SYDNEY GRAMMAR SCHOOL

NAME _____ MASTER _____

2019 Annual Examination

Form IV Mathematics

Friday 1st November 2019

General Instructions

- Working time — 2 hours
- Attempt all questions
- Write using black pen
- Calculators approved by NESA may be used

Total Marks: 108

- Relevant mathematical reasoning and calculations are required
- Record your answers on the writing paper provided

Collection

- Arrange your solutions in order
- Staple the sheets of writing paper together
- Write your name and master on this page and on the first page of your writing paper
- Place everything inside this question booklet

4A: RR	4B: REJ	4C: LYL	4D: OMD
4E: BR	4F: BDD	4G: PKH	4H: SDP
4I: LL	4J: NJL		

Checklist

- 14 sheets of writing paper per boy
- Candidature: 183 pupils

RR

Question	Marks
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
TOTAL:	

QUESTION ONE (12 marks) **Marks**

(a) Use your calculator to evaluate $\cos 36^\circ$ correct to three decimal places. **1**

(b) Simplify $3x^2 - 1 + 2x^2$. **1**

(c) Write the number 0·0045 in scientific notation. **1**

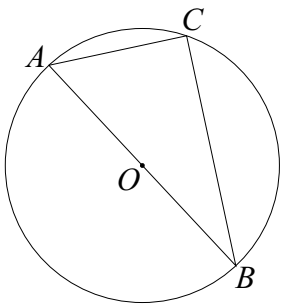
(d) Expand and simplify $(x - 3)(x + 3)$. **1**

(e) Simplify $\frac{x}{3} + \frac{x}{5}$. **1**

(f) Evaluate $\log_3 81$. **1**

(g) Factorise $x^2 - x - 2$. **1**

(h) **1**



The circle above has centre O and a diameter of AB. What is the size of $\angle ACB$?

(i) If $P(x) = x^3 + 2x$, evaluate $P(5)$. **1**

(j) Write down the exact value of $\sin 60^\circ$. **1**

(k) Write down the gradient of the line $3y = x + 4$. **1**

(l) Determine the median of the following numbers: **1**

3, 3, 5, 7, 9, 10, 14, 18, 21

QUESTION TWO (12 marks) **Marks**

(a) Find the exact value of $\cos 135^\circ$. **2**

(b) Write the following expressions in the form x^k:

(i) $\dfrac{1}{x^3}$ **1**

(ii) $x^2 \times \sqrt{x}$ **1**

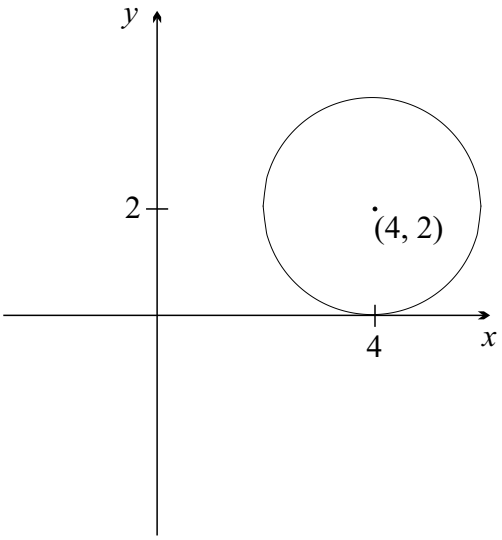
(c) Express $\dfrac{1}{\sqrt{3}+1}$ with a rational denominator. **2**

(d) Simplify the following:

(i) $\log_5 \sqrt{5}$ **1**

(ii) $\log_5 15 - \log_5 3$ **1**

(e) **2**



Find the equation of the circle in the diagram above with centre $(4, 2)$ and radius 2 units.

(f) Determine the vertex of the parabola $y = x^2 - 4x - 5$. **2**

QUESTION THREE (12 marks) **Marks**

(a) Solve $\tan\theta = \sqrt{3}$ for $0° \leq \theta \leq 360°$. **2**

(b) The numbers 1 to 10 are written down on separate pieces of paper and placed in a hat. One number is picked out at random.

(i) Write down the probability that this number is:

(α) A multiple of two **1**

(β) A multiple of three **1**

(ii) Are these two events mutually exclusive? Explain your answer. **1**

(c) Let $P(x) = 3x^2 + 2$ and $Q(x) = 5 - x^2$. Find simplified expressions for:

(i) the sum $P(x) + Q(x)$ **1**

(ii) the difference $P(x) - Q(x)$ **1**

(iii) the product $P(x) \times Q(x)$ **1**

(d) Solve the following pair of simultaneous equations: **2**

$$\begin{aligned} 3x + 2y &= 8 \\ 2x - y &= 10 \end{aligned}$$

(e) In the last two months Alyssa Healy has played six one-day international cricket matches for Australia. Her mean run score for the six matches is 72 runs. **2**

If she scored 112, 71, 8, 61, and 58 runs in the first five matches respectively, how many runs did she score in the sixth match?

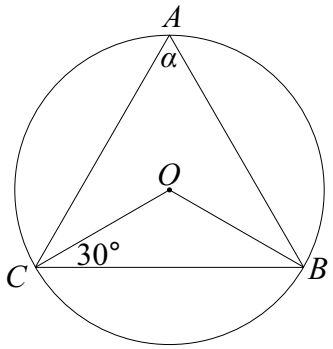
QUESTION FOUR (12 marks) **Marks**

(a) Solve $3^x = 47$. Give your answer to 3 decimal places. **2**

(b) (i) Find the gradient of the line through the points $A(2, -1)$ and $B(5, 3)$. **1**

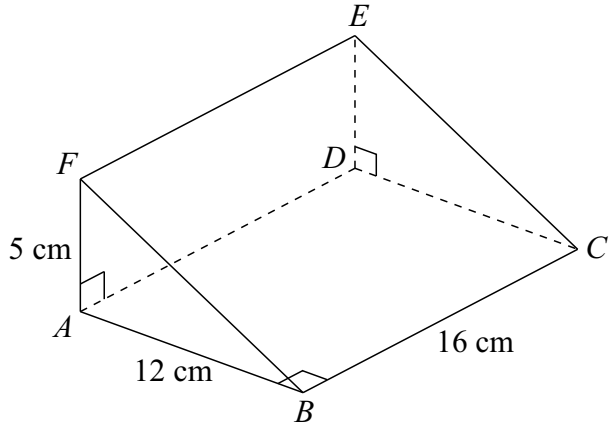
(ii) Hence find the equation of the line through A and B. **1**

(c) **2**



Find the value of the pronumeral α in the diagram above, giving full geometric reasoning.

(d)



A right triangular prism is shown in the diagram above, with $AB = 12\,\text{cm}$, $BC = 16\,\text{cm}$, and $AF = 5\,\text{cm}$.

