FILE

Sydney Technical High School

Name: .....

Maths Class: .....

Year 10

Mathematics

Yearly Examination

November, 2017

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided

Section 1	Multiple Choice Questions 1-10 10 Marks
Section II	Questions 11-15 75 Marks

SECTION 1 (10 Marks)

Use the multiple choice answer sheet for Questions 1-10

1. Which of the following is an expression for $x^2 - 4y^2$?

   (A) $(x + 2y)(x - 2y)$

   (B) $(x - 2y)^2$

   (C) $(x + 2y)^2$

   (D) $(x + 4y)(x - 4y)$

2. What is $\frac{2}{3-\sqrt{2}}$ as a fraction with a rational denominator?

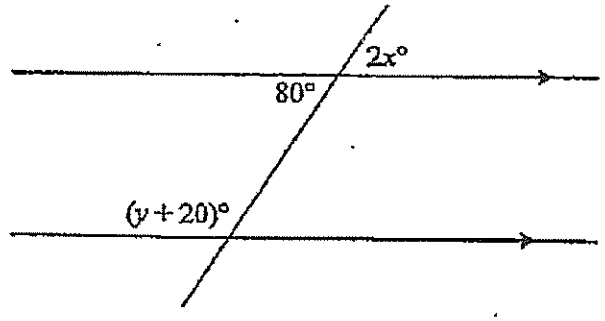
   (A) $6 - 2\sqrt{2}$

   (B) $6 + 2\sqrt{2}$

   (C) $\frac{6-2\sqrt{2}}{7}$

   (D) $\frac{6+2\sqrt{2}}{7}$

3. What are the values of x and y?



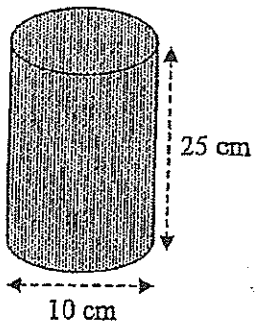
   (A) $x = 40°, y = 60°$

   (B) $x = 40°, y = 80°$

   (C) $x = 80°, y = 60°$

   (D) $x = 80°, y = 80°$

4. What is the total surface area (in cm^2) of this closed cylinder?



(A) 175π

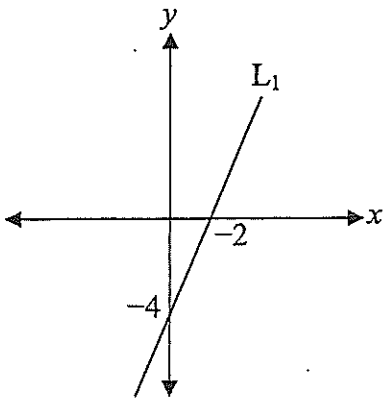
(B) 300π

(C) 625π

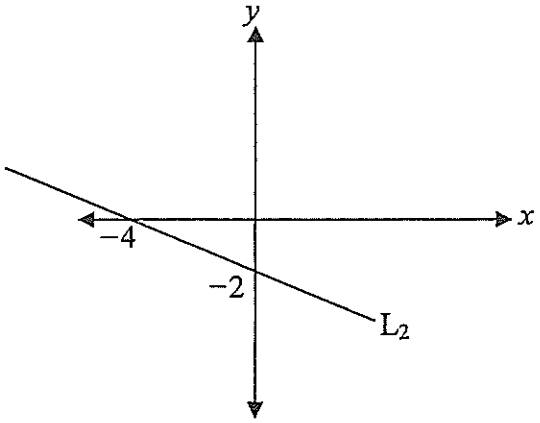
(D) 700π

5. Which of the lines shown in the diagrams has equation $y = -2x - 4$?

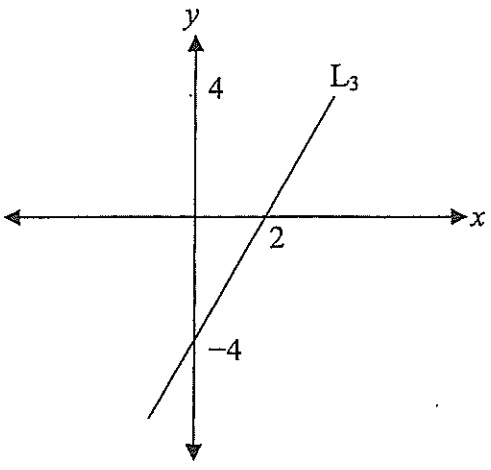
(A)



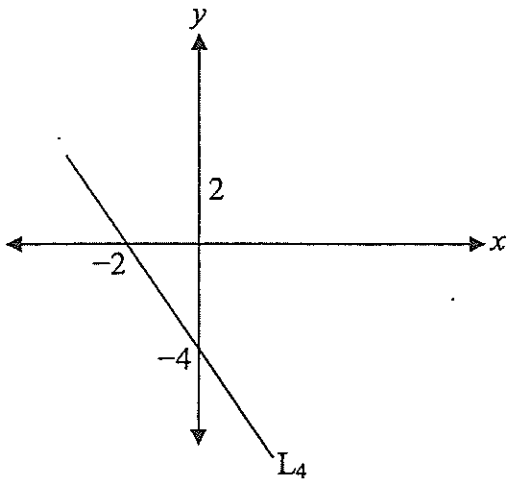
(B)



