NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL



2018 Annual Examination

FORM IV

MATHEMATICS

Wednesday 7th November 2018

General Instructions

- Writing time — 2 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 108 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

4A: WJM	4B: RDP	4C: LYL	4D: BDD
4E: RR	4F: LRP	4G: LL	4H: SDP
4I: DWH	4J: REJ		

Checklist

- Writing paper — approx. 18 sheets
- Candidature — 181 boys

Examiner

SDP

QUESTION ONE (12 marks) Start a new page. **Marks**

(a) Write 0·045 in scientific notation. **1**

(b) Evaluate $\log_8 1$. **1**

(c) When rolling two six-sided dice, what is the probability of obtaining two sixes? Express your answer as a fraction. **1**

(d) Write down the exact value of $\cos 45°$. **1**

(e) If $P(x) = x^3 - 2x^2 + 3x - 1$, find $P(3)$. **1**

(f) Find the y-intercept of $y = 3^{-x}$. **1**

(g) Determine the related angle for 315°. **1**

(h) Express $\dfrac{5}{\sqrt{6}}$ with a rational denominator. **1**

(i) Calculate the value of $\log_3 20$. Give your answer correct to 2 decimal places. **2**

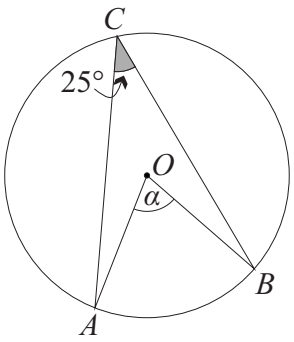
(j) Find the gradient of the line $2x + 3y + 2 = 0$. **1**

(k) Find the x-intercepts of $y = x^2 + 6x + 5$. **1**

Examination continues next page ...

QUESTION TWO (12 marks) Start a new page. **Marks**

(a)

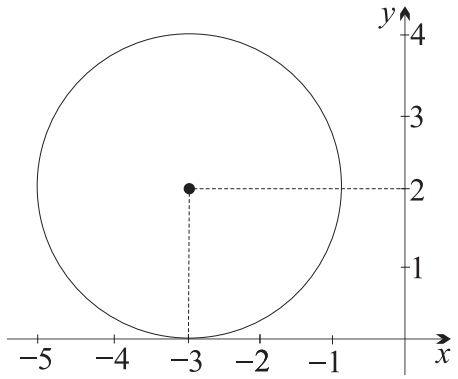


1

The diagram above shows a circle with centre O. Find the value of α, giving a clear reason.

(b) Evaluate $\left(4 \times 10^{-4}\right) \div \left(8 \times 10^{10}\right)$. Give your answer in scientific notation. **2**

(c) **2**



Find the equation of the circle shown above.

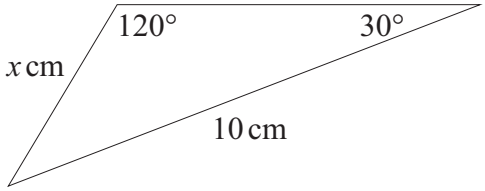
(d) When the value of the square of an integer is reduced by 31, you get 113. Let x be the integer.

(i) Form an equation, using the information above. **1**

(ii) Hence find the possible values of the integer. **1**

(e) Suppose $P(x) = x^4 - x^3 + x^2 - ax + 6$ and $P(2) = 12$. Find the value of a. **2**

(f)



(i) State the exact value of $\sin 120°$. **1**

(ii) Use the sine rule to find the exact value of x in the diagram above. **2**

Examination continues overleaf ...

QUESTION THREE (12 marks) Start a new page. **Marks**

(a) Simplify:

(i) $\left(\frac{x}{7}\right)^{-2}$ **1**

(ii) $\left(\frac{64}{y^3}\right)^{\frac{2}{3}}$ **1**

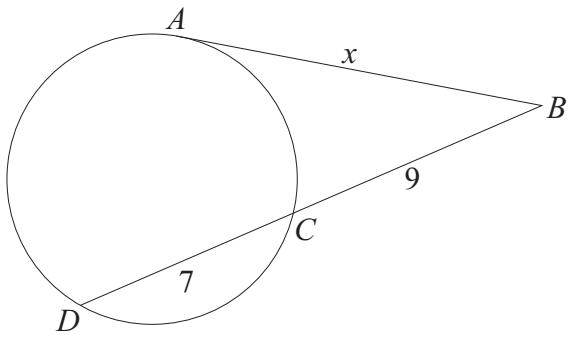
(b) Simplify:

(i) $\log_a 11 + \log_a 3$ **1**

(ii) $\log_x 6y - \log_x 3y$ **1**

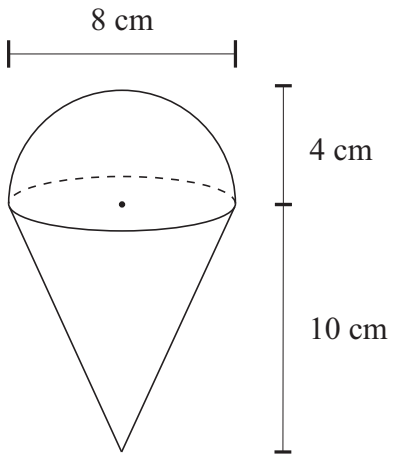
(c) Solve $\tan\theta = -\frac{1}{\sqrt{3}}$ for $0^\circ \leq \theta \leq 360^\circ$. **2**

(d) **2**



In the diagram above, AB is a tangent to the circle. Find the value of x, giving a clear reason.

(e) **3**



The solid above consists of a hemisphere and a cone. Calculate the volume of the solid. Give your answer correct to 2 decimal places.

(f) Find the coordinates of the vertex of the parabola $y = x^2 - 12x + 36$. **1**

Examination continues next page ...

QUESTION FOUR (12 marks) Start a new page. **Marks**

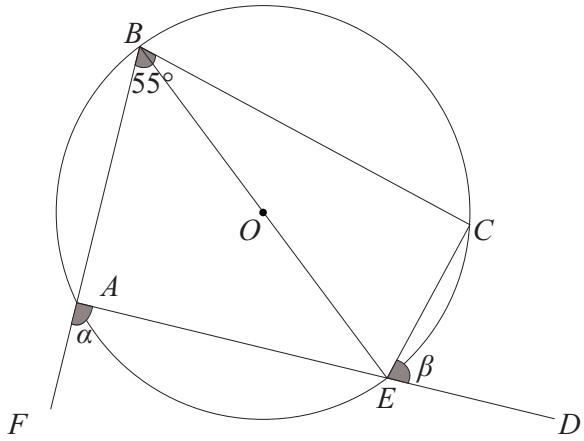
(a) (i) Find the surface area of a rectangular prism with dimensions 3 cm by 4 cm by 5 cm. **1**

(ii) All the lengths in this rectangular prism are doubled. What is the new surface area? **1**

(b) Given $\sin\theta = \dfrac{1}{3}$ and θ is obtuse, find the exact value of $\tan\theta$, in simplified form. **2**

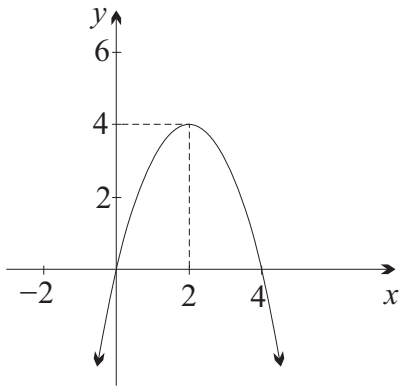
(c) The area of a triangle is $10\,\text{cm}^2$ and two of the sides measure 4 cm and 6 cm. Find the possible sizes of the included angle. Give your answer correct to the nearest degree. **3**

(d) **3**



Find the values of α and β in the diagram above, giving reasons.