(i) Find the volume of the prism. **1**

(ii) Calculate the surface area of the prism. **3**

(iii) Determine the size of the angle between the diagonals AC and CF. Give your answer correct to the nearest degree. **2**

QUESTION FIVE (12 marks) **Marks**

(a) Sketch the following graphs on separate number planes. Make sure to include all important features and intercepts. Make each graph one-third of a page in size.

(i) $y = 2^x - 1$ **2**

(ii) $y = (x-1)^2$ **2**

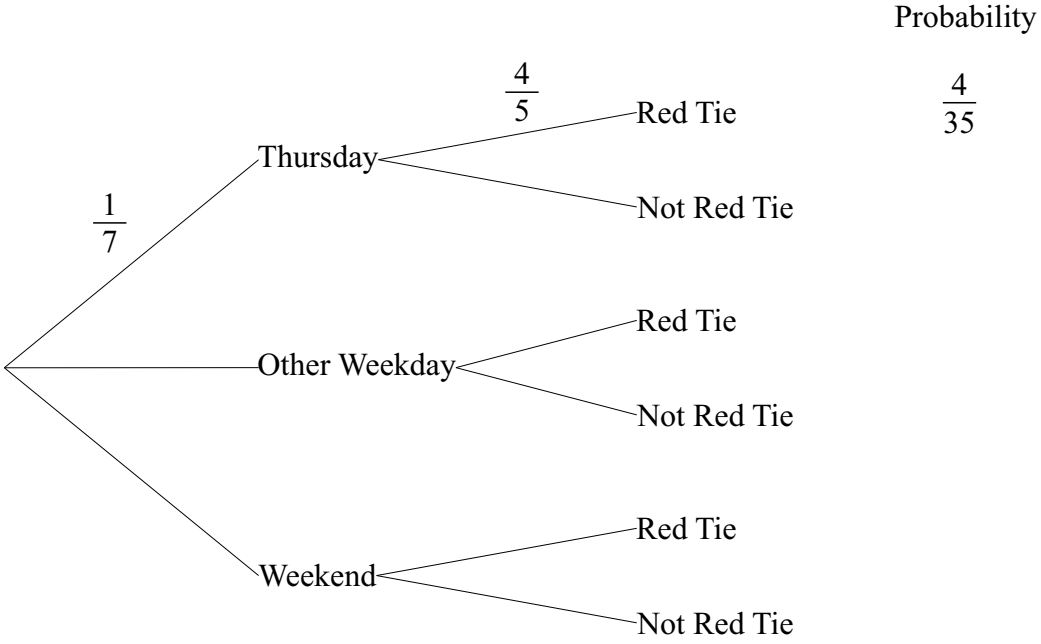
(iii) $y = (x-1)^2(x+2)^3$ **2**

(b) (i) Sketch $y = \sin\theta$ for $0^\circ \leq \theta \leq 360^\circ$. **2**

(ii) Hence solve $\sin\theta = 0$ for $0^\circ \leq \theta \leq 360^\circ$. **1**

(c) A certain teacher likes to match his ties to the day of the week. On Thursdays, there is an 80% chance that he is wearing a red tie. On other weekdays, there is a 10% chance that he is wearing a red tie. He doesn't wear a tie on weekends.

(i) Copy and complete the following tree diagram. **1**



(ii) What is the probability that he wears a red tie? **1**

(iii) One day he is spotted wearing a red tie. What is the probability that the day on which he was spotted was a Thursday? **1**

QUESTION SIX (12 marks) **Marks**

(a) Shannon and Arjun are playing a game which requires rolling two dice and taking the sum of the top faces. Shannon is lazy so he rolls one die and then doubles the value instead. Their results after twelve turns each are as follows:

Shannon	2	2	4	4	6	6	8	8	10	10	12	12
Arjun	2	3	5	6	7	7	7	8	8	9	10	12

(i) Determine the median, mean, and range of both sets of data. **2**

(ii) By considering other statistical measures of spread, give an example of how these data sets are different. **1**

(b) Consider the polynomial $P(x) = x^3 - 8x^2 + 21x - 18$.

(i) Show that $x - 2$ is a factor of $P(x)$. **1**

(ii) Hence fully factorise $P(x)$. **2**

(c) The number of koalas on Moreton Island has been rapidly decreasing. At the beginning of 2012 there were 1650 koalas and this figure has been decreasing by 30% per year on average.

(i) Let t be the number of years from the beginning of 2012. Write down a formula for K, the number of koalas on the island, in terms of t. **1**

(ii) Determine the first year in which there will be fewer than 100 koalas left on the island. **2**

(d) Odysseus sets sail from his home city of Ithica. He sails on a bearing of S60°E for 15 km before stopping at point A. He then sails on a bearing of S60°W for 30 km to point B.

(i) Draw a diagram showing this information. **1**

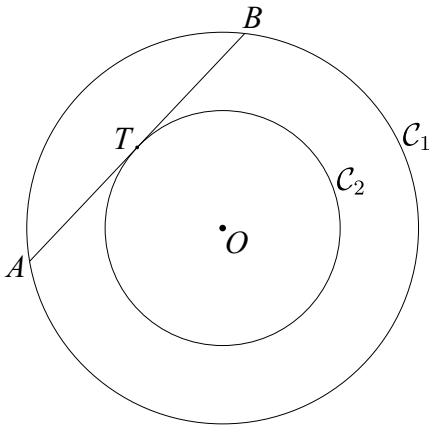
(ii) How far is Odysseus from Ithica? Give your answer correct to the nearest kilometre. **2**

QUESTION SEVEN (12 marks) **Marks**

(a) Find the points of intersection between the circle $x^2 + y^2 = 20$ and the hyperbola $y = \dfrac{6}{x}$. **3**

(b) Given that $\log_a 4 = 2{\cdot}8808$ and $\log_a 3 = 2{\cdot}2830$ correct to four decimal places, evaluate $\log_{a^2} 72$. **3**

(c)



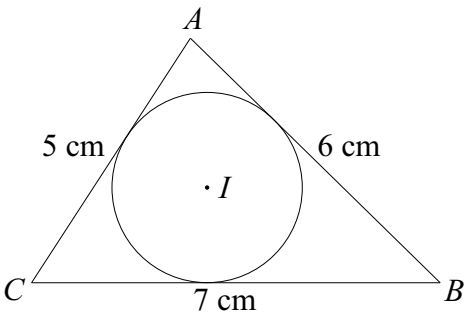
$AB = 10$ cm

Two concentric circles $\mathcal{C}_1$ and $\mathcal{C}_2$ have common centre O. AB is a chord of C_1 and a tangent to C_2 at T. The length of AB is 10 cm.

