NAME MASTER

SYDNEY GRAMMAR SCHOOL

2017 Annual Examination

FORM IV

MATHEMATICS

Wednesday 8th November 2017

General Instructions

- Writing time — 2 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 108 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

4A: DWH	4B: LYL	4C: CMDB	4D: NJL/WJM
4E: RPL	4F: BR/SDP	4G: FMW	4H: RCF
4I: DNW	4J: LRP		

Checklist

- Writing paper required.
- Candidature — 174 boys

Examiner

DNW

QUESTION ONE (12 marks) Start a new page. **Marks**

(a) Evaluate 4^0. **1**

(b) Find the y-intercept of the line with equation $3x + 4y = 24$. **1**

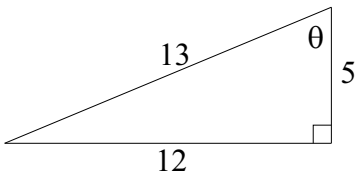
(c) Use your calculator to evaluate $\log_{10} 23$ correct to two decimal places. **1**

(d) Simplify $5x + 6 - 7x - 2$. **1**

(e) Evaluate $P(-1)$ if $P(x) = x^4 - 2x$. **1**

(f) Solve $3x + 8 = 2$. **1**

(g) **1**



Write down the value of $\tan\theta$ for the triangle above.

(h) What is the related angle for 285°? **1**

(i) What is the surface area of the cube with edge length 4 cm? **1**

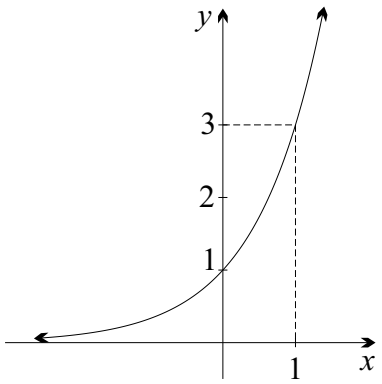
(j) Write the equation $y = \log_5 x$ in index form. **1**

(k) Factorise $x^2 + 5x - 14$. **1**

(l) Write down the solutions of $(x + 2)(x - 3) = 0$. **1**

QUESTION TWO (12 marks) Start a new page. **Marks**

(a) **1**



The diagram shows the graph of $y = a^x$.
What is the value of the base a?

Examination continues next page ...

(b) (i) Draw a neat sketch of $y = \cos\theta$ for $0° \leq \theta \leq 360°$. **2**

(ii) Hence or otherwise solve $\cos\theta = -1$ for $0° \leq \theta \leq 360°$. **1**

(c) Four cards, numbered 1 to 4, are placed in a bag. A card is drawn at random, the result noted and the card replaced. This is then repeated. The result of the experiment is the total of the two draws.

(i) Copy and complete the following table of possible outcomes. **1**

+	1	2	3	4
1	2	3		
2		4		
3				
4				

(ii) What is the probability that the total is divisible by 3? **1**

(iii) What is the probability that the total is divisible by 3 and that it is even? **1**

(d) Determine the exact value of $\sin 240°$. **2**

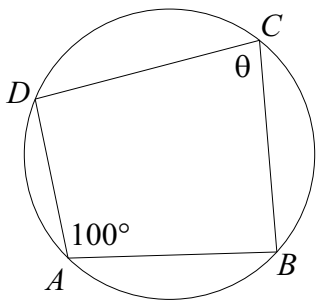
(e) (i) Copy and complete the following table of values for $y = \dfrac{1}{x}$. **1**

x	-4	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	4
y					$*$				

(ii) Hence sketch the graph of $y = \dfrac{1}{x}$, clearly showing any significant features. **2**

QUESTION THREE (12 marks) Start a new page. **Marks**

(a) **2**



The diagram above shows cyclic quadrilateral $ABCD$ with $\angle BAD = 100°$. Find θ. Give a reason for your answer.

(b) Show that $P(x) = 4x^3 - 21x + 10$ is divisible by $(x - 2)$. **1**

Examination continues overleaf ...