(e) **2**



Find the equation of the parabola graphed above.

Examination continues overleaf ...

QUESTION FIVE (12 marks) Start a new page. **Marks**

(a) Two cards are drawn randomly **without replacement** from a standard deck of playing cards. It is recorded whether each card is a diamond or not.

(i) Draw a probability tree diagram for this experiment, showing all probabilities. **2**

(ii) Find the probability of drawing no diamonds. **1**

(iii) Given that the first card chosen is a diamond, calculate the probability that both cards drawn will be diamonds. **1**

(iv) Find the probability that both cards drawn will be diamonds, given that at least one card is a diamond. **2**

(b) The population of Sydney was 5 million at the beginning of the year 2000. Assume a constant rate of population increase of 2% per annum.

(i) Write down a formula for P, the population of Sydney in year t, where t is the number of years **after** the year 2000. **1**

(ii) During which year does this model predict that the population of Sydney reached 6 million? **2**

(c) Divide $P(x) = 2x^3 + 5x^2 + 5x + 10$ by $D(x) = x + 2$. Express the result in the form $P(x) = D(x)Q(x) + R(x)$, where the degree of $R(x)$ is less than the degree of $D(x)$. **3**

QUESTION SIX (12 marks) Start a new page. **Marks**

(a) Sketch $y = 1 + \log_2 x$. **2**

(b) Sketch the graph of $y = -\dfrac{1}{x-2}$, showing intercepts with the axes, asymptotes and any other key features. **2**

(c) (i) Sketch $y = \cos\theta$, for $0° \leq \theta \leq 720°$. **2**

(ii) Solve $\cos\theta = 1$, for $0° \leq \theta \leq 720°$. **1**

(d) (i) Fully factorise $P(x) = x^3 - 4x^2 - 3x + 18$. **3**

(ii) Sketch the graph of $y = P(x)$. **2**

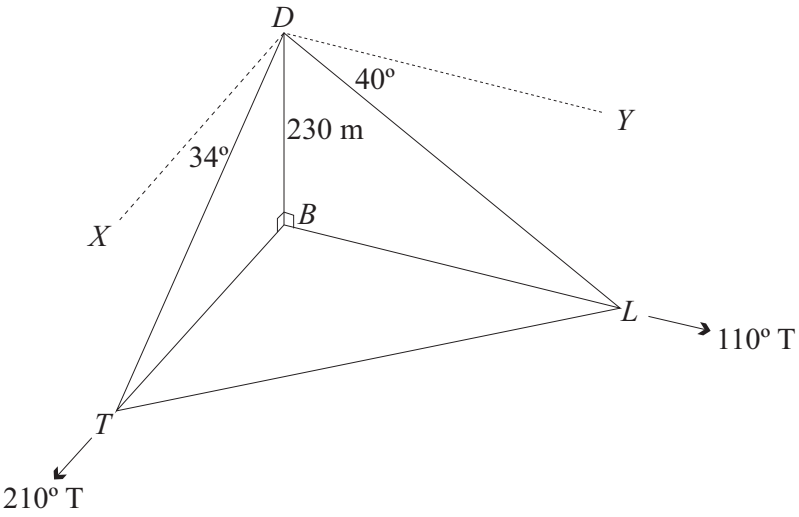
Examination continues next page ...

QUESTION SEVEN (12 marks) Start a new page. **Marks**

(a) Solve $\tan 2\theta = -0{\cdot}8$, where $0° \leq \theta \leq 360°$. Give your answers correct to 1 decimal place. **3**

(b) Show that the line $12x - y - 7 = 0$ meets the graph $y = 3x^2 + 5$ at only one point. **3**

(c)



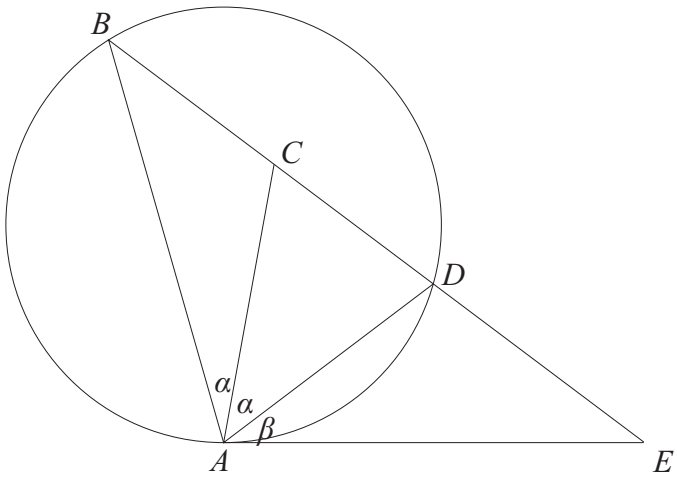
The observation deck D of City Tower is 230 m directly above the base B of the tower. A tourist looking down from the observation deck observes a lorry, L, on a bearing of 110°T at an angle of depression of 40°. She also sees a train T on a bearing of 210°T at an angle of depression of 34°. Assume the lorry and the train are on flat ground.

(i) Find $\angle TBL$. **1**

(ii) Find the distance BL. Give your answer correct to the nearest metre. **1**

(iii) Hence, find the distance between the lorry and the train. Give your answer correct to the nearest metre. **1**

(d) **3**



In the diagram above AE is a tangent to the circle. Points B and D lie on the circumference of the circle such that AC bisects $\angle BAD$. Line BD is produced to meet the tangent at E. Let $\angle CAB = \alpha$ and $\angle DAE = \beta$.

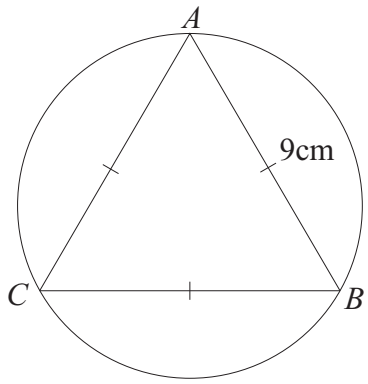
Prove that $EC = EA$.

Examination continues overleaf ...

QUESTION EIGHT (12 marks) Start a new page. **Marks**

(a) Solve $9^{x-2} = \dfrac{1}{27\sqrt{3}}$. **2**

(b) **2**



In the above diagram ABC is an equilateral triangle with side length 9 cm. The circumcircle ABC passes through all three vertices of $\triangle ABC$ and has an area $k\pi\,\text{cm}^2$. Find the value of k.