(i) Using geometric reasoning, explain why $TB = 5$ cm. **1**

(ii) Hence find the difference between the areas of the two circles. Express your answer as a multiple of π. **1**

(d)



A circle is inscribed within a triangle ABC with side lengths 5 cm, 6 cm, 7 cm and is tangent to each of the sides. Let I be the centre of the circle.

(i) Evaluate $\cos B$. **1**

(ii) Hence find the exact area of $\triangle ABC$. **2**

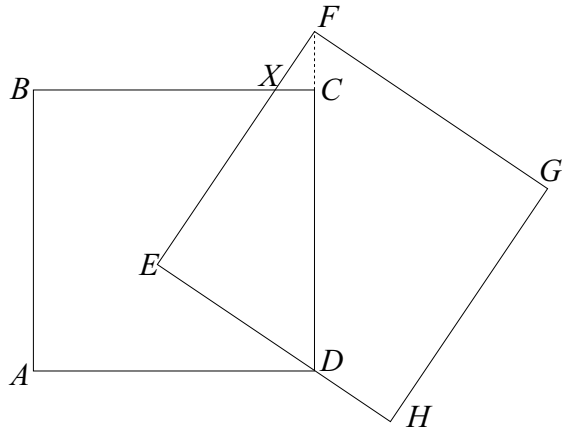
(iii) By considering the areas of $\triangle AIB, \triangle BIC$, and $\triangle CIA$, determine the exact radius of the circle. **1**

QUESTION EIGHT (12 marks) **Marks**

(a) Solve $2^{3x+1} = 27$. Give your answer correct to four decimal places. **2**

(b) Solve $2\cos 2\theta = 1$ for $0° \leq \theta \leq 360°$ given that $\cos\theta < 0$. **3**

(c)



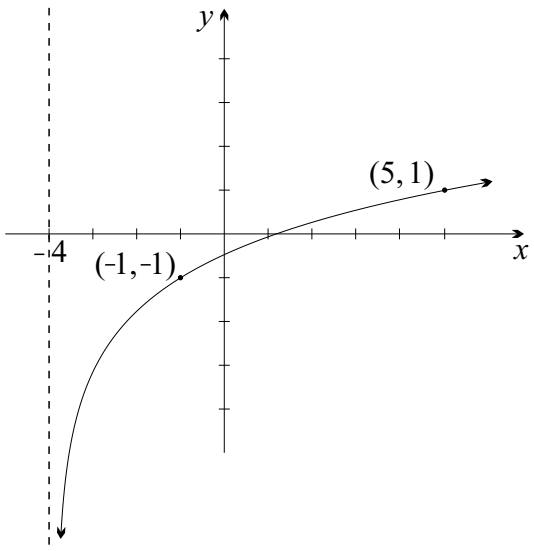
Two congruent squares $ABCD$ and $EFGH$ of side length 25 cm are arranged so that D lies on EH and F is collinear with CD. Let the point of intersection of BC and EF be called X. The length FX is 6 cm.

(i) Justify that the overlapping shape $EXCD$ forms a cyclic quadrilateral. **1**

(ii) Hence, or otherwise, determine the length of CF. **2**

(iii) Determine the area of the overlapping shape $EXCD$. **1**

(d) **3**



The above graph is of the form $y - k = \log_a(x - h)$.
It has a vertical asymptote of $x = -4$, and passes through $(-1, -1)$ and $(5, 1)$.
Determine the equation of the logarithm.

QUESTION NINE (12 marks) **Marks**

(a) Consider the cubic polynomial $P(x) = x^3 - (k+3)x - 2$, where k is a positive real number. Do NOT attempt to find the zeroes to this polynomial.

(i) By considering $P(0)$ and the graph of the polynomial $y = P(x)$, explain in fifty words or fewer why the cubic polynomial must have at least one zero which is positive. **1**

(ii) By considering $P(-1)$ and the graph of the polynomial $y = P(x)$, explain in fifty words or fewer why the cubic must have exactly one zero which is a positive. **2**

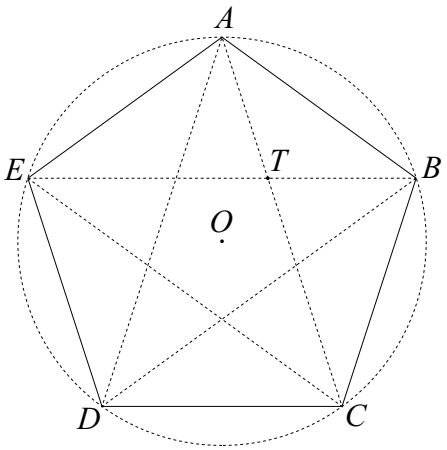
(b) A particular graph is given by all points on the Cartesian plane which are solutions to the equation $x^4 - y^4 = 4x^2 - 4y^2$.

(i) Show that all points where $y^2 = x^2$ satisfy the equation. **1**

(ii) By factorising the equation, find the set of points for which $x^4 - y^4 = 4x^2 - 4y^2$ if $x^2 \neq y^2$. Describe the familiar curve on which these points lie. **1**

(iii) Hence graph the equation $x^4 - y^4 = 4x^2 - 4y^2$, making sure to consider both parts above. **2**

(c)



Consider the regular pentagon $ABCDE$ with side length 1 unit. Since it is regular, it is also cyclic. Thus all five vertices of the pentagon lie on a circle with centre O.

(i) Explain using circle geometry why $\angle EAD = \angle DAC = \angle CAB = 36°$. **1**

(ii) Let AC intersect BE at T and let the length $BE = x$ units. Given $\triangle BAT \,|||\, \triangle BEA$, show that the length of the diagonal BE satisfies the equation **2**

$$x^2 - x - 1 = 0\,.$$

(iii) Hence find the exact value of $\cos 36°$. **2**

END OF PAPER

B L A N K P A G E

SGS Annual 2019 - Form IV Mathematics - Solutions

Question One

a) $\cos 36^\circ \doteq 0.809$ ✓ (Penalise rounding here)

b) $3x^2 - 1 + 2x^2 = 5x^2 - 1$ ✓

c) $0.0045 = 4.5 \times 10^{-3}$ ✓

d) $(x-3)(x+3) = x^2 - 9$ ✓

e) $\frac{x}{3} + \frac{x}{5} = \frac{8x}{15}$ ✓

f) $\log_3 81 = 4$ ✓

g) $x^2 - x - 2 = (x-2)(x+1)$ ✓

h) $\angle ACB = 90^\circ$ ✓ (angle in a semi-circle) (No penalty for lack of reason)

