SYDNEY BOYS HIGH SCHOOL

2025 | YEAR 10 STAGE 5 TERM 4 YEARLY

MATHEMATICS

General Instructions: Time Allowed: 10 minutes reading
120 minutes working

- All questions from Section A to Section F may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using a black non-erasable pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet. Indicate that you have done this. Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Any answer that totally relies on calculator technology may not necessarily be awarded full marks.

PUBLISHER'S NOTES:

- This is a recreation of the Sydney Boys High School 2025 Year 10 Stage 5 Yearly.
  This is not the official document.
- The examined topics were:
  - Further Equations,
  - Functions and Other Graphs,
  - Logarithms,
  - Circle Geometry, and
  - Working Mathematically (the non-routine usage of the above topics)

  The Polynomials I topic was removed.
- **THIS EXAM IS VERY DIFFICULT. PLEASE BE WARNED.**
- **Question 2 (c) was removed from the official exam - it was worth 3 marks. All students were eligible for the marks as a result.**

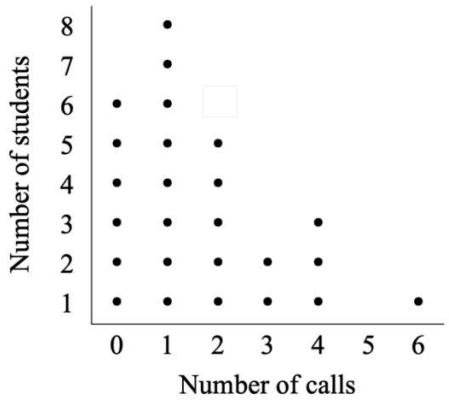
SECTION A (___/19)

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1 | What are the solutions to the equation $x^2 + 2x - 7 = 0$?

A] $x = -1 \pm \sqrt{2}$
B] $x = -2 \pm \sqrt{2}$
C] $x = -2 \pm 2\sqrt{2}$
D] $x = -1 \pm 2\sqrt{2}$

2 | The number of phone calls made on Wednesday by 25 students is shown in the dot plot.



Another student makes two phone calls on Wednesday.
Which of these will change if these calls are included in the dot plot?

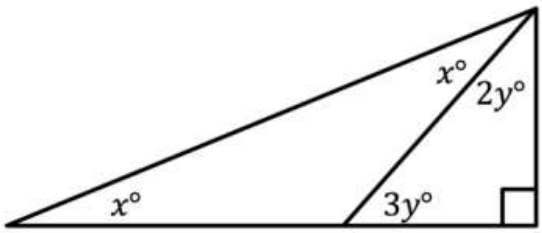
A] Mean
B] Median
C] Mode
D] Range

3 | Which expression is the correct factorisation of $27x^3 - 1$?

A] $(3x + 1)(9x^2 - 3x + 1)$
B] $(3x - 1)(9x^2 - 3x + 1)$
C] $(3x - 1)(9x^2 + 3x + 1)$
D] $(3x - 1)(9x^2 + 6x + 1)$

4 | What is the value of x?

A] 18
B] 27
C] 36
D] 45

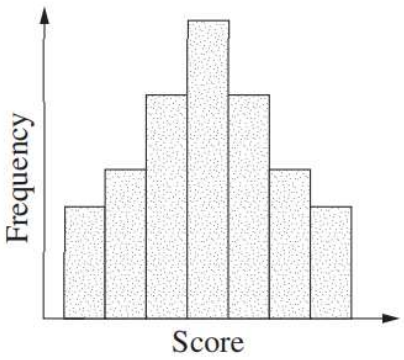


5 | The size of each interior angle in a particular regular polygon is 150° more than the size of each exterior angle. How many sides does the polygon have?

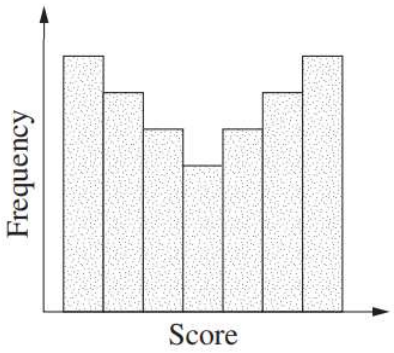
A] 12
B] 16
C] 20
D] 24

6 | Which of the frequency histograms shows data that could be normally distributed?

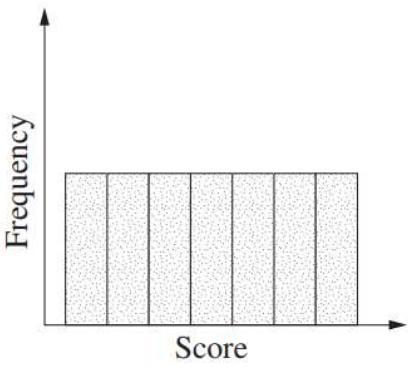
A]



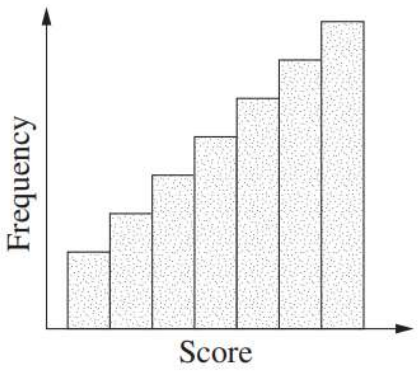
B]



C



D]



7 | Consider $f(x) = \frac{6}{x}$ and $g(x) = 2x + 4$. For which values of x is $f(x) = g(x)$?

A] $x = -1$ or $x = 3$
B] $x = -1$ or $x = -3$
C] $x = 1$ or $x = 3$
D] $x = 1$ or $x = -3$

8 | An object is in the form of a sphere.
If it is remolded and changed into a cylinder, what is the effect on the volume?

A] $V_C = V_S \times \frac{4}{3}$
B] Not enough information.
C] $V_C = V_S \times \frac{3}{4}$
D] $V_C = V_S$

9 | Given $\left(\frac{1}{2}\right)^x \geq \frac{1}{1024}$, which of the following is correct?

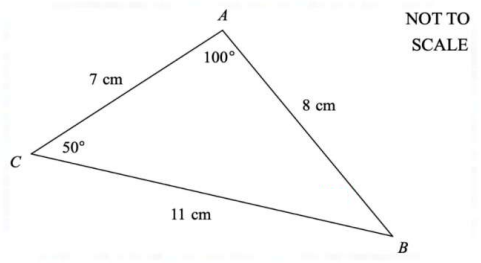
A] $log_{10}x \leq 1$
B] $log_{10}x \geq 1$
C] $x \geq 10$
D] $x \leq 10$

10 | Consider $f(x) = \left|\frac{x}{2}\right| - \frac{x}{2}$.
What are the coordinates of the midpoint of the line joining the following points?
$H[f(4),\ f(-4)],\ K[f(6),\ f(-6)]$

A] $(0, -1)$
B] $(0, 5)$
C] $(4, 6)$
D] $(5, 5)$

11 | Which of the following is the correct expression for the area of ΔABC?

A] $A = \frac{1}{2} \times 7 \times 8 \times sin(30°)$
B] $A = \frac{1}{2} \times 7 \times 8 \times sin(50°)$
C] $A = \frac{1}{2} \times 8 \times 11 \times sin(30°)$
D] $A = \frac{1}{2} \times 8 \times 11 \times sin(50°)$



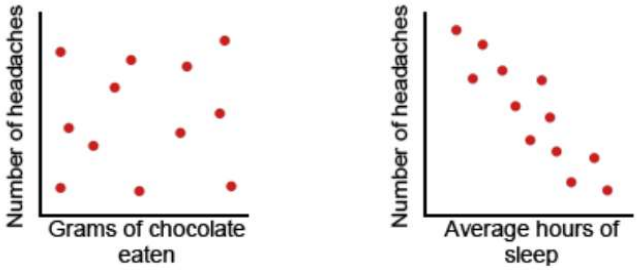
12 | What is the maximum value of $y = 1 - 2cos(x)$?

A] -1
B] 1
C] 2
D] 3

13 | A set of data is normally distributed with a mean of 48 and a standard deviation of 3. Approximately what percentage of the scores lies between 39 and 45?

A] 15.85%
B] 31.70%
C] 47.50%
D] 49.85%

14 | Marlo suspects that his headaches are related to the amount of chocolate he eats or the hours of sleep he gets.
Over several weeks he records data and the plots it to create the following graphs:



What conclusion can he make?
A] Chocolate seems to be related to his headaches but sleep does not.
B] Sleep seems to be related to his headaches but chocolate does not.
C] Neither chocolate nor sleep appear to be related to his headaches.
D] Both chocolate and sleep appear to be related to his headaches.

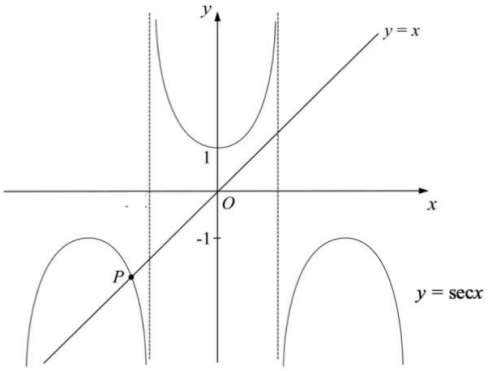
15 | The table shows the compounded values of $1 at different interest rates over different periods:

Compounded values of $1

Number of periods	*Interest rate per period*				
	1%	2%	3%	4%	5%
2	1.0201	1.0404	1.0609	1.0816	1.1025
4	1.0406	1.0824	1.1255	1.1699	1.2155
6	1.0615	1.1262	1.1941	1.2653	1.3401
8	1.0829	1.1717	1.2668	1.3686	1.4775
10	1.1046	1.2190	1.3439	1.4802	1.6289
12	1.1268	1.2682	1.4258	1.6010	1.7959

Becky hopes to have $21,000 in 2 years to buy a car. She opens an account today which pays interest of 4% p.a., compounded quarterly.
Using the table, which expression calculates the minimum single sum that Becky needs to invest today to ensure she reaches her savings goal?
A] 21000×1.0816
B] $21000 \div 1.0816$
C] 21000×1.0829
D] $21000 \div 1.0829$

16 | The graphs of $y = x$ and $y = sec(x)$ are shown below.



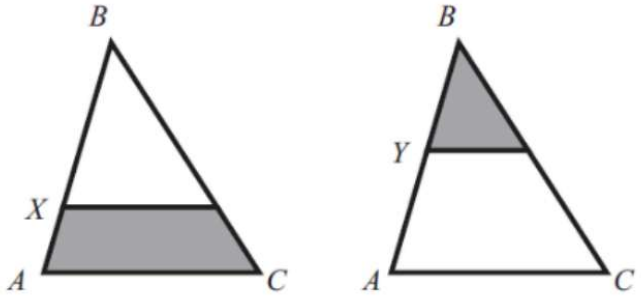
Which equation can be used to find the x-coordinate of the point of intersection of the two graphs?

A] $x = sin(x)$
B] $x = cos(x)$
C] $1 = xcos(x)$
D] $1 = xsin(x)$

17 | Zoe opened an account to save for a new car. Six months after opening the account, she made a deposit of \$1200 and continued depositing \$1200 at the end of each six month period. Interest was paid at 3% per annum, compounded half-yearly.
How much was in Zoe's account two years after first opening it?

A] \$4909.08
B] \$4982.72
C] \$5018.16
D] \$5094.55

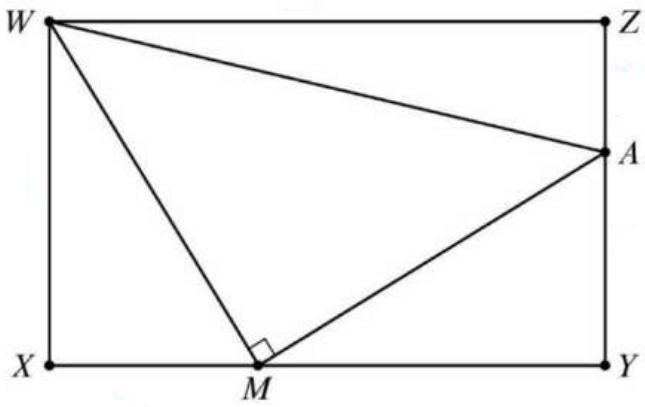
18 |



In the triangle ABC, we can draw a line parallel to its base AC, through X or Y. The areas of the shaded regions are the same. The ratio $BX: BA = 4: 1$.
What is the ratio $BY: YA$?

A] 2 : 1
B] 3 : 1
C] 3 : 2
D] 4 : 3

19 | $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies on XY, point A lies on YZ, and $\angle WMA$ is a right angle. The areas of ΔWXM and ΔWAZ are equal.
What is the area of ΔWMA?



A] 13
B] 14
C] 15
D] 16

End of Section A

SECTION B (___/23)

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1 | What is the gradient of the line $2x + 3y + 4 = 0$? **1**

2 | Find the domain of $y = \frac{1}{\sqrt{3-x}}$. **1**

3 | Solve $2^x = 5$, correct to 3 significant figures. **2**

4 | If $ax + 3 = bx$, make x the subject. **2**

5 | If $cos(\theta) = \frac{1}{2}$ and $sin(\theta) < 0$, find the exact value of $tan(\theta)$. **2**

6 | A function is defined as:
If $0 \leq x \leq 3$, then $f(x) = 2x - 1$, but if $3 < x \leq 5$, then $f(x) = \frac{1}{3}x + 4$.