(c) The remainder when $P(x) = x^5 + 3x^4 - 11x^3 - 27x^2 + ax + b$ is divided by $(x^2 - 1)$ is $(4x + 6)$. Find the values of a and b. **3**

(d) A thief broke into a wine cellar to steal wine. On his first visit at midnight, he stole 15 L of wine from a barrel and replaced it with 15 L of water to cover his tracks. He returned at 3 am, stole a further 15 L of the diluted wine from the same barrel and added a further 15 L of water to top it back up. In the morning, the ratio of wine to water in the barrel was 169 : 87. Let the original amount of wine in the barrel be x Litres.

(i) Write an equation involving x using this information. **2**

(ii) By solving this equation, find how much wine was in the barrel before the thief visited. **1**

(e) Find the value of x in the following equation. **2**

$$10^x = \left(10^{734} + 25\right)^2 - \left(10^{734} - 25\right)^2$$

Examination continues next page ...

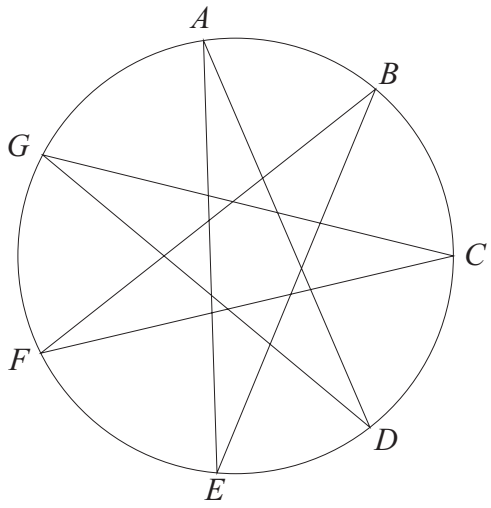
QUESTION NINE (12 marks) Start a new page. **Marks**

(a) Find the two points of intersection for the following pair of circles. **3**

$$x^2 + y^2 + 6x + 8y = 0 \text{ and } x^2 + y^2 - 5x - 25y = 0$$

(b) Simplify $\log_{\sqrt{x}} \left(\sqrt{x\sqrt{x\sqrt{x}}} \right)$, where $x > 0$. **1**

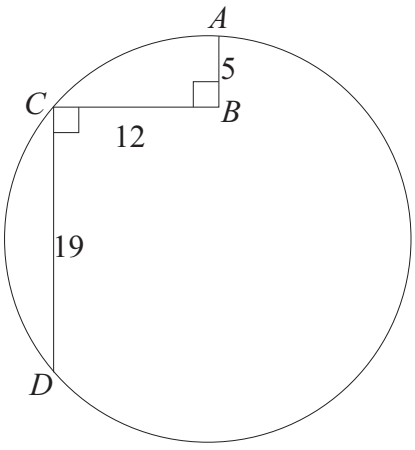
(c) **2**



The points A, B, C, D, E, F, G all lie on a circle, as shown above. Showing reasoning, find the value of:

$$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G.$$

(d) **3**



In the diagram above, $AB = 5$, $BC = 12$ and $CD = 19$. Find the exact value of the diameter of the circle.

(e) (i) Expand $\left(x^2 + y^2\right)^4$. **1**

(ii) Hence, or otherwise, find the values of $\sin\theta\cos\theta$, where $\sin^8\theta + \cos^8\theta = \dfrac{1}{8}$. **2**

END OF EXAMINATION

Form 4 Annual Solutions

Question 1

a) 4.5×10^{-2} ✓

b) 0 ✓

c) $P(\text{one six}) = \frac{1}{6}$

$P(\text{two sixes}) = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$ ✓

d) $\cos 45^\circ = \frac{1}{\sqrt{2}}$ ✓

or $\frac{\sqrt{2}}{2}$

e) $P(3) = 3^3 - 2 \times 3^2 + 3 \times 3 - 1$

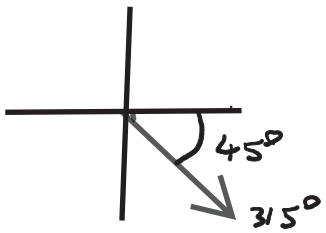
$= 17$ ✓

f) y-intercept when $x = 0$

$y = 3^{-0}$

$y = 1$ or y-intercept $(0, 1)$ ✓

g)



related angle is 45° ✓

h) $\frac{5}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$ ✓

i) $\log_3 20 = \frac{\log_{10} 20}{\log_{10} 3}$ ✓

$= 2.73$ (2dp) ✓

1j) $2x + 3y + 2 = 0$

$3y = -2x - 2$

$y = -\frac{2}{3}x - \frac{2}{3}$

gradient is $-\frac{2}{3}$ ✓

k) x-intercepts when $y = 0$

$0 = x^2 + 6x + 5$

$0 = (x+5)(x+1)$

$x+5=0$ or $x+1=0$

$x = -5$ or $x = -1$ ✓ both

x-intercepts $(-5, 0)$ and $(-1, 0)$

Question 2

a) $d = 2 \times 25^\circ$

$= 50^\circ$

(angle at centre is double angle at circumference standing on the same arc) ✓ with reason

b) $\frac{4 \times 10^{-4}}{8 \times 10^{10}} = 0.5 \times 10^{-14}$ ✓

$= 5 \times 10^{-15}$ ✓

c) Centre $(-3, 2)$, radius 2

$(x+3)^2 + (y-2)^2 = 2^2$

$(x+3)^2 + (y-2)^2 = 4$ ✓ ✓

di) $x^2 - 31 = 113$ ✓

ii) $x^2 = 144$

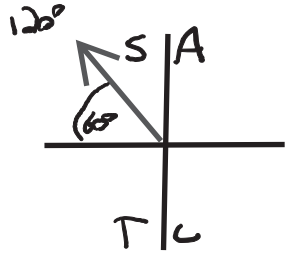
$x = 12$ or $x = -12$ ✓ both

e) $P(2) = 2^4 - 2^3 + 2^2 - 2a + 6$ ✓

$12 = 18 - 2a$

$2a = 6$

$a = 3$ ✓

f i) 

$\sin 120° = \sin 60°$

$= \frac{\sqrt{3}}{2}$ ✓

ii) $\frac{x}{\sin 30} = \frac{10}{\sin 120}$ ✓

$x = \frac{10 \sin 30}{\sin 120}$

$x = \frac{10 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$x = 5 \times \frac{2}{\sqrt{3}}$

$x = \frac{10}{\sqrt{3}}$ ✓

or $x = \frac{10\sqrt{3}}{3}$

Question 3

a i) $\left(\frac{x}{7}\right)^{-2} = \frac{49}{x^2}$ ✓

ii) $\left(\frac{64}{y^3}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{64}{y^3}}\right)^2$

