SYDNEY GRAMMAR SCHOOL

2015 Annual Examination

FORM IV
MATHEMATICS

Thursday 5th November 2015

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 108 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name and master on this question paper and submit it with your answers.

4A: KWM	4B: DNW	4C: NL
4D: SG	4E: DWH	4F: PKH
4G: FMW	4H: SJE	4I: SO

Checklist

- Writing paper required.
- Candidature — 176 boys

Examiner
DNW

QUESTION ONE (12 marks) Start a new page. **Marks**

(a) What is the gradient of the line $y = -2x + 5$? 1

(b) Expand $(x + 2)(x + 5)$. 1

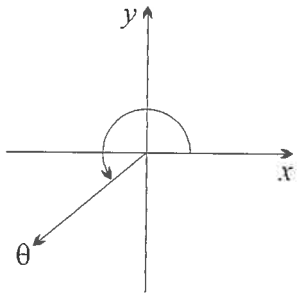
(c) Factorise $x^2 - 25$. 1

(d) Solve $(2x + 1)(x - 3) = 0$. 1

(e) Use a calculator to evaluate $\dfrac{5}{\sin 55^\circ}$ correct to one decimal place. 1

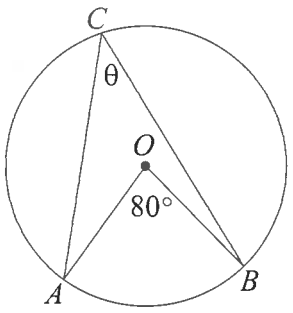
(f) A card is drawn at random from a deck of cards. What is the probability it is a heart? 1

(g) 1



The angle θ is shown above. State whether $\sin\theta$ is positive or negative.

(h) 1



What is the value of θ in the above diagram? Give a reason for your answer.

(i) What is the exact volume of a sphere with radius 3 cm? 1

(j) Express $\dfrac{1}{\sqrt{3}}$ with a rational denominator. 1

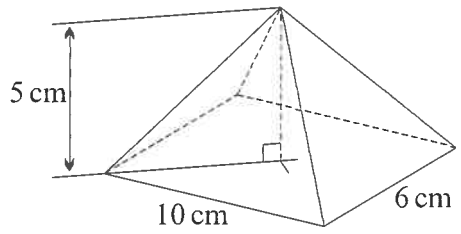
(k) Write $y = 3^x$ in logarithmic form. 1

(l) Find the exact value of $\tan 30^\circ$. 1

Exam continues next page ...

QUESTION TWO (12 marks) Start a new page. **Marks**

(a)



1

The pyramid shown above has a rectangular base with dimensions 10 cm by 6 cm. It has a perpendicular height of 5 cm. What is the volume of the pyramid?

(b) A circle has equation $(x-4)^2+(y+3)^2=9$.

(i) Write down its radius. 1

(ii) Write down the coordinates of its centre. 1

(iii) How many times does this circle intersect the x-axis? 1

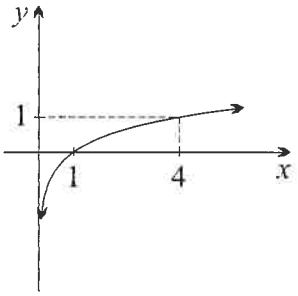
(c) Write the fraction $\dfrac{2}{\sqrt{7}-\sqrt{3}}$ with a rational denominator, in simplest form. 2

(d) A coin is tossed twice.

(i) Draw a tree diagram showing the various possible outcomes. 1

(ii) Determine the probability that there are more heads than tails. 1

(e) 1



The diagram above shows the graph of $y=\log_a x$.

What is the value of the base a?

(f) (i) Copy and complete the following table of values for $y=2^{-x}$. 1

x	-2	-1	0	1	2
y					

(ii) Hence sketch the graph of $y=2^{-x}$. 2

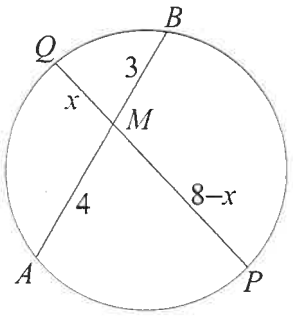
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QUESTION THREE (12 marks) Start a new page. **Marks**

(a) Find the equation of the line parallel with $y = 2x + 3$ which passes through $(1, -1)$. Give your answer in the form $y = mx + b$. **2**

(b) Solve $x^2 - 3x - 2 = 0$ by the quadratic formula. **2**

(c) **3**



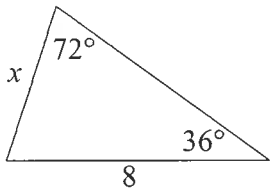
Determine the possible values of x in the diagram above.
Give a reason for your answer.