(a) What is the range of this function? **1**

(b) Find the value of $f(4) - f(2)$. **2**

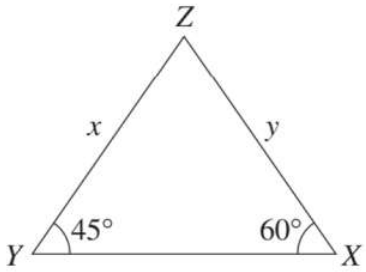
7 | Consider $f(x) = x^2 - 4$ and $g(x) = 4 - 2x$. Solve $f(g(x)) = 0$. **2**

8 | A curve has the equation $y = f(x)$.
The coordinates of the only turning point is $(6, 5)$.
Write down the coordinatesof the turning point on the curve with the equation:

(a) $y = f(x - 4)$. **1**

(b) $y = f(3x)$. **1**

9 | In the diagram, XYZ is a triangle where $\angle ZYX = 45°$ and $\angle ZXY = 60°$.
Find the exact value of $\frac{x}{y}$. **2**



NOT TO SCALE

10 | The surface area of a sphere of radius R is equal to the total surface area of 10 closed cylinders of radius r and height $4r$.
Find an expression for R in terms of r. **2**

11 | A game is played in which two coloured dice are thrown once.
The six faces of the red die are numbered 1, 3, 5, 7, 9 and 11.
The six faces of the white die are numbered 2, 4, 6, 8, 10 and 12.
The player wins if the number of the red die is larger than the number on the white die.

(a) Show that the probability of the player winning a game is $\frac{5}{12}$. **1**

(b) Find the probability that a player wins exactly once in 2 successive games. **2**

(c) Find the probability that a player wins at least once in 2 successive games. **1**

End of Section B

SECTION C (___/18)

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1 | Tung Tung sits a Physics test and a Biology test.

(a) Tung Tung's mark in the Physics test is 67.
The mean mark for this test is 41 and the standard deviation is 7.
Calculate the z-score for Tung Tung's mark in this test. **1**

(b) In the biology test, the mean mark is 76 and the standard deviation is 3.5.
Tung Tung's z-score is the same in both the Physics and the Biology test.
What is his mark in the Biology test? **2**

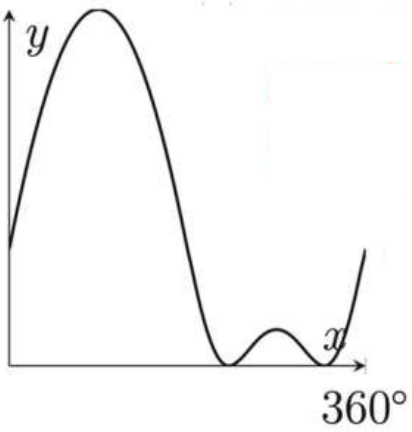
2 | Solve $2sin(\theta) + \frac{1}{sin(\theta)} = 3$ for $\theta \in [0, 360°]$. **3**

3 | (a) Write $6u^2 - 8u + 3$ in the form $6(u - b)^2 + c$, where b and c are real. **2**

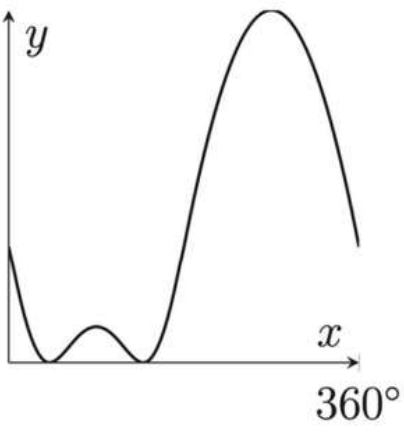
(b) Consider $cos^2(x) + sin^2(x) = 1$.
Rewrite $9 - 8sin(x) - 6cos^2(x)$ without cosines. **2**

(c) Which of the following graphs shows $y = log_2\left(9 - 8sin(x) - 6cos^2(x)\right)$ for $x \in [0, 360°]$? Circle the correct response. **1**

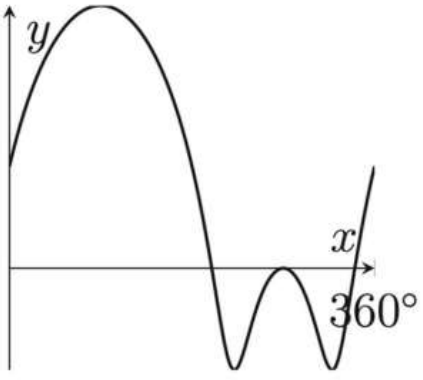
A]



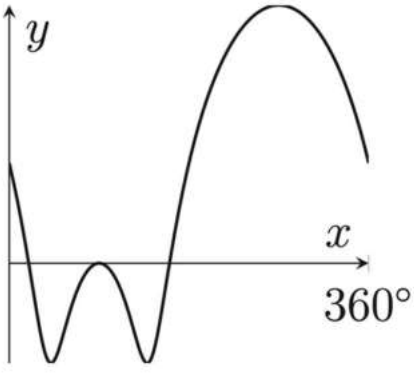
B]



C]



D]



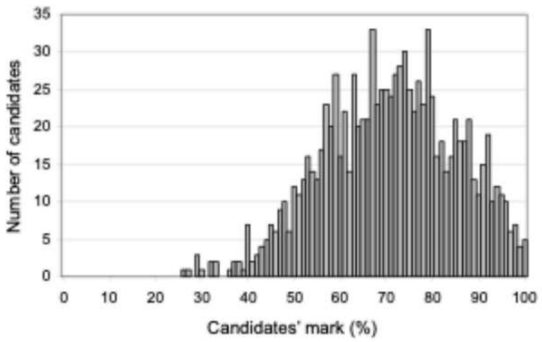
4 | Let x be a positive real number, where

$$log_3 x + log_9 x + log_{27} x + log_{81} x = 10.$$

Solve for x, in the form 3^r where $r \in \mathbb{Q}$. **3**

5 | Consider the circle $(x - x_1)^2 + (y + 4)^2 = 25$. Find the values of x_1 for which the point $M(4, -1)$ will lie inside the circle. **3**

6 | In an ideal examination, in order to maximise the discrimination between candidates, there should be a range of questions such that: the more difficult questions are answered correctly by only a small number of the best candidates and the easier questions are answered correctly by all but a small number of the weaker candidates.
The graph below shows the distribution of results for a given examination. **1**



A teacher makes a comment "the easier questions were too easy". Do you agree or disagree? Briefly justify.

End of Section C

SECTION D (___/18)

1 | Find the values of x such that **2**

$$\frac{10^{4x} \times 2^{3(x^2-5x)} \times 5^{2(1-2x)}}{20^2} = 1$$

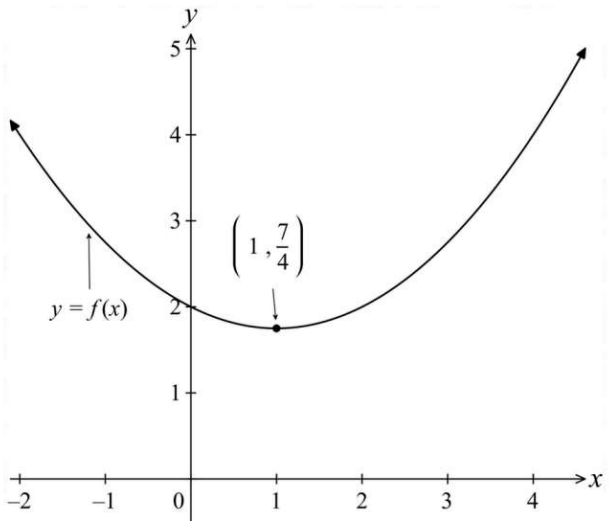
Show clear algebraic working.

2 | Fully factorise **3**

$$\left(1 + \frac{y^2+z^2-x^2}{2yz}\right) \div \left(1 - \frac{x^2+y^2-z^2}{2xy}\right)$$

Show clear algebraic working.

3 | The graph of $f(x) = \frac{1}{4}[(x - 1)^2 + 7]$ is given below.



(a) State the largest domain containing $x = 3$ for an inverse function to exist. **1**

(b) Sketch the graph of $y = f^{-1}(x)$ on the provided graph. **1**

(c) State the domain of the inverse function. **1**

(d) Find $f^{-1}(f(-1))$. **2**

(e) Find the coordinates of the point of intersection of $y = f(x)$ and $y = f^{-1}(x)$, if any exist. Show clear working. **2**

4 | Solve for x and y, where both x and y belong in the first quadrant: **3**

$$2sin(x + y) = \sqrt{3}$$
$$3tan(x - y) = \sqrt{3}$$

5 | Given x satisfies $x^2 - 17x + 71 = 0$, without solving for x, find the value of: **3**

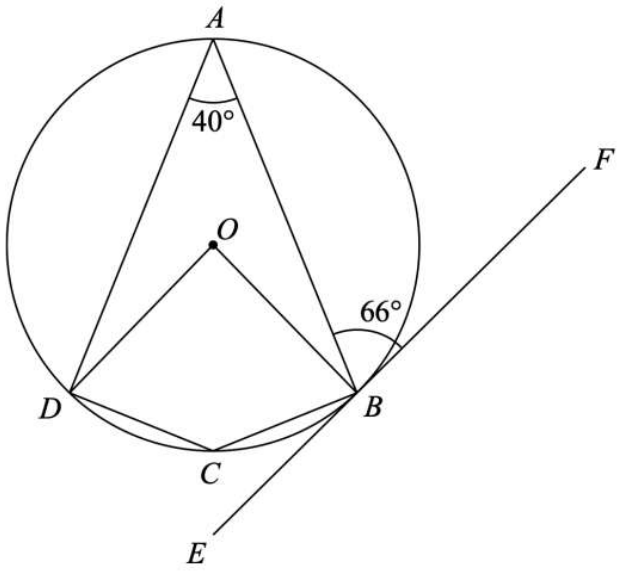
$$\frac{(x-9)^2}{1+(x-9)^4}$$

Note: Answers that solve for x to get the result will **NOT** receive any marks.

End of Section D

SECTION E (___/14)

1 | $A,\ B,\ C$ and D are points on a circle with centre O.
EBF is a tangent to the circle at B.



(a) Find $\angle DCB$, giving geometric reasons. **2**

(b) Find $\angle ADO$, giving geometric reasons. **3**

2 | Tree Gong draws ΔABC with $AB = 3\text{cm}$, $BC = 4\text{cm}$, and $\angle ABC = 90°$.
Polly Gone then draws ΔDBF on top of ΔABC so that D lies on AB, F lies on the extension of BC, $DB = 2\text{cm}$, and sides AC and DF meet at E.
If $AE = 3\text{cm}$ and $EC = 2\text{cm}$, determine the length of CF. **3**

3 | Sina drew square $ABCD$ with side length 6 cm on a piece of paper and passed the paper to Theo.
Theo drew a circle on top of the square so that the circle passes through A and D, and the circle is tangent to side BC at point P.
Determine the radius of the circle. **3**

4 | The first term in a sequence is 24.

We can determine the next terms in the sequence as follows:

- If a term is even, then divide it by 2 to get the next term.
- If a term is odd, then multiply it by 3 and add 1 to get the next term.

By doing this, we determine that the first three terms in the sequence are 24, 12, and 6.

Shweta writes the first n terms in this sequence and notices that the sum of these terms is a four-digit number.

How many different possible values of n are there? **3**

End of Section E

SECTION F (___/13)

PUBLISHER'S NOTE: Question 2 (c) was removed from the official exam - it was worth 3 marks. All students were eligible for the marks as a result.

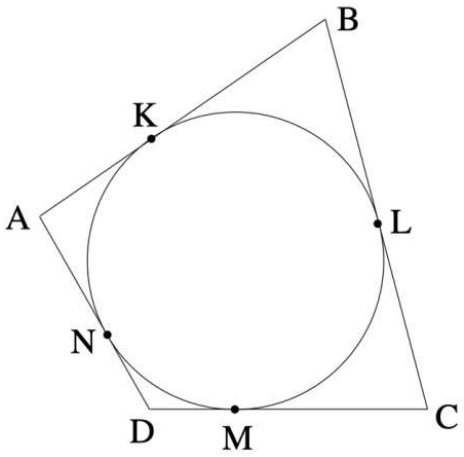
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1 | The Governor's Centre (TGC) is a tall building on flat ground in SBHS.
Three students Luke, Billy and Jack are observing TGC from the ground. Luke is due North of TGC, Jack is due East of TGC and Billy is on the line of sight from Luke to Jack. The angles of elevation to the top of TGC from Luke, Billy and Jack are 26°, 28° and 30° respectively.

(a) Draw a diagram, clearly labelling all the information. **2**

(b) Find the bearing of Billy from TGC. **3**

2 |



(a) $ABCD$ is a quadrilateral with all angles less than 180°. The sides of $ABCD$ are tangent to the circle at points K, L, M and N. Show that: **2**

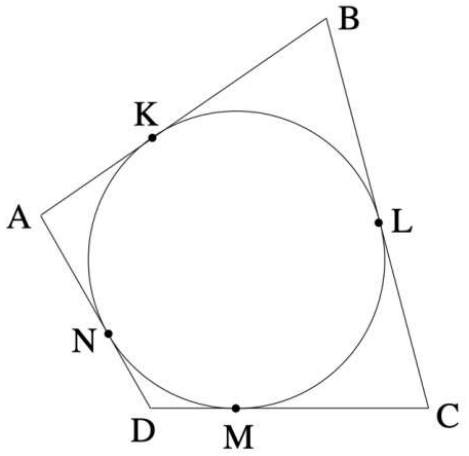
$$AB + CD = AD + BC$$

(b) Let X be the point of intersection of the angle bisectors of $\angle ABC$ and $\angle BCD$. Prove that X is the centre of a circle to which AB, BC and CD are tangent. **3**

(c) $ABCD$ is a quadrilateral, with all angles less than 180°. Given that

$$AB + CD = AD + BC,$$

show that there exists a circle to which all sides of $ABCD$ are tangent. **3**



End of Section F and End of Examination

SYDNEY BOYS HIGH SCHOOL

2025

YEAR 10
STAGE 5 TERM 4
YEARLY

Sample Solutions

NOTE: This process of checking your mark is about reading the solutions and the comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.
Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

Multiple Choice Answers

1	2	3	4	5
D	**A**	**C**	**B**	**D**
6	7	8	9	10
A	**D**	**D**	**D**	**B**
11	12	13	14	15
C	**D**	**A**	**B**	**D**
16	17	18	19	
C	**A**	**C**	**C**	

Section A Solutions

1 What are the solutions to the equation $x^2 + 2x - 7 = 0$? **1**

A. $x = -1 \pm \sqrt{2}$

B. $x = -2 \pm \sqrt{2}$

C. $x = -2 \pm 2\sqrt{2}$

D. $x = -1 \pm 2\sqrt{2}$

It is A or D by looking at the coefficient of the x term:

$$x^2 + 2x - 7 = 0 \Rightarrow \left(x + \frac{2}{2}\right)^2 = 7 + \left(\frac{2}{2}\right)^2$$

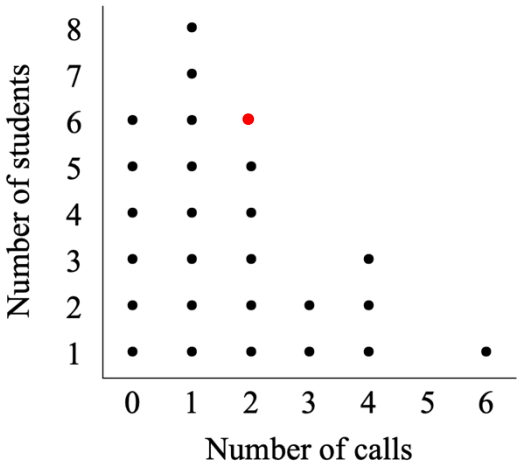
$$\therefore (x+1)^2 = 8 \Rightarrow x = -1 \pm \sqrt{8}$$

OR

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times (-7)}}{2} = \frac{-2 \pm \sqrt{32}}{2} = \frac{-2 \pm 4\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$$

A	20
B	6
C	11
D	167

2 The number of phone calls made on Wednesday by 25 students is shown in the dot plot. **1**



A	195
B	7
C	1
D	1

Another student makes two phone calls on Wednesday.
Which of the mean, median, mode or range will change if these calls are included in the dot plot?