$= \left(\frac{4}{y}\right)^2$

$= \frac{16}{y^2}$ ✓

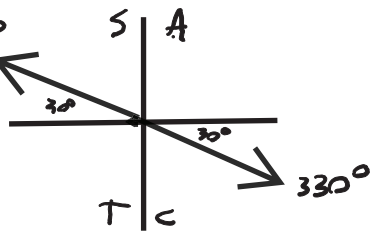
bi) $\log_a 11 + \log_a 3 = \log_a 33$ ✓

ii) $\log_x 6y - \log_x 3y = \log_x \frac{6y}{3y}$

$= \log_x 2$ ✓

c) $\tan\theta = -\frac{1}{\sqrt{3}}$

related angle is $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$ ✓



$\theta = 150^\circ$ or 330° ✓ both

d) $AB^2 = PB \times CB$ (tangent and secant from an external point) ✓ reason

$x^2 = 16 \times 9$

$x^2 = 144$

$x = 12$ ✓ (not -12 as length is positive)

e) Radius $= 4$cm

Volume hemisphere $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 4^3$

$= \frac{128}{3}\pi$ cm^3 (134.04cm^3) ✓

Volume cone $= \frac{1}{3} \times \pi \times 4^2 \times 10$

$= \frac{160}{3}\pi$ cm^3 (167.55cm^3) ✓

Total Volume $= 96\pi$ cm^3

≈ 301.59 cm^3 (2dp) ✓

Penalise 1 mark for no units

f) $y = x^2 - 12x + 36$

$y = (x-6)^2$

vertex $(6, 0)$ ✓

OR

axis of symmetry $= -\frac{b}{2a}$

$x = -\frac{-12}{2\times 1}$

$x = 6$

sub in $y = 6^2 - 12\times 6 + 36$

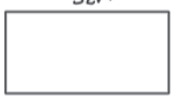
$y = 0$

$(6, 0)$ ✓

Question 4

ai) 2X 

Area $= 12cm^2$

2X 

Area $= 15cm^2$

2X 

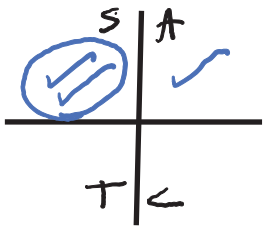
Area $= 20cm^2$

Total Surface Area $= 2\times 12 + 2\times 15 + 2\times 20$
$= 94cm^2$ ✓

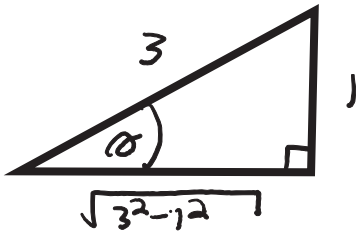
aii) Length Scale Factor $= 2$
so Area Scale Factor $= 2^2$
$= 4$

So resulting Surface Area $= 94cm^2 \times 4$
$= 376cm^2$ ✓

b) $\sin\theta = \frac{1}{3}$



✓

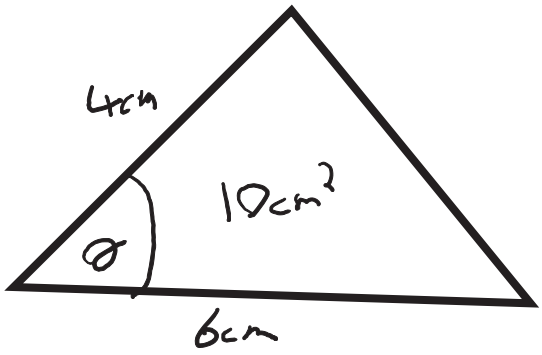


$\sqrt{3^2 - 1^2}$
$= \sqrt{8}$
$= 2\sqrt{2}$ ✓

so $\tan\theta = -\frac{1}{2\sqrt{2}}$ ✓

or $\tan\theta = -\frac{\sqrt{2}}{4}$

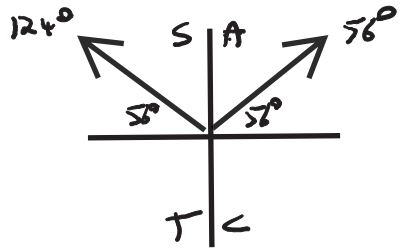
c)



$10 = \frac{1}{2} \times 4 \times 6 \times \sin\theta$ ✓

$\sin\theta = \frac{5}{6}$

related angle is $\sin^{-1}\left(\frac{5}{6}\right) = 56°$ ✓ (nearest degree)



$\theta = 56°$ or $124°$ ✓

penalise one mark for rounding

d) $\angle BAE = 90°$ (Thales' Theorem)

$d = 180° - 90°$ (adjacent angles on straight line)

$d = 90°$ ✓ with reason

$\angle CEA = 180° - 55°$ (opposite angles in cyclic quadrilateral sum to 180°)

$= 125°$ ✓

$\beta = 180° - 125°$ (adjacent angles on straight line)

$= 55°$ ✓ with reason

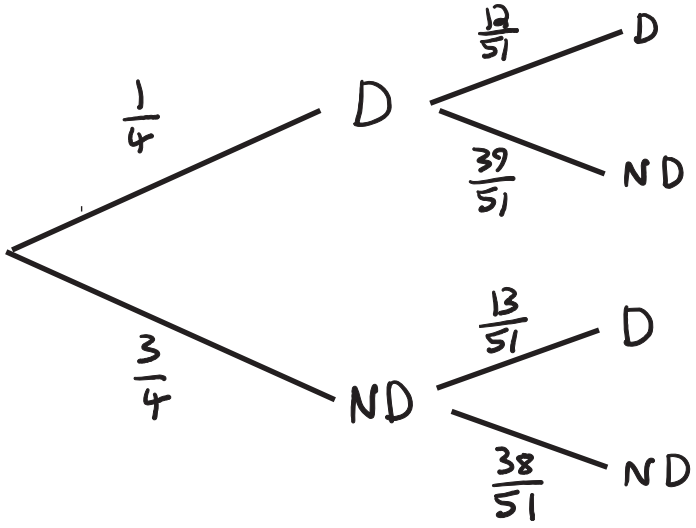
e) Parabola; concave down, vertex at (2, 4)

