SYDNEY BOYS HIGH SCHOOL

2024

YEAR 10 STAGE 5
TERM 4
YEARLY EXAM

Mathematics

Time Allowed: 90 minutes
Reading Time: 5 minutes

General Instructions:

- All questions may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using a black non-erasable pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet. Indicate that you have done this. Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Any answer that totally relies on calculator technology may not necessarily be awarded full marks.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: RCW

Name: _____

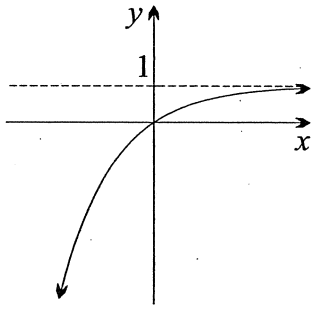
Class	Teacher	
10Ma**A**	A Wang	
10Ma**B**	M Zheng	
10Ma**C**	J Tao	
10Ma**P**	W Beare	
10Ma**L**	R Wang	
10Ma**U**	A Ward & J James	
10Ma**S**	J Chan	

Question	Marks
A	/18
B	/15
C	/15
D	/15
E	/12
F	/10
Total	**/85**

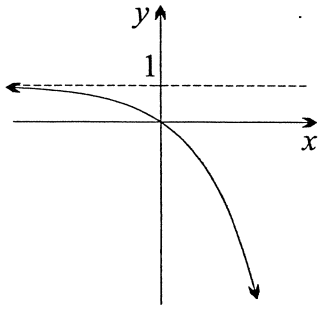
Section A

1) Which of the following graphs could have the equation $f(x) = 1 - 2^{-x}$? **1**

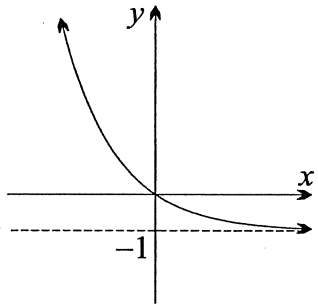
A)



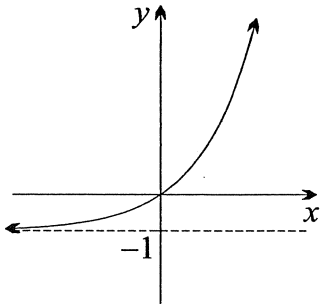
B)



C)



D)



2) Solve $x(3 - x) = 0$ **1**

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3) Evaluate $\log_3 81$. **1**

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4) Solve $\sin\theta = -0.5$ for $0° \le \theta \le 360°$. **2**

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5) What is the angle that the line $y = -x$ makes with the positive direction of the x-axis? **1**

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6) Let M denote a length in metres
N denote a length in inches
F denote the length in feet

The conversion from metres to feet is given by $F = 3.28084M$.
The conversion from feet to inches is given by $N = 12F$
Let $f(x) = 3.28084x$ and $g(x) = 12x$.

i) Explain what $f^{-1}(x)$ represents for the units above. **1**

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ii) Explain what $g(f(x))$ represents for the units above. **1**

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iii) Find the length in inches of a rope of 3.5 m. **1**
Give your answer correct to 5 significant figures.

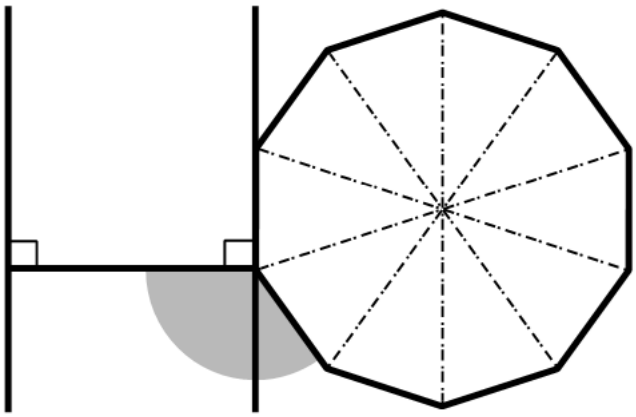
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7) A logo consists of an H shape and a regular decagon. **2**

The diagram below shows the logo.



State the size of the shaded angle. Geometric reