SYDNEY GRAMMAR SCHOOL

NAME _____ MATHEMATICS MASTER _____

2020 Annual Examination

Form IV Mathematics

Friday 30th October 2020

8:39 am

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.

Total Marks: 108

Questions 1–9

- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.

Collection

- Arrange your solutions in order.
- Staple the sheets of writing paper together.
- Write your name and master on this page and on each page of writing paper.
- Place everything inside this question booklet.

4A: SDP	4B: AEA	4C: NJL	4D: CWG
4E: YH	4F: LL	4G: DWH-GMC	4H: LRP
4I: WJM	4J: BR		

Checklist

- 14 sheets of writing paper per boy
- Candidature: 198 pupils

Writer: NJL

	Marks
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
TOTAL	

QUESTION ONE (12 marks) **Marks**

(a) Evaluate $\log_{10} 100$. 1

(b) (i) Write $0{\cdot}002\,41$ in scientific notation. 1

(ii) Write $8{\cdot}34 \times 10^3$ in decimal notation. 1

(c) Solve $5^x = \dfrac{1}{25}$. 1

(d) Let $P(x) = 6x^4 - 3x + 2$. State:

(i) the degree of $P(x)$, 1

(ii) the leading term of $P(x)$. 1

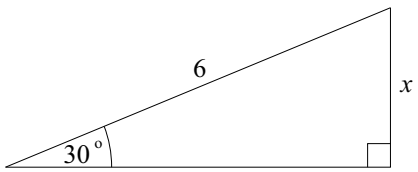
(e) Express $\dfrac{1}{\sqrt{2}}$ as a fraction with a rational denominator. 1

(f) Evaluate:

(i) $64^{\frac{3}{2}}$ 1

(ii) $14x^{12} \div 2x^4$ 1

(g) 1



Find the value of x in the triangle above.

(h) Draw a quadrant diagram to represent $480°$ and state its related angle. 2

QUESTION TWO (12 marks) **Marks**

(a) Solve $(x-8)(2x+3)=0$. 2

(b) Express the following as single logarithms:

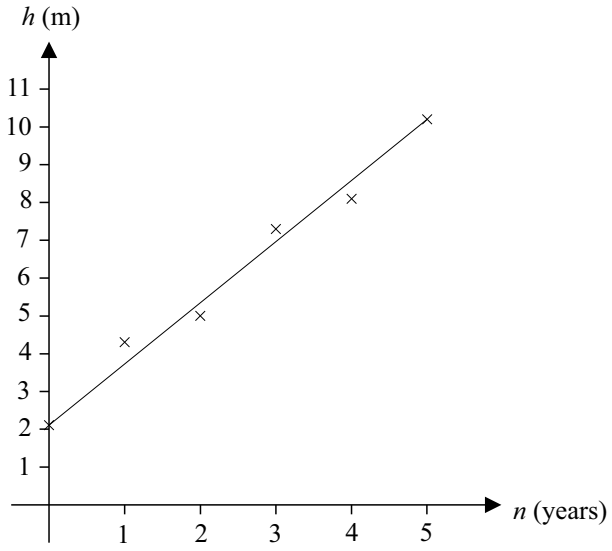
(i) $\log_{10} 8 - \log_{10} 4$ 1

(ii) $2\log_{10} 3 + \log_{10} 2$ 2

(c) A cylinder has a circular base of radius 2 cm and perpendicular height of 10 cm. Calculate the volume of the cylinder. Give your answer in exact form. 2

(d) Solve $3^x = 37$. Give your answer correct to 2 decimal places. 2

(e)



Years After Planting (n)	Height (h)
0	2·1
1	4·3
2	5·0
3	7·3
4	8·1
5	10·2

Michael plants a tree and measures its height every year for the next five years.
Let h be the height of the tree in metres and n be the number of years after planting.
His table of values, scatter plot and line of best fit for the height of the tree, n years after planting, are shown above.

(i) Using the table, state the height of the tree when it was planted, and five years after it was planted. 1

(ii) Using your answer to part (i), show that the line of best fit drawn on the graph has equation $h = 1{\cdot}62n + 2{\cdot}1$. 1

(iii) Hence estimate the height of the tree 10 years after planting. 1

QUESTION THREE (12 marks) **Marks**

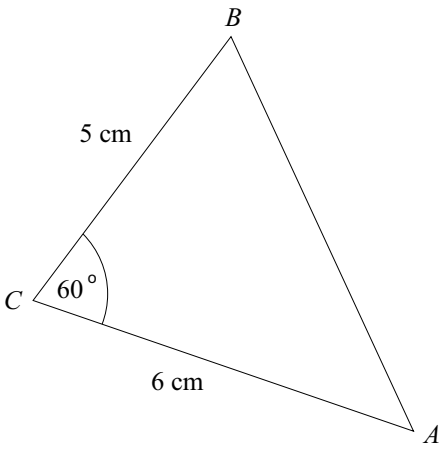
(a) (i) Copy and complete the table below for the equation $y = 2^x + 1$. [1]

x	-1	0	1	2	3
y					

(ii) Hence or otherwise, sketch the graph of $y = 2^x + 1$ for $-1 \leq x \leq 3$. [2]

(iii) State the equation of the horizontal asymptote of the graph of $y = 2^x + 1$. [1]

(b)



In $\triangle ABC$ above, $\angle ACB = 60°$, $BC = 5\,\text{cm}$ and $AC = 6\,\text{cm}$.

(i) Use the cosine rule to calculate the exact length of side AB. [2]

(ii) Calculate the exact area of $\triangle ABC$. [2]

(c) The clone population P of an otherwise unpopulated planet, n years after the arrival of a single clone ship, is given by $P = 700 \times 1{\cdot}15^n$.

(i) How many clones arrived on the ship? [1]

(ii) Find the expected population, to the nearest whole number of clones, after 40 years. [1]

(iii) During which year after the arrival of the ship will the population exceed 1×10^9 clones? [2]

QUESTION FOUR (12 marks) **Marks**

(a) Factorise $x^2 - 3x - 10$. [2]

(b) A red bag contains five red disks numbered one to five. A yellow bag contains four yellow disks numbered one to four. One disk is randomly selected from each bag and the sum of the numbers recorded.

(i) List the sample space for this experiment. [1]

(ii) What is the probability that a sum of six is recorded? [2]

(iii) What is the probability that a sum greater than seven is recorded? [1]

(c) A hiker walks 8 km along a straight track from point A to point B on a bearing of 260°T. At point B he turns and walks due north to point C. At point C the hiker's compass tells him that point A is now on a bearing of 120°T.

(i) Draw a diagram to illustrate the walker's path, showing all the information given above. [1]

(ii) Calculate the bearing of A from B. [1]

(iii) Calculate the distance AC. Give your answer correct to the nearest metre. [2]

(d) State the value of a if $P(x) = x^3 - 3x + a$ and $P(1) = -4$. [2]

QUESTION FIVE (12 marks) **Marks**

(a) Find the exact value of $\tan(240°)$. 2

(b) Show that $x^2 + 4x + 30 = 0$ has no real solutions. 1

(c) (i) Complete the square to express $y = x^2 - 8x + 10$ in the form $y = (x - h)^2 + k$. 1

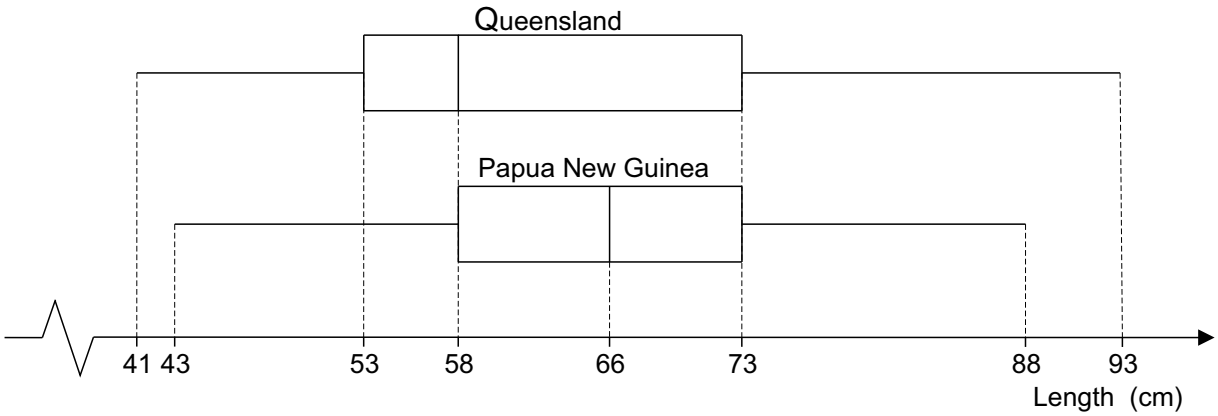
(ii) Hence or otherwise:

(α) State the coordinates of the vertex of $y = x^2 - 8x + 10$. 1

(β) State the x-intercepts of $y = x^2 - 8x + 10$ in exact form. 2

(d)

Boxplot of Lengths of Flatback Sea Turtles in Queensland and Papua New Guinea



The parallel boxplots above show the distribution of lengths of adult flatback sea turtles sampled by scientists in Queensland and Papua New Guinea.

(i) State the two medians and interquartile ranges for the lengths of flatback sea turtles in Queensland and Papua New Guinea. 2

(ii) A healthy adult flatback is considered to have a minimum length of 58 cm. State the percentage of flatbacks measured which were healthy in:

(α) Queensland; 1

(β) Papua New Guinea. 1

(iii) Compare the lengths of flatbacks found in Queensland and Papua New Guinea. References to statistical measures of spread and central tendency are required. 1

QUESTION SIX (12 marks) **Marks**

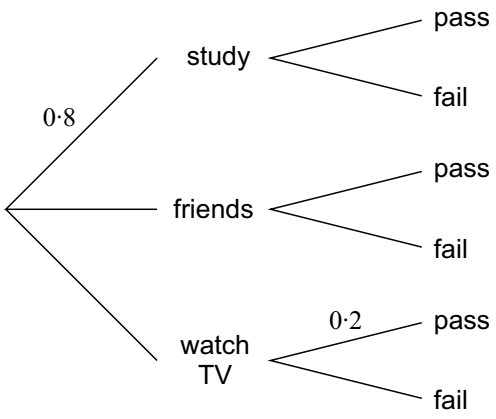
(a) Consider the data set $6, 8, 12, 14, 15$.
Find the:

(i) mean. 1

(ii) standard deviation. Give your answer correct to 2 decimal places. 2

(b) Nigel has a test on Friday morning. On Thursday night he can either (a) study, (b) go out with friends or (c) watch TV. The probability that he elects these choices are 0·8, 0·1 and 0·1 respectively. The probability that he passes the test given (a), (b) or (c) is 0·7, 0·1 and 0·2 respectively.

(i) 1



Copy the tree diagram above onto your answer paper. Show the probabilities on each branch.

(ii) What is the probability that Nigel goes out with friends and fails his test? 1

(iii) What is the probability that Nigel passes his test? 1

(iv) What is the probability that Nigel watches TV, given that he passes his test? 1

(c) (i) Sketch the graph of $y = \cos\theta$ for $0^\circ \leq \theta \leq 360^\circ$. 2

(ii) Hence solve $\cos\theta = 0$ for $0^\circ \leq \theta \leq 360^\circ$. 1

(d) Solve $\sin\theta = -\dfrac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$. 2

QUESTION SEVEN (12 marks) **Marks**

(a) Let $P(x) = x^4 - 5x^3 - x^2 + 11x - 30$.

(i) Use the factor theorem to show that $(x - 5)$ is a factor of $P(x)$. 1

(ii) Find another linear factor of $P(x)$. 1

(b) If $\cos\theta = \dfrac{2}{\sqrt{5}}$, find the possible values of $\sin\theta$. 2

(c) Solve the following pair of simultaneous equations: 3

$$y = \frac{3}{x+3}$$
$$y = 2x + 5$$

(d) Use long division to divide $6x^4 + 4x^3 + 19x^2 + 14x - 7$ by $2x^2 + 7$. 2

(e) Sketch the graph of $y = -x(x+4)^2(x-4)^3$. Use approximately one third of a page. 3

QUESTION EIGHT (12 marks) **Marks**

(a) Sketch the following graphs on separate number planes. Include any intercepts and asymptotes on your graph.

(i) $y = \log_{10}(x+1)$ 2

(ii) $(x+3)^2 + (y+4)^2 = 9$ 2

(iii) $y = \dfrac{1}{x} + 1$ 2

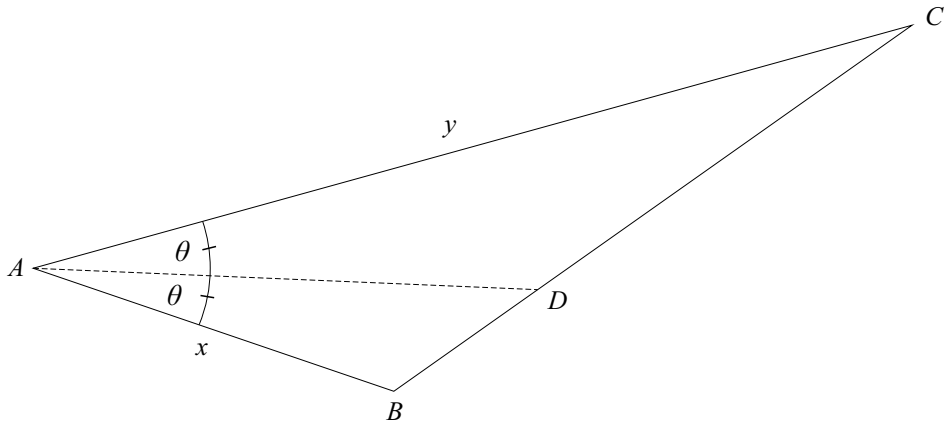
(b) Solve the following pair of simultaneous equations: 3

$$x^2 - y^2 = 11$$
$$\log_{10} x - \log_{10} y = 1$$

(c) Solve $\tan(\alpha - 45^\circ) = -\dfrac{1}{\sqrt{3}}$ for $0^\circ \leq \alpha \leq 360^\circ$. 3

QUESTION NINE (12 marks) **Marks**

(a) 3



In triangle ABC above $\angle BAC = 2\theta$, $AB = x$ and $AC = y$. The angle bisector of $\angle BAC$ meets side BC at D.