A. Mean

B. Median

C. Mode

D. Range

C and D are obviously wrong.
Check the median i.e. the 13th score (1) and so A is the answer.

3 Which expression is the correct factorisation of $27x^3 - 1$? **1**

A. $(3x + 1)(9x^2 - 3x + 1)$

B. $(3x - 1)(9x^2 - 3x + 1)$

C. $(3x - 1)(9x^2 + 3x + 1)$

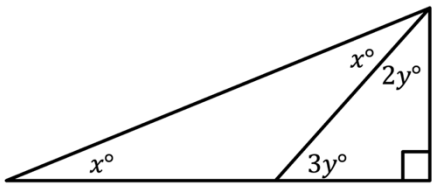
D. $(3x - 1)(9x^2 + 6x + 1)$

A	3
B	10
C	175
D	17

$$27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)\left[(3x)^2 + (3x)(1) + (1)^2\right]$$

A is just wrong.
B is a common mistake because students forget that the signs switch in the brackets.
D is wrong as students mistakenly think of the perfect square expansion.

4 What is the value of x ? **1**



A. 18

B. 27

C. 36

D. 45

A	2
B	196
C	2
D	5

$5y = 90$ (∠ sum of Δ)
$\therefore y = 18$
$2x = 3y$ (exterior ∠ of Δ)
$\therefore x = 27$

5 The size of each interior angle in a particular regular polygon is 150° more than the size of each exterior angle. How many sides does the polygon have? **1**

A. 12

B. 16

C. 20

D. 24

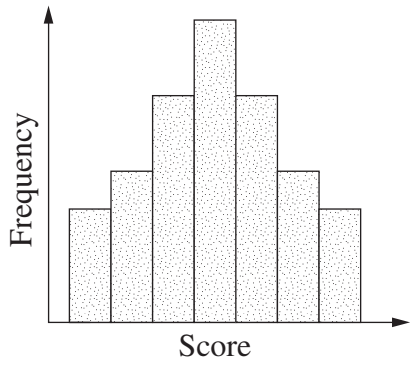
A	23
B	21
C	23
D	138

Interior angle + exterior angle = 180°
Interior angle – exterior angle = 150°
$\therefore$ exterior angle = 15°

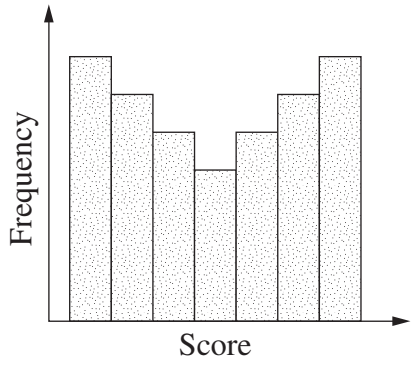
$\therefore$ Sides $= \dfrac{360}{15} = 24$

6 Which of the following frequency histograms shows data that could be normally distributed? **1**

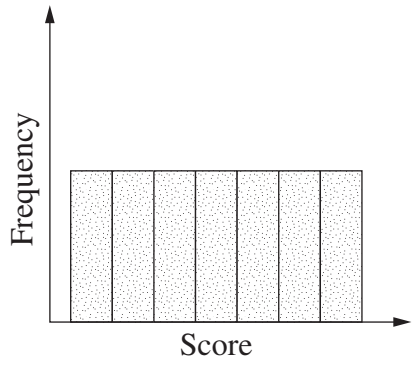
A.



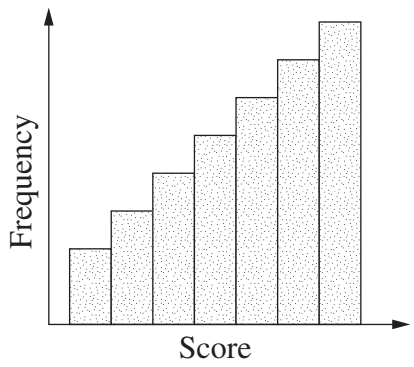
B.



C.



D.



A	187
B	1
C	13
D	4

7 Consider $f(x) = \frac{6}{x}$ and $g(x) = 2x + 4$. For what values of x is $f(x) = g(x)$? **1**

A. $x = -1$ or $x = 3$

B. $x = -1$ or $x = -3$

C. $x = 1$ or $x = 3$

D. $x = 1$ or $x = -3$

A	4
B	1
C	1
D	199

Check $x = 1$: ✓ So C or D only

Check $x = 3$: × So D

8 An object is in the form of a sphere. If it is remolded and changed into a cylinder, what is the effect on the volume? **1**

A. The volume increases by a factor of $\frac{4}{3}$.

B. There is not enough information.

C. The volume decreases by a factor of $\frac{3}{4}$.

D. The volume remains unchanged.

A	1
B	32
C	5
D	167

Your Science teachers have been informed!

9 Given $\left(\frac{1}{2}\right)^x \geq \frac{1}{1024}$, which of the following is correct? **1**

A. $\log_{10} x \leq 1$

B. $\log_{10} x \geq 1$

C. $x \geq 10$

D. $x \leq 10$

A	6
B	9
C	66
D	124

$\left(\frac{1}{2}\right)^x \geq \frac{1}{1024} \Rightarrow 2^x \leq 2^{10}$

$\therefore x \leq 10$

10 Consider $f(x) = \left|\frac{x}{2}\right| - \frac{x}{2}$. **1**

What are the coordinates of the midpoint of the line joining the points $H[f(4), f(-4)]$ and $K[f(6), f(-6)]$?

A. $(0, -1)$

B. $(0, 5)$

C. $(4, 6)$

D. $(5, 5)$

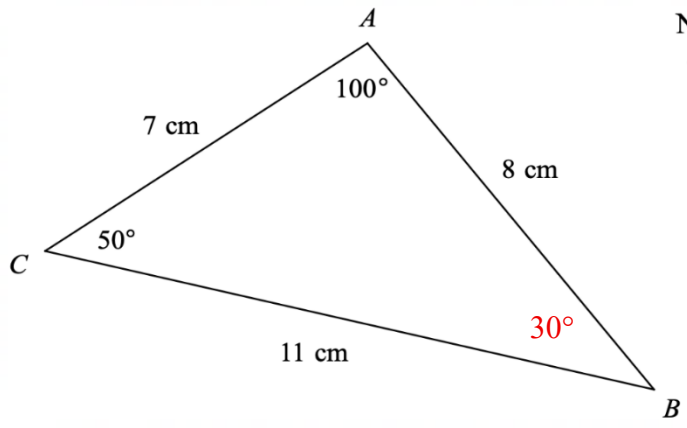
A	8
B	175
C	9
D	12

Note: $f(a) = 0$ if $a > 0$ and $f(a) = -a$ if $a < 0$

$\therefore H$ is $(0, 4)$ and K is $(0, 6)$

$\therefore$ the midpoint is $(0, 5)$

11 Which of the following is the correct expression for the area of ΔABC ? **1**



NOT TO SCALE

A. $A = \frac{1}{2} \times 7 \times 8 \times \sin 30°$

B. $A = \frac{1}{2} \times 7 \times 8 \times \sin 50°$

C. $A = \frac{1}{2} \times 8 \times 11 \times \sin 30°$

D. $A = \frac{1}{2} \times 8 \times 11 \times \sin 50°$

A	4
B	4
C	187
D	10

12 What is the maximum value of $y = 1 - 2\cos x$? **1**

A. -1

B. 1

C. 2

D. 3

$-1 \leq \cos x \leq 1 \Rightarrow 2 \geq -2\cos x \geq -2$

$\therefore -1 \leq 1 - 2\cos x \leq 3$

A	15
B	23
C	19
D	147

13 A set of data is normally distributed with a mean of 48 and a standard deviation of 3. **1**
Approximately what percentage of the scores lies between 39 and 45?

A. 15.85%

B. 31.7%

C. 47.5%

D. 49.85%

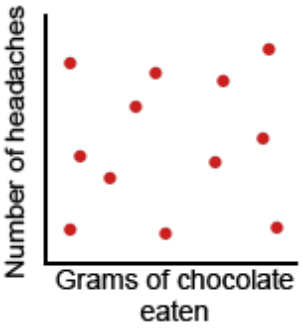
$39 = 48 - 3 \times 3 \ (\bar{x} - 3\sigma)$

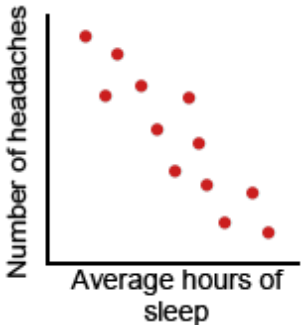
$45 = 48 - 1 \times 3 \ (\bar{x} - \sigma)$

A	144
B	25
C	14
D	21

$\bar{x} - 3\sigma \leq x \leq \bar{x} - \sigma$: $49.85\% - 34\% = 15.85\%$

14 Marlo suspects that his headaches are related to the amount of chocolate he eats or the hours of sleep he gets. Over several weeks he records data and the plots it to create the following graphs. **1**





What conclusion can he make?

A. Chocolate seems to be related to his headaches but sleep does not.

B. Sleep seems to be related to his headaches but chocolate does not.

C. Neither chocolate nor sleep appear to be related to his headaches.

D. Both chocolate and sleep appear to be related to his headaches.

A	0
B	202
C	3
D	0

15 The table shows the compounded values of $1 at different interest rates over different periods. **1**

Compounded values of $1

Number of periods	*Interest rate per period*				
	1%	2%	3%	4%	5%
2	1.0201	1.0404	1.0609	1.0816	1.1025
4	1.0406	1.0824	1.1255	1.1699	1.2155
6	1.0615	1.1262	1.1941	1.2653	1.3401
8	1.0829	1.1717	1.2668	1.3686	1.4775
10	1.1046	1.2190	1.3439	1.4802	1.6289
12	1.1268	1.2682	1.4258	1.6010	1.7959

Becky hopes to have $21 000 in 2 years to buy a car. She opens an account today which pays interest of 4% p.a., compounded quarterly.

Using the table, which expression calculates the minimum single sum that Becky needs to invest today to ensure she reaches her savings goal?

A. 21000×1.0816

B. $21000 \div 1.0816$

C. 21000×1.0829

D. $21000 \div 1.0829$

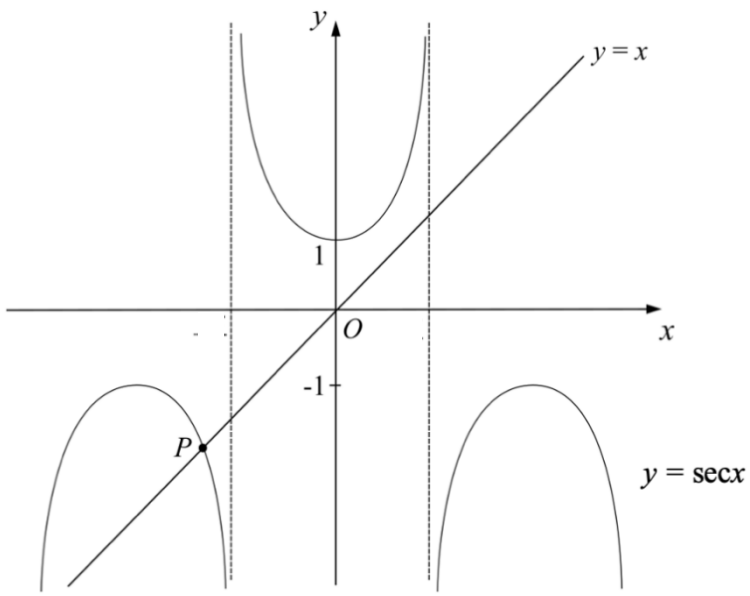
$$21\,000 = P\left(1 + \frac{4\%}{4}\right)^{2 \times 4}$$

$$\therefore 21\,000 = P \times 1.0829$$

A	18
B	60
C	31
D	96

16 The graphs of $y = x$ and $y = \sec x$ are shown below. **1**

Note: $\sec x = \dfrac{1}{\cos x}$



Which equation can be used to find the x-coordinate of the point of intersection of the two graphs?

A. $x = \sin x$

B. $x = \cos x$

C. $1 = x\cos x$

D. $1 = x\sin x$

$x = \sec x \Leftrightarrow x = \dfrac{1}{\cos x}$

$\therefore x\cos x = 1$

A	8
B	24
C	166
D	7

17 Zoe opened an account to save for a new car. Six months after opening the account, she made a deposit of $1200 and continued depositing $1200 at the end of each six month period. Interest was paid at 3% per annum, compounded half-yearly. **1**

How much was in Zoe's account two years after first opening it?