$y - 4 = -(x-2)^2$ ✓ for $-x^2$ ✓ for -2 and -4

OR $y = -x^2 + 4x$

Question 5

i) Let D be event of Diamond
ND be event of Not Diamond

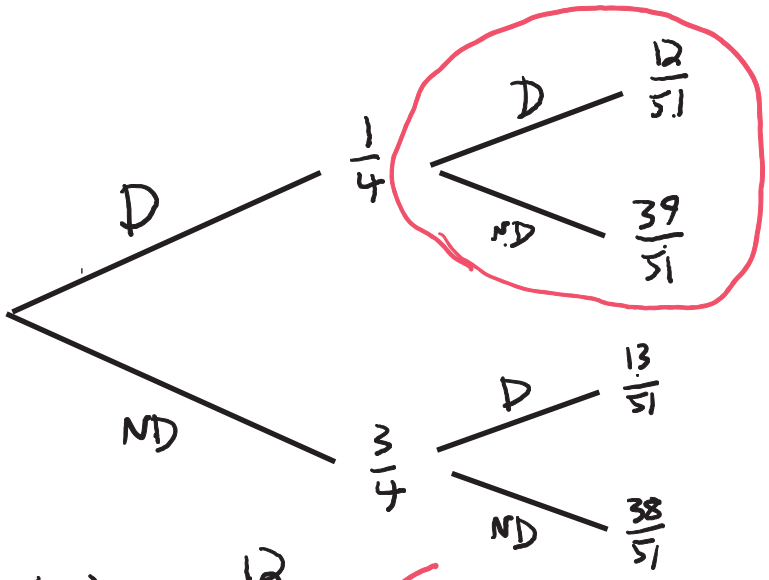


✓ shape
✓ probabilities

ii) $P(ND, ND) = \frac{3}{4} \times \frac{38}{51}$

$= \frac{19}{34}$ or 0·56 (2dp) ✓

iii) If first card chosen is a diamond, need next card to be a diamond



$P(D) = \frac{12}{51}$ ✓

or $P(D) = \frac{4}{17}$

iv) $P(\text{at least one diamond}) = 1 - P(\text{no diamonds})$

$= 1 - \frac{19}{34}$

$= \frac{15}{34}$ ✓

$P(\text{2 diamonds}) = \frac{1}{4} \times \frac{13}{51}$

$= \frac{1}{17}$

$P(\text{2 diamonds} \mid \text{at least one diamond}) = \frac{1}{17} \div \frac{15}{34}$

$= \frac{2}{15}$ ✓

bi) $P = 5,000,000 \times 1.02^t$ ✓

ii) $6,000,000 = 5,000,000 \times 1.02^t$

$\frac{6}{5} = 1.02^t$

$\log_{1.02} \frac{6}{5} = t$ ✓

$t \simeq 9.206$ years (3dp)

in year 2009 the population reached 6 million ✓

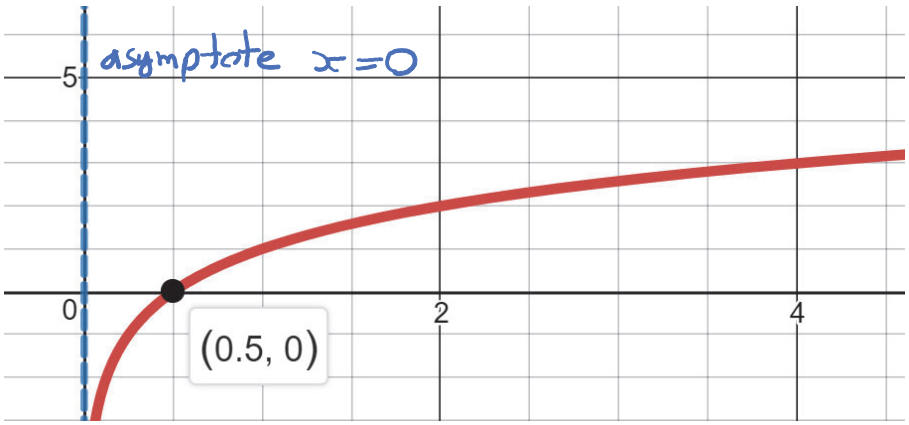
c)

$$\begin{array}{r} 2x^2 + x + 3 \\ x+2 \overline{)\,2x^3 + 5x^2 + 5x + 10} \\ -(2x^3 + 4x^2) \\ \hline x^2 + 5x \\ -(x^2 + 2x) \\ \hline 3x + 10 \\ -(3x + 6) \\ \hline 4 \end{array}$$ ✓

$P(x) = (x+2)(2x^2 + x + 3) + 4$ ✓

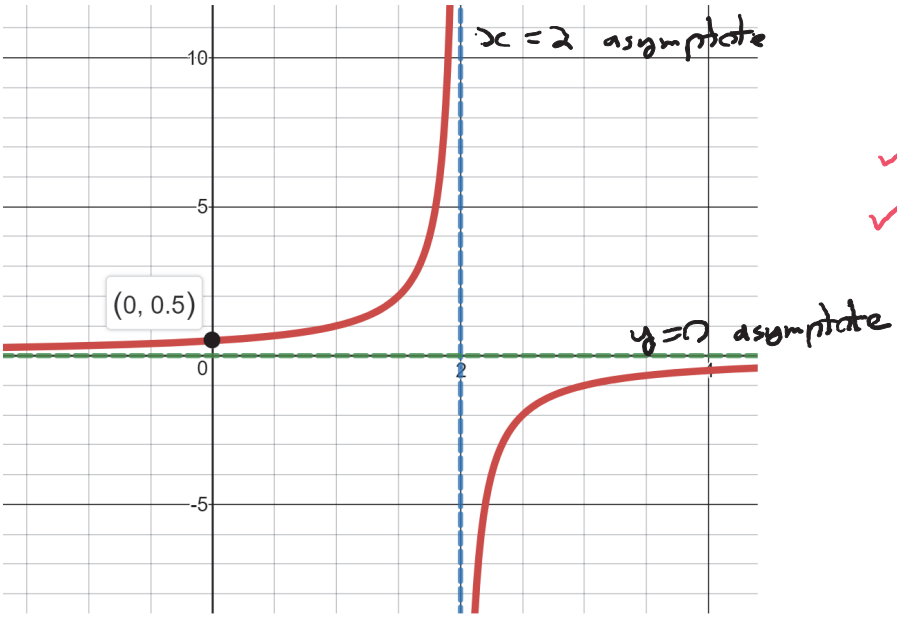
Question 6

a)



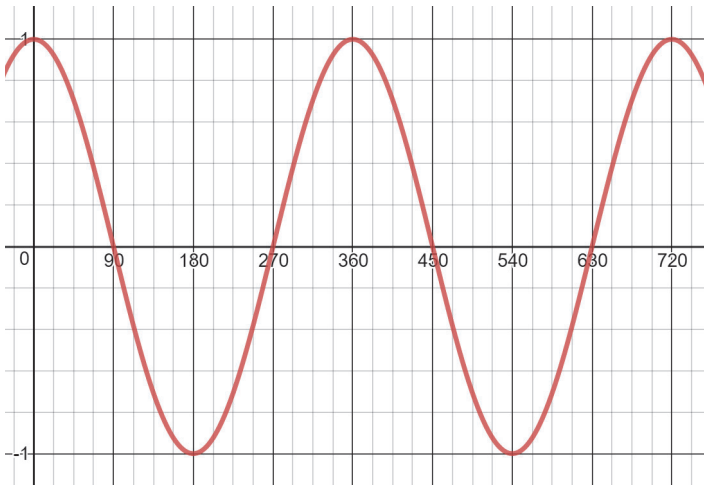
✓shape ✓x-intercept
✓asymptote drawn

b)



✓shape ✓intercept
✓asymptotes

c)i)