Show that $\dfrac{BD}{DC} = \dfrac{x}{y}$.

(b) The circle with equation $x^2+(y-6)^2 = r^2$ with $r > 0$, and the parabola with equation $y = -\dfrac{1}{4}(x-10)(x+10)$ intersect in exactly two distinct points. 3

Calculate the possible values of r.

(c) Let $P(x) = x^{82} - 3x^{81} + 2x^{80} + x^{14} - 3x^{13} + 2x^{12} + 7x$. When $P(x)$ is divided by $x(x-1)(x-2)$, the quotient is $Q(x)$ and the remainder is $R(x)$, such that:

$$x^{82} - 3x^{81} + 2x^{80} + x^{14} - 3x^{13} + 2x^{12} + 7x = x(x-1)(x-2)Q(x) + R(x).$$

(i) State the maximum value of the degree of $R(x)$. 1

(ii) Hence, without using long division, state the remainder when $x^{82} - 3x^{81} + 2x^{80} + x^{14} - 3x^{13} + 2x^{12} + 7x$ is divided by $x(x-1)(x-2)$. 2

(d) Two thirds of the pupils at a school either play an instrument, play rugby or play both. Three quarters of the pupils at the school either play an instrument, play volleyball or play both. None of the rugby players at the school happen to play volleyball and only one twelfth of the pupils at the school play none of these three things. Playing rugby and playing an instrument are independent events. Playing volleyball and playing an instrument are also independent events.

(i) Draw a Venn diagram to illustrate this situation for the pupils at this school. 1

(ii) Calculate the probability that a pupil chosen at random plays rugby. 2

END OF PAPER

B L A N K P A G E

B L A N K P A G E

SQS Form IV Annual Examination 2020

1) a) 2 ✓

b) i) 2.41×10^{-3} ✓

ii) 8340 ✓

c) $x = -2$ ✓

d) i) 4 ✓

ii) $6x^4$ ✓

e) $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓

f) i) $\left(\sqrt{64}\right)^3 = 8^3$

$= 512$ ✓

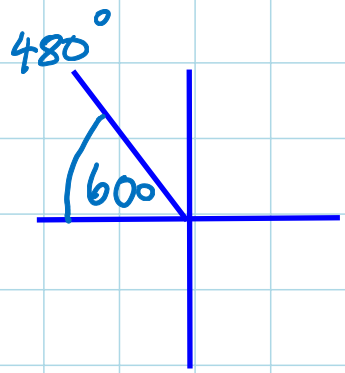
ii) $\frac{14x^{12}}{2x^4} = 7x^8$ ✓

g) $\sin 30 = \frac{x}{6}$

$x = 6 \times \frac{1}{2}$

$x = 3$ ✓

h)



Correct quadrant ✓

60° ✓

2.) a) $x-8=0$ or $2x+3=0$

$x=8$ ✓ or $2x=-3$

$x=-\frac{3}{2}$ ✓

b) i) $\log_{10}8-\log_{10}4=\log_{10}(8/4)$

$=\log_{10}(2)$ ✓

ii) $2\log_{10}3+\log_{10}2=\log_{10}3^2+\log_{10}2$ ✓

$=\log_{10}9+\log_{10}2$

$=\log_{10}(9\times2)$

$=\log_{10}(18)$ ✓

c) $V=\pi r^2h$

$=\pi\times2^2\times10$

$=40\pi\ cm^3$ ✓✓ (penalise units here)

d) $3^x=37$

$x=\log_3 37$ ✓

$=\frac{\log_{10}37}{\log_{10}3}$ ✓

$=3.29$ (to 2 d.p)

e) i) when planted $h = 2.1$ m
5 years after $h = 10.2$ m } ✓

ii) $(0, 2.1)$ $(5, 10.2)$

$$m = \frac{10.2 - 2.1}{5 - 0}$$

$$m = 1.62$$ ✓

hence $y - y_1 = m(x - x_1)$

$$y - 2.1 = 1.62(n - 0)$$

$$y = 1.62n + 2.1$$

iii) $n = 10$, $y = 1.62 \times 10 + 2.1$

$$y = 16.2 + 2.1$$

$$y = 18.3$$

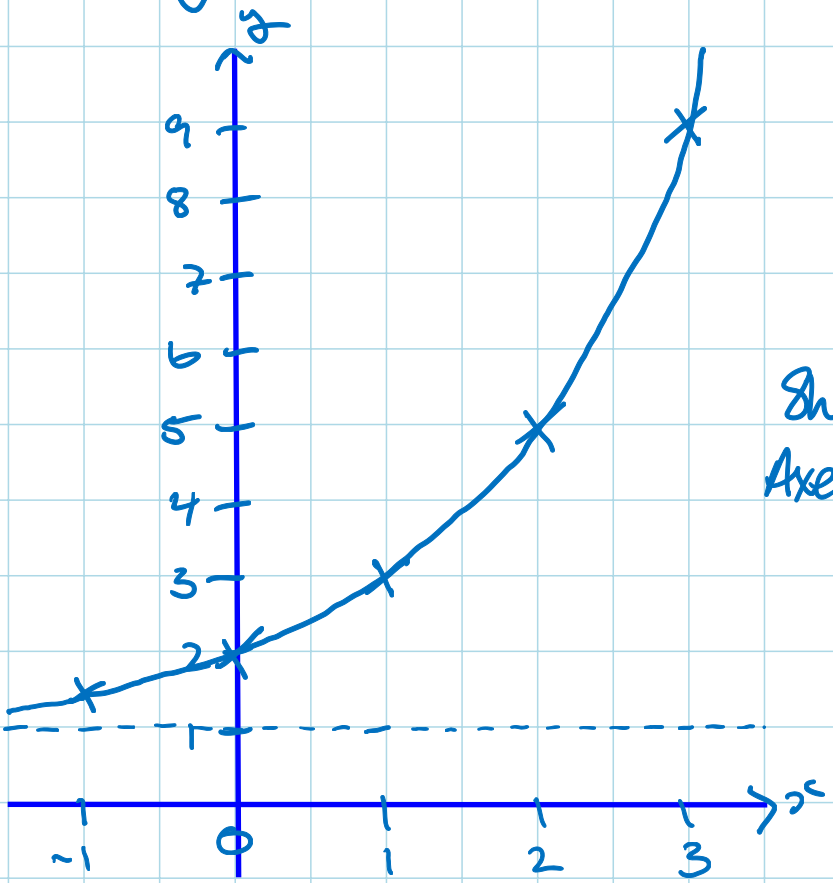
The tree will be 18.3m high. ✓

3) a) i)

x	-1	0	1	2	3
y	1.5	2	3	5	9

✓

ii)