i) $P(x) = x^3 + 2x$

$P(5) = 5^3 + 2 \times 5$

$= 135$ ✓

j) $\sin 60^\circ = \frac{\sqrt{3}}{2}$ ✓

k) $3y \doteq x + 4$

$y = \frac{1}{3}x + \frac{4}{3}$

gradient is $\frac{1}{3}$ ✓

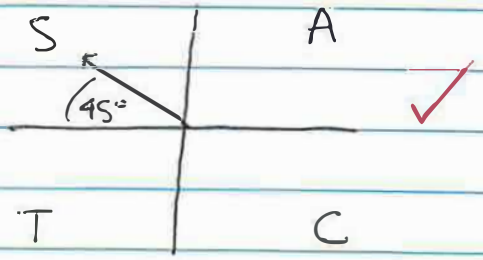
l) Median = 9 ✓

Question Two

a) $\cos 135^\circ$

$= -\cos(45^\circ)$

$= -\frac{1}{\sqrt{2}}$ ✓



✓

b) i) $\frac{1}{x^3} = x^{-3}$ ✓

ii) $x^2 \times \sqrt{x} = x^2 \times x^{\frac{1}{2}}$

$= x^{\frac{5}{2}}$ ✓

c) $\frac{1}{\sqrt{3}+1} = \frac{1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$ ✓

$= \frac{\sqrt{3}-1}{2}$ ✓

d) i) $\log_5 \sqrt{5} = \frac{1}{2}$ ✓

ii) $\log_5 15 - \log_5 3 = \log_5 5$

$= 1$ ✓

e) $(x-4)^2 + (y-2)^2 = 4$ ✓centre ✓radius

f) $y = x^2 - 4x - 5$

$= x^2 - 4x + 4 - 9$

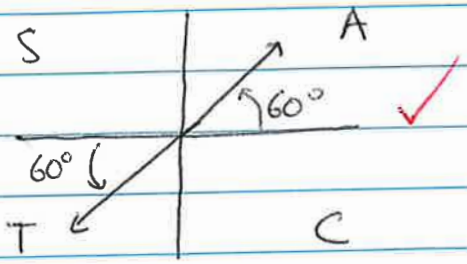
$= (x-2)^2 - 9$ ✓

Vertex at $(2, -9)$ ✓

Question Three

a) $\tan\theta = \sqrt{3}$

$\theta = 60°, 240°$ ✓



✓

b) i) α) $P(\text{Multiple of } 2) = \frac{5}{10}$

$= \frac{1}{2}$ ✓

β) $P(\text{Multiple of } 3) = \frac{3}{10}$ ✓

ii) These events are not mutually exclusive as the number drawn could be 6 which is a multiple of both 2 and 3. ✓

c) $P(x) = 3x^2 + 2 \qquad Q(x) = 5 - x^2$

i) $P(x) + Q(x) = 2x^2 + 7$ ✓

ii) $P(x) - Q(x) = 4x^2 - 3$ ✓

iii) $P(x) \times Q(x) = (3x^2+2) \times (5-x^2)$

$= 15x^2 - 3x^4 + 10 - 2x^2$

$= -3x^4 + 13x^2 + 10$ ✓

d) $3x + 2y = 8 \quad ①$

$2x - y = 10 \quad ②$

$2 \times ②$: $4x - 2y = 20 \quad ③$

$① + ③ \quad 7x = 28$

$x = 4$ ✓

sub into ①: $12 + 2y = 8$

$2y = -4$

$y = -2$ ✓

so $x = 4$ and $y = -2$

Question Three (cont.)

e) Let x_6 be the unknown score.

$$\bar{x} = \frac{112 + 71 + 8 + 61 + 58 + x_6}{6}$$ ✓

$$72 = \frac{310 + x_6}{6}$$

$$432 = 310 + x_6$$

$$x_6 = 122$$

Alyssa scored 122 runs in the sixth match. ✓

Question Four

a) $3^x = 47$

$x = \log_3 47$ ✓

$= \dfrac{\log_{10} 47}{\log_{10} 3}$

$\doteq 3.505$ ✓

b) i) $m_{AB} = \dfrac{3-(-1)}{5-2}$

$= \frac{4}{3}$ ✓

ii) l: $y-(-1) = \frac{4}{3}(x-2)$

$3y + 3 = 4x - 8$

l: $4x - 3y - 11 = 0$ ✓

c) $OB = OC$ (radii)

so $\angle OBC = 30°$ (equal base angles of isosceles $\triangle BOC$)

$\angle BOC = 120°$ (angle sum of $\triangle BOC$) ✓reasoning throughout

$\alpha = 60°$ ✓(angles at circumference and centre on minor arc BC)

d) i) $V = \frac{1}{2} \times 5 \times 12 \times 16$

$= 480\,cm^3$ ✓ (no penalty for incorrect units)

ii) $BF^2 = 5^2 + 12^2$

$BF = 13$ ✓

$S.A. = \frac{1}{2} \times 5 \times 12 \times 2 + 5 \times 16 + 12 \times 16 + 13 \times 16$ ✓

$= 540cm^2$ ✓ (penalise units here only)