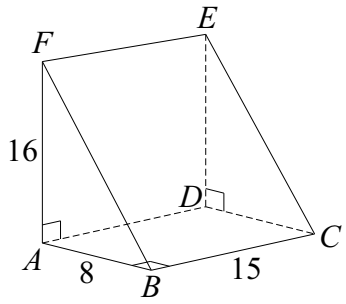
(c) A parabola has equation $y - 1 = (x - 2)^2$.

(i) State the concavity of this parabola. **1**

(ii) Write down the coordinates of its vertex. **1**

(iii) How many times does this parabola intersect the x-axis? **1**

(d)



A right triangular prism $ABCDEF$ is shown in the diagram above, with $AB = 8\,\text{cm}$, $BC = 15\,\text{cm}$ and $AF = 16\,\text{cm}$.

(i) Find $\angle ABF$, the angle between the base and the sloping face, correct to the nearest degree. **1**

(ii) Find the angle between the face diagonal CF and the base $ABCD$. Give your answer correct to the nearest degree. **2**

(e) Use long division to find the quotient and remainder when $P(x) = 2x^2 + 3x - 4$ is divided by $(x + 1)$. **3**

QUESTION FOUR (12 marks) Start a new page. **Marks**

(a) Triangle PQR has sides of length $PQ = 2{\cdot}1\,\text{m}$ and $PR = 3{\cdot}2\,\text{m}$, and $\angle QPR = 40°$. Find the area of this triangle correct to four significant figures. **2**

(b) Express $\dfrac{3}{\sqrt{7} - 1}$ with a rational denominator in simplest form. **2**

(c) Use the quadratic formula to solve $x^2 + 2x - 5 = 0$. Give your answer in exact form. **3**

(d) A parabola has equation $y = (x - 3)(x + 1)$.

(i) Find the y-intercept. **1**

(ii) Write down the x-intercepts. **1**

(iii) Find the equation of the axis of symmetry. **1**

(iv) Determine the coordinates of the vertex. **1**

(v) Sketch the parabola, showing all this information. **1**

Examination continues next page ...

QUESTION FIVE (12 marks) Start a new page. **Marks**

(a) In a particular game, a coin is tossed three times. Harry wins the game if there are more heads, otherwise Tracy wins the game.

(i) Draw a tree diagram showing the possible outcomes. **1**

(ii) What is the probability that Harry wins? **1**

(iii) What is the probability that Harry wins with exactly two heads? **1**

(iv) What is the probability that Tracy wins given that the first toss is a tail? **1**

(b) Solve $\tan\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$. **2**

(c) The horizontal line $y = k$ intersects the circle $(x-2)^2 + (y+1)^2 = 16$.

(i) Sketch the circle. Do NOT find the intercepts with the axes. **2**

(ii) What values may k take? **1**

(d) Solve:

(i) $\log_3 x = 2$ **1**

(ii) $\log_x 16 = 2$ **1**

(iii) $x = \log_5 \sqrt{5}$ **1**

QUESTION SIX (12 marks) Start a new page. **Marks**

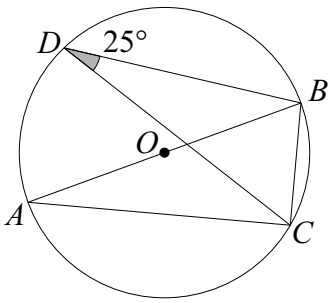
(a) Given that $\log_7 2 = \alpha$ and $\log_7 3 = \beta$, express $\log_7 12$ in terms of α and β. **2**

(b) Suppose that $\sin\theta = -\frac{1}{2}$ and that $\tan\theta > 0$, where $0^\circ \leq \theta \leq 360^\circ$. Find θ. **3**

(c) Solve $2^x = 3$. Give your answer correct to three decimal places. **2**

(d) Triangle KLM has sides $KL = 7\,\text{cm}$, $LM = 13\,\text{cm}$ and $MK = 8\,\text{cm}$. Use the cosine rule to find the size of $\angle MKL$. **2**

(e)



The circle with centre O has diameter AB, which is intersected by another chord CD. The angle subtended by BC at D is 25°.