A. $4909.08

B. $4982.72

C. $5018.16

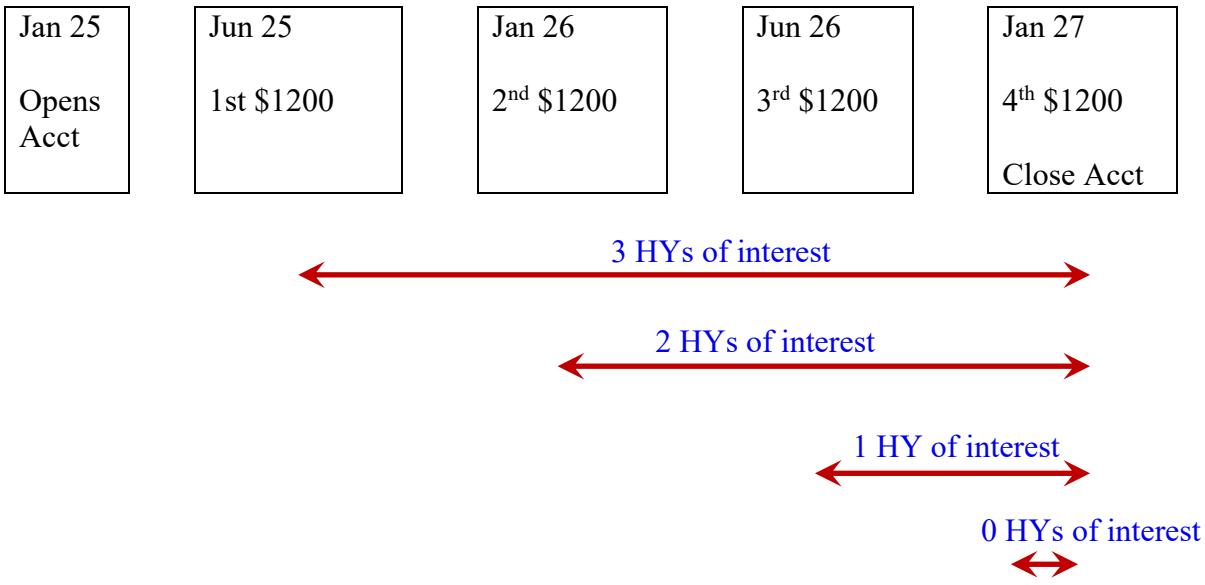
D. $5094.55

A	65
B	95
C	26
D	18

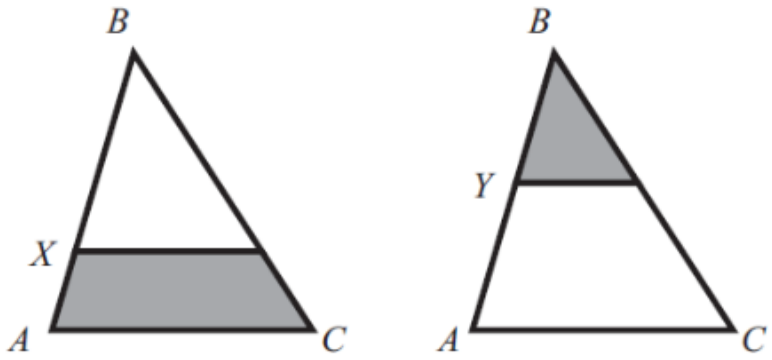
3% p.a. = 1.5% per HY and so 1 + 1.5% = 1.0015

Two years after opening means 4 deposits, but the first deposit would only get 3 HYs of interest, until the last deposit would get no interest.

$$A = 1200\left(1.0015^3 + 1.0015^2 + 1.0015 + 1\right)$$
$$= 4909.08$$



18 **1**



In the triangle ABC, we can draw a line parallel to its base AC, through X or Y.
The areas of the shaded regions are the same.
The ratio $BX : XA = 4 : 1$.

What is the ratio $BY : YA$?

A. 2 : 1

B. 3 : 1

C. 3 : 2

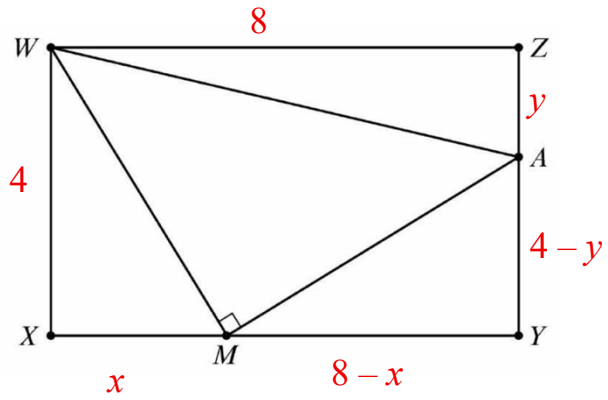
D. 4 : 3

A	30
B	15
C	133
D	26

In the first diagram, $BX : BA = 4 : 5$, and so the area of top : bottom = 16 : 25.
So the shaded area is 9 : 25.

In the second diagram, the area is 9 : 25 and so $BY : BA = 3 : 5$
$\therefore BY : YA = 3 : 2$

19 $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies on XY, point A lies on YZ, and $\angle WMA$ is a right angle. The areas of ΔWXM and ΔWAZ are equal. What is the area of ΔWMA? **1**



A	17
B	59
C	48
D	78

$\frac{1}{2} \times 4 \times x = \frac{1}{2} \times 8 \times y \Rightarrow x = 2y$

$WM^2 = 16 + x^2$

$AM^2 = (8 - x)^2 + (4 - y)^2$

$WA^2 = 64 + y^2$

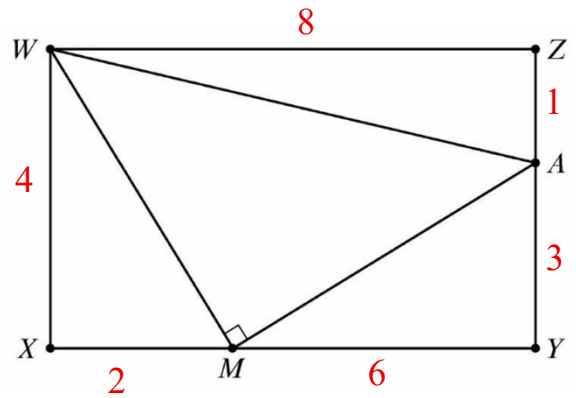
$\therefore 16 + x^2 + (8 - x)^2 + (4 - y)^2 = 64 + y^2$

$\therefore 16 + x^2 + 64 - 16x + x^2 + 16 - 8y + y^2 = 64 + y^2$

$\therefore 2x^2 - 16x + 32 - 8y = 0 \Rightarrow 2x^2 - 16x + 32 - 4x = 0 \quad [8y = 4x]$

$\therefore x^2 - 10x + 16 = 0 \Rightarrow (x - 2)(x - 8) = 0$

$\therefore x = 2 \ (x \neq 8)$



$\therefore$ Area $= 32 - 4 - 4 - 9 = 15$

A. 13 B. 14 C. 15 D. 16

End of Section A solutions

Distribution of MC Marks

SCORE	FREQUENCY
19	0
18	11
17	23
16	35
15	37
14	37
13	16
12	13
11	9
10	14
9	8
8	1
7	0
6	1
5	0
4	0
3	0
2	0
1	0
0	0

Section B Solutions — 23 Marks

1 What is the gradient of the line $2x + 3y + 4 = 0$? **1**

$3y = -2x - 4$

$y = -\frac{2}{3}x - \frac{4}{3}$

$m = -\frac{2}{3}$

Comments: Generally well done.

Common mistakes:

- Not using the correct algebra to get to $y = mx + c$,
- Not identifying / labelling the gradient from $y = mx + c$

2 Find the domain of $y = \frac{1}{\sqrt{3-x}}$. **1**

den. cannot be 0, ~~not~~ and a $\sqrt{\ } > 0$

$\therefore 3 - x > 0$

$3 > x$

$x < 3$

Comments: Generally well done.

Common mistakes:

- The inequality was pointing the wrong way
- The inequality included 0.

3 Solve $2^x = 5$, correct to 3 significant figures. **2**

method (1): rearrange

$x = \log_2 5$

(change base f.) $= \frac{\ln 5}{\ln 2}$

$= 2.321928095$

~ 2.32 (3SF)

(m.2): ln both sides

$\ln 2^x = \ln 5$

$x \ln 2 = \ln 5$

$x = \frac{\ln 5}{\ln 2}$

Comments: Very well done

Common mistake:

- Giving more than 3 sig. fig.

4 If $ax + 3 = bx$, make x the subject. **2**

$ax - bx = -3$ (move all x's to one side)

$x(a-b) = -3$ (factorise)

$x = \frac{-3}{a-b}$ or $\frac{3}{b-a}$

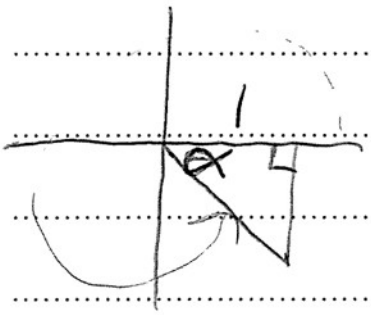
Comments: Very well done

Most common mistake:

- Not using the correct algebra to move all the x terms to one side.

5 If $\cos\theta = \frac{1}{2}$ and $\sin\theta < 0$, find the exact value of $\tan\theta$. **2**

4th Quad.



$\cos\alpha = \frac{1}{2}$

$\alpha = 60^\circ$

$\theta = 360 - 60$

$= 300^\circ$

$\tan 300^\circ = -\tan 60^\circ$

$= -\sqrt{3}$

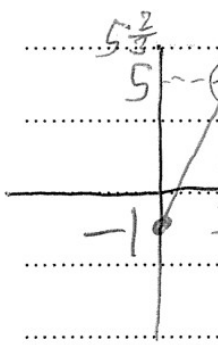
Comments: *Most common mistake*:

- Many students neglected to identify that the triangle had to be in the 4th quadrant due to cos being positive and sin being negative, hence they forgot the (–) sign.

6 A function is defined as:

If $0 \le x \le 3$ then $f(x) = 2x - 1$, but if $3 < x \le 5$ then $f(x) = \frac{1}{3}x + 4$.

(a) What is the range of this function? **1**



$f(0) = 2 \times 0 - 1 = -1$

$f(3) = 2 \times 3 - 1 = 5$ from below

& $f(3) = \frac{1}{3}(3) + 4 = 5$ from above } same

$f(5) = \frac{5}{3} + 4 = 5\frac{2}{3}$

Range of fn. (left) $= [-1, \frac{17}{3}]$ or $-1 \le y \le 5\frac{2}{3}$

Comments: Most students got full marks.

Common mistake:

- Problems occurred when students got the endpoints wrong (arithmetic mistakes).

Note: A complete solution requires that you show that the 2 piecewise functions share a common middle point at $f(3)$. Very few students achieved this.

(b) Find the value of $f(4) - f(2)$. **2**

$f(4) = \frac{1}{3}(4) + 4 = 5\frac{1}{3}$

$f(2) = 2(2) - 1 = 3$

$f(4) - f(2) = 5\frac{1}{3} - 3 = 2\frac{1}{3}$ or $\frac{7}{3}$

Comments: Most students got full marks.

Most common mistake:

- Some students forgot to put brackets around $f(2)$ so they subtracted the wrong number.

7 Consider $f(x) = x^2 - 4$ and $g(x) = 4 - 2x$. Solve $f(g(x)) = 0$. **2**

sub $g(x)$ into $f(x)$, ie

$(4-2x)^2 - 4 = 0$

$16 - 16x + 4x^2 - 4 = 0$

$4x^2 - 16x + 12 = 0$

$(\div 4)\quad x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ or 1

Comments: Not well done

Common mistakes:

- Lots of students subbed the wrong way: $g(x)$ has to go into $f(x)$
- Some students didn't have the algebra skills to go all the way to the solution:
  - they forgot to include the –4 in the subsequent line
  - they didn't get the middle term $-16x$ right
  - they thought $16 - 4 = -12$

Any issues will, of course, give you the wrong quadratic to factorise.

8 A curve has equation $y = f(x)$.

The coordinates of the only turning point is (6, 5).

Write down the coordinates of the turning point on the curve with equation:

(a) $y = f(x - 4)$. **1**

4 units to the right $\longrightarrow$ (10, 5)

ie $x - 4 = 6$

$x = 10$

Comments: Not well done. There were no (very few) half marks.

Common mistakes:

- Giving (2, 5) instead (going the wrong way).
- Changing the y-value when we have only moved the turning point horizontally.

(b) $y = f(3x)$. **1**

$3x = 6$

$x = 2 \longrightarrow (2, 5)$

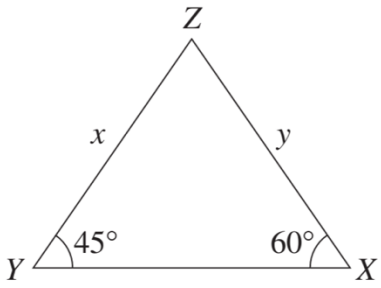
Comments: Not well done. There were no (very few) half marks.

Common mistakes:

- Giving (18, 5) instead (going the wrong way).
- Changing the y-value.

9 In the diagram, XYZ is a triangle where $\angle ZYX = 45°$ and $\angle ZXY = 60°$. **2**

Find the exact value of $\frac{x}{y}$.



NOT TO SCALE

$\frac{x}{\sin 60} = \frac{y}{\sin 45}$ (sin rule)

$\frac{x}{y} = \frac{\sin 60}{\sin 45}$

$= \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}}$

$= \frac{\sqrt{3}}{\sqrt{2}}$ or $\sqrt{\frac{3}{2}}$ or $\sqrt{1.5}$ or $\frac{\sqrt{6}}{2}$

Comments: Generally well done.

Most students successfully identified the sine rule, or some equivalent process to get the equation.

Common mistakes:

- Some students did not have the algebra skills to make $\frac{x}{y}$ the subject.
- Some students don't remember the exact values of sin 60°, and sin 45°.
- Some students couldn't fully simplify the final answer.

10 The surface area of a sphere of radius R is equal to the total surface area of 10 closed cylinders of radius r and height $4r$. **2**

Find an expression for R in terms of r.