iii) $AC^2 = 12^2 + 16^2$

$AC = 20$ ✓

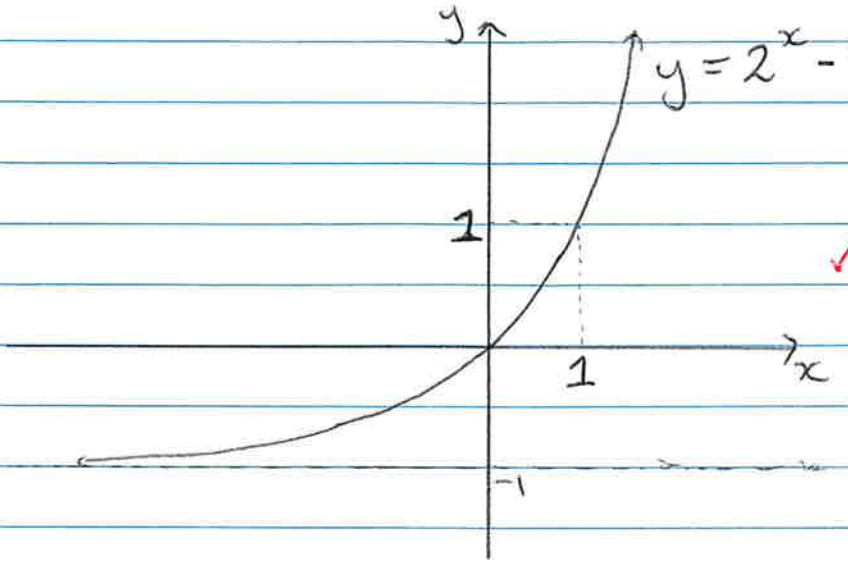
$\tan(\angle ACF) = \frac{5}{20}$

$\angle ACF = \tan^{-1}\frac{1}{4}$

$\doteq 14°$ ✓

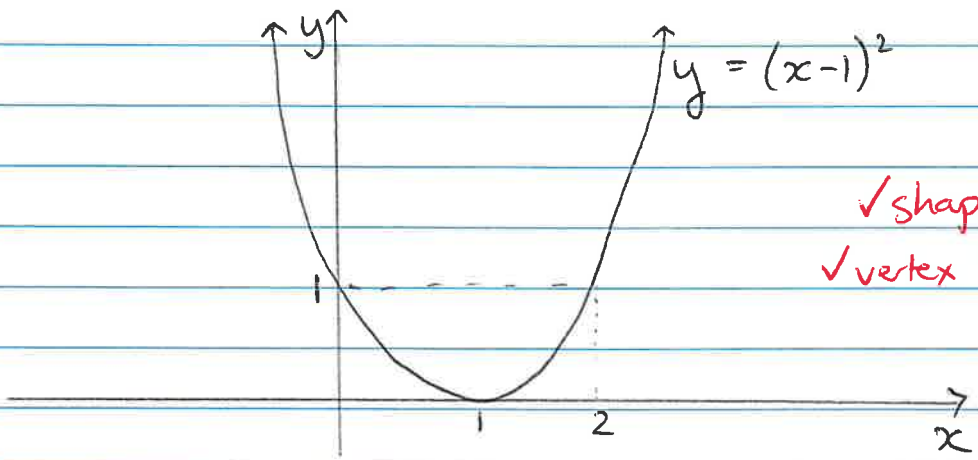
Question Five

a) i)



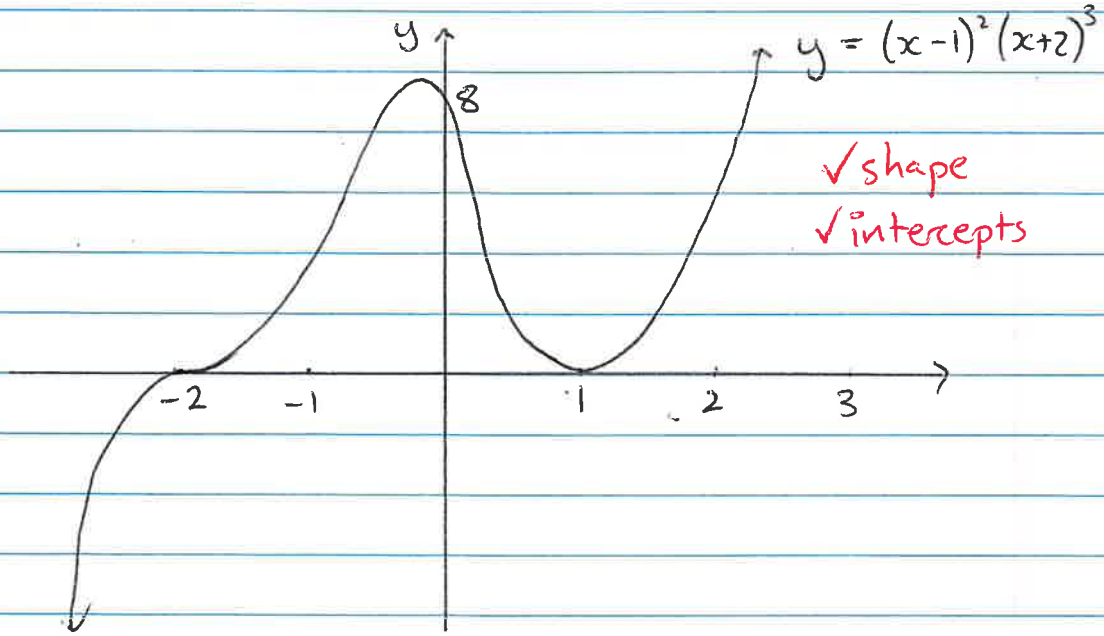
✓shape
✓asymptote & intercept

ii)



✓shape
✓vertex & intercepts

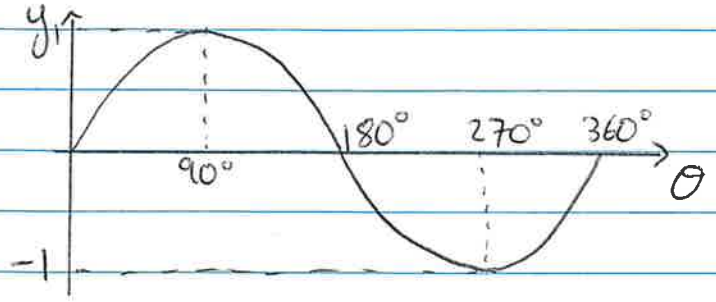
iii)



✓shape
✓intercepts

Question Five (cont.)

b) i)

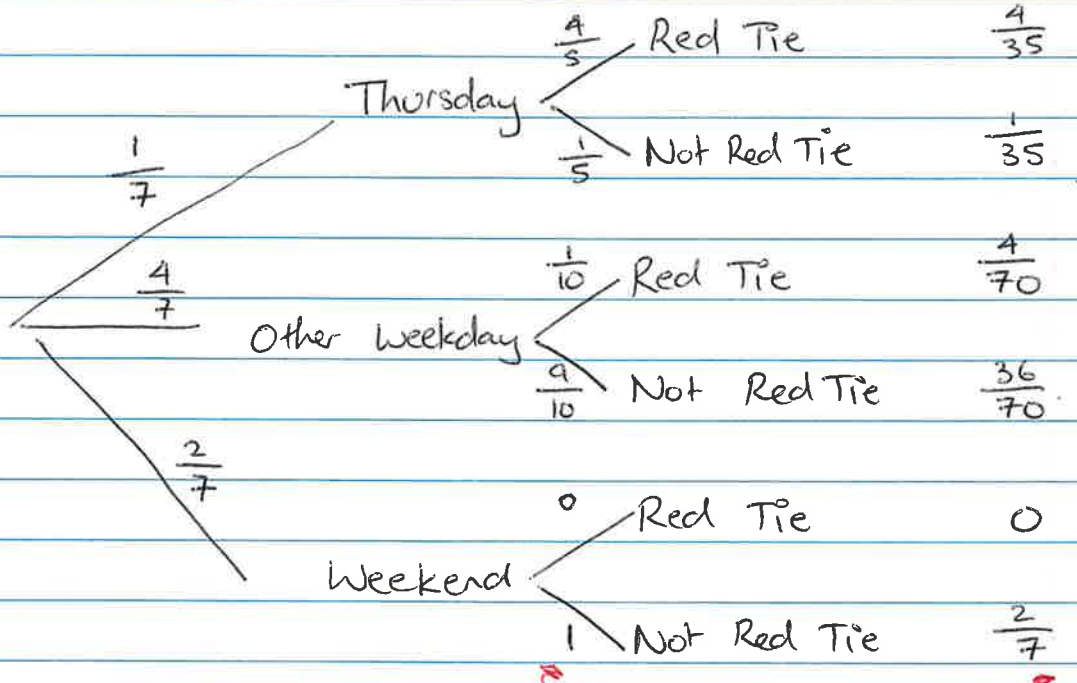


✓shape
✓intercepts and max/min.

