: _____ Teacher Name: _____

(b) $y = x^2 + 1$ ... ①

$6x - y = 8$ ... ②

Substitute ① into ②:

$6x - x^2 - 1 = 8$

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

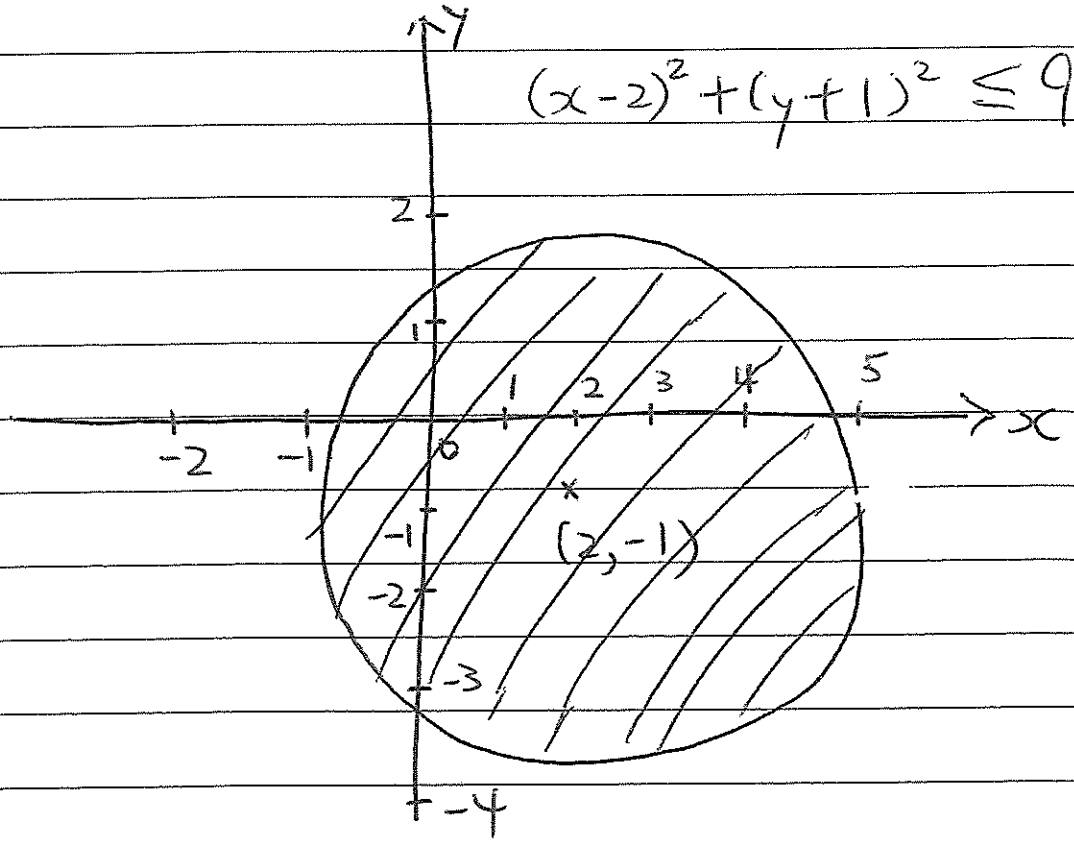
$x = 3$

Substitute $x = 3$ into ①

$y = 10$

$\begin{cases} x = 3 \\ y = 10 \end{cases}$

(c) Radius = 3 Centre : $(2, -1)$



Student Name: _____ Teacher Name: _____

Q10(a)