$4\pi R^2 = 10(2\pi r^2 + 2\pi r(4r))$

$= 10(10\pi r^2)$

$4\pi R^2 = 100\pi r^2$

$R^2 = 25r^2$

$R = \pm 5r$ but distance $\neq (-)$

$\therefore R = 5r$

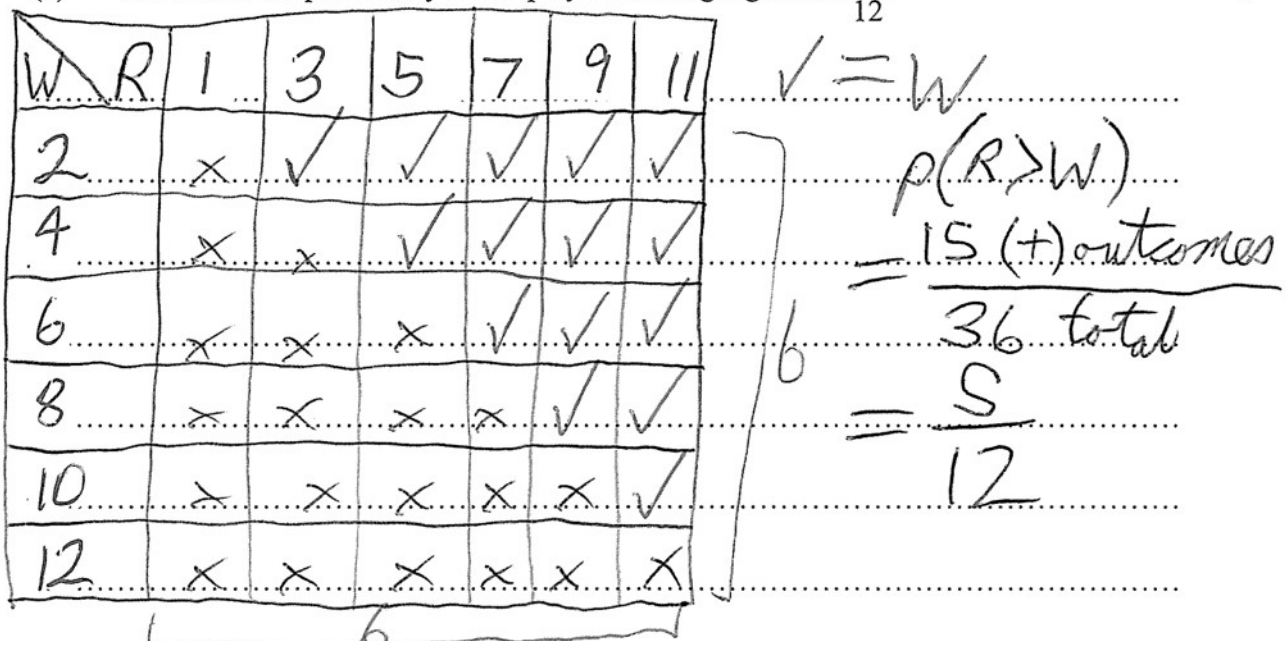
Comments: Poor.

Common mistakes:

- Students did not remember surface area formulae (both sphere and/or cylinder, especially the cylinder).
- Students did incorrect multiplications i.e. they couldn't multiply $2 \times \pi \times r \times 4 \times r$ to get $8\pi r^2$.
- Students forgot their Y7 work on like terms i.e. $8\pi r^2 + 2\pi r^2 = 10\pi r^2$.
- Students forgot that there were 10 cylinders.

11 A game is played in which two coloured dice are thrown once.
The six faces of the red die are numbered 1, 3, 5, 7, 9 and 11.
The six faces of the white die are numbered 2, 4, 6, 8, 10 and 12.
The player wins if the number of the red die is larger than the number on the white die.

(a) Show that the probability of the player winning a game is $\frac{5}{12}$. **1**



Comments: Seemed okay for most students.

Some students were unable to articulate clearly how and why they got $\frac{5}{12}$ using either a counting process or valid algebraic working.

Saying there are 5 winning combos out of 12 numbers is neither adequate nor convincing.

(b) What is the probability that the player wins exactly once in two successive games? **2**

$P(W \times 1) = P(WL) + P(LW)$
$= 2 \times \frac{5}{12} \times \frac{7}{12}$
$= \frac{70}{144}$
$= \frac{35}{72}$

Comments: Seemed okay for most students.

Most common mistake: Forgetting to multiply by 2.

Question 11 continues on page B8

(c) What is the probability that the player wins at least once in two successive games? **1**

$P(W \text{ at } t) = 1 - P(LL)$ or $P(WW) + P(WL) + P(LW)$

$= 1 - \left(\frac{7}{12}\right)^2$ part (b)

$= \frac{95}{144}$

Comments: Seemed okay for most students.

Most common mistakes:

- subtracting the wrong case (for complementary method)
- forgetting to include 3 cases for the positive cases method.

End of Section B solutions

Section C Solutions — 18 marks

1 Tung Tung sits a Physics test and a Biology test.

(a) Tung Tung's mark in the Physics test is 67. **1**
The mean mark for this test is 41 and the standard deviation is 7.
Calculate the z-score for Tung Tung's mark in this test.

$z = \frac{67 - 41}{7}$

$= \frac{26}{7}$

- Generally well done.
- Some students rounded, answers should always be left in exact form, unless otherwise specified.

(b) In the Biology test, the mean mark is 76 and the standard deviation is 3.5. **2**
Tung Tung's z-score is the same in both the Physics test and the Biology test.
What is his mark in the Biology test?

$\frac{26}{7} = \frac{x - 76}{3.5}$

$13 = x - 76$

$x = 89$

$\therefore$ He got 89 in his Biology test.

Common mistake: $\frac{26}{7} = \frac{76 - x}{3.5}$

z-score: $\frac{x - \mu}{\sigma}$

2 Solve $2\sin\theta + \frac{1}{\sin\theta} = 3$ for $\theta \in [0, 360^\circ]$. **3**

$2\sin^2\theta + 1 = 3\sin\theta$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$\sin\theta = \frac{1}{2}$ $\quad$ $\sin\theta = 1$

$\theta = 30^\circ, 150^\circ$ $\quad$ $\theta = 90^\circ$

$\therefore \theta = 30^\circ, 90^\circ, 150^\circ$

- Students needed to multiply both sides by $\sin\theta$ and turn the equation into a quadratic in terms of $\sin\theta$.

3 (a) Write $6u^2 - 8u + 3$ in the form $6(u - b)^2 + c$, where b and c are real numbers. **2**

$6\left(u^2 - \frac{4}{3}u\right) + 3 = 6\left(u^2 - \frac{4}{3}u + \frac{4}{9} - \frac{4}{9}\right) + 3$

$= 6\left(u - \frac{2}{3}\right)^2 - \frac{24}{9} + 3$

$= 6\left(u - \frac{2}{3}\right)^2 + \frac{1}{3}$

Common mistake:

$6\left(u - \frac{2}{3}\right)^2 - 6\left(\frac{4}{9}\right) + 3 = 6\left(u - \frac{2}{3}\right)^2 + \frac{23}{9}$

(b) Consider $\cos^2 x + \sin^2 x = 1$. Rewrite $9 - 8\sin x - 6\cos^2 x$ without cosines. **2**

$9 - 8\sin x - 6(1 - \sin^2 x) = 9 - 8\sin x - 6 + 6\sin^2 x$

$= 6\sin^2 x - 8\sin x + 3$

Some students didn't fully simplify.

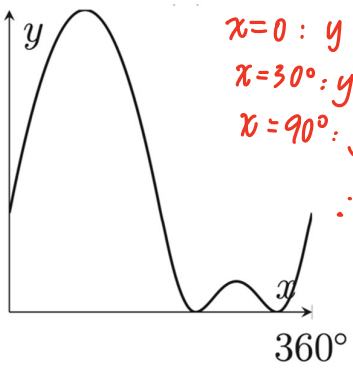
(c) Which of the following graphs shows $y = \log_2(9 - 8\sin x - 6\cos^2 x)$ for $x \in [0°, 360°]$? **1**

Circle the correct response.

$y = \log_2\left[6\left(\sin x - \frac{2}{3}\right)^2 + \frac{1}{3}\right]$

- Poorly done.
- Use a calculator to test values.

(A)



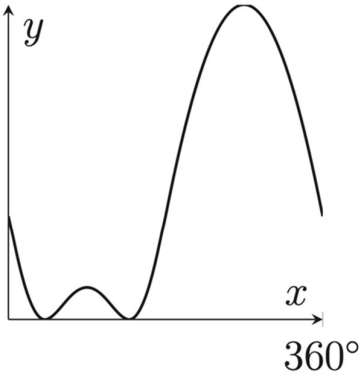
$x = 0$: $y = \log_2 3 > 0$

$x = 30°$: $y = \log_2 \frac{1}{2} = -1 < 0$

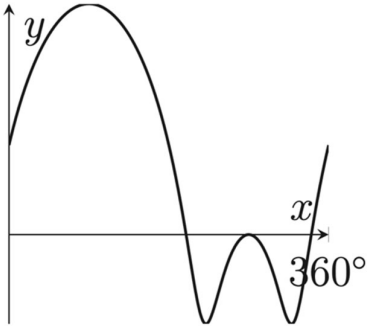
$x = 90°$: $y = \log_2 1 = 0$

∴ Must be D.

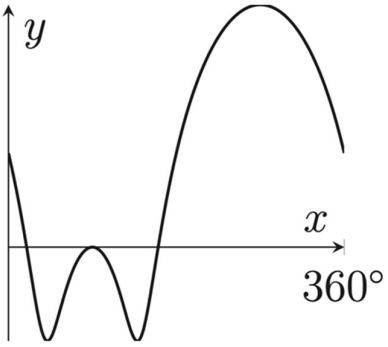
(B)



(C)



(D)



4 Let x be a positive real number, where **3**

$$\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = 10 .$$

Solve for x, give your answer in the form 3^r where $r \in \mathbb{Q}$.

Method 1

$\log_3 x + \frac{\log_3 x}{\log_3 9} + \frac{\log_3 x}{\log_3 27} + \frac{\log_3 x}{\log_3 81} = 10$

$\log_3 x \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right] = 10$ $\quad (\because \log_3 9 = \log_3 3^2 = 2\log_3 3 = 2)$

$\log_3 x \left(\frac{25}{12}\right) = 10$

$\log_3 x = \frac{120}{25}$

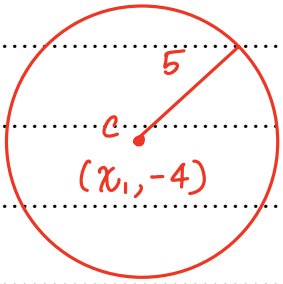
$\therefore x = 3^{\frac{24}{5}}$

Poorly done. Students needed to change the base: $\log_a b = \frac{\log_c b}{\log_c a}$.

See back page for Method 2.

5 Consider the circle $(x - x_1)^2 + (y + 4)^2 = 25$. Find the values of x_1 for which the point $M(4, -1)$ will lie inside the circle. **3**

Method 1



Since M lies inside the circle, $CM < 5$.

$\sqrt{(4 - x_1)^2 + (-1 + 4)^2} < 5$

$(4 - x_1)^2 + 9 < 25$

$(4 - x_1)^2 < 16$

$(x_1 - 4)^2 < 16$

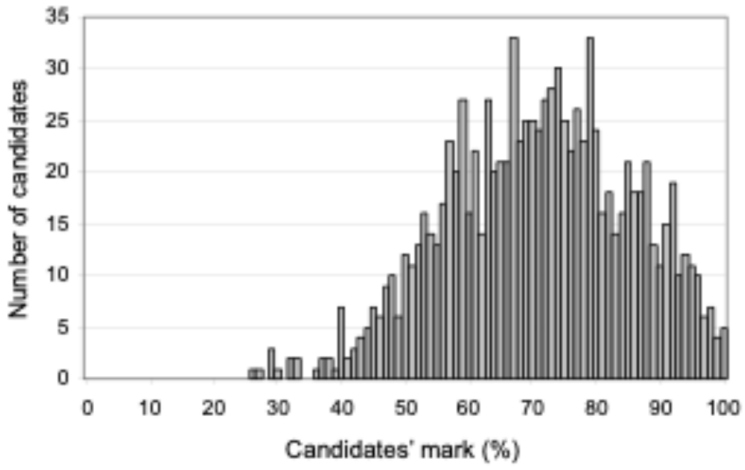
$-4 < x_1 - 4 < 4$

$\therefore 0 < x_1 < 8$

- Poorly done.
- Many students had $x_1 = 0, 8$. Equality occurs when M lies on the circle, not inside.
- See back page for Method 2 and further comments.

6 In an ideal examination, in order to maximise the discrimination between candidates, there should be a range of questions such that: the more difficult questions are answered correctly by only a small number of the best candidates and the easier questions are answered correctly by all but a small number of the weaker candidates. **1**

The graph below shows the distribution of results for a given examination.



A teacher makes a comment "the easier questions were too easy". Do you agree or disagree? **Briefly** justify.

Method 1

Agree, no one got 0% - 20%.

Method 2

Disagree, there are still people getting 20 - 30%.

- Generally well done.
- Students had different interpretations (agree / disagree), hence the mark was allocated for the justification.
- Some students didn't directly address the question and were vague in their response.
- Some students agreed, but their justification suggested otherwise, and vice versa.

End of Section C

Extra Working Page for Section C only

Question 4 (Method 2)

$\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = 10$

$\therefore x = 3^r$

$\log_3 3^r + \log_9 3^r + \log_{27} 3^r + \log_{81} 3^r = 10$

$r\log_3 3 + r\log_9 3 + r\log_{27} 3 + r\log_{81} 3 = 10$

$r\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = 10$

Note: $\log_9 3 = \frac{\log_3 3}{\log_3 9} = \frac{1}{2}$

$r\left(\frac{25}{12}\right) = 10$

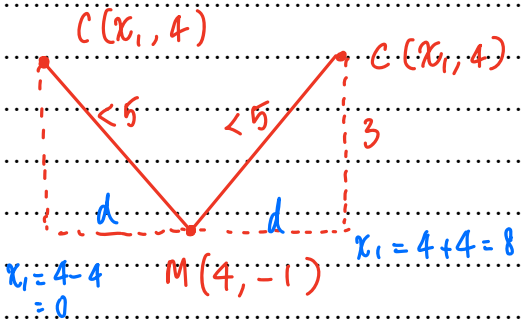
$r = \frac{120}{25} = \frac{24}{5}$

$\therefore x = 3^{\frac{24}{5}}$

Question 5 (Method 2)

CM needs to be < 5 for M to lie within circle.