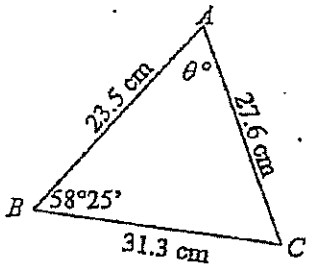
(C)



(D)



6. Which of the following is **NOT** a correct expression involving θ in triangle ABC?



Not to scale

(A) $31.3^2 = 27.6^2 + 23.5^2 - 2 \times 27.6 \times 23.5 \cos\theta$

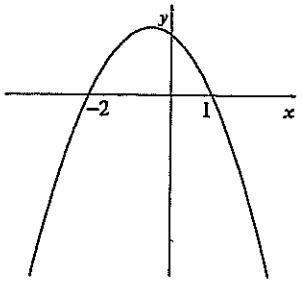
(B) $\cos\theta = \frac{23.5^2+27.6^2-31.3^2}{2x23.5x27.6}$

(C) $\frac{31.3}{sin\theta} = \frac{27.6}{sin58°25'}$

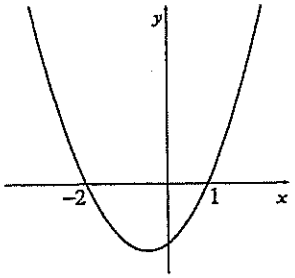
(D) $\frac{sin\theta}{31.3} = \frac{sin58°25'}{23.5}$

7. Which graph best represents $y = x^2 + x - 2$?

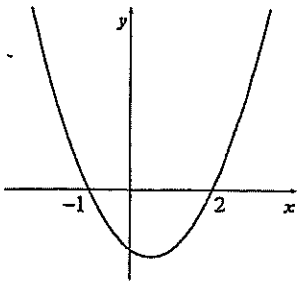
(A)



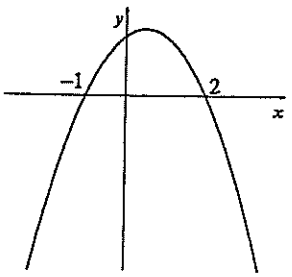
(B)



(C)



(D)



8. Which of the following correctly expresses p as the subject of $q = \frac{4}{9p^2}$?

(A) $p = \frac{\pm 2q}{3}$

(B) $p = \frac{\pm 2}{3q}$

(C) $p = \frac{\pm 2\sqrt{q}}{3}$

(D) $p = \frac{\pm 2}{3\sqrt{q}}$

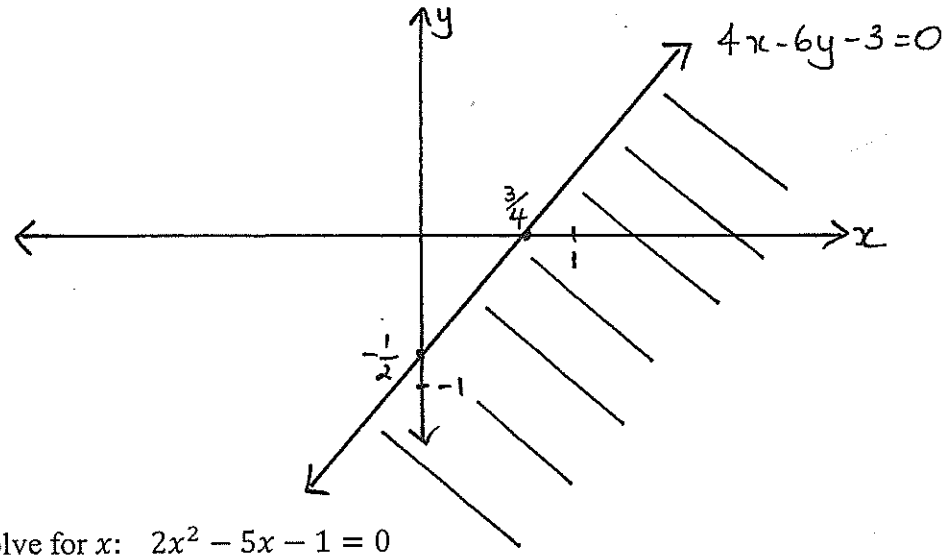
9. The shaded region is where

(A) $4x - 6y - 3 \leq 0$

(B) $4x - 6y - 3 \geq 0$

(C) $4x - 6y - 3 < 0$

(D) $4x - 6y - 3 > 0$



10. Solve for x: $2x^2 - 5x - 1 = 0$

(A) $x = \frac{5 \pm \sqrt{17}}{4}$

(B) $x = \frac{-5 \pm \sqrt{17}}{4}$

(C) $x = \frac{5 \pm \sqrt{33}}{4}$

(D) $x = \frac{-5 \pm \sqrt{33}}{4}$

SECTION 2 (75 Marks)

Answer all questions, starting each question on a new sheet of paper in your answer booklet with the question number at the top of the page.