Shape ✓
Axes, labels ✓

(Do not penalise missing asymptote).

iii) $y = 1$ ✓

b) i) $a^2 = b^2 + c^2 - 2bc\cos A$

$AB^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos 60$ ✓

$AB = \sqrt{31}$ cm ✓

ii) Area $= \frac{1}{2} \times 6 \times 5 \times \sin 60°$ ✓

$= \frac{15\sqrt{3}}{2}$ cm^2 ✓

Only penalise exact value once.

c) i) 700 ✓

ii) $P = 700 \times 1.15^{40}$

$= 187504$ ✓

iii) $1000,000,000 = 700 \times 1.15^{n}$

$\frac{10,000,000}{7} = 1.15^{n}$

$n = \log_{1.15}\left(\frac{10,000,000}{7}\right)$

$n = \frac{\log_{10}\left(\frac{10,000,000}{7}\right)}{\log_{10}(1.15)}$

$n = 101.4023221$ ✓

∴ During 102nd year ✓

(Trial and error is fine)

4) a) $x^2 - 3x - 10 = (x+2)(x-5)$

numbers ✓

signs ✓

b) i) Red = {1, 2, 3, 4, 5}

Yellow = {1, 2, 3, 4}

ξ = {2, 3, 4, 5, 6, 7, 8, 9} ✓ (Accept a list)

ii)

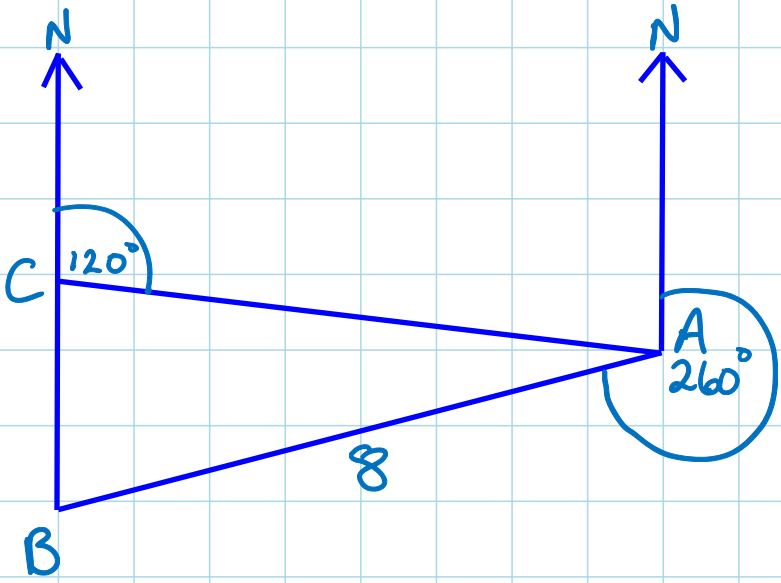
	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9

Any valid working ✓

$\frac{4}{20} = \frac{1}{5}$ ✓ (Simplified)

iii) $\frac{3}{20}$ ✓

c) i)



✓ (All info needed)

ii) $\angle CBA + 100^\circ = 180^\circ$

$\angle CBA = 80^\circ$ ✓

iii) $\angle ACB = 60^\circ$

$\frac{AC}{\sin 80} = \frac{8}{\sin 60}$ ✓

$AC = 9.097$ km ✓

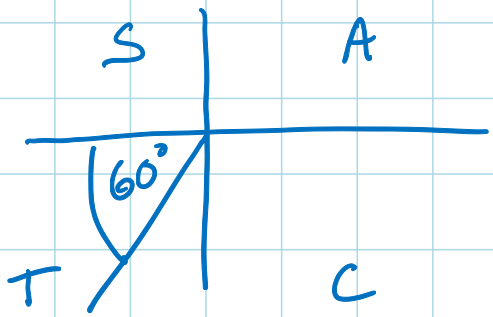
(to nearest m)

d) $P(x) = x^3 - 3x + a$

$-4 = 1^3 - 3\times 1 + a$ ✓

$a = -2$ ✓

5.) a) $\tan(240^\circ)$



$\tan(240) = \tan(60)$ ✓

$= \sqrt{3}$ ✓

b) $\Delta = b^2 - 4ac$

$= 4^2 - 4 \times 1 \times 30$

$= -104.$ ✓

$\Delta < 0$ ∴ No real solutions.

c) i) $y = x^2 - 8x + 10$

$y = (x-4)^2 - 16 + 10$

$y = (x-4)^2 - 6$ ✓

ii) α) $(4, -6)$ ✓

β) $(x-4)^2 - 6 = 0$ ✓

$(x-4)^2 = 6$

$x - 4 = \pm\sqrt{6}$

$x = 4 \pm \sqrt{6}$ ✓

$x = 4+\sqrt{6}$ or $x = 4-\sqrt{6}$

d) i) medians : Queensland 58 cm
P.N.G 66 cm } ✓

I.Q.R : Qunslnd : 73 − 53 = 20
P.N.G : 73 − 58 = 15 } ✓

ii) α) 50% ✓ β) 75% ✓

iii) With an I.Q.R of 20, Queensland flatbacks have a wider spread of lengths compared to P.N.G (with an I.Q.R of only 15).

Although they have a wider spread of lengths, Queensland flatbacks have a lower median (of 58) compared to those in P.N.G (of 66) meaning that they are, on average, shorter.