C could be left/right of M.



$d < \sqrt{5^2 - 3^2} = 4$

$x_1 = 4 \pm 4$

$= 0, 8$

$\therefore 0 < x_1 < 8$

Some students did $x_1 = 4 \pm 5$

$= -1, 9$

These students didn't find the horizontal distance $d = 4$. They could only receive a maximum score of 1.

Those that did 5 ± 4 did not receive any marks.

Section D Solutions — 18 marks

1 Find the values of x such that **2**

$$\frac{10^{4x}\times 2^{3\left(x^2-5x\right)}\times 5^{2(1-2x)}}{20^2}=1$$

Show clear algebraic working.

$$\frac{10^{4x}\times 2^{3\left(x^2-5x\right)}\times 5^{2(1-2x)}}{20^2}=\frac{(2\times 5)^{4x}\times 2^{3x^2-15x}\times 5^{2-4x}}{(4\times 5)^2}$$

$$=\frac{2^{4x}\times 5^{4x}\times 2^{3x^2-15x}\times 5^{2-4x}}{2^4\times 5^2}$$

$$=2^{4x+3x^2-15x-4}\times 5^{4x+2-4x-2}$$

$$=2^{3x^2-11x-4}\times 5^0$$

$$=2^{3x^2-11x-4}$$

$$=1$$

$$=2^0$$

$$\therefore 3x^2-11x-4=0$$

$$(3x+1)(x-4)=0$$

$$x=-\frac{1}{3},\ 4$$

Comments

This question was surprisingly poorly done, enen though Index Laws have been taught since Year 7.

To achive full marks, students needed to correctly simplify the left-hand side expression and solve the quadratic equation.

Students who attempted to use Logarithm Laws were unable to solve this question successfully.

A significant number of students either did not correctly apply the Index Laws or made careless mistakes.

2 Fully factorise: **3**

$$\left(1+\frac{y^2+z^2-x^2}{2yz}\right)\div\left(1-\frac{x^2+y^2-z^2}{2xy}\right)$$

$$\left(1+\frac{y^2+z^2-x^2}{2yz}\right)\div\left(1-\frac{x^2+y^2-z^2}{2xy}\right)=\left(\frac{2yz+y^2+z^2-x^2}{2yz}\right)\div\left(\frac{2xy-x^2-y^2+z^2}{2xy}\right)$$

$$=\left(\frac{y^2+2yz+z^2-x^2}{2yz}\right)\div\left(\frac{z^2-\left(x^2-2xy+y^2\right)}{2xy}\right)$$

$$=\frac{(y+z)^2-x^2}{2yz}\times\frac{2xy}{z^2-(x-y)^2}$$

$$=\frac{(y+z+x)(y+z-x)}{z}\times\frac{x}{(z+x-y)(z-x+y)}$$

$$=\frac{x(x+y+z)(y+z-x)}{z(x-y+z)(y+z-x)}$$

$$=\frac{x(x+y+z)}{z(x-y+z)}$$

Comments

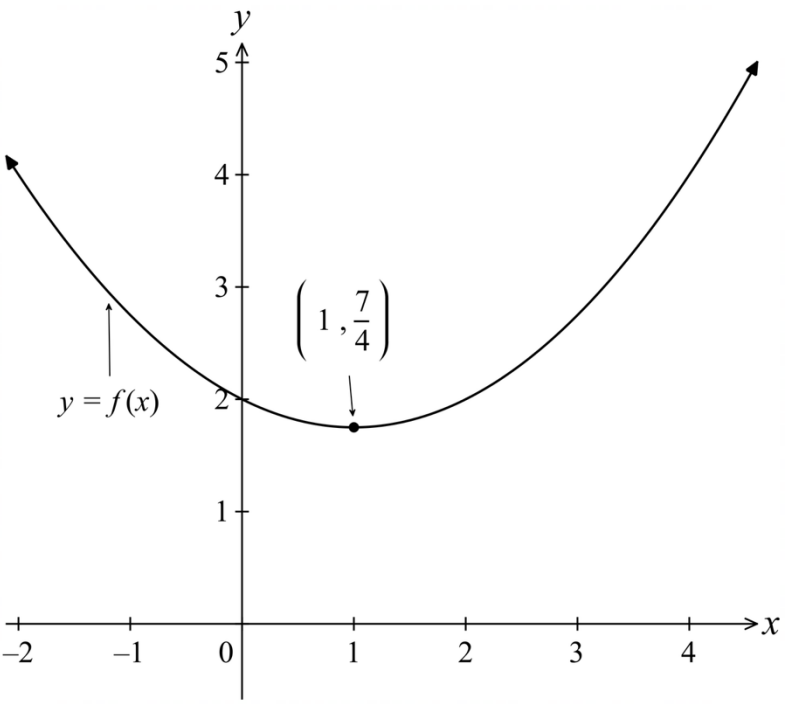
Poorly done

Only a few students achieved full marks on this question.

Students were unable to:

- *manipulate simple expression involving negative signs (see the first line)*
- *recognise perfect squares (see the second and third lines)*
- *recognise the difference of two squares (see the third and fourth lines)*
- *cancel like terms in division*

3 The graph of $f(x) = \frac{1}{4}[(x-1)^2 + 7]$ is given below.



(a) State the largest domain containing $x = 3$ for an inverse function to exist. **1**

For an inverse function to exist, the function must be one-to-one.

$\because x = 3$ needs to be included

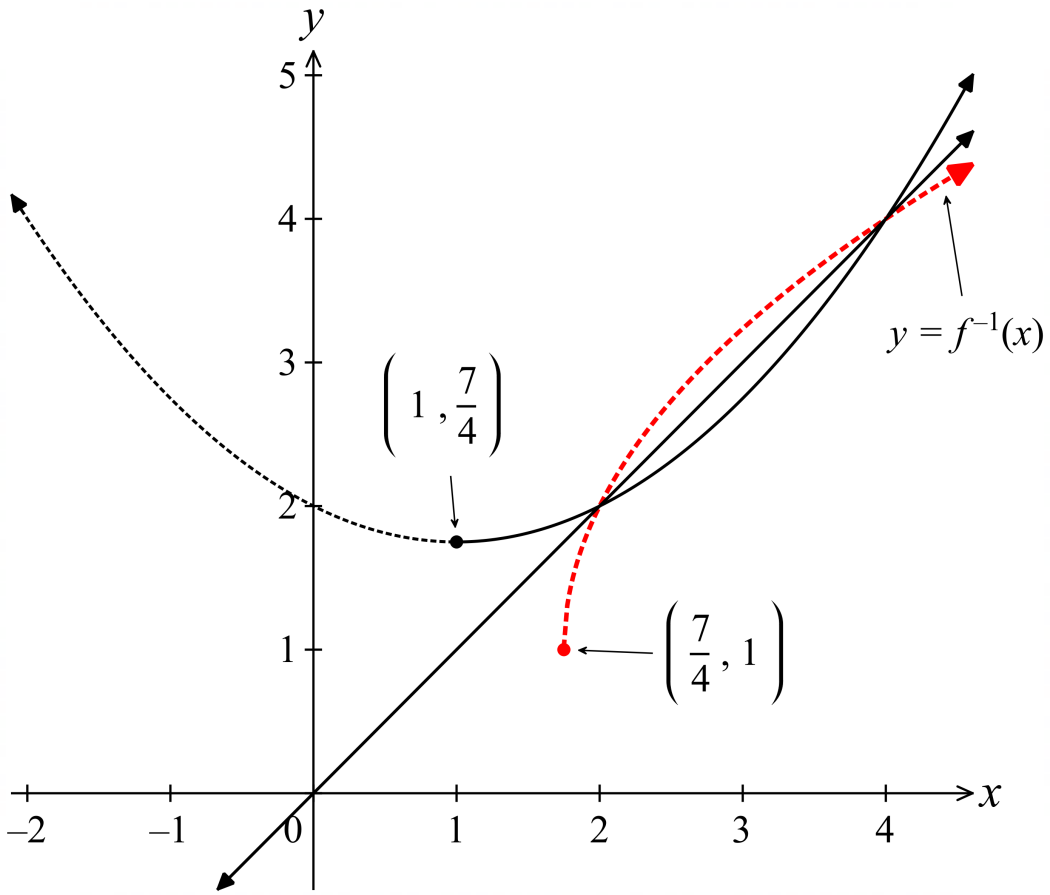
$\therefore [1, \infty)$ is the largest domain

Comments

Generally well done.
No half marks
Common mistakes: *all real x,* $x \leq 1$

Question 3 is continued on page D4

3 (b) Sketch the graph of $y = f^{-1}(x)$ on the template below. **1**



Comments

Generally well done.
This question was very generously marked:
1 mark was awarded if the inverse function graph was the reflection of the graph of f(x) across the line y = x.

For the correct answer, please refer to the answer above.

(c) State the domain of the inverse function. **1**

$\left[\frac{7}{4}, \infty\right)$

Comments

Well done.
No half marks

Question 3 is continued on page D5

3 (d) Find $f^{-1}(f(-1))$. **2**

Method 1:

Since $x = 1$ is the axis of symmetry of the parabola

$$\therefore f(-1) = f\left(1+\left(1-(-1)\right)\right) = f(3)$$

$$f^{-1}\left(f(-1)\right) = f^{-1}\left(f(3)\right) = 3 \qquad \left[f^{-1}\left(f(x)\right) = x\right]$$

Method 2:

$$x = \frac{1}{4}\left[(y-1)^2 + 7\right]$$

$$y = \pm\sqrt{4x-7} + 1$$

The range of the inverse function is $[1, \infty)$

$$\therefore y = \sqrt{4x-7} + 1$$

$$f(-1) = \frac{1}{4}\left[(-1-1)^2 + 7\right] = \frac{11}{4}$$

$$f^{-1}\left(f(-1)\right) = f^{-1}\left(\frac{11}{4}\right)$$

$$= \sqrt{4\times\frac{11}{4} - 7} + 1$$

$$= 3$$

Comments

Poorly done.

Students who attempted to use the Method 1 did not recognise that –1 is not in the domain.
Students who used the Method 2 were unable to correctly find the expression of the inverse function

Question 3 is continued on page D6

3 (e) Find the coordinates of the points of intersection of $y = f(x)$ and $y = f^{-1}(x)$, if any. Show clear working. **2**

The points of the intersrions of $y = f(x)$ and $y = f^{-1}(x)$ must be on the line $y = x$.

$$\therefore x = \frac{1}{4}\left[(x-1)^2 + 7\right]$$

$$4x = x^2 - 2x + 1 + 7$$

$$\therefore x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$$\therefore x = 2, 4$$

$$\therefore y = 2, 4$$

Therefore, the coordinates of the points of intersection are (2, 2) and (4, 4).

Comments

Generally well done.

To achieve full marks, students needed to show their full working.

Students who set $f(x) = f^{-1}(x)$ struggled to simplify and solve the equation $\frac{1}{4}\left[(x-1)^2 + 7\right] = \sqrt{4x-7} + 1$

4 Solve for x and y, where both x and y belongs to the first quadrant: **3**

$$2\sin(x+y)=\sqrt{3}$$
$$3\tan(x-y)=\sqrt{3}$$

$\because$ Both x and y belong to the first quadrant

$\therefore x+y\in(0^\circ,180^\circ),\quad x-y\in(-90^\circ,90^\circ)$

$\sin(x+y)=\dfrac{\sqrt{3}}{2}\Rightarrow x+y=60^\circ \text{ or } 120^\circ$

$\tan(x-y)=\dfrac{\sqrt{3}}{3}\Rightarrow x-y=30^\circ$

$x+y=60^\circ$
$x-y=30^\circ$

$\therefore x=45^\circ,\ y=15^\circ$

$x+y=120^\circ$
$x-y=30^\circ$

$\therefore x=75^\circ,\ y=45^\circ$

Comments

Generally well done.

Most students only provided one set of answers as they did not include $x + y = 120^\circ$

5 Given x satisfies $x^2 - 17x + 71 = 0$, without solving for x, find the value of: **3**

$$\frac{(x-9)^2}{1+(x-9)^4}.$$

Note: Answers that solve for x to get the result will **NOT** receive any marks.

Method 1:

$$\begin{aligned}(x-9)^2 &= x^2-18x+81\\ &= x^2-17x-x+71+10\\ &= x^2-17x+71+10-x\\ &= 10-x\end{aligned}$$

$$\begin{aligned}\frac{(x-9)^2}{1+(x-9)^4} &= \frac{10-x}{1+(10-x)^2}\\ &= \frac{10-x}{1+100-20x+x^2}\\ &= \frac{10-x}{x^2-20x+101}\\ &= \frac{10-x}{x^2-17x-3x+71+30}\\ &= \frac{10-x}{30-3x}\\ &= \frac{1}{3}\end{aligned}$$

Method 2:

Let $t = x-9$

$\therefore x = t+9$

$\because x^2-17x+71=0$

$\therefore (t+9)^2-17(t+9)+71=0$

Expand brackets and simplify it: $t^2+t-1=0$

$\therefore t^2 = 1-t$

$$\begin{aligned}\frac{(x-9)^2}{1+(x-9)^4} &= \frac{t^2}{1+t^4}\\ &= \frac{1-t}{1+(t^2)^2}\\ &= \frac{1-t}{1+(1-t)^2}\\ &= \frac{1-t}{1+1-2t+t^2}\\ &= \frac{1-t}{2-2t+1-t}\\ &= \frac{1-t}{3(1-t)}\\ &= \frac{1}{3}\end{aligned}$$

Question 5 is continued on page D9

Method 3:

$$x^2-17x+71+1=1$$
$$(x-8)(x-9)=1$$
$$\therefore x-8=\frac{1}{x-9}$$
$$\therefore x-9=\frac{1}{x-9}-1$$
$$x-9-\frac{1}{x-9}=-1$$
$$\left(x-9-\frac{1}{x-9}\right)^2=1$$
$$(x-9)^2-2+\frac{1}{(x-9)^2}=1$$
$$\frac{(x-9)^4+1}{(x-9)^2}=3$$

$$\therefore \frac{(x-9)^2}{1+(x-9)^4}=\frac{1}{3}$$

Comments

Extremely poorly done.