(i) Find the size of $\angle BAC$. Give a reason for your answer. **1**

(ii) Hence or otherwise find $\angle CBA$. Give reasons for your answer. **2**

Examination continues overleaf ...

QUESTION SEVEN (12 marks) Start a new page. **Marks**

(a) Simplify $\dfrac{x^2-25}{x^2+2x-15}$. **2**

(b) (i) Sketch the graph of $y = 2^x$. **1**

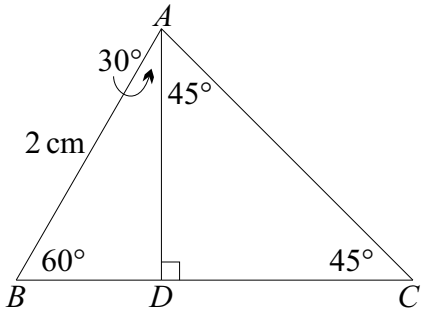
(ii) Use your knowledge of shifting to sketch $y+2 = 2^{x+1}$. Clearly mark any intercepts with the axes, and clearly show any asymptotes. **2**

(c) Two of the zeroes of the polynomial $P(x) = x^3 - 4x^2 - 3x + 18$ are $x = -2$ and $x = 3$. That is, $P(x) = (x+2)(x-3)(x-\alpha)$.

(i) By considering the value of $P(0)$, find the remaining zero. **1**

(ii) Hence sketch $y = P(x)$. **2**

(d)



The diagram shows $\triangle ABC$ with $\angle ABC = 60°$ and $\angle ACB = 45°$. The altitude through A meets BC at D, with $\angle BAD = 30°$ and $\angle CAD = 45°$. The length of AB is 2 cm.

(i) Write down the exact lengths of AD and BD. **1**

(ii) Hence find the exact length of BC. **2**

(iii) Use the sine rule in $\triangle ABC$ to determine the exact value of $\sin 75°$. **1**
[There is no need to rationalise the denominator in your answer.]

Examination continues next page ...

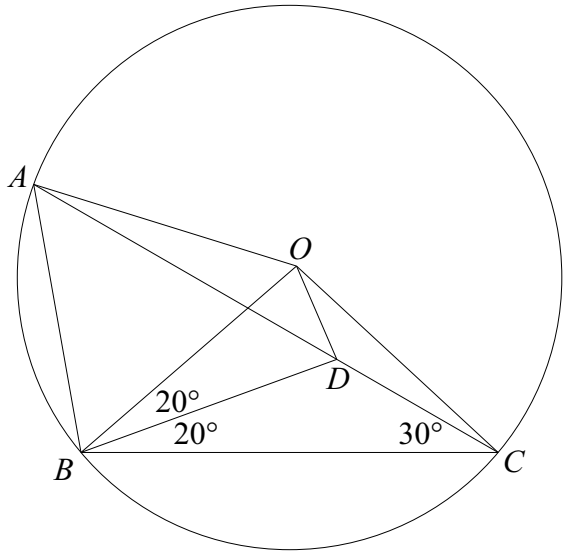
QUESTION EIGHT (12 marks) Start a new page. **Marks**

(a) Consider the equation $\sin 2\theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$.

(i) Solve this trigonometric equation. **3**

(ii) Which of your solutions also satisfy $\sin\theta < 0$? **1**

(b)



The diagram above shows $\triangle ABC$ with its vertices on the circumference of the circle with centre O. The bisector of $\angle OBC$ intersects AC at D. It is found that $\angle BCA = 30^\circ$ and that $\angle OBD = \angle CBD = 20^\circ$. You may assume that both $\triangle AOB$ and $\triangle BOC$ are isosceles.