(d) Simplify:

(i) $\log_{10} 50 - \log_{10} 5$ **1**

(ii) $\log_6 12 + \log_6 3$ **1**

(e) **2**



Use the sine rule to determine x correct to one decimal place.

(f) What is the acute related angle for $500°$? **1**

QUESTION FOUR (12 marks) Start a new page. **Marks**

(a) A parabola has equation $y = (x - 1)(x - 5)$.

(i) Find the y-intercept. **1**

(ii) Write down the x-intercepts. **1**

(iii) Find the equation of the axis of symmetry. **1**

(iv) Determine the coordinates of the vertex. **1**

(v) Sketch the parabola, showing all this information. **1**

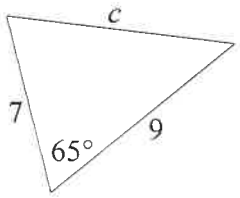
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(b) Simplify $\dfrac{2x^2 - x - 1}{x^2 - 5x + 4}$. **2**

(c) (i) Sketch the graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$. **2**

(ii) Hence solve $\sin\theta = 0$ for $0° \leq \theta \leq 360°$. **1**

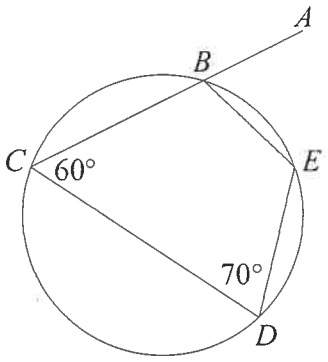
(d) **2**



Use the cosine rule to find the value of c in the triangle above.
Give your answer correct to two decimal places.

QUESTION FIVE (12 marks) Start a new page. **Marks**

(a) **1**



Determine the size of $\angle ABE$ in the diagram above. Give a reason for your answer.

(b) In a group of patients, 14 had caught influenza in the last year, 17 had caught a cold and 22 had suffered either one or both. A patient is chosen at random from this group. Using a Venn diagram or otherwise, determine the probability that the patient had caught only one disease. **3**

(c) Solve:

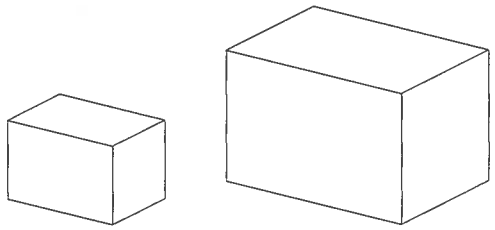
(i) $\log_3 x = -2$ **1**

(ii) $\log_x 216 = 3$ **1**

(d) Find the coordinates of the intersection points of $y = 3x - 1$ with $y = \dfrac{2}{x}$. **3**

Exam continues overleaf ...

(c)



The two rectangular prisms shown above are similar. They have surface areas $360\,\text{cm}^2$ and $1000\,\text{cm}^2$.

(i) Determine the enlargement factor. **1**

(ii) Given that the volume of the smaller prism is $108\,\text{cm}^3$, find the volume of the larger prism. **2**

QUESTION SIX (12 marks) Start a new page. **Marks**

(a) Complete the square to find the vertex of the parabola with equation $y = x^2 - 2x - 2$. **2**

(b) Consider the hyperbola with equation $y = \dfrac{1}{x+1}$.

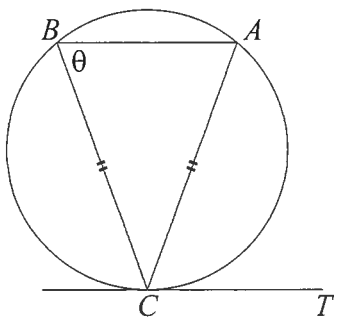
(i) The horizontal asymptote has equation $y = 0$. What is the equation of the vertical asymptote? **1**

(ii) Find the y-intercept. **1**

(iii) Sketch the hyperbola. Show the coordinates of at least one point on each branch. **2**

(c) It is known that $\sin\theta = \dfrac{8}{17}$ and that $\tan\theta < 0$. Find the exact value of $\cos\theta$. **3**

(d) **3**



The diagram above shows isosceles $\triangle ABC$ with its vertices on the circumference of a circle. The line CT is tangent to the circle at C. Let $\angle ABC = \theta$.

Prove that $AB \parallel CT$.