(Must discuss both spread and central tendency for 1 mark). ✓

Q6) a) i) $\bar{x} = \frac{6+8+12+14+15}{5}$

$\bar{x} = 11$ ✓

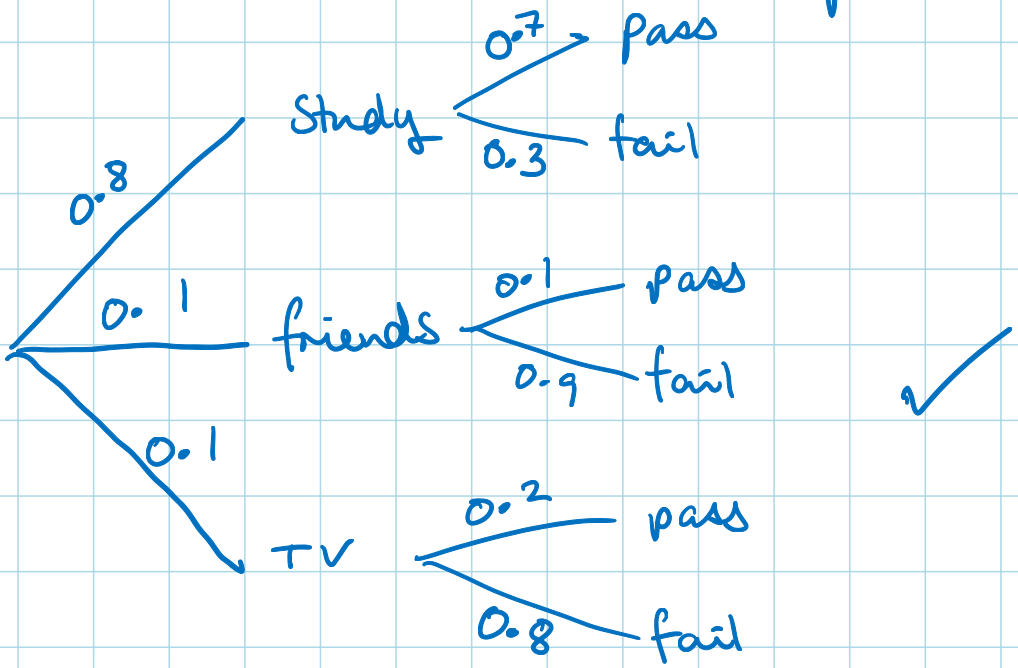
ii) $\sigma^2 = \frac{(6-11)^2 + (8-11)^2 + (12-11)^2 + (14-11)^2 + (15-11)^2}{5}$ ✓

$\sigma^2 = 12$

$\sigma = \sqrt{12}$

$= 3.46$ ✓ (to 2 d.p)

b) i)



✓

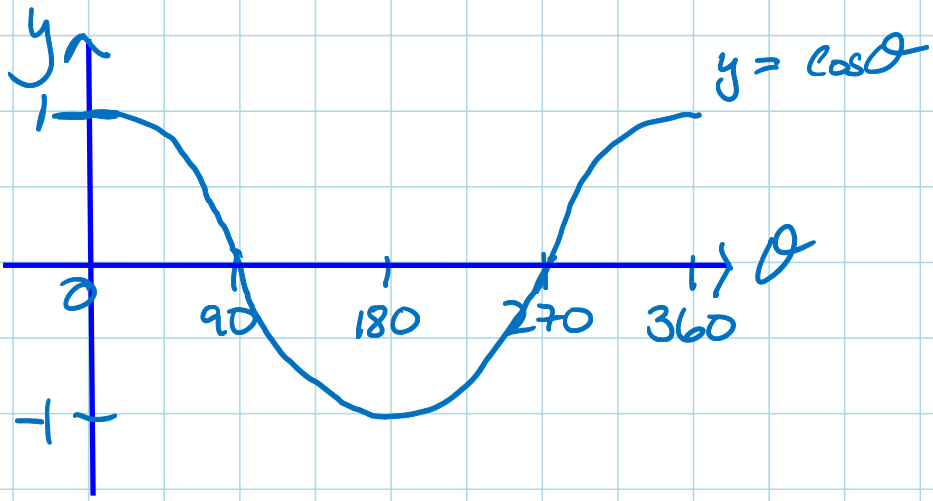
ii) $P(\text{friends}) \times P(\text{fails}) = 0.1 \times 0.9 = 0.09$ ✓

iii) $0.8 \times 0.7 + 0.1 \times 0.1 + 0.1 \times 0.2 = 0.59$ ✓

iv) $P(TV \mid P) = \frac{P(TV \cap P)}{P(P)} = \frac{0.1 \times 0.2}{0.59}$

$= \frac{2}{59}$ ✓

c) i)



general shape and max/min -1 and 1 ✓

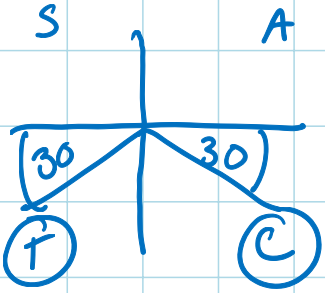
x-intercepts and labelled axes ✓

ii) $\theta = 90^\circ$ or $\theta = 270^\circ$ ✓ (both needed).

d) $\sin\theta = -\frac{1}{2}$

$\theta_r = \sin^{-1}(1/2)$

$\theta_r = 30^\circ$ ✓



$\theta = 210^\circ$ or $\theta = 330^\circ$ ✓ (Both)

7) a) $P(x) = x^4 - 5x^3 - x^2 + 11x - 30$

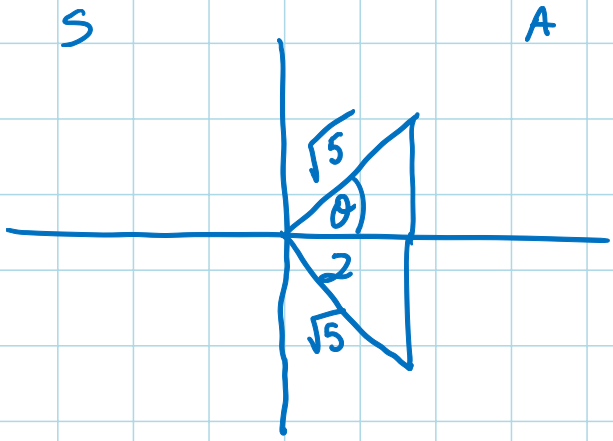
i) $P(5) = 5^4 - 5\times5^3 - 5^2 + 11\times5 - 30$ ✓

$= 0$

ii) 30: 1×30, 2×15, 3×10, 5×6

$P(-2) = 0$ ∴ $(x+2)$ ✓

b) $\cos\theta = \frac{2}{\sqrt{5}}$ $\left(\frac{A}{H} = \frac{2}{\sqrt{5}}\right)$