Question 11 (15 marks)

(a) Write $a^{-2/3}$ without fractional or negative indices 1

(b) Find the volume of a sphere with a radius of 2.4cm correct to 1 decimal place. 1

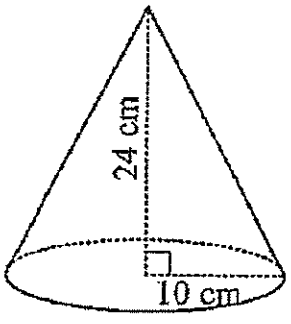
(c) A photocopying machine purchased for \$42500 depreciates at a rate of 14% p.a.
Find the value of the photocopying machine after 6 years. 2

(d) If $cos\theta = 0.7569$ and $0° \leq \theta \leq 360°$, find the 2 values of θ to the nearest degree. 2

(e) Solve these simultaneous equations

$$x + y = 8$$
$$x = y + 2$$ 2

(f) The cone shown has a perpendicular height of 24cm and the base has a radius of 10cm.



NOT TO SCALE

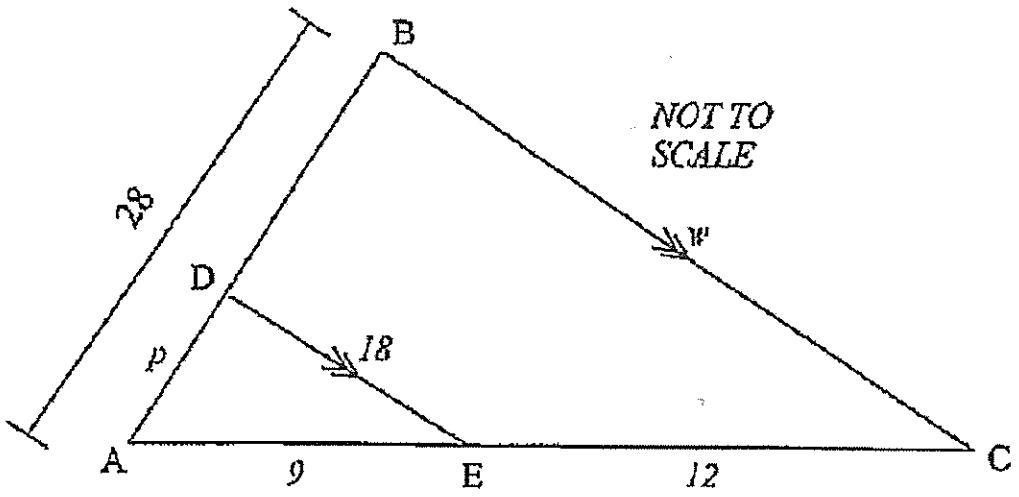
(i) Find the slant height of the cone 1

(ii) Find the surface area of the whole cone correct to 2 decimal places 2

QUESTION 11 Continued over the page

(g) Solve $x = \frac{2x+8}{x}$ 2

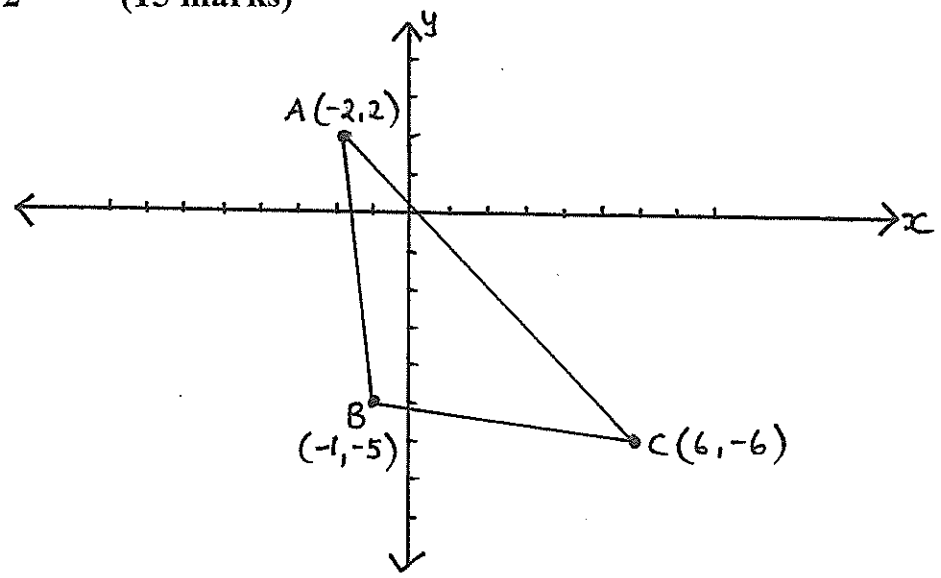
(h) Given $\Delta ABC \,|||\, \Delta ADE$, find the values of p and w.