(i) Determine $\angle BDA$. **1**

(ii) Find $\angle BAC$ and hence show that $\triangle ADB$ is isosceles. **3**

(iii) Prove that $\triangle AOB$ is equilateral. **2**

(iv) Find $\angle BOD$. **2**

EXAMINATION CONTINUES OVERLEAF

QUESTION NINE (12 marks) Start a new page. **Marks**

(a) The point $P(x, y)$ lies in the first quadrant. The coordinates of P satisfy the equation:

$$\frac{\log_{10} x}{y - x} = \frac{\log_{10} y}{x - y}\,.$$

(i) Find the simplest relationship between x and y that does not involve logarithms. **3**

(ii) Hence graph the familiar curve that P lies on, carefully noting any restrictions. **1**

(b) (i) Write down the formula for the expansion of $(A - B)^2$. **1**

(ii) Hence or otherwise expand and simplify $(A - B)^3$. **1**

(c) The cubic polynomial $C(x) = x^3 - 32x + 24$ has three zeroes.

Do NOT attempt to find these zeroes.

It is known that one of the zeroes can be written as $x = p - \sqrt{q}$, where both p and q are positive integers (whole numbers) and where $\sqrt{q}$ is not rational. That is, $\sqrt{q}$ is neither an integer nor a fraction.

(i) Show that $3p^2 + q - 32 = 0$. **3**

(ii) Hence show that $x = p + \sqrt{q}$ is also a zero of $C(x)$. **2**

(iii) In forty words or less, explain why the third zero of $C(x)$ must be rational. **1**

END OF EXAMINATION

QUESTION ONE (12 marks)

(a) $4^0 = 1$ √

(b) at $x = 0$, $y = 6$ √
so the y-intercept is $(0, 6)$.

(c) $\log_{10} 23 \doteqdot 1{\cdot}36$ √
[Penalise incorrect rounding.]

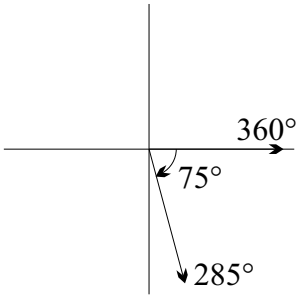
(d) $5x + 6 - 7x - 2 = 4 - 2x$. √

(e) $P(-1) = (-1)^4 - 2 \times (-1)$
$= 3$. √

(f) $3x + 8 = 2$
$x = -2$. √

(g) $\tan\theta = \frac{12}{5}$ √

(h) Related angle $= 75°$. √
[Accept just a diagram.]



(i) area of one face $= 4 \times 4\,\text{cm}^2$
Total area $= 6 \times 4 \times 4$
$= 96\,\text{cm}^2$ √

(j) $x = 5^y$ √

(k) $x^2 + 5x - 14 = (x + 7)(x - 2)$ √

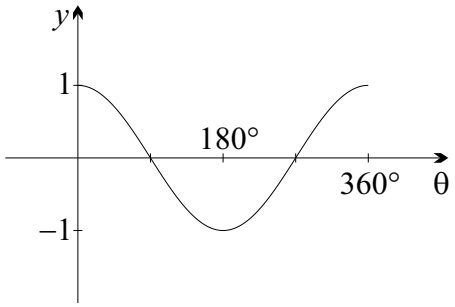
(l) $x = -2$ or 3 √

Total for Question 1: 12 Marks

QUESTION TWO (12 marks)

(a) $a = 3$ (y-coordinate at $x = 1$) √

(b) (i) Amplitude: √



shape and x-intercepts: √

(ii) $\theta = 180^\circ$ √

(c) (i) √

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

(ii) $P(\text{divisible by 3}) = \frac{5}{16}$ (two 3s and three 6s) √

(iii) $P(\text{even and divisible by 3}) = \frac{3}{16}$ (just the 6s) √

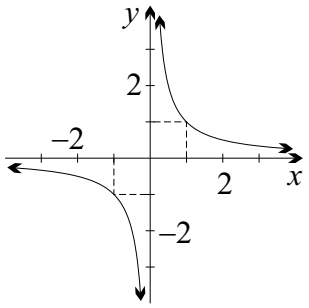
(d) $\sin 240^\circ = -\sin 60^\circ$ √

$= -\frac{\sqrt{3}}{2}$ √

(e) (i)

x	-4	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	4
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	-2	$*$	2	1	$\frac{1}{2}$	$\frac{1}{4}$