A significant number of students did not attempt this question.

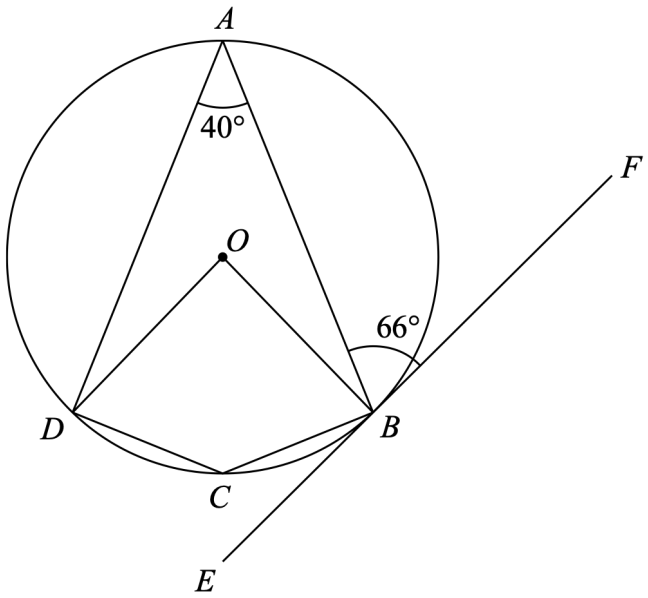
Method 1 was the most popular approach.
However, many students were unable to continue manipulating the expression $x^2-20x+101$.

Students who attempted to use the substitution method were also unsuccessful in solving this problem.

End of Section D solutions

Section E Solution — 14 marks

1 A, B, C and D are points on a circle with centre O.
EBF is a tangent to the circle at B.



(a) Find $\angle DCB$, giving reasons. **2**

$\angle DCB = 180° - \angle DAB$ (opp. $\angle$s of cyclic quad)
$= 180° - 40°$
$= 140°$

1 mark for the correct answer
1 mark for providing correct geometric reasoning

Done very well.
A common error was students treating $DOBC$ as a cyclic quadrilateral when it clearly isn't.

Question 1 continues on E2

1 (b) Find $\angle ADO$, giving reasons. **3**

Method 1.

$\angle OBF = 90°$	(tangent to radius)
$\angle OBA = 24°$	(adj. $\angle$s)
$\angle OAB = 24°$	(base $\angle$s in isos. Δ)
$\angle OAD = 16°$	(adj. $\angle$s)
$\angle ODA = 16°$	(base $\angle$s in isos. Δ)

Method 2.

$\angle ADB = 66°$	(alt. seg. thm)
$\angle DOB = 80°$	($\angle$ at centre is twice the $\angle$ at circumference)
$\angle ODB = 50°$	(base $\angle$s in isos. Δ, $\angle$ sum on Δ)
$\angle ADO = 16°$	(adj. $\angle$s)

Method 3.

$\angle OBF = 90°$	(tangent to radius)
$\angle OBA = 24°$	(adj. $\angle$s)
$r\angle DOB = 280°$	($\angle$ at centre is twice the $\angle$ at circumference)
$\angle ODA = 16°$	($\angle$sum of quad. $ODAB$)

1 mark for the correct answer
1 mark for providing correct geometric reasoning
1 mark for showing all necessary steps of work

Note: annotations on the diagram will be noted as working

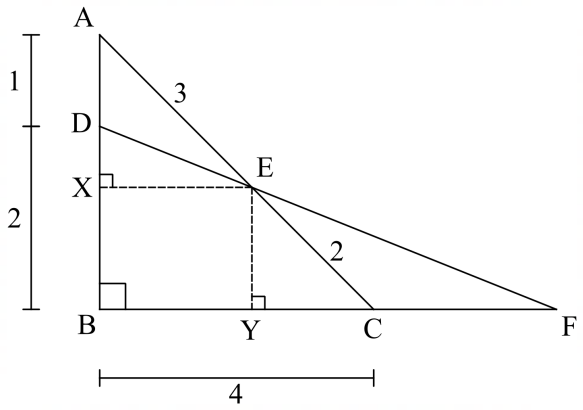
Generally done well.
Method 3 was the most popular method amongst students.

The common errors arose from incorrect assumptions, either assuming that AC is a diameter, or assuming that AO is the angle bisector. Neither of these are necessarily true.

2 Tree Gong draws ΔABC with $AB = 3$ cm, $BC = 4$ cm, and $\angle ABC = 90°$. **3**

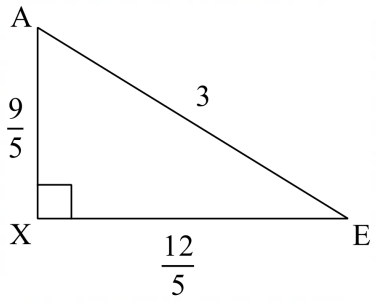
Polly Gone then draws ΔDBF on top of ΔABC so that D lies on AB, F lies on the extension of BC, $DB = 2$ cm, and sides AC and DF meet at E.

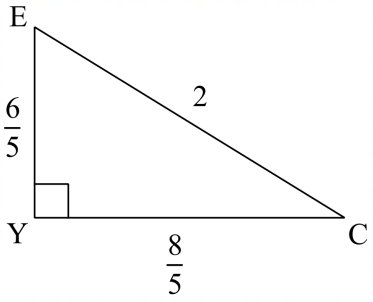
If $AE = 3$ cm and $EC = 2$ cm, determine the length of CF.



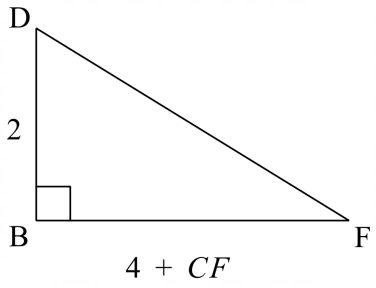
In addition to the construction of line DEF on triangle ABC, we also construct EX and EY parallel to the two legs of the triangle.

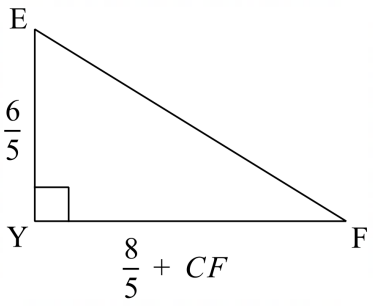
Using corresponding angles on parallel lines, we can use AA similarity to find two sets of similar triangles, $\Delta AXE \sim \Delta EYC \sim \Delta ABC$ and $\Delta DBF \sim \Delta EYF$.
The first set of similar triangles are in the ratio 3:2:5, which gives us the following side lengths.





Using these measurements, we can now compare the second set of similar triangles.





Question 2 continues on E4

2 Using corresponding ratios in similar triangles, we then construct an equation in terms of CF.

$$\frac{\left(\frac{6}{5}\right)}{2} = \frac{\left(\frac{8}{5}+CF\right)}{(4+CF)} \qquad \text{(corresp. ratios in sim. } \Delta\text{s)}$$

$$\left(\frac{6}{5}\right)(4+CF) = 2\left(\frac{8}{5}+CF\right)$$

$$\frac{24}{5}+\frac{6}{5}CF = \frac{16}{5}+2CF$$

$$24+6CF = 16+10CF$$

$$8 = 4CF$$

$$CF = 2$$

1 mark for drawing a correct diagram
1 mark for using similarity to find relevant lengths
1 mark for the correct answer

Generally done poorly.
Most students were able to secure a mark for drawing a valid diagram.

Some students chose to position point F closer to B on the extension of BC which is technically a valid interpretation of the question but is physically impossible if the diagram were drawn to scale.

Some students attempted to use ADE and ECF as similar triangles, but they are not.

Many students attempted to solve this problem using either trigonometry or Pythagoras' theorem without any success.
Very fortunately for the marker, these approaches do not work without making the additional constructions noted in the solution above.

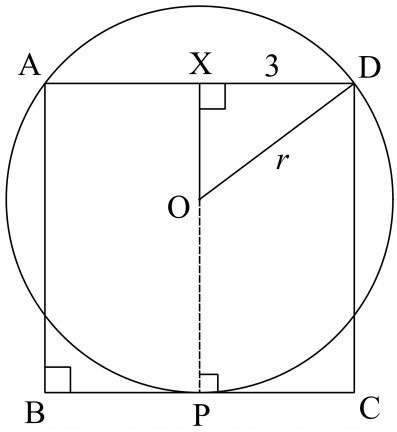
One student was able to brute force the problem by translating the diagram onto a coordinate plane and solving using coordinate geometry. Elegant? No. But effective? Yes.

3 Sina drew square $ABCD$ with side length 6 cm on a piece of paper and passed the paper to Theo. **3**

Theo drew a circle on top of the square so that the circle passes through A and D, and the circle is tangent to side BC at point P.

Determine the radius of the circle.

Method 1. (Pythagoras' Theorem)



In addition to the construction of the appropriate circle with respect to square $ABCD$, we also construct the radius OD and let X be the point on AD such that ODX is a right triangle. Note that X is the midpoint of AD, thus $XD = 3$. (proof in italics, not required)

Technically, to prove that $XD = 3$, we need to recognise that $OP \perp BC$ (tangent to radius), then construct X as the extension of OP to AD.
Since XOP is collinear by construction, we get that $OX \perp AD$ (perp. to parallel lines).
Therefore, OX bisects AD (perpendicular radius to a chord) and thus $XD = 3$.

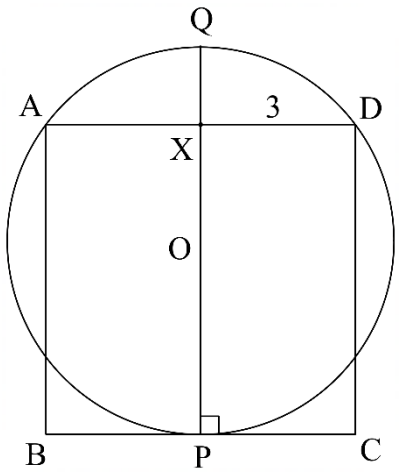
We also note that $OX = 6 - r$ (since OP is a radius).

In ΔODX we can use relate the side lengths using Pythagoras' theorem and solve for r.

$$OX^2 + XD^2 = OD^2 \qquad \text{(pythag. thm)}$$
$$(6-r)^2 + 3^2 = r^2$$
$$36 - 12r + r^2 + 9 = r^2$$
$$45 = 12r$$
$$r = \frac{15}{4}$$

Question 3 continues on E6

3 **Method 2. (Intersecting Chords)**

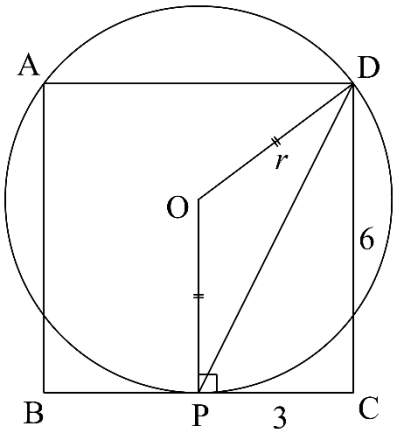


Construct the diameter from P, meeting AD at X and the circle at Q.
By the same reasoning as in Method 1, we know that PQ is a perpendicular bisector of AD.

If the radius is r, then $QX = 2r - 6$.
Therefore:

$$PX \cdot XQ = AX \cdot XD \qquad \text{(intersecting chords)}$$
$$6(2r-6) = 3 \times 3$$
$$12r - 36 = 9$$
$$12r = 45$$
$$r = \frac{15}{4}$$

Method 3. (Trigonometry)



Construct the radii OD and OP, and the chord PD.

$$PD = \sqrt{6^2 + 3^2} = \sqrt{45} \qquad \text{(pythag.)}$$
$$\angle OPD = \angle PDC = \tan^{-1}\left(\frac{1}{2}\right) \qquad \text{(alt. } \angle\text{s on parallel lines)}$$
$$\angle POD = 180° - 2\tan^{-1}\left(\frac{1}{2}\right) \qquad (\angle \text{ sum of isos. } \Delta)$$

With this, we can use cosine rule to solve for the radius:

$$\sqrt{45}^2 = r^2 + r^2 - 2r^2 \cos\left(180° - 2\tan^{-1}\left(\frac{1}{2}\right)\right)$$
$$45 = r^2\left(2 - 2\cos\left(180° - 2\tan^{-1}\left(\frac{1}{2}\right)\right)\right)$$
$$r = \sqrt{\frac{45}{2 - 2\cos\left(180° - 2\tan^{-1}\left(\frac{1}{2}\right)\right)}}$$
$$= \frac{15}{4}$$

Question 3 continues on E7

3 **1 mark** for drawing a correct diagram (recognising that OX bisects AD)
1 mark for using Pythagoras' theorem
1 mark for the correct answer

Poorly done.
Many students failed to draw a diagram that was accurate and contained the necessary information.

The most common answer was $r = 3$ which is obviously wrong (since AD cannot be a diameter).
The other most common mistake was assuming that O is also the centre of the square (it is not).

4 The first term in a sequence is 24. **3**
We can determine the next terms in the sequence as follows:

- If a term is even, then divide it by 2 to get the next term.
- If a term is odd, then multiply it by 3 and add 1 to get the next term.

By doing this, we can determine that the first three terms in the sequence are 24, 12, and 6.
Shweta writes the first n terms in this sequence and notices that the sum of these terms is a four-digit number.

How many different possible values of n are there?

First, continue the sequence to find where the terms start looping.

24, 12, 6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, etc.

We can see that the sequence starts looping through 4-2-1 which has a length of three terms and a sum of 7.
The sum of the first eight terms (before the sequence starts looping) is 84.