$(\sqrt{5})^2 = 2^2 + x^2$

$5 - 4 = 1$ ✓

$\sin\theta = \pm\frac{1}{\sqrt{5}}$ (both) ✓

c) $y = \frac{3}{x+3}$ $y = 2x+5$

$2x+5 = \frac{3}{x+3}$

$(2x+5)(x+3) = 3$ ✓

$2x^2 + 11x + 15 = 3$

$2x^2 + 11x + 12 = 0$

$(2x+3)(x+4) = 0$

$2x + 3 = 0$ or $x + 4 = 0$

$2x = -3$

$x = -3/2$

$y = 2$ ✓

$x = -4$

$y = -3$ ✓

d)

$$2x^2 + 7 \;\overline{)\; 6x^4 + 4x^3 + 19x^2 + 14x - 7}$$

$3x^2 + 2x - 1$ ✓

$6x^4 + 21x^2$ ✓

$4x^3 - 2x^2 + 14x - 7$

$4x^3 + 14x$

$-2x^2 - 7$

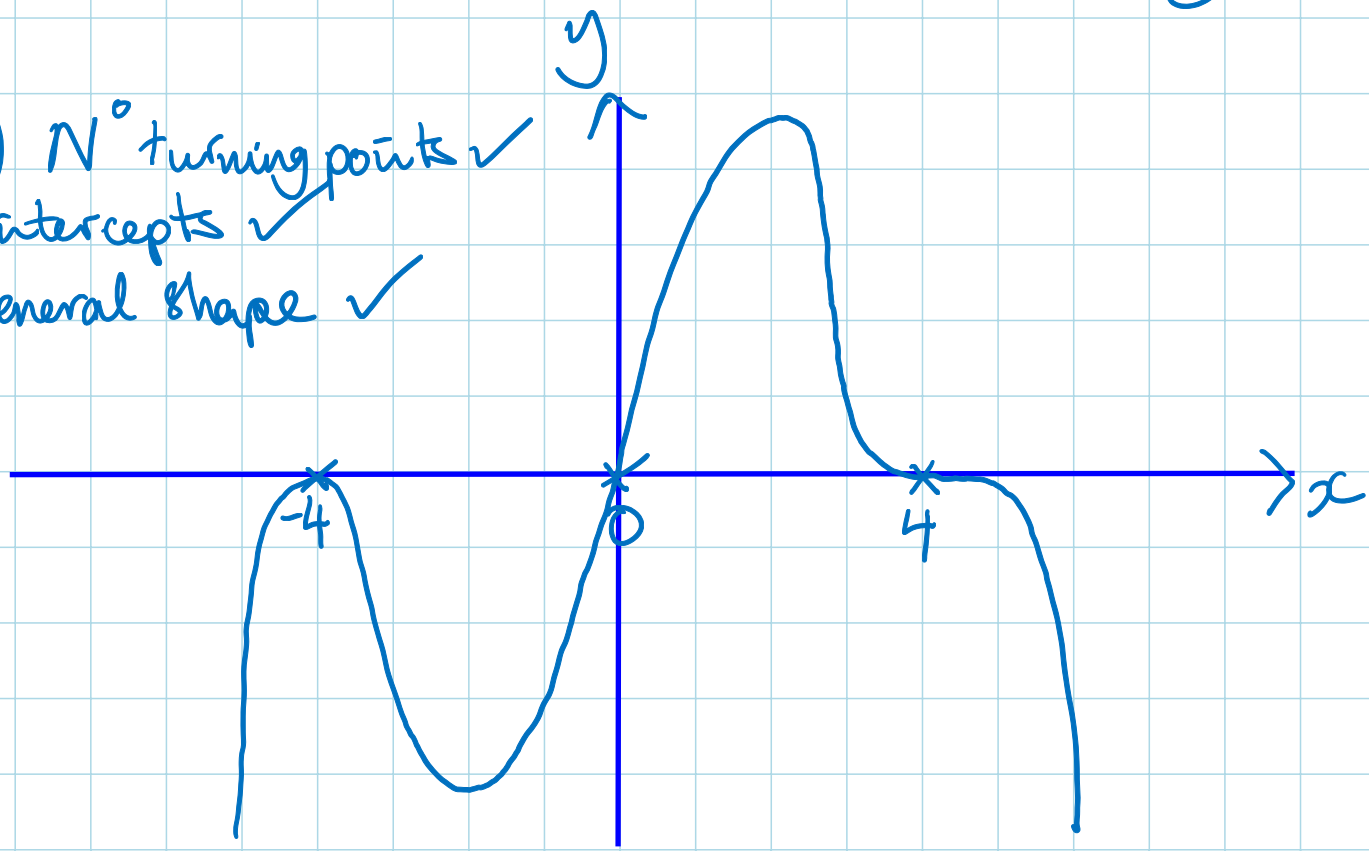
$-2x^2 - 7$

0

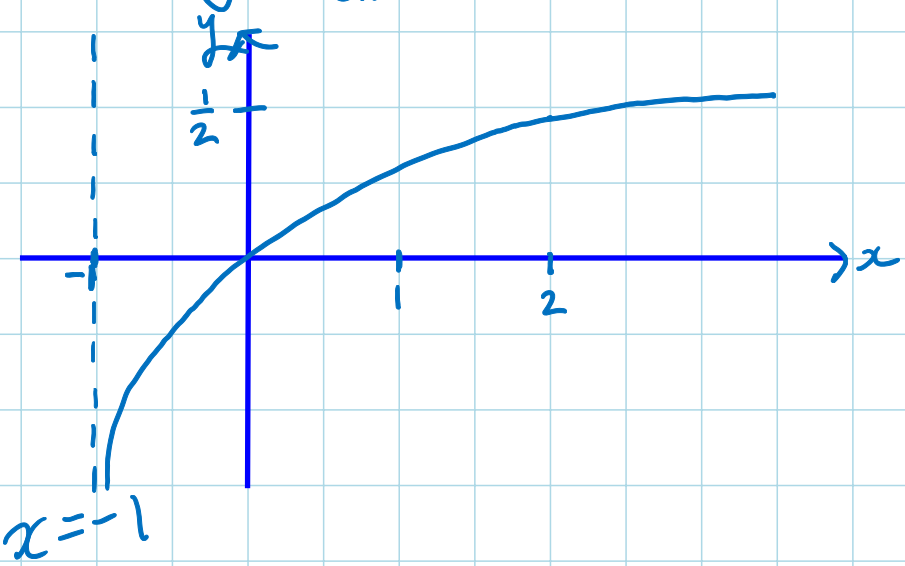
e) N° turning points ✓

intercepts ✓

General shape ✓



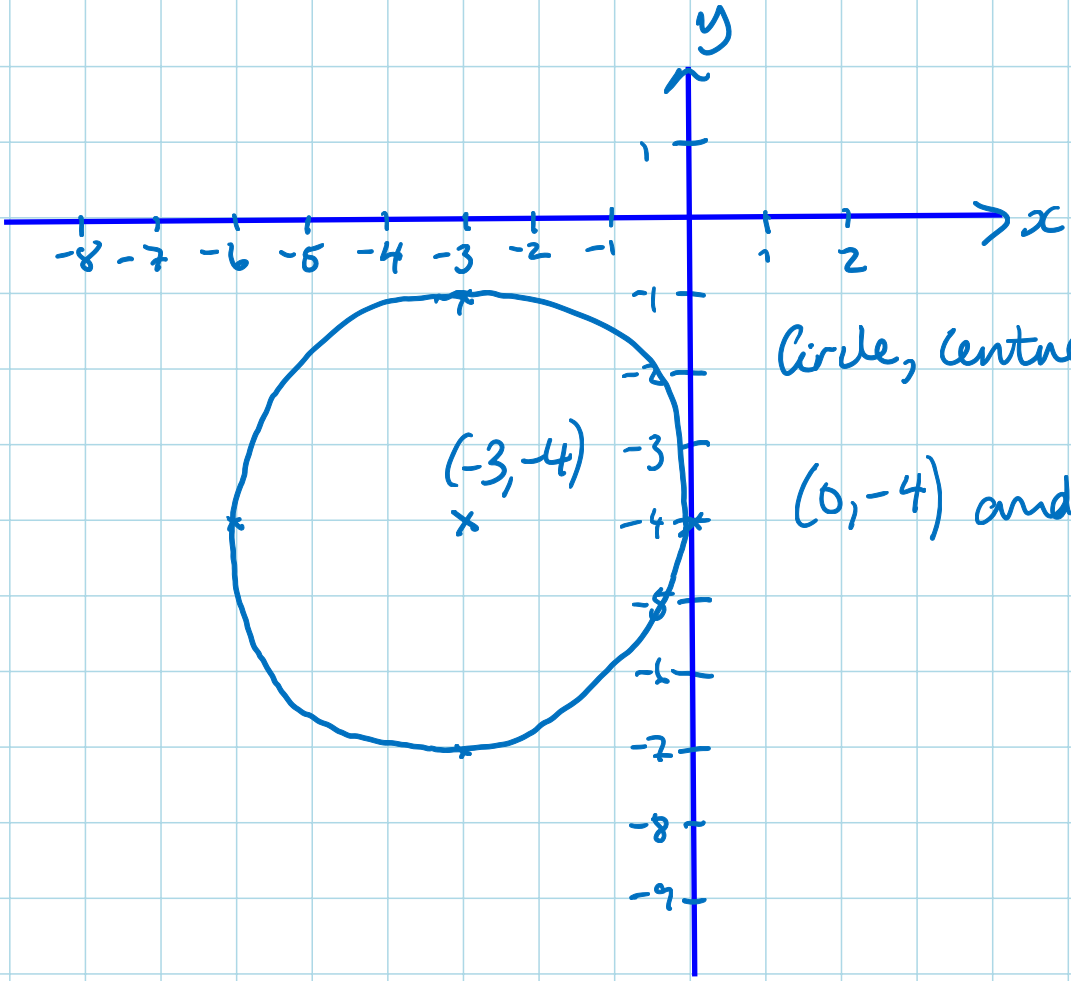
8) a) ii) $y = \log_{10}(x+1)$



Shape + axes ✓

(0,0) and asymptote ✓

ii) $(x+3)^2 + (y+4)^2 = 9$

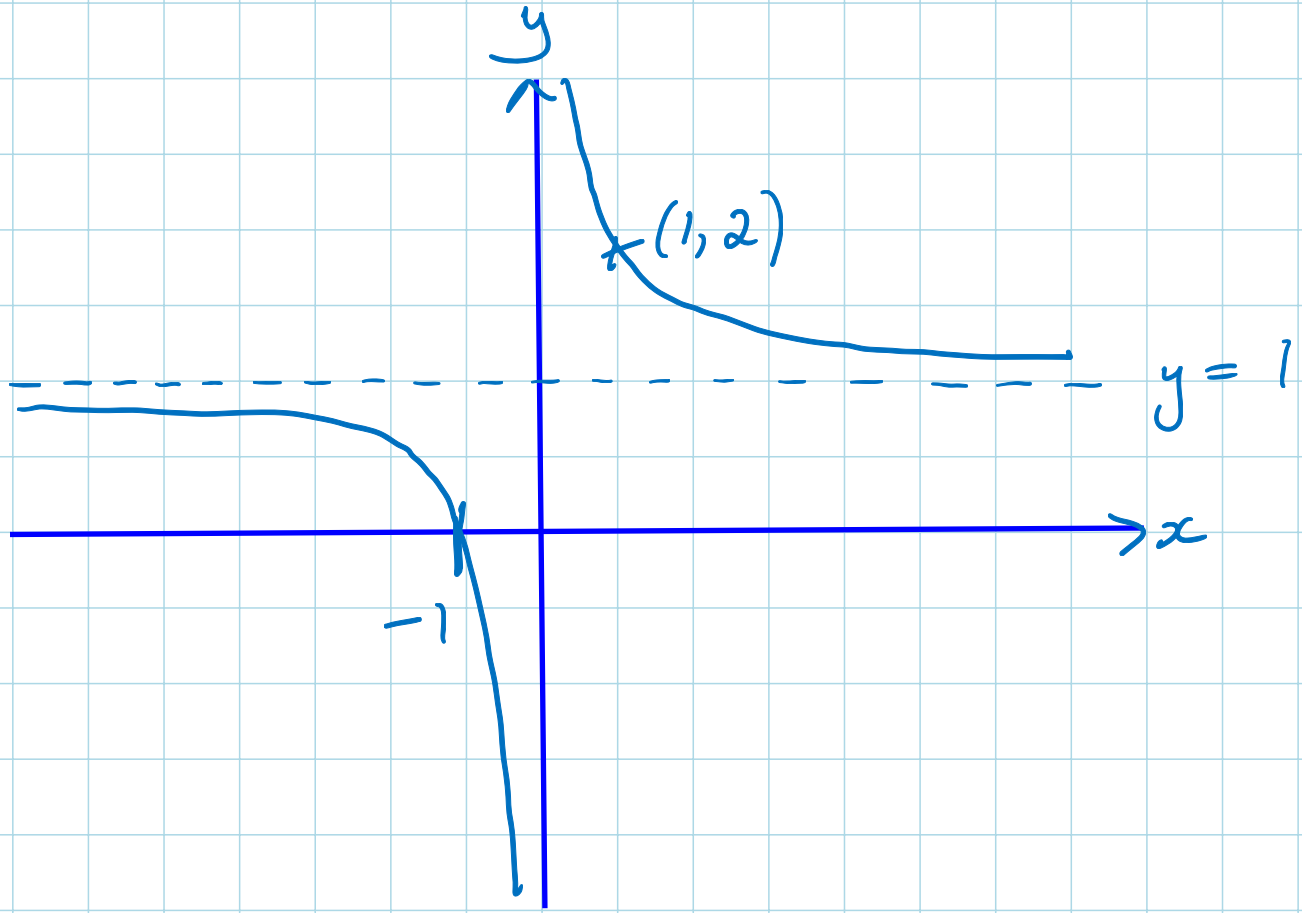


Circle, centre (-3,-4) ✓

(0,-4) and axes ✓

iii) $y = \frac{1}{x} + 1$

$y - 1 = \frac{1}{x}$



Shape and asymptote ✓

x-intercept and labelled axes ✓

b) $x^2 - y^2 = 11$ ① $\log_{10} x - \log_{10} y = 1$ ②

② $\log_{10}\left(\frac{x}{y}\right) = 1$ ✓

$\frac{x}{y} = 10$

$x = 10y$

① $(10y)^2 - y^2 = 11$ ✓

$100y^2 - y^2 = 11$

$99y^2 = 11$

$y^2 = \frac{1}{9}$

$y = \pm 1/3$

$y > 0$

$\therefore y = 1/3$

$x^2 - \frac{1}{9} = 11$

$x^2 = \frac{100}{9}$

$x > 0$

$x = \frac{10}{3}$

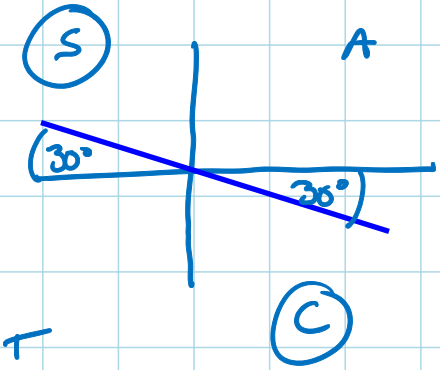
} both (✓)