✓shape
✓intercepts

6 (ii) $\cos\theta = 1$, when $\theta = 0°, 360°, 720°$ ✓all

d i) $P(x) = x^3 - 4x^2 - 3x + 18$

$P(1) = 12$
$P(-1) = 16$
$P(2) = 4$
$P(-2) = 0$ so $(x+2)$ a factor ✓

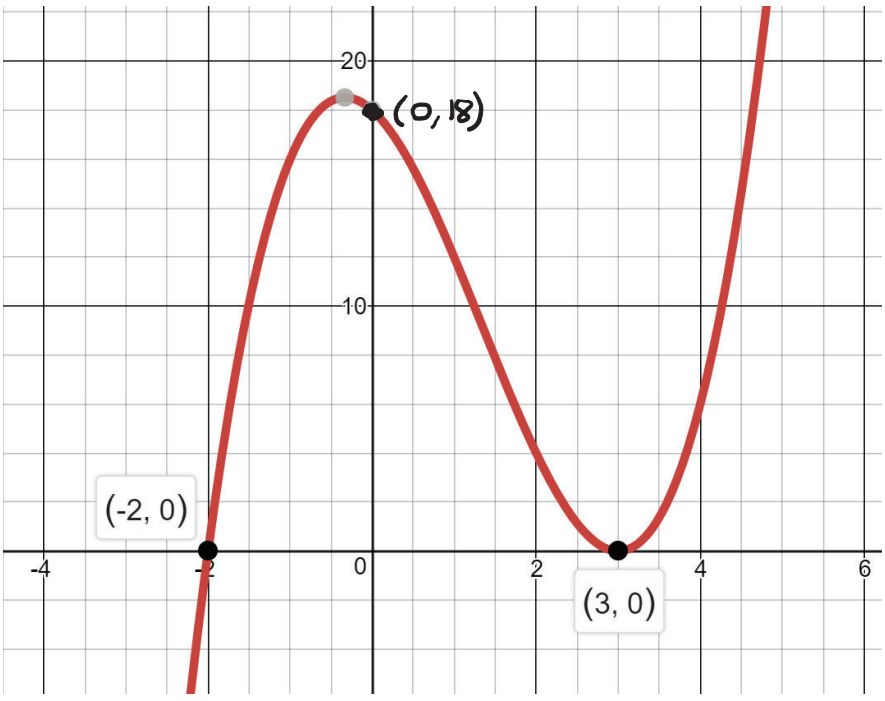
$$x+2 \;\overline{)\; x^3 - 4x^2 - 3x + 18}$$

$x^2 - 6x + 9$ ✓

$-(x^3 + 2x^2)$
$-6x^2 - 3x$
$-(-6x^2 - 12x)$
$9x + 18$
$-(9x + 18)$
0

$P(x) = (x+2)(x^2 - 6x + 9)$
$= (x+2)(x-3)^2$ ✓

ii)

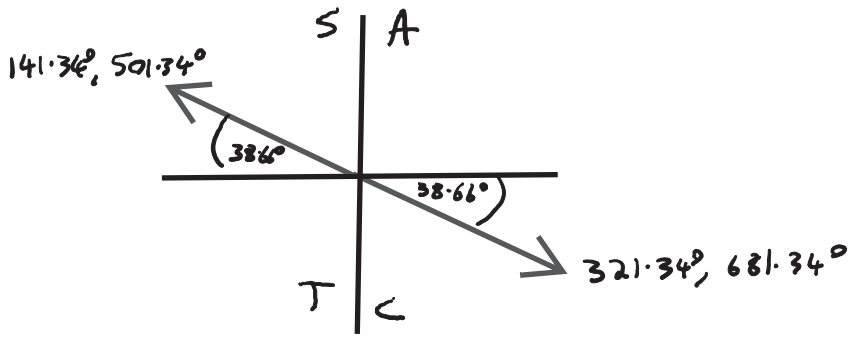


✓shape
✓intercepts and double zero

Question 7

a) $\tan 2\theta = -0.8$ $\quad 0° \leq 2\theta \leq 720°$

related angle $\tan^{-1}(0.8) = 38.66°$ ✓ (2dp)



$2\theta = 141.34°, 321.34°, 501.34°, 681.34°$ ✓

$\theta = 70.7°, 160.7°, 250.7°, 340.7°$ ✓

b) $12x - y - 7 = 0$ $\quad y = 3x^2 + 5$ (2)

$y = 12x - 7$ (1)

(1) = (2) $\quad 12x - 7 = 3x^2 + 5$ ✓

$\div 3$ $\quad 0 = 3x^2 - 12x + 12$

$0 = x^2 - 4x + 4$ ✓

$b^2 - 4ac = (-4)^2 - 4 \times 1 \times 4$

$= 0$ so only one solution

so tangent ✓

OR

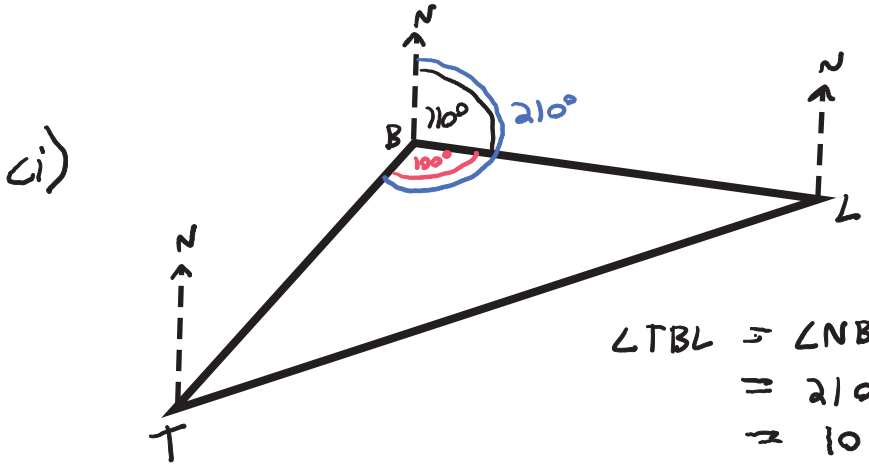
$0 = (x-2)^2$

$x = 2$

only one solution

so tangent ✓

c i)

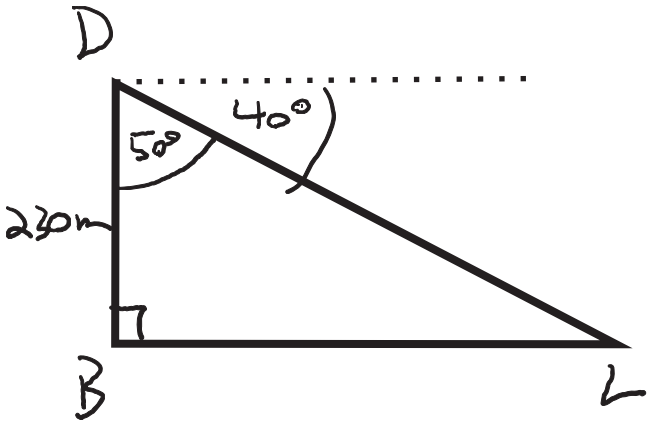


$\angle TBL = \angle NBT - \angle NBL$ (adjacent angles)