2

Question 12 (15 marks)

(a)



(i) Calculate the midpoint P of AC 1

(ii) Calculate the gradient of AC 1

(iii) Find the exact value of the length of AC 2

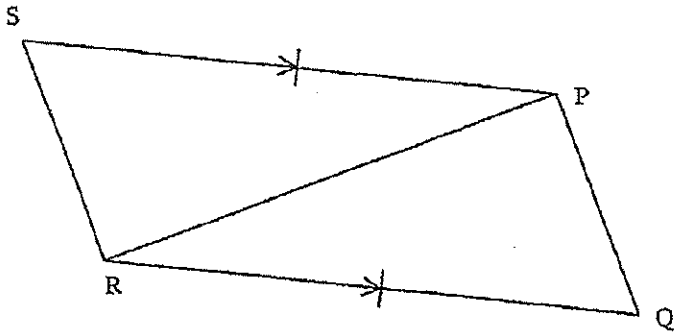
(iv) Find the gradient of BP and show that AC and BP are perpendicular 2

(v) Find the length of BP 1

(vi) Find the area of ΔABC 1

(viii) Find the equation of line AC 2

(b) In the diagram, PS || QR and PS=RQ



(i) Prove that ΔPSR≡ ΔPQR 3

(ii) Hence prove that SR = QP 1

(c) Simplify $\frac{2a}{b} - \frac{a}{2b}$ 1

Question 13 (15 marks)

(a) Solve these simultaneous equations using the elimination method

$$2x - y = -3$$
$$5x + 2y = 15$$

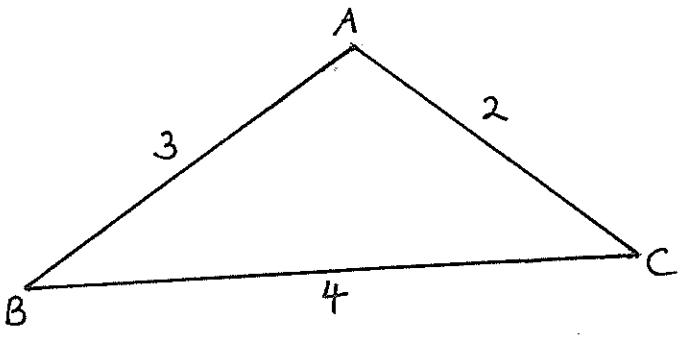
2

(b) Solve for x

$$\frac{2x-1}{5} = \frac{x}{3} - 1$$

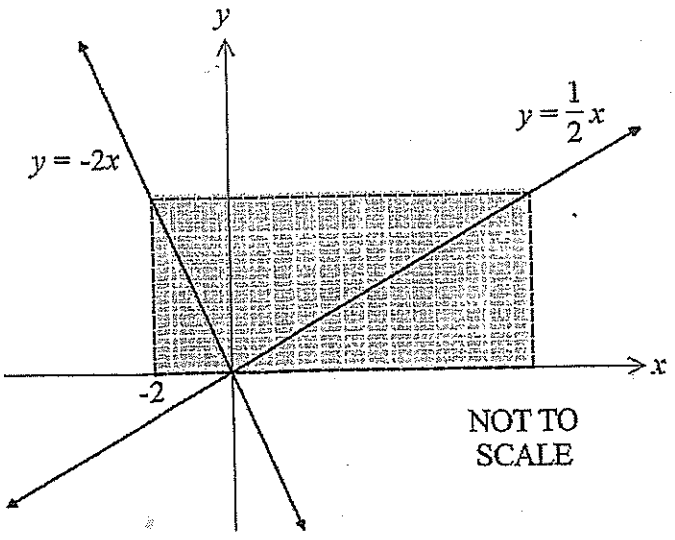
2

(c) Use the cosine rule to find $\angle BAC$ to the nearest minute



2

(d) Calculate the shaded area (in square units) on the diagram below.



2

Question 13 continues over the page

(e) Sketch the following functions showing:

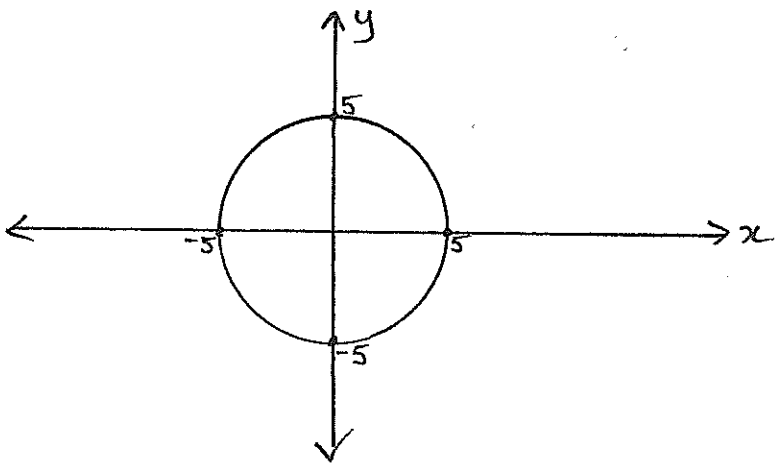
(i) the y-intercepts or asymptotes where appropriate