c) $\tan(\alpha - 45^\circ) = -\frac{1}{\sqrt{3}}$ for $0^\circ \leq \alpha \leq 360^\circ$

$\tan\theta = -\frac{1}{\sqrt{3}}$

$-45^\circ \leq \alpha - 45 \leq 315^\circ$

$-45^\circ \leq \theta \leq 315^\circ$



$\theta_r = 30^\circ$ ✓

$\theta = (180 - 30)^\circ, -30^\circ$

$\theta = 150^\circ, -30^\circ$ ✓

$\alpha - 45 = 150^\circ, -30^\circ$

$\alpha = 195^\circ, 15^\circ$ ✓

9) a) $\frac{BD}{\sin\theta} = \frac{x}{\sin\alpha}$ ✓ where $\alpha = \angle ADB$

$\frac{DC}{\sin\theta} = \frac{y}{\sin\beta}$ where $\beta = \angle ADC$

In general, $\sin(\phi) = \sin(180 - \phi)$

and since $\alpha + \beta = 180°$ (straight angle)

$\alpha = 180° - \beta$

$\sin(\alpha) = \sin(\beta)$ ✓

hence $\frac{BD}{x} = \frac{\sin\theta}{\sin\alpha} = \frac{DC}{y} = \frac{\sin\theta}{\sin\beta}$ ✓

$\therefore \frac{BD}{DC} = \frac{x}{y}$

b) $x^2 + (y-6)^2 = r^2$ $\quad y = -\frac{1}{4}(x-10)(x+10)$

$x^2 + \left(\left[-\frac{1}{4}(x-10)(x+10)\right] - 6\right)^2 = r^2$ ✓

$x^2 + \left(-\frac{1}{4}(x^2-100) - 6\right)^2 = r^2$

$x^2 + \left(-\frac{1}{4}x^2 + 25 - 6\right)^2 = r^2$

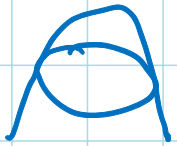
$x^2 + \left(-\frac{1}{4}x^2 + 19\right)^2 = r^2$

$x^2 + \frac{1}{16}x^4 - \frac{19}{2}x^2 + 361 = r^2$

$\frac{1}{16}x^4 - \frac{17}{2}x^2 + (361 - r^2) = 0$

let $u = x^2$

$\frac{1}{16}u^2 - \frac{17}{2}u + (361 - r^2) = 0$