√



(ii) Clear asymptote: √

Overall shape / points plotted: √

Total for Question 2: 12 Marks

QUESTION THREE (12 marks)

(a) $\theta = 180^\circ - 100^\circ$ (opposite angles of cyclic quadrilateral $ABCD$) √

$= 80^\circ$ √

[The first mark is for the reason.]

(b) $P(2) = 4 \times 2^3 - 21 \times 2 + 10$

$= 0$ √

Hence $P(x)$ is divisible by $(x-2)$.

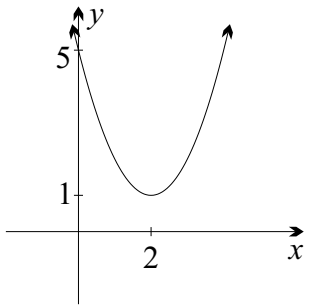
[In fact $P(x) = (x-2)(4x^2+8x-5)$, though this is not needed.]

(c) (i) Concave up ($a = 1$) √

(ii) $V = (2, 1)$ √

(iii) No x-intercepts √

[Diagram is not required.]



(d) (i) $\tan \angle ABF = \frac{16}{8} = 2$

so $\angle ABF \doteqdot 63^\circ$ (to the nearest degree) √

(ii) $AC = 17$ (by Pythagoras) √

[Do not penalise omitted reason.]

$\tan \angle ACF = \frac{16}{17}$

so $\angle ACF \doteqdot 43^\circ$ √

(e) (i)

$$
\begin{array}{r}
2x+1 \\
(x+1)\overline{)\,2x^2+3x-4} \\
\underline{2x^2+2x} \\
x-4 \\
\underline{x+1} \\
-5
\end{array}
$$

[First mark for at least one correct iteration of division algorithm.] √

[Second mark for division correctly completed.] √

Hence $Q(x) = 2x + 1$

and $R(x) = -5$ √

[or $2x^2 + 3x - 4 = (x+1)(2x+1) - 5$.]

Total for Question 3: 12 Marks

QUESTION FOUR (12 marks)

(a) Area $= \frac{1}{2} \times 2{\cdot}1 \times 3{\cdot}2 \times \sin 40^\circ$ √

$\doteq 2{\cdot}160\,\text{m}^2$ √

[Penalise incorrect significant figures, including 2·16 which is three sig. fig.]

(b) $$\frac{3}{\sqrt{7}-1} = \frac{3}{\sqrt{7}-1} \times \frac{\sqrt{7}+1}{\sqrt{7}+1}$$ √