(ii) the x-intercepts

(α) $y = 2^x + 1$ 2

(β) $y = \frac{1}{x-4}$ 2

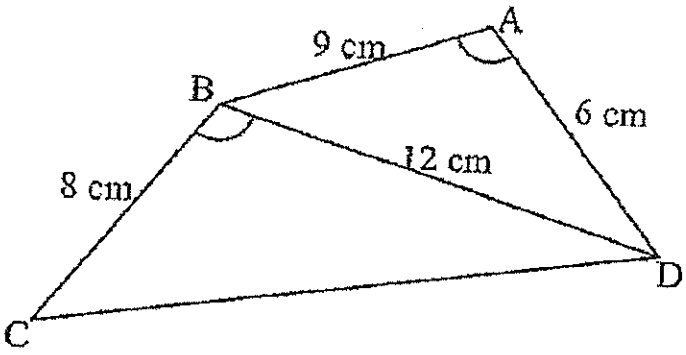
(f) Write down the equation of the following graph 2



(g) Simplify $\frac{6g^{10}h^4}{9g^2h^5}$ 1

Question 14 (15 marks)

(a) In the diagram, $ABCD$ is a quadrilateral and BD is a diagonal. $CB = 8$ cm, $AB = 9$ cm and $BD = 12$ cm. $\angle DAB = \angle CBD$.



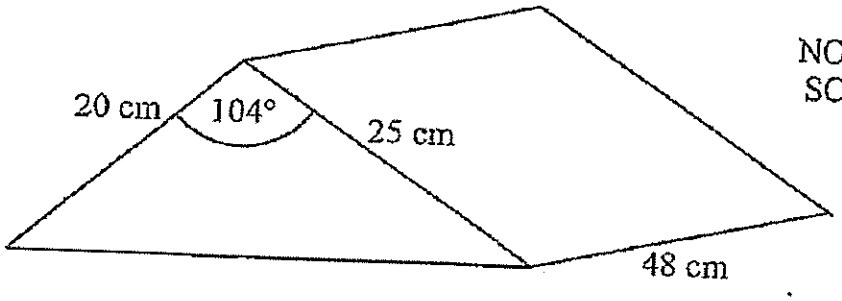
NOT TO SCALE

(i) Prove that $\Delta ABD \,|||\, \Delta BDC$ 3

(ii) Find the length of CD 1

(iii) Prove that AB and CD are parallel 1

(b) Find the volume of the triangular prism shown below. Give your answer correct to 2 decimal places. 2



NOT TO SCALE

(c) By completing the square, find the exact solution to $x^2 + 4x - 3 = 0$ 2

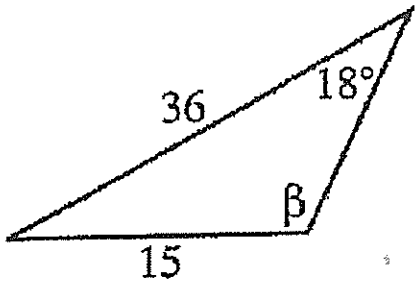
(d) Express $(3 + \sqrt{2})^2$ as $a + \sqrt{b}$ where and b are positive integers. 2

Question 14 continued over page

(e) The lines with equations $2x - 4y + 1 = 0$ and $y = 2mx$ are perpendicular.

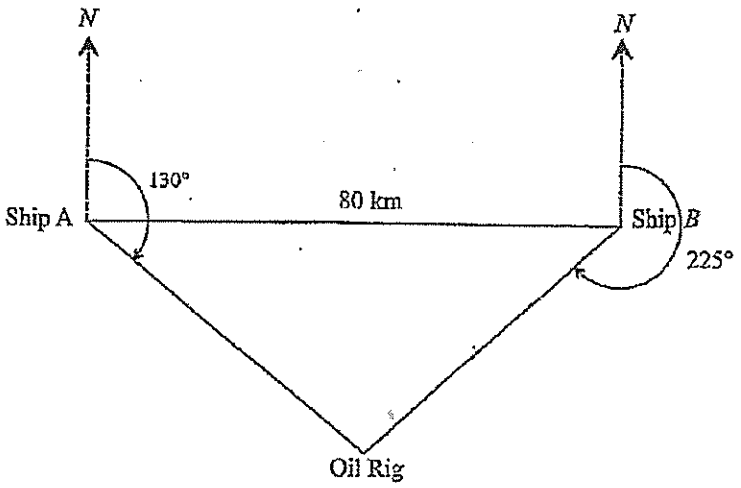
What is the value of m? 2

(f) Find the obtuse angle β, correct to the nearest degree. 2



Question 15 (15 marks)

(a) Two ships are 80 km apart with Ship B due east of Ship A. From Ship B, the bearing of an oil rig is 225° and from Ship A the bearing is 130°.