ii) $\sin\theta = 0$

$\theta = 0°, 180°, 360°$ ✓

c) i)



✓for all branches or all final prob.

ii) $P(\text{Red Tie}) = \frac{4}{35} + \frac{4}{70}$

$= \frac{6}{35}$ ✓

iii) $P(\text{Thursday} \mid \text{Red Tie}) = \frac{\frac{4}{35}}{\frac{6}{35}}$

$= \frac{2}{3}$ ✓

Question Six

a) i) Shannon: Median $= \frac{6+8}{2}$

$= 7$

Mean $= \frac{2+2+9+4+\cdots+12}{12}$

$= 7$

Range $= 12-2$

$= 10$

Arjun: Median $= \frac{7+7}{2}$

$= 7$

Mean $= \frac{2+3+5+\cdots+12}{12}$

$= 7$

Range $= 12-2$

$= 10$

✓ one pair correct ✓ all correct

ii) IQR: Shannon $= 10-4$

$= 6$

Arjun $= 8.5 - 5.5$

$= 3$ ✓ both

(Shannon's results are more spread out)

Alternatively: $\sigma_{Shannon} = 3.42$ (calc)

$\sigma_{Arjun} = 2.68$ (calc) ✓ both

b) i) $P(x) = x^3 - 8x^2 + 21x - 18$

$P(2) = 2^3 - 8\times2^2 + 21\times2 - 18$

$= 0$

Hence by the factor theorem $x-2$ is a factor of $P(x)$ ✓

ii)

$$x-2\overline{)\,x^3-8x^2+21x-18}$$

$x^3 - 2x^2$

$-6x^2 + 21x$ ✓ process

$-6x^2 + 12x$

$9x - 18$

$9x - 18$

0

$P(x) = (x-2)(x^2-6x+9)$

$= (x-2)(x-3)^2$ ✓

Alternatively:

$P(3) = 3^3 - 8\times3^2 + 21\times3 - 18$

$= 0$ ✓

$P(x) = (x-2)(x-3)(x-\alpha)$

by inspection $\alpha = 3$

$P(x) = (x-2)(x-3)^2$ ✓

Question Six (cont.)

c) i) $K = 1650 \times 0.7^t$ ✓

ii) Let $K = 100$

$100 = 1650 \times 0.7^t$

$0.7^t = \frac{100}{1650}$

$t = \log_{0.7} \frac{100}{1650}$ ✓

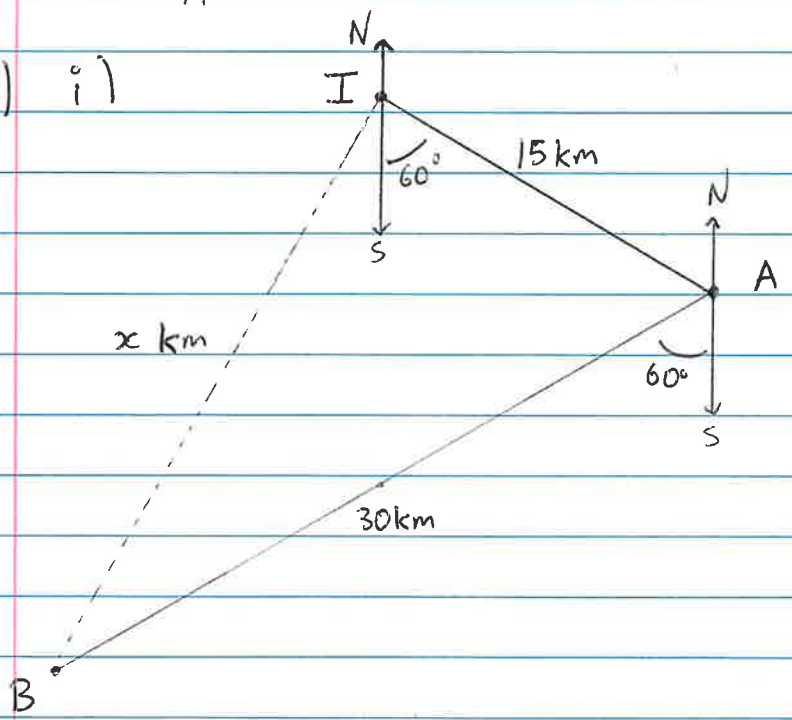
$\doteq 7.86$ years

Hence in 2019 (.86) there will be fewer than 100 koalas. ✓

(Approx. Nov. 10th 2019) (Accept 2020)

N.B. While this is only a model and has limits (e.g. discreteness of koalas), we should assume a steady rate of decrease throughout the year. Hence the answer of 2019

d) i)



✓angles and lengths shown

ii) $\angle IAS = 120°$ (cointerior angles on parallel lines)

$\therefore \angle IAB = 60°$ (adjacent angles)

$x^2 = 15^2 + 30^2 - 2 \times 15 \times 30 \times \cos(60°)$ ✓

$= 675$

$x \doteq 25.98\ldots$

$x = 26$ km (nearest km) ✓

Question Seven

a) $x^2 + y^2 = 20$ ①

$y = \frac{6}{x}$ ②

sub ② into ①:

$x^2 + \left(\frac{6}{x}\right)^2 = 20$

$x^4 + 36 = 20x^2$ ✓

$x^4 - 20x^2 + 36 = 0$

$(x^2 - 18)(x^2 - 2) = 0$

$x^2 = 18$ or $x^2 = 2$

$x = \pm 3\sqrt{2}$ $x = \pm\sqrt{2}$ ✓

sub into ② to get

$y = \pm\frac{6}{3\sqrt{2}}$ or $y = \pm\frac{6}{\sqrt{2}}$

$= \pm\sqrt{2}$ $= \pm 3\sqrt{2}$

Points of intersection are $(-3\sqrt{2}, -\sqrt{2})$, $(-\sqrt{2}, -3\sqrt{2})$, $(\sqrt{2}, 3\sqrt{2})$, $(3\sqrt{2}, \sqrt{2})$ ✓

b) $\log_{a^2} 72 = \frac{\log_a 72}{\log_a a^2}$

$= \frac{\log_a 72}{2}$ ✓

$= \frac{\log_a 8 + \log_a 9}{2}$ ✓

$= \frac{\frac{3}{2}\log_a 4 + 2\log_a 3}{2}$

$= 4.4436$ ✓

Question Seven (cont.)