$$= \frac{\sqrt{7}+1}{2}$$ √

(c) For the quadratic equation $x^2 + 2x - 5 = 0$

$\Delta = 2^2 - 4 \times 1 \times (-5)$

$= 24$ √

so $x = \dfrac{-2 \pm \sqrt{24}}{2}$ √

$= -1 \pm \sqrt{6}$ √

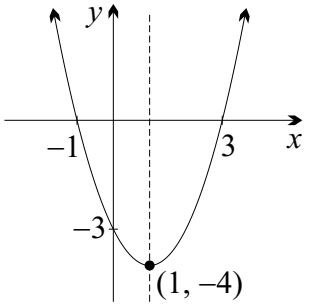
(d) (i) y-intercept: $y(0) = -3$ √

(ii) x-intercepts: $x = -1, 3$ √

(iii) axis: $x = \dfrac{3+(-1)}{2} = 1$ √

(iv) Vertex: $V = (1, -4)$ [Accept just $y = -4$.] √

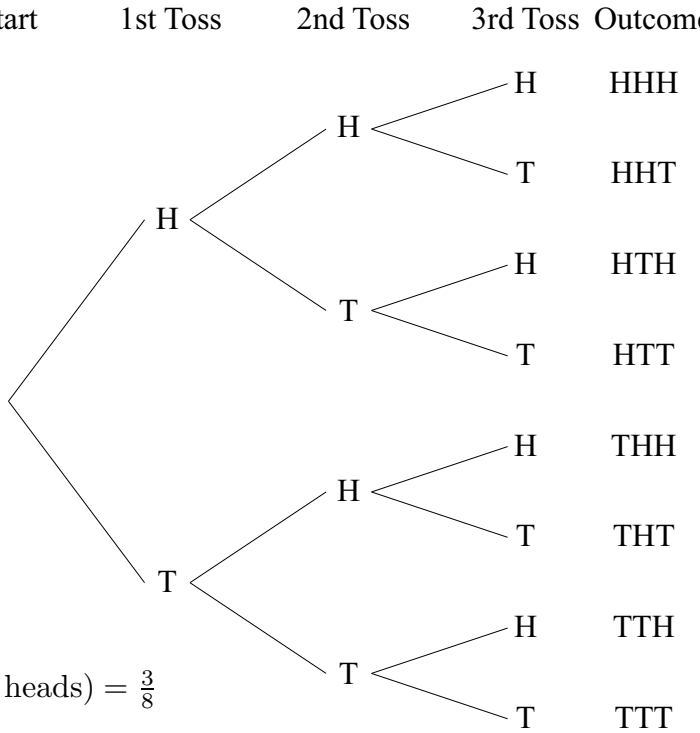
(v) Sketch showing all this. √



Total for Question 4: 12 Marks

QUESTION FIVE (12 marks)

(a) (i) [Tree at right ...
Accept omission of the "Outcome" column, but grumble.]



√

(ii) $P(\text{Harry wins}) = \frac{4}{8} = \frac{1}{2}$ √

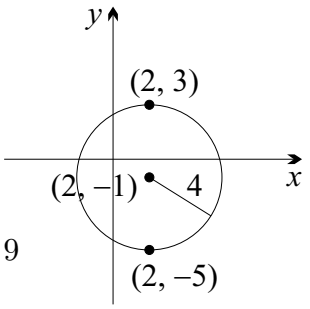
(iii) $P(\text{Harry wins with two heads}) = \frac{3}{8}$ √

(iv) $P(\text{Tracy wins given 1st toss is tails}) = \frac{3}{4}$ √

(b) $\tan\theta = 1$
so θ is in quadrants 1 or 3
related angle is $45°$ √
hence $\theta = 45°, 225°$ √

(c) (i) Centre: √
Radius: √

(ii) $-5 \le k \le 3$ √



(d) (i) In index form $x = 3^2 = 9$ √

(ii) In index form $x^2 = 16$
so $x = 4$ (base must be positive) √

(iii) $x = \frac{1}{2}$ √

Total for Question 5: 12 Marks

QUESTION SIX (12 marks)

(a) $\log_7 12 = \log_7 \left(2^2 \times 3\right)$ √

[or any other appropriate first step.]

so $\log_7 12 = 2\log_7 2 + \log_7 3$

$= 2\alpha + \beta$ √

(b) $\sin\theta < 0$ and $\tan\theta > 0$

thus θ is in quadrant 3. √

Related angle $= 30°$. √

Hence $\theta = 210°$. √

(c) $2^x = 3$

so $x = \log_2 3$ √

[or other appropriate logarithmic form]

thus $x = \dfrac{\log_{10} 3}{\log_{10} 2}$

$\div 1{\cdot}585$ √

[Grumble if not correctly rounded, but no penalty: $1{\cdot}58496\ldots$]

(d) $\cos\theta = \dfrac{7^2 + 8^2 - 13^2}{2 \times 7 \times 8}$ √

[or other appropriate use of cosine rule]

so $\cos\theta = -\frac{1}{2}$.