$2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

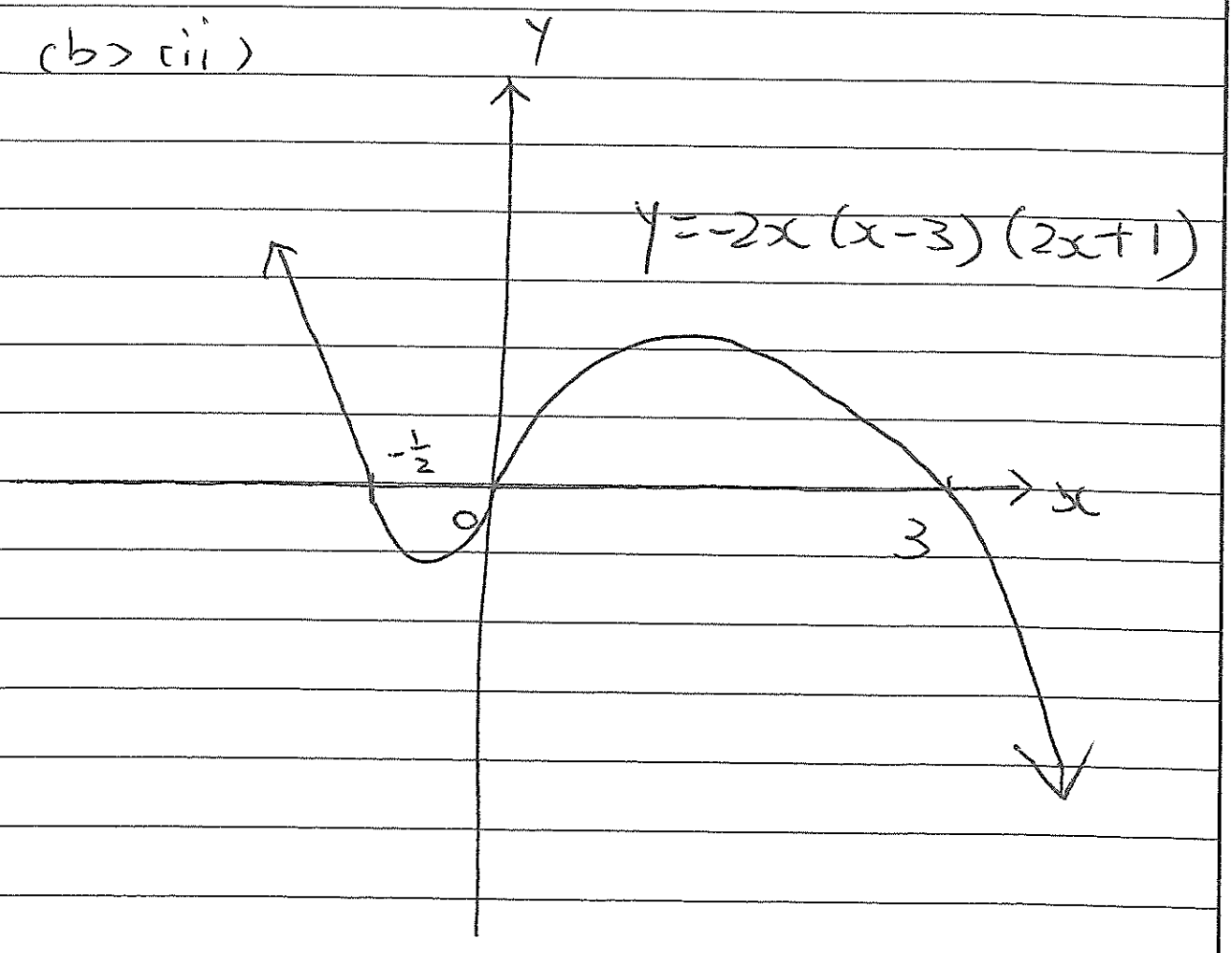
Acute angle $\theta = 30°$

S	A
T	C

$\theta = 150°, 210°$

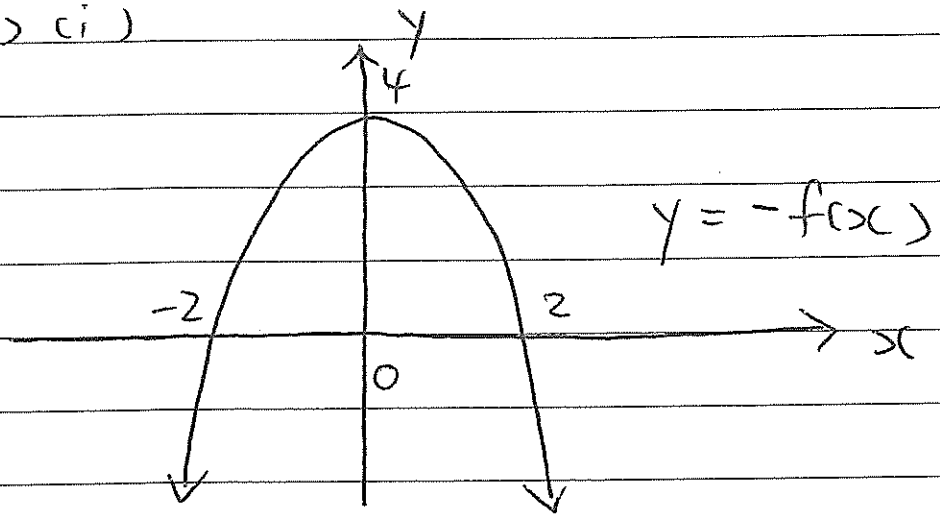
(b)(i) x-intercepts are $0, 3, -\frac{1}{2}$

(b)(ii)

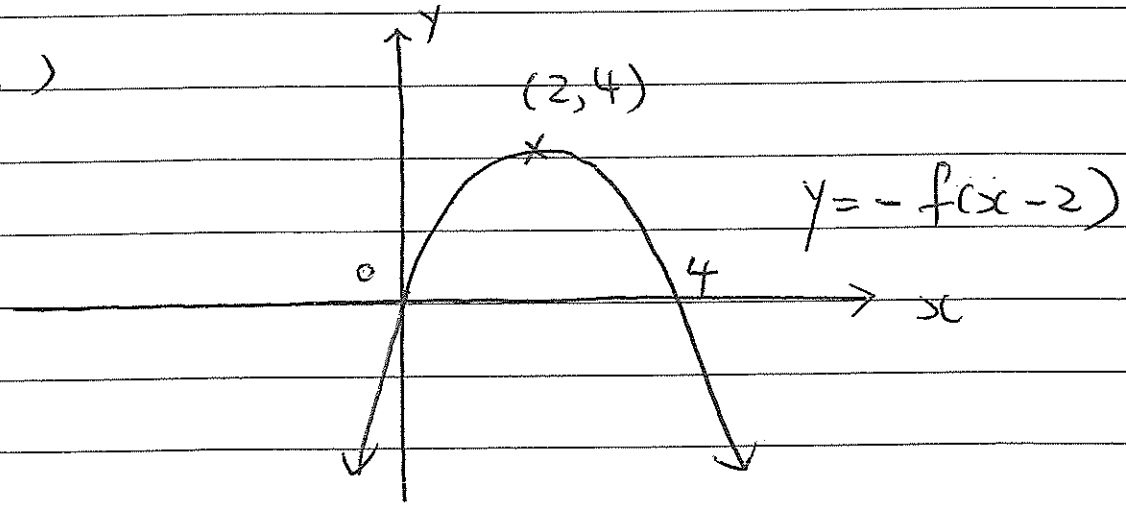


Student Name: _____ Teacher Name: _____

10 (c) (i)



(ii)



(d) $3x^4 - 10x^2 + 8 = 0$

$(3x^2 - 4)(x^2 - 2) = 0$

$x^2 = \frac{4}{3}$ or $x^2 = 2$

$x = \pm\sqrt{\frac{4}{3}}$ or $x = \pm\sqrt{2}$

$x = \pm\frac{2}{\sqrt{3}}$ or $x = \pm\sqrt{2}$

(e) LHS $= \cos^2\theta - \sin^2\theta$

$= (1 - \sin^2\theta) - \sin^2\theta$

$= 1 - 2\sin^2\theta = $ RHS