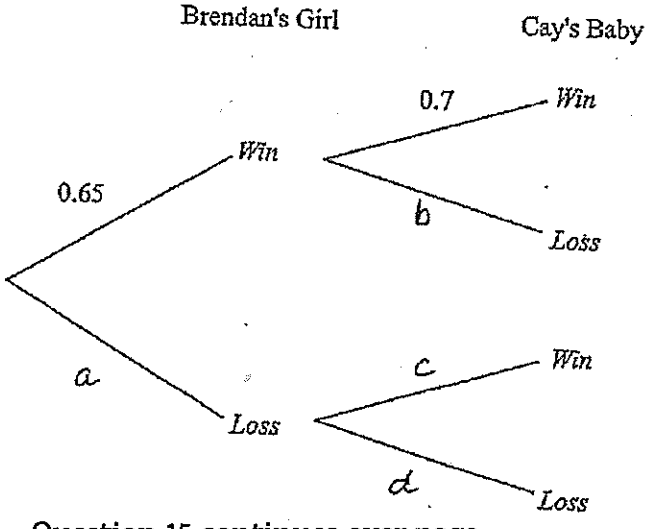
Find the distance of the oil rig from Ship A correct to 2 decimal places. 3

(b) Stefanie is training two horses, Brendan's Girl and Cay's Baby.

She has determined that the probability that Brendan's Girl wins a race is 0.65, while the probability that Cay's Baby wins is 0.7.

Next weekend, both horses will compete in separate races at a carnival.

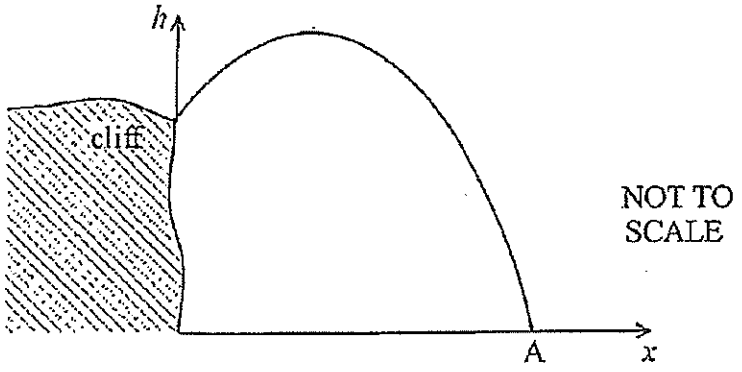
Find the values of a, b, c, d then determine the probability that only one of Stefanie's horses will win their race next weekend.



2

Question 15 continues over page

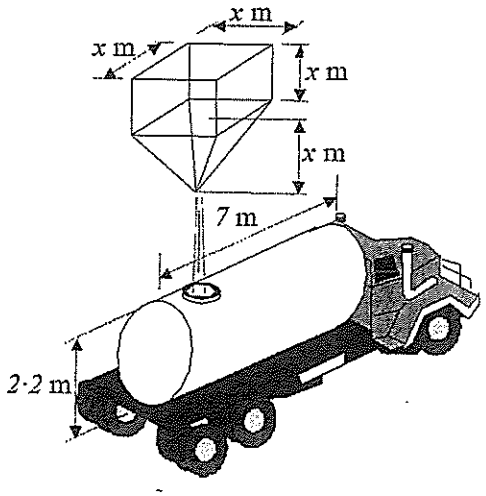
(c) A ball is thrown from the top of a cliff and hits the ground at A. The path of the ball is the parabola $h = 20 + 8x - x^2$, where x and h are in metres.



(i) Find how far the ball lands from the base of the cliff. 2

(ii) What is the maximum height of the ball above the ground? 2

(d) A "hopper" is a hollow storage container. This hopper is made by joining a cube and a square pyramid, each of height $x\ m$ it is used to fill a cylindrical tank of diameter 2.2 m and length 7 m.