$= 210° - 110°$

$= 100°$ ✓

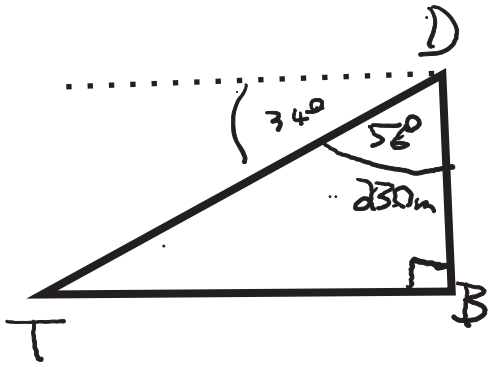
7.(ii)



$\tan 50 = \frac{BL}{230}$

$BL = 230 \tan 50$

$BL \approx 274\text{m}$ (nearest m) ✓

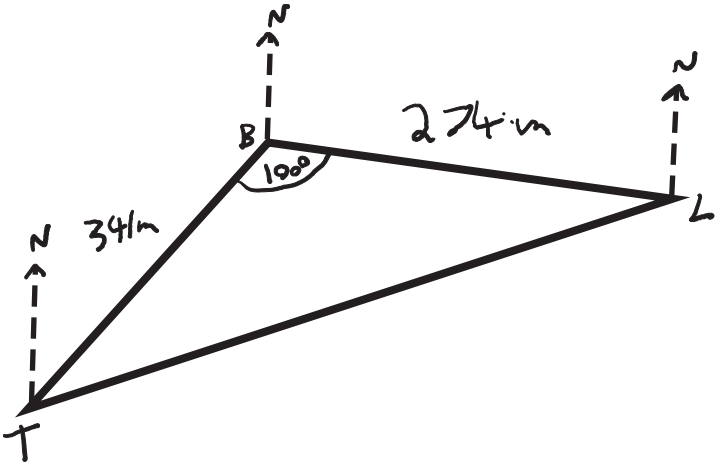


$\tan 56 = \frac{BT}{230}$

$BT = 230 \tan 56$

$BT \approx 341\text{m}$ (nearest m)

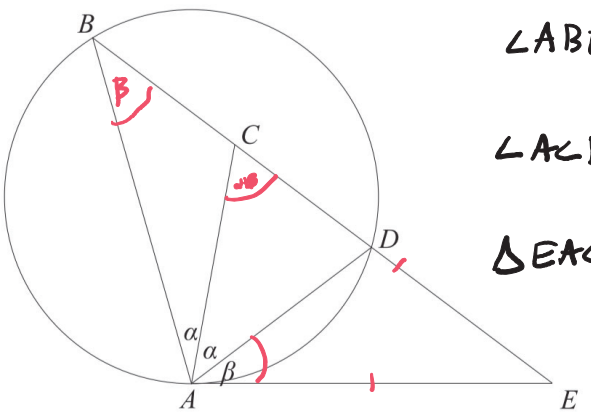
iii)



$TL^2 = 274^2 + 341^2 - 2 \times 274 \times 341 \times \cos 100°$

$TL \approx 473\text{m}$ (nearest m) ✓

d)



$\angle ABE = \beta$ (alternate segment theorem) ✓

$\angle ACD = \alpha + \beta$ (exterior angle on a Δ) ✓

ΔEAC is isosceles as base angles are equal.
So $EC = EA$ ✓

Question 8

a) $(3^3)^{x-2} = \frac{1}{3^3 \times 3^{\frac{1}{2}}}$

$3^{2x-4} = \frac{1}{3^{3\frac{1}{2}}}$

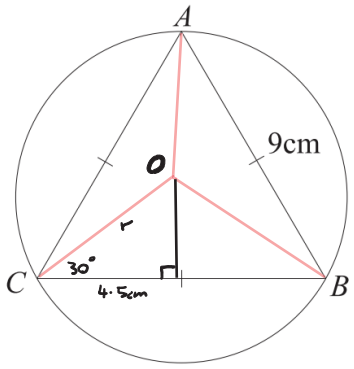
$3^{2x-4} = 3^{-3\frac{1}{2}}$ ✓

$2x-4 = -3\frac{1}{2}$

$2x = \frac{1}{2}$

$x = \frac{1}{4}$ ✓

b)



$\angle ACB = 60°$ (angles in equilateral triangle)

$\angle OCB = 30°$ (OC bisects $\angle ACB$)

$\cos 30 = \frac{4.5}{r}$ ✓

$r = \frac{4.5}{\sqrt{3}/2}$

$= 3\sqrt{3}$

Area circle $= \pi \times (3\sqrt{3})^2$

$= 27\pi \text{ cm}^2$

so $k = 27$ ✓

c) $P(x) = x^5 + 3x^4 - 11x^3 - 27x^2 + ax + b$

Divide by $x^2 - 1 = (x-1)(x+1)$

when $x = 1$,

$P(1) = -34 + a + b$ ✓ sub in (-1) or 1 and $4x + 6 = 10$

$-34 + a + b = 10$

$a + b = 44$ (1)

$P(-1) = -14 - a + b$ and $4x + 6 = 2$

$-14 - a + b = 2$

$-a + b = 16$ (2)

8c continued...) ① + ③ $2b = 60$

$b = 30$ ✓

so $a = 14$ ✓

OR

$$x^2 - 1 \;\overline{)}\; x^5 + 3x^4 - 11x^3 - 27x^2 + ax + b$$

$x^3 + 3x^2 - 10x - 24$ ✓

$-(x^5 + 0x^4 - x^3)$

$3x^4 - 10x^3 - 27x^2$

$-(3x^4 + 0x - 3x^2)$

$-10x^3 - 24x^2 + ax$

$-(-10x^3 + 0x + 10x)$

$-24x^2 + (a-10)x + b$

$-(-24x^2 + 0x + 24)$

$(a-10)x + (b-24)$

So $(a-10)x + (b-24) = 4x + 6$

$a - 10 = 4$

$a = 14$ ✓

$b - 24 = 6$

$b = 30$ ✓

di) After midnight $(x-15)$ L wine, 15 L water

so fraction of wine in mixture is $\frac{x-15}{x}$ ✓

at 3am 15L of mixture taken

$\left(\frac{x-15}{x} \times 15\right)$ L wine taken

so $\left(x-15-\frac{x-15}{x} \times 15\right)$ L wine left

$$\frac{x-15-\frac{x-15}{x} \times 15}{x} = \frac{169}{256}$$ ✓

ii) $\times x$

$$256\left(x-15-\frac{15x-225}{x}\right) = 169x$$

$$256(x^2-15x-15x+225) = 169x^2$$

$$256x^2-7680x+57600 = 169x^2$$

$$87x^2-7680x+57600=0$$

$$x = \frac{-(-7680) \pm \sqrt{(-7680)^2-4\times87\times57600}}{2\times87}$$

$x = 80$ ✓ or 8.27

must discard 8.27

not applicable as 15L taken, not possible.