Exam continues next page ...

QUESTION SEVEN (12 marks) Start a new page. **Marks**

(a) Find the exact value of $\cos 225°$. 1

(b) On a bench is a bag which contains 6 blue discs and 4 red discs. A disc is taken at random from the bag. The colour is noted and it is then placed on the bench beside the bag. This process is then carried out again.

(i) Draw a probability tree diagram for this experiment. 1

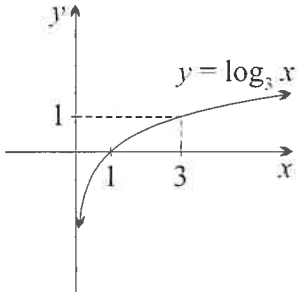
(ii) What is the probability of choosing two red discs? 1

(iii) The experiment is performed by someone else and you do not know the outcome. What is the probability that both discs are red given that the person tells you that at least one disc is red? 1

(c) A certain solid cone has a base area of 2π cm^2 and a curved surface area of 6π cm^2. Determine its exact height. 3

(d) Solve $2\tan\theta - 4 = 1$, correct to the nearest degree, for $0° \leq \theta \leq 360°$. 3

(e) 2



The diagram above shows the graph of $y = \log_3 x$. Use it and your knowledge of shifted graphs to sketch $y = \log_3(x + 1) - 1$. Clearly show any asymptotes and intercepts.

Exam continues overleaf ...

QUESTION EIGHT (12 marks) Start a new page. **Marks**

(a) Let $G(x) = x^3 - 13x - 12$

(i) Use the factor theorem to show that $(x-4)$ is a factor of $G(x)$. **1**

(ii) Use long division to determine the quotient when $G(x)$ is divided by $(x-4)$. **2**

(iii) Hence, or otherwise, solve $x^3 - 13x - 12 = 0$. **2**

(b) Let $F(x) = x^2(x+1)(x-2)^3$.

(i) Sketch the graph of $y = F(x)$, clearly showing the intercepts with the axes. **3**

(ii) Hence solve $x^2(x+1)(x-2)^3 > 0$. **1**

(c) Let $P(x) = x^3 - 2x^2 - 2x + 2$ and let the zeroes of the polynomial be at $x = \alpha, \beta$ and γ. Do NOT attempt to find α, β or γ.

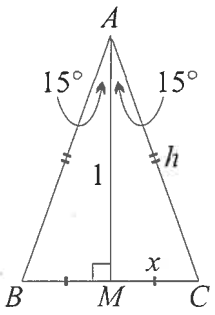
(i) Write $P(x)$ as a product of its factors in terms of its zeroes α, β and γ. **1**

(ii) Hence evaluate $(2+\alpha)(2+\beta)(2+\gamma)$. **2**

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QUESTION NINE (12 marks) Start a new page. **Marks**

(a)

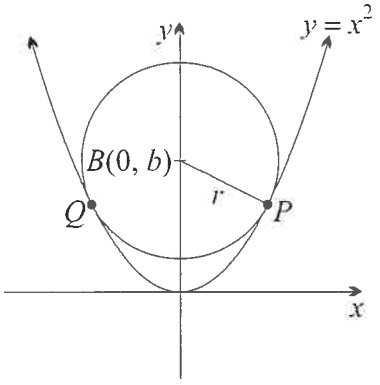


In the diagram above, $\triangle ABC$ is isosceles with $AB = AC$ and $\angle BAC = 30°$. The altitude AM has length 1, and it bisects both the base and the apex angle. Let $AC = h$ and $MC = x$, where $x < 1$.

(i) Use the area of $\triangle ABC$ to show that $h^2 = 4x$. **2**

(ii) Use the result of part (i) and $\triangle AMC$ to help show that $\tan 15° = 2 - \sqrt{3}$. **3**

(b)



In the diagram above the circle with centre $B(0, b)$ and radius r touches the parabola $y = x^2$ at the points P and Q only.

(i) Write down the equation of the circle. **1**

(ii) Show that the y-coordinates of P and Q satisfy the equation **1**

$$y^2 + (1 - 2b)y + (b^2 - r^2) = 0\,.$$

(iii) Use the geometry of the situation to explain why there is only one solution to the quadratic equation in part (ii). **1**

(iv) Hence show that $r^2 = b - \frac{1}{4}$. **2**

(v) Determine the y-coordinate of P in terms of b. **1**

(vi) What is the radius of the largest circle, with its centre on the positive y-axis, which touches the parabola only once? **1**

END OF EXAMINATION