(i) Find the volume of the cylindrical tank, correct to 2 decimal places. 2

(ii) Show that the volume (V) of the hopper can be given by $V = \frac{4}{3}x^3$ 2

(iii) A full load of the hopper exactly fills the tank. Find the value of x correct to 1 decimal place. 2

END OF PAPER

YEAR 10 YEARLY SOLUTIONS FILE

Section I

1. $(x-2y)(x+2y)$ (A)

2. $\frac{2}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{2(3+\sqrt{2})}{7}$ (D)

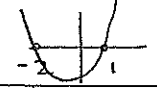
3. $x = 40 \quad y = 80$ (B)

4. $SA = 2\pi rh + 2\pi r^2$
$= 2 \times \pi \times 5 \times 25 + 2\pi \times 5^2$ (B)
$= 300\pi$

5. $y = -2x - 4$ (D)

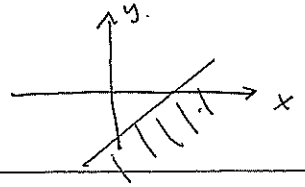
6. (D)

7. $y = (x+2)(x-1)$ (B)



8. $9p^2 = \frac{4}{q}$
$p^2 = \frac{4}{9q}$ $p = \pm\frac{2}{3\sqrt{q}}$ (D)

9. (B)



10. $x = \frac{5 \pm \sqrt{25 - 4(2)(-1)}}{4}$
$x = \frac{5 \pm \sqrt{33}}{4}$ (C)

SECTION 2

11 (a) $a^{-2/3} = \frac{1}{a^{2/3}}$
$= \frac{1}{\sqrt[3]{a^2}}$

(b) $V = \frac{4}{3}\pi r^3$
$V = \frac{4}{3} \times \pi \times 2.4^3$
$V = 57.9\,cm^3$

(c) $C = 42500\left(1 - \frac{14}{100}\right)^6$
$= \$17194.11$

(d) $\cos\theta = 0.7569$
$\theta = 41^\circ, 319^\circ$.

(e) $x + y = 8$
$x = y + 2$
$y + 2 + y = 8$
$2y + 2 = 8$
$2y = 6$
$y = 3$
$x = 3 + 2$
$x = 5$. $\therefore x = 5, y = 3$

(f) (i) $s^2 = 24^2 + 10^2$

$s^2 = 676$

$s = 26$.

(ii) $SA = \pi r^2 + \pi r s$

$= \pi \times 10^2 + \pi \times 10 \times 26$

$= 1130.97 cm^2$

(g) $x = \frac{2x+8}{x}$

$x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

(h) $\frac{w}{18} = \frac{21}{9}$

$w = 42$

$\frac{p}{28} = \frac{9}{21}$

$p = 12$

12 (a) (i) $M_{AC} = \left(\frac{-2+6}{2}, \frac{2+-6}{2}\right)$

$P = (2, -2)$

(ii) $m_{AC} = \frac{-6-2}{6--2}$

$m_{AC} = \frac{-8}{8}$

$m_{AC} = -1$

(iii) $d_{AC} = \sqrt{(6--2)^2 + (-6-2)^2}$

$= \sqrt{64 + 64}$

$= \sqrt{128}$

$= 8\sqrt{2}$

(iv) $m_{BP} = \frac{-5--2}{-1-2}$

$= \frac{-3}{-3}$

$= 1$

$m_{AC} \times m_{BP} = 1 \times -1$

$= -1$ $\therefore$ AC $\perp$ BP. as $m_1 m_2 = -1$

(v) $d_{BP} = \sqrt{(-1-2)^2 + (-5--2)^2}$ $(-1, -5)$ $(2, -2)$

$= \sqrt{9 + 9}$

$= \sqrt{18}$

(vi) $A = \frac{1}{2} \times 8\sqrt{2} \times \sqrt{18}$

$= \frac{1}{2} \times 8\sqrt{2} \times 3\sqrt{2}$

$= 24 u^2$

(vi) Eqn AC

$y+6 = -1(x-6)$

$y+6 = -x+6$

$x+y = 0$.

(b) (i) In ΔPSR and ΔPQR

PR is common side

SP = RQ (given)

$\angle SPR = \angle PRQ$ (alternate angles equal in parallel lines SP ∥ RQ)

$\therefore \Delta PSR \equiv \Delta PQR$ by SAS.

(ii) SR = QP corresponding sides of congruent triangles

(c) $\frac{2a}{b} - \frac{a}{2b}$

$= \frac{4a-a}{2b}$

$= \frac{3a}{2b}$.

13(a) $2x-y=-3 \quad \times 2$

$5x+2y=15$

$4x-2y=-6$

$+ \quad 5x+2y=15$

$9x = 9$

$x = 1$

$2-y=-3$

$-y=-5$

$y=5$.

(b) $\frac{1}{3} \times \frac{2x-1}{5} = \frac{x}{3} \times \frac{1}{5} \quad \times 15$