e) Let $a = 10^{734}$ $\quad b = 25$

$$10^x = (a+b)^2 - (a-b)^2$$

$$10^x = (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$$

$$10^x = 4ab$$ ✓

$$10^x = 4 \times 10^{734} \times 25$$

$$10^x = 100 \times 10^{734}$$

$$10^x = 10^{736}$$

$$x = 736$$ ✓

Question 9

a) ① $x^2 + y^2 + 6x + 8y = 0$ $\quad$ ② $x^2 + y^2 - 5x - 25y = 0$

① − ② $\quad 11x + 33y = 0$

$$x + 3y = 0$$

$$x = -3y \quad ③$$ ✓

so solution lies on line $x = -3y$

③ into ① $\quad (-3y)^2 + y^2 + 6(-3y) + 8y = 0$

$$10y^2 - 10y = 0$$ ✓ or similar

$$y^2 - y = 0$$

$$y(y-1) = 0$$

$$y = 0 \quad \text{or} \quad y = 1$$

sub into ③ $\quad x = 0 \quad$ or $\quad x = -3$

$(0, 0)$ or $(-3, 1)$ ✓ both

b) $\sqrt{x\sqrt{x\sqrt{x}}} = \sqrt{x\sqrt{x \times x^{\frac{1}{2}}}}$

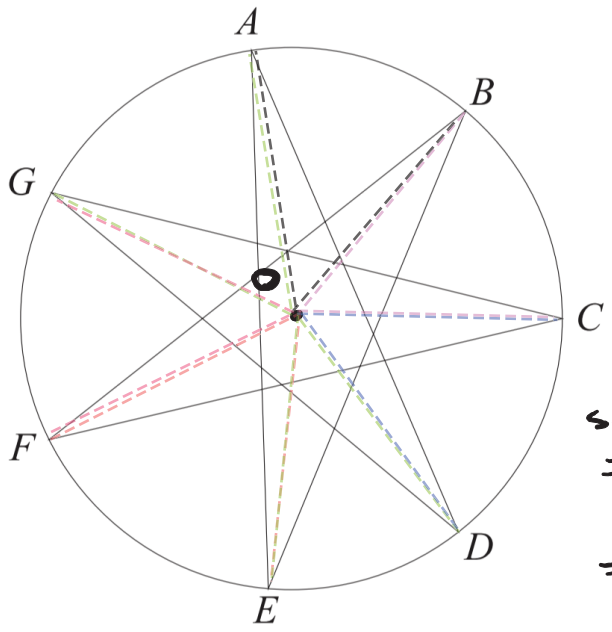
$= \sqrt{x\sqrt{x^{\frac{3}{2}}}}$

$= \sqrt{x \times x^{\frac{3}{4}}}$

$= \sqrt{x^{\frac{7}{4}}}$

so $\log_{\sqrt{x}} \sqrt{x^{\frac{7}{4}}} = \frac{7}{4}$ ✓

c)



$\angle A = \frac{1}{2}\angle EOD$ (angle at centre double angle at circumference)

$\angle B = \frac{1}{2}\angle FOE$

etc ✓ reason

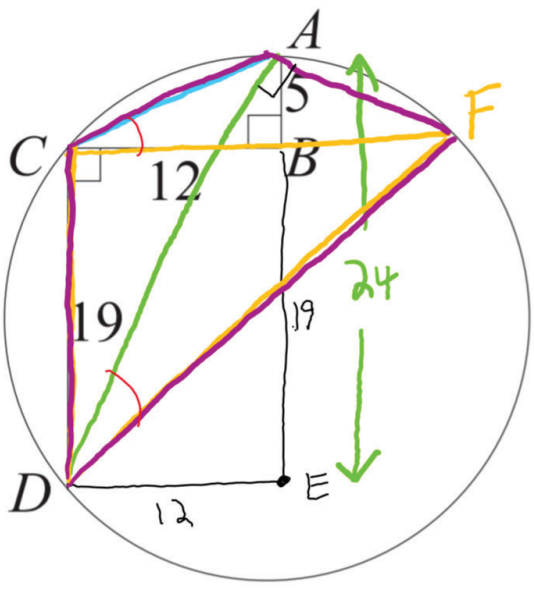
so $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle D$

$= \frac{1}{2}$ (angles at centre)

$= \frac{1}{2} \times 360°$

$= 180°$ ✓

d)



9i) Create point E in the circle so that BCDE is a rectangle

So DE = 12 and BE = 19

$AE = AB + BE$
$= 5 + 19$
$= 24$

and $AD = \sqrt{12^2 + 24^2}$
$= 12\sqrt{5}$ ✓

$AC = \sqrt{12^2 + 5^2}$
$= 13$

Extend CB to meet circle at F
so DF is the diameter as $\angle DCF = 90°$ (angles in semicircle)

$\angle DAF = 90°$ (angles in semi-circle)

$\angle ACF = \angle ADF$ (angles subtended by arc AF)

So $\triangle CBA \;|||\; \triangle DAF$ (AA) ✓

So $\frac{DF}{DA} = \frac{CA}{CB}$

$\frac{DF}{12\sqrt{5}} = \frac{13}{12}$

$DF = 13\sqrt{5}$

the diameter measures $13\sqrt{5}$ ✓

9ei) $(x^2+y^2)^4 = (x^2+y^2)(x^2+y^2)(x^2+y^2)(x^2+y^2)$

$= (x^4+2x^2y^2+y^4)(x^2+y^2)(x^2+y^2)$

$= (x^6+3x^4y^2+3x^2y^4+y^6)(x^2+y^2)$

$= x^8+4x^6y^2+6x^4y^4+4x^2y^6+y^8$ ✓

ii) $(\sin^2\theta+\cos^2\theta)^4 = 1^4$

Let $s=\sin\theta$, $c=\cos\theta$

$(s^2+c^2)^4 = 1$

$s^8+4s^6c^2+6s^4c^4+4s^2c^6+c^8 = 1$

$s^8+c^8+4s^2c^2(s^4+c^4)+6s^4c^4 = 1$

$\frac{1}{8}+4s^2c^2((s^2+c^2)^2-2s^2c^2)+6s^4c^4 = 1$

$4s^2c^2(1-2s^2c^2)+6s^4c^4 = \frac{7}{8}$

$4s^2c^2-8s^4c^4+6s^4c^4 = \frac{7}{8}$

$-2s^4c^4+4s^2c^2-\frac{7}{8} = 0$

$\div -2$ ↓ $s^4c^4-2s^2c^2+\frac{7}{16} = 0$ ✓

$s^2c^2 = \frac{2\pm\sqrt{(-2)^2-4\times1\times\frac{7}{16}}}{2\times1}$

$s^2c^2 = \frac{1}{4}$ or $s^2c^2 = \frac{7}{4}$

$\sin\theta\cos\theta = \pm\frac{1}{2}$ or $\sin\theta\cos\theta = \pm\frac{\sqrt{7}}{2}$

but max/min $\sin\theta$ and $\cos\theta$ is $1/-1$

so $-1 \le \sin\theta\cos\theta \le 1$ so $\sin\theta\cos\theta = \pm\frac{1}{2}$ ✓

only