To determine the possible numbers of terms such that the sequence has a four-digit sum, we need to determine the first time the sequence gets a four-digit sum, the first time for a five-digit sum, and the number of loops that can fit in between them as well.

$(1000-84)\div 7 = 130 \text{ r } 6$, so there are 130 loops of 4-2-1 to reach a sum of 994.
We continue the sequence sum: 994, 998, 1000, … so the loops from 1000 onwards are 1-4-2.

$(9999-1000)\div 7 = 1285 \text{ r } 4$, so there are 1285 loops of 1-4-2 to reach 9995.
Again, we continue the sequence sum: 9995, 9996, 10000, … which means that we can only fit one more term into the sequence (after our loops) before reaching a five-digit sum.

Therefore, the times when the sequence has a four-digit sum are:

- Once, when the sequence has a sum of 1000
- Once, when the sequence has a sum of 9996
- And three times for each of the 1285 loops in between.

Thus, the total number of different possible values for n is $1285\times 3+2=3857$.

1 mark for continuing the sequence to find the loop
1 mark for determining when sequence sum first reaches four digits
1 mark for finding the number of loops that give a four-digit sum

Done poorly. Generally not attempted.
Most students that attempted this problem were able to find the loop for 1 mark.
A disturbing number of students were unable to identify 6 and 8 as even numbers.

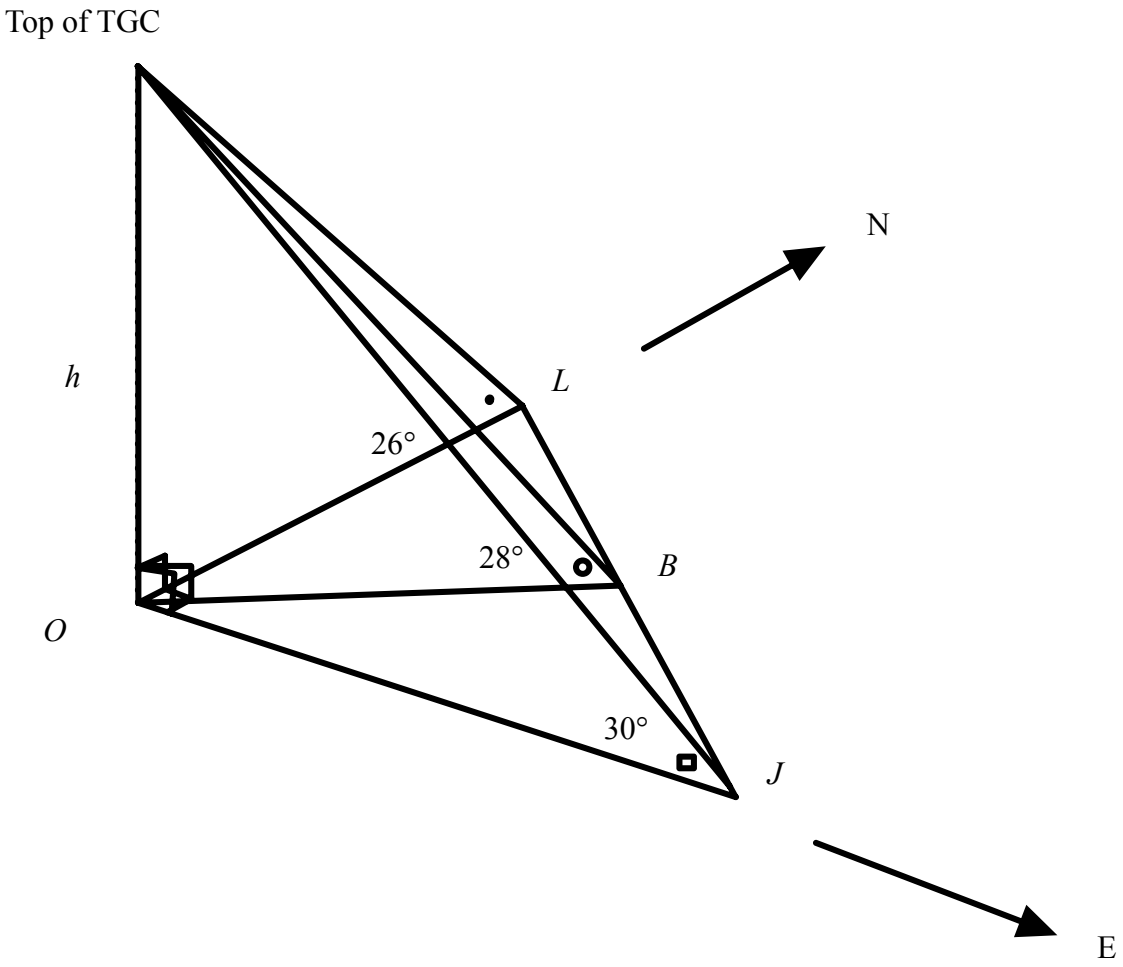
A handful of students were able to solve the problem in its entirety, which was nice to see.

End of Part E solutions

Section F Solutions — 13 marks

1 The Governor's Centre (TGC) is a tall building on flat ground in SBHS. Three students Luke, Billy and Jack are observing TGC from the ground. Luke is due North of TGC, Jack is due East of TGC and Billy is on the line of sight from Luke to Jack. The angles of elevation to the top of TGC from Luke, Billy and Jack are 26°, 28° and 30° respectively.

(a) Draw a diagram, clearly labelling all the information. **2**



Comment: Pupils needed to show the angles, apply all right angles, indicate directions and show Billy was on the line between Luke and Jake.

(b) Find the bearing of Billy from TGC. **3**

$$\tan 64^\circ = \frac{OL}{h}; \tan 62^\circ = \frac{OB}{h}; \tan 60^\circ = \frac{OJ}{h}$$

$$OL = h\tan 64^\circ, OB = h\tan 62^\circ; OJ = h\tan 60^\circ$$

$$\begin{aligned}\angle OLJ &= \tan^{-1}\left(\frac{OJ}{OL}\right)\\ &= \tan^{-1}\left(\frac{h\tan 60^\circ}{h\tan 64^\circ}\right)\\ &= \tan^{-1}\left(\frac{\sqrt{3}}{\tan 64^\circ}\right)\\ &\approx 40^\circ 11'\end{aligned}$$

$$\frac{\sin\angle OBL}{OL} = \frac{\sin\angle OLJ}{OB}$$

$$\begin{aligned}\angle OBL &= \sin^{-1}\left(\sin\angle OLJ \times \frac{OL}{OB}\right)\\ &= \sin^{-1}\left(\sin\left(\tan^{-1}\left(\frac{\sqrt{3}}{\tan 64^\circ}\right)\right) \times \frac{\tan 64^\circ}{\tan 62^\circ}\right)\\ &\approx 44^\circ 43' \quad \text{or} \quad 180^\circ - 44^\circ 13' = 135^\circ 17'\end{aligned}$$

$$\angle BOL = 180^\circ - (\angle OBL + \angle OLB) \quad \text{or} \quad \angle OBL - \angle OLB$$

$$\angle BOL = \begin{cases} 180^\circ - \left(\tan^{-1}\left(\dfrac{h\tan 64^\circ}{h\tan 60^\circ}\right) + \sin^{-1}\left(\sin\left(\tan^{-1}\left(\dfrac{\tan 64^\circ}{\tan 60^\circ}\right)\right) \times \dfrac{\tan 60^\circ}{\tan 62^\circ}\right)\right) \\ \sin^{-1}\left(\sin\left(\tan^{-1}\left(\dfrac{\sqrt{3}}{\tan 64^\circ}\right)\right) \times \dfrac{\tan 64^\circ}{\tan 62^\circ}\right) - \tan^{-1}\left(\dfrac{\sqrt{3}}{\tan 64^\circ}\right)\end{cases}$$

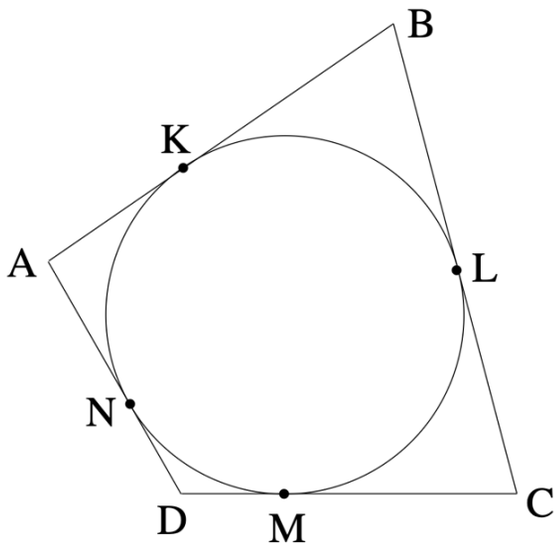
$$\approx 95^\circ 6' \quad \text{or} \quad 4^\circ 44'$$

Billy is between Luke & Jack. Therefore, the angle must be the smaller of the two.

Billy is N4°44'E of TGC.

Comment: No pupil got any further than the first sine rule solution and that student did not consider the ambiguous case. This was very poorly done.

2



(a) *ABCD* is a quadrilateral with all angles less than 180°. The sides of *ABCD* are tangent to the circle at points *K*, *L*, *M*, and *N*. Show that: **2**

$$AB + CD = AD + BC$$

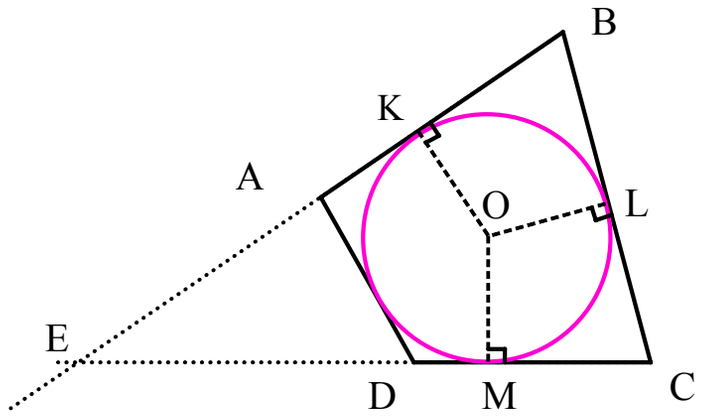
$$\left.\begin{array}{l} NA = AK \\ BL = KB \\ LC = CM \\ \underline{DN = MD} \end{array}\right\} + \quad \text{tangents from an external point.}$$

$NA + BL + LC + DN = AK + KB + CM + MD$
$AN + ND + BL + LC = AB + CD$
Therefore: $AD + BC = AB + CD$

Comment: Most pupils were able to do this. Some tried other methods, but these mostly were unsuccessful.

Question 2 continues on page F4

(b) Let X be the point of intersection of the angle bisectors of $\angle ABC$ and $\angle BCD$. **3**
Prove that X is the centre of a circle to which AB, BC and CD are tangent.



A circle may be inscribed in any triangle. Extend BA & CD such that they meet at point E.

Let the inscribed circle of triangle BCE touch BC at L, CE at M, EB at K and O be the centre.

$BK = BL$ & $CL = CM$. (Tangents from the same point are equal)

$BLOK$ & $CMOL$ are cyclic quads since K, L & M are all tangents (converse of opposite angles in a cyclic quad).

OB & OC are the diameters of the cyclic quads respectively (converse of angle in a semicircle).

Then triangles BKO & BLO are congruent (SSS).

Also, angles KBO & LBO are equal (corresponding angles in congruent triangles).

Therefore, OB bisects the angle at B.

Similarly, triangles CLO & CMO are congruent (SSS).

Also, angles LCO & MCO are equal (corresponding angles in congruent triangles).

Therefore, OC bisects the angle at C.

Then X lies on the line that has both points O and B & the line with points O and C.

X must be O, which is the centre of the circle.

Comment: Solutions to this were full of assumptions or statements that needed to be proven. For example, some pupils stated that the bisector of tangents from an external point goes through the centre of the circle. This is not a theorem and there is a decent amount of work to prove it. This was typical of most answers.

Question 2 continues on page F5

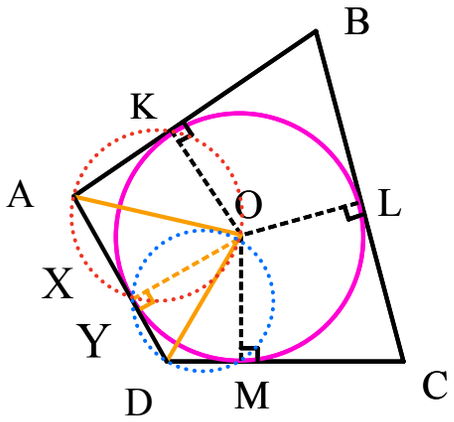
(c) The information below may be useful in solving the following problem.

If the points P, Q, R lie on a straight line, in that order and T is any point not on the line, then you may assume that $QT - QP > RT - RP$. [Do NOT prove].

$ABCD$ is a quadrilateral, with all angles less than 180°. Given that **3**

$$AB + CD = AD + BC,$$

show that there exists a circle to which all sides of $ABCD$ are tangent.



It was shown it part b that a circle may be inscribed in quad $ABCD$ such that it is tangent to AB, BC & CD.

Recall: $AB + CD = BC + DA$

$$AK + KB + CM + MD = BL + LC + AD$$
$$\Rightarrow AK + MD = AD$$

ΔAKO & ΔDMO are each in a semicircle (angle in a semicircle)

Let X be the point at which the circle which circumscribes AKO meets the circle inscribed in $ABCD$.

Let Y be the point at which the circle which circumscribes DMO meets the circle inscribed in $ABCD$.

AX & DY must be tangent to the inscribed circle of $ABCD$ and $AX = AK$ & $DY = DM$ (converse of angle in a semicircle & targets from an external point).

$$AK + DM = AD$$

Therefore a circle may be inscribed in any quadrilateral $ABCD$, where $AB + CD = BC + DA$.

Comment: The point of this question was to prove that AD is tangent to the circle that has tangents on AB, BC and CD, where $AB + CD = BC + AD$. Most students assumed that the circle was tangential to AD at some point N. This is assuming the result and not a valid proof.

This question was deleted after the fact, since the diagram given led most students to believe that they could assume that AD is tangent to the circle at N. Those that did assume this, did NOT read the question properly.

End of solutions