Thus $\theta = 120°$ √

(e) (i) $\angle BAC = \angle BDC$ (angles in the same segment)

$= 25°$ √

(ii) $\angle ACB = 90°$ (angle in a semicircle) √

so $\angle CBA = 65°$ (angle sum of $\triangle ABC$) √

Total for Question 6: 12 Marks

QUESTION SEVEN (12 marks)

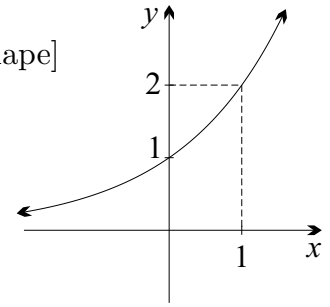
(a) $$\frac{x^2-25}{x^2+2x-15} = \frac{(x+5)(x-5)}{(x+5)(x-3)}$$ √

[First mark for either numerator or denominator correctly factored.]

$$= \frac{x-5}{x-3}$$ √

[Second mark only if completely correct.]

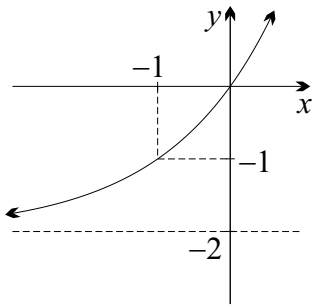
(b) (i) [must show asymptote, y-intercept and correct shape] √



(ii) Shift left 1, down 2 √

[or graphical evidence]

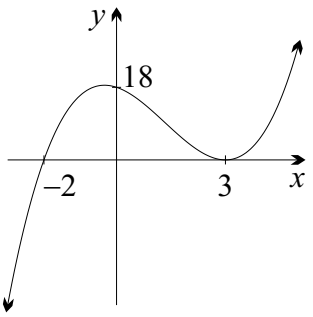
[must show asymptote, y-intercept and correct shape for 2nd mark] √



(c) (i) $P(0) = 2 \times (-3) \times (-\alpha)$

so $18 = 6\alpha$

thus $\alpha = 3$ √



(ii) Note that $P(x) = (x+2)(x-3)^2$.

Correct shape at one x-intercept: √

Remainder of graph correct (incl. y-intercept): √

(d) (i) $AD = \sqrt{3}\,\text{cm}$, $BD = 1\,\text{cm}$ ($60°$, $30°$ special triangle) √

(ii) $DC = AD$ (isosceles $\triangle ADS$)

$= \sqrt{3}\,\text{cm}$ √

[Reason not required.]

Hence $BC = 1 + \sqrt{3}\,\text{cm}$ √

(iii) $$\frac{\sin 75°}{BC} = \frac{\sin 45°}{BA}$$

so $$\sin 75° = \frac{1+\sqrt{3}}{2\sqrt{2}}$$ √

Total for Question 7: 12 Marks

QUESTION EIGHT (12 marks)

(a) (i) $\sin 2\theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$

Let $\omega = 2\theta$ in this equation, then

$\sin \omega = \frac{1}{2}$ for $0^\circ \leq \omega \leq 720^\circ$ √

For ω: the related angle is 30°, in quadrants 1 or 2

$\omega = 30^\circ, 150^\circ, 390^\circ, 510^\circ$ √

so $\theta = 15^\circ, 75^\circ, 195^\circ, 255^\circ$ √

(ii) Now $\sin \theta < 0$ so require solutions in 3rd or 4th quadrants.

$\theta = 195^\circ, 255^\circ$ √

(b) (i) $\angle BDA = 50^\circ$ (exterior angle of $\triangle BDC$) √

(ii) $\angle BCO = 40^\circ$ (base angles of isosceles $\triangle BOC$) √

$\angle BOC = 100^\circ$ (angle sum of $\triangle BOC$) √

$\angle BAC = 50^\circ$ (angles at centre and circumference on arc BC) √

(iii) $\angle BOA = 60^\circ$ (angles at centre and circumference on arc AB) √

$\angle A = \angle B = \frac{1}{2}(180^\circ - 60^\circ)$ (angle sum of isosceles $\triangle BOA$)

$= 60^\circ$ √

hence all angles are equal and $\triangle BOA$ is equilateral.