c) i) $OT \perp AB$ (tangent is perpendicular to radius)

OT bisects AB ($OT \perp AB$ and passes through centre). ✓

Hence $TB = 5cm$

ii) Let $OB = r_1$, $OT = r_2$

From $\triangle OTB$

$$r_2^2 + 5^2 = r_1^2$$

$$25 = r_1^2 - r_2^2$$

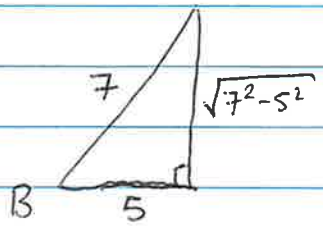
So $\pi r_1^2 - \pi r_2^2 = \pi (r_1^2 - r_2^2)$

$= 25\pi \ cm^2$ ✓

d) i) $\cos B = \dfrac{6^2 + 7^2 - 5^2}{2 \times 6 \times 7}$

$= \dfrac{5}{7}$ ✓

ii)



$\sin B = \dfrac{\sqrt{24}}{7}$

$= \dfrac{2\sqrt{6}}{7}$ ✓

$\text{Area}_{ABC} = \frac{1}{2} \times 6 \times 7 \times \dfrac{2\sqrt{6}}{7}$

$= 6\sqrt{6} \ cm^2$ ✓

iii) $\text{Area}_{AIB} = \frac{1}{2} \times 6 \times r$

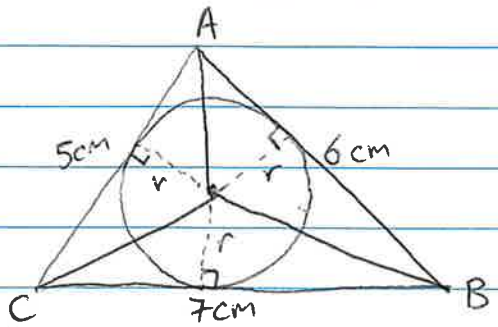
$= 3r$

$\text{Area}_{BIC} = \frac{1}{2} \times 7 \times r$

$= 3.5r$

$\text{Area}_{CIA} = \frac{1}{2} \times 5 \times r$

$= 2.5r$



Adding: $6\sqrt{6} = 9r$

$r = \dfrac{2\sqrt{6}}{3} \ cm$ ✓

Question Eight

a) $2^{3x+1} = 27$

$3x+1 = \log_2 27$

$x = \dfrac{\log_2 27 - 1}{3}$ ✓

$= \dfrac{\frac{\log_{10} 27}{\log_{10} 2} - 1}{3}$

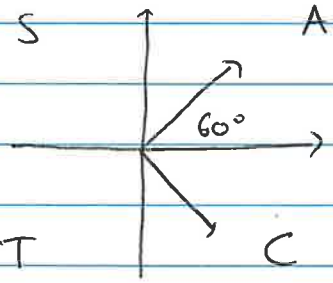
$\doteq 1.2516$ ✓

b) $2\cos 2\theta = 1$ $\quad 0° \leq \theta \leq 360°$

$\cos 2\theta = \frac{1}{2}$ $\quad 0° \leq 2\theta \leq 720°$ ✓

$2\theta = 60°, 300°, 420°, 660°$ ✓

$\theta = 30°, 150°, 210°, 330°$



but $\cos\theta < 0$, so

$\theta = 150°, 210°$ only ✓

c) i) $\angle XCD = 90°$ } (Squares)
$\angle DEX = 90°$ }

so $\angle XCD + \angle DEX = 180°$

hence EXCD is a cyclic quadrilateral as opposite angles are supplementary. ✓

ii) Let CF = x cm

then $25 \times 6 = (25+x)x$ (intersecting secants) ✓

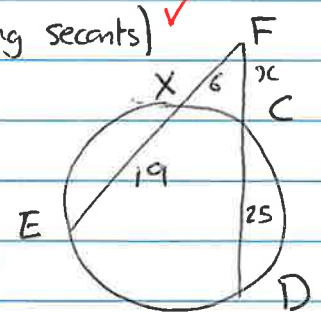
$150 = 25x + x^2$

$x^2 + 25x - 150 = 0$

$(x+30)(x-5) = 0$

$x = 5$ as $x > 0$

so CF is 5 cm long ✓



Question Eight (cont.)

c) iii) In $\triangle CFX$, $5^2 + CX^2 = 6^2$

$CX^2 = 11$

$CX = \sqrt{11}$

Hence $\text{Area}_{CFX} = \frac{5\sqrt{11}}{2}$

In $\triangle DEF$, $25^2 + ED^2 = 30^2$

$ED^2 = 275$

$ED = 5\sqrt{11}$

Hence $\text{Area}_{DEF} = \frac{125\sqrt{11}}{2}$

Thus $\text{Area}_{EXCD} = \frac{125\sqrt{11}}{2} - \frac{5\sqrt{11}}{2}$

$= 60\sqrt{11}$ cm^2 ✓

d) Vertical asymptote at $x = -4$ so $h = -4$

$y - k = \log_a(x+4)$ ✓

sub. $(-1,-1)$ and $(5,1)$ into this equation to get

$-1 - k = \log_a(3)$ ①

$1 - k = \log_a(9)$ ②

② − ①: $2 = \log_a(9) - \log_a(3)$

$= \log_a(3)$

$\Rightarrow a = \sqrt{3}$ ✓

∴ sub into ①

$-1 - k = \log_{\sqrt{3}}(3)$

$= 2$

$k = -3$ ✓

So $y + 3 = \log_{\sqrt{3}}(x+4)$

Question Nine

a) i) $P(x) = x^3 - (k+3)x - 2$

$P(0) = -2$

Consider the graph $y = P(x)$.

When $x = 0$, the curve is below the x-axis but as $P(x)$ is a monic cubic we know that for large positive x-values it must be above.

At some point the curve must cross, giving a positive zero. ✓

ii) $P(-1) = (-1)^3 - (k+3)(-1) - 2$

$= -1 + k + 3 - 2$

$= k$

The curve passes through $(-1, k)$ which is above the x-axis.

Thus it crosses at some x-value between -1 and 0.