$6x-3 = 5x-15$

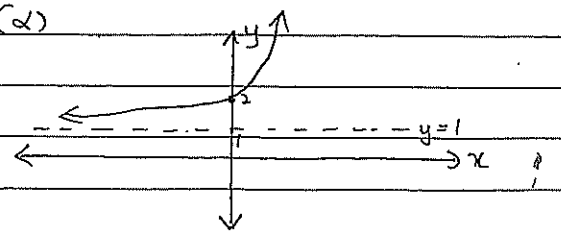
$x = -12$

(c) $\cos A = \frac{2^2+3^2-4^2}{2\times2\times3}$

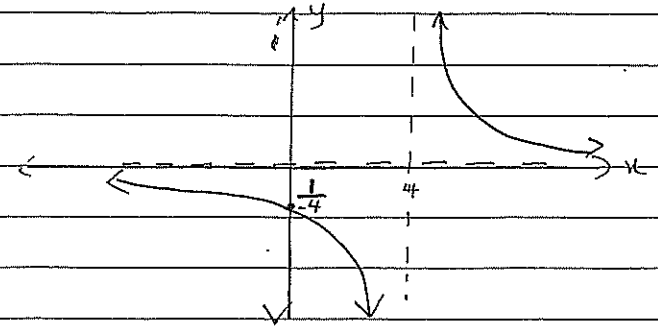
$A = 104^\circ 29'$

(d) $A = 4 \times 10 = 40u^2$

(e) (α)



(β)



f. $x^2+y^2=25$

(g) $\frac{6g^{\cancel{2}}\,h^{\cancel{4}}}{3\cancel{9}\,g^2h^{\cancel{5}}}$

$= \frac{2g^8}{3h}$

Q14(a)(i) In ΔABD and ΔBDC

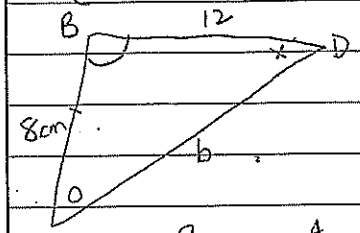
$\angle CBD = \angle BAD$ (given)

$\frac{AB}{BD} = \frac{9}{12} = \frac{3}{4}$.

$\frac{AD}{BC} = \frac{6}{8} = \frac{3}{4}$

$\therefore \Delta ABD \,|||\, \Delta BDC$ 2 sides in same ratio and included angle equal

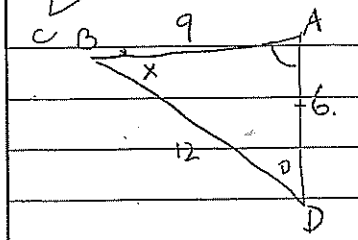
(ii)



$\frac{CD}{BD} = \frac{4}{3}$.

$\frac{CD}{12} = \frac{4}{3}$

$CD = 16$ cm.



(iii) $\angle ABD = \angle BDC$ corresponding angles in similar triangles / alternate angles equal

$\therefore AB \parallel CD$

(b) $V = \left(\frac{1}{2} \times 20 \times 25 \times \sin 104\right) \times 48$

$V = 11643.5487\ cm^3$

$V = 11643.55 cm^3$

(c) $x^2 + 4x + 4 = 3 + 4$.

$(x+2)^2 = 7$

$x + 2 = \pm\sqrt{7}$

$x = \pm\sqrt{7} - 2$

(d) $(3+\sqrt{2})^2$

$= 9 + 6\sqrt{2} + 2$

$= 11 + 6\sqrt{2}$

$= 11 + \sqrt{72}$

$a = 11 \quad b = 72$

(e) $2x - 4y + 1 = 0$

$4y = 2x + 1$

$y = \frac{1}{2}x + \frac{1}{4}$

$m_1 = \frac{1}{2}$

$y = 2mx$

$m_2 = 2m$

$\frac{1}{2} \times 2m = -1$

$m = -1$

(f) $\frac{\sin 18}{15} = \frac{\sin B}{36}$.

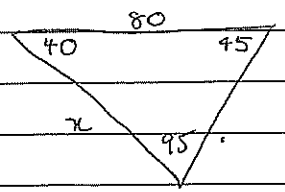
$\sin B = \frac{36 \sin 18}{15}$

$B = 48^\circ, 132^\circ$

$\therefore B = 132^\circ$..

Q15

(a)



$\frac{x}{\sin 45} = \frac{80}{\sin 95}$

$x = \frac{80 \sin 45}{\sin 95}$ ~~3~~

$x = 56.7846 km$

(b) $a = 0.35$

$b = 0.3$

$c = 0.7$

$d = 0.3$

$P(WL) + P(LW)$

$= (0.65 \times 0.3) + (0.35 \times 0.7)$

$= 0.44$

~~2~~

(c) (i) Ball lands when $h=0$

$20+8x-x^2=0$

$x^2-8x-20=0$

$(x-10)(x+2)=0$

$x=10, -2$

Ball lands 10m from cliff 2

(ii) $x=\frac{-b}{2a}$

$x=\frac{8}{2}$

$x=4$

When $x=4$ $h=36$

∴ Max height is 36m . 2

(d) (i) $V=\pi \times 1.1^2 \times 7$

$V=26.61m^3$ 2

(ii) $V=x^3+\frac{1}{3}\times x^2 \times x$

$V=x^3+\frac{1}{3}x^3$

$V=\frac{4}{3}x^3$. 2

(iii) $26.61=\frac{4}{3}x^3$

$x^3=19.9575$

$x=2.7$ 2