$\Delta = 0$ if  and touches in 2 places.

$\therefore b^2 - 4ac = 0$

$\left(\frac{17}{2}\right)^2 - 4\left(\frac{1}{16}\right)(361 - r^2) = 0$

$r^2 = 72$

$r = \sqrt{72}$ ✓

OR if , $r > 19$ ✓

c) i) 2 ✓ ($R(x)$ at most 1 degree less than the divisor).

ii) $P(x) = x^{82} - 3x^{81} + 2x^{80} + x^{14} - 3x^{13} + 2x^{12} + 7x$

$P(0) = 0$ $\quad P(0) = R(0)$

if $R(x) = ax^2 + bx + c$

$R(0) = 0 \quad \therefore \quad c = 0$

$P(1) = 7$ $\quad R(1) = a + b$

$P(2) = 14$ $\quad R(2) = 4a + 2b$

$\therefore$ ① $7 = a + b$

② $14 = 4a + 2b$ } ✓

② $- 2 \times$ ① $\quad 14 = 4a + 2b$

$- (14 = 2a + 2b)$

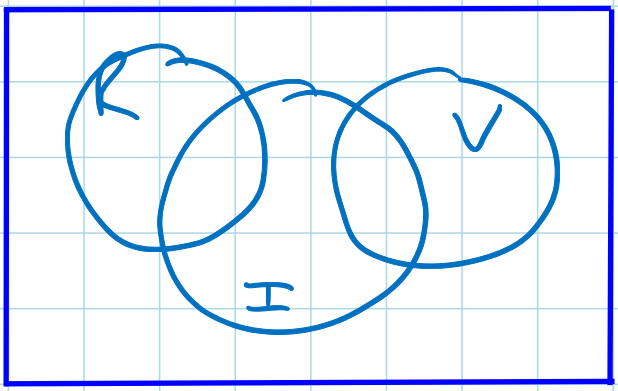
$0 = 2a$

$a = 0$

$\therefore \quad b = 7$

$\therefore \quad R(x) = 7x$ ✓

d)



✓

ii) Since independent events $P(R \cap I) = P(R) \times P(I)$

and $P(V \cap I) = P(V) \times P(I)$

$\therefore P(R \cup I) = P(R) + P(I) - P(R \cap I)$

$\frac{2}{3} = P(R) + P(I) - P(R)P(I)$ ①

Similarly $\frac{3}{4} = P(V) + P(I) - P(V)P(I)$ ②

Also $\frac{11}{12} = P(I) + P(R) + P(V) - P(R)P(I) - P(V)P(I)$ ③

① + ② − ③

$\frac{2}{3} + \frac{3}{4} - \frac{11}{12} = P(R) + P(I) - P(R)P(I)$
$+ P(V) + P(I) - P(V)P(I)$
$- P(R) - P(V) - P(I) + P(R)P(I)$
$+ P(V)P(I)$

$\frac{1}{2} = P(I)$ ✓

① $\frac{2}{3} = P(R) + \frac{1}{2} - \frac{1}{2} \times P(R)$

$\frac{1}{6} = \frac{1}{2} P(R)$

$P(R) = \frac{1}{3}$ ✓