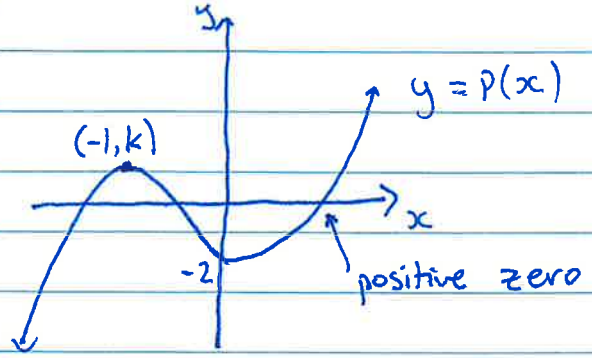
Also, for large negative x values, y is negative. So there is one solution before $x = -1$. ✓

As there are only three zeroes of a cubic, exactly one zero of $P(x)$ is positive. ✓

Allow up to eighty words per part.

Do not penalise mixing of terminology regarding zeroes, roots, and intercepts.

. For reference, the curve must look like this



Question Nine (cont.)

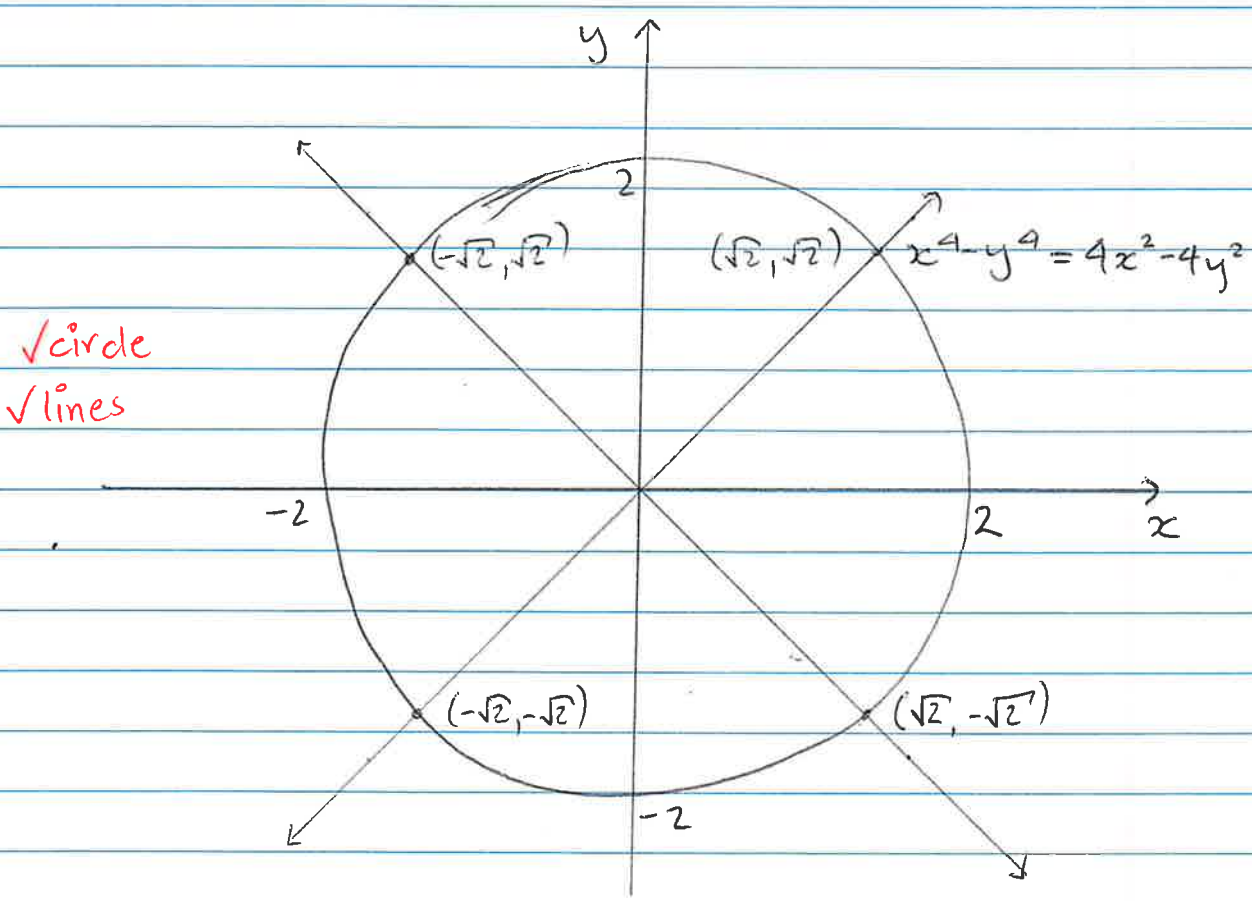
b) i) $LHS = x^4 - y^4$
$= x^4 - (x^2)^2$
$= x^4 - x^4$
$= 0$

$RHS = 4x^2 - 4y^2$
$= 4x^2 - 4x^2$
$= 0$
$= LHS$ ✓

Hence all points where $y^2 = x^2$ are solutions.

ii) $x^4 - y^4 = 4x^2 - y^2$
$(x^2 - y^2)(x^2 + y^2) = 4(x^2 - y^2)$
$x^2 + y^2 = 4$ if $x^2 \neq y^2$.
These points lie on a circle centred at the origin, radius 2 ✓

iii) Note: $y^2 = x^2$
$\Rightarrow y = \pm x$



✓circle
✓lines

Question Nine (cont.)

c) i) Note $\angle AOB = \angle BOC = \angle COD = \angle DOE = \angle EOA = 72°$ (revolution, ABCDE regular)

so $\angle EAD = \frac{1}{2}\angle EOD = 36°$ (angles at the centre and circumference on minor arc ED)

and similarly

$\angle DAC = \frac{1}{2}\angle DOC = 36°$

$\angle CAB = \frac{1}{2}\angle COB = 36°$ ✓

ii) Note $\angle ABE = \angle BEA = 36°$ (base angles of isosceles $\triangle ABE$)

so $\angle BAT = \angle ABT = 36°$ (matching angles in similar triangles)

$\angle ATE = 72°$ (exterior angle of $\triangle BAT$)

since $\angle EAT = \angle ATE = 72°$, $\triangle EAT$ is isosceles (equal base angles)

Thus $ET = 1$ ✓reasoning required

Let $BE = x$, then $BT = x - 1$

By similarity $\frac{BA}{BT} = \frac{BE}{BA}$

$\Rightarrow \frac{1}{x-1} = \frac{x}{1}$ ✓

$1 = x^2 - x$

so $x^2 - x - 1 = 0$

iii) First note $x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{5}}{2}$

so $x = \frac{1+\sqrt{5}}{2}$ $(x > 0)$ ✓

Let OA meet BE at X

OA bisects BE so $EX = \frac{1+\sqrt{5}}{4}$

In $\triangle AXE$, $\angle XEA = 36°$, $EX = \frac{1+\sqrt{5}}{4}$, $AE = 1$

Hence $\cos 36° = \frac{1+\sqrt{5}}{4}$ ✓