(iv) $BO = BA$ (equilateral $\triangle ABO$)

$= BD$ (isosceles $\triangle ABD$) √

Thus $\triangle BOD$ is isosceles and hence

$\angle BOD = 80^\circ$ (angle sum of isosceles $\triangle BOD$) √

Total for Question 8: 12 Marks

QUESTION NINE (12 marks)

(a) (i) There are several ways to do this. Here is one:

Let $\dfrac{\log_{10} x}{y-x} = \dfrac{\log_{10} y}{x-y} = z$. Then

$$\log_{10} x = z(y-x)$$

and $$\log_{10} y = z(x-y)$$ √

Add these equations to get

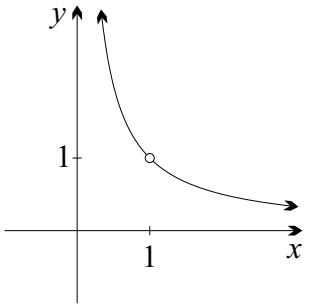
$$\log_{10} x + \log_{10} y = 0$$ √

so $$\log_{10}(xy) = 0$$

or $$xy = 1$$ √

(ii) That is, $y = \dfrac{1}{x}$ with $y \neq x$.

The graph of the hyperbola, 1st quadrant only, excluding $(1,1)$. √



[The restriction MUST be clearly indicated in some way.]

(b) (i) $(A-B)^2 = A^2 - 2AB + B^2$ √

(ii) $(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$ √

(c) Note that $C(x) = (x+6)(x-3+\sqrt{5})(x-3-\sqrt{5})$ in this case.
However, the factorisation is irrelevant to the question.

(i) It is given that $p - \sqrt{q}$ is a zero, thus

$$C(p-\sqrt{q}) = 0.$$

So $$\text{LHS} = (p-\sqrt{q})^3 - 32(p-\sqrt{q}) + 24$$ √

$$= p^3 - 3p^2\sqrt{q} + 3pq - q\sqrt{q} - 32p + 32\sqrt{q} + 24$$

$$= (p^3 + 3pq - 32p + 24) - (3p^2 + q - 32)\sqrt{q}\,.$$ √

That is $$(p^3 + 3pq - 32p + 24) - (3p^2 + q - 32)\sqrt{q} = 0. \qquad (\dagger\dagger)$$

Now, by way of contradiction, assume that $(3p^2 + q - 32) \neq 0$. Then

$$\sqrt{q} = \frac{p^3 + 3pq - 32p + 24}{3p^2 + q - 32}$$

which is rational. This is a contradiction.

Hence $3p^2 + p - 32 = 0$. √

[Or any other valid argument.]

(ii) Since $3p^2 + p - 32 = 0$, it follows from equation (††) that
$(p^3 + 3pq - 32p + 24) = 0$ also. ☑

Next, $C(p + \sqrt{q}) = (p + \sqrt{q})^3 - 32(p + \sqrt{q}) + 24$
$= (p^3 + 3pq - 32p + 24) + (3p^2 + q - 32)\sqrt{q}$
$= 0 + 0 \times \sqrt{q}$
$= 0$ ☑

That is, $x = p + \sqrt{q}$ is also a zero of $C(x)$.

(iii) There are several ways to argue this. Here is one:

Parts (i) and (ii) demonstrate that irrational zeroes must come in pairs.

[Technically, those zeroes that involve quadratic surds come in pairs.]

There are three zeroes and hence there can be at most one irrational pair.
The product of the irrational pair is rational.
The remaining zero is thus rational.

[This or any other valid argument. Accept up to 80 words.] ☑

Total for Question 9: 12 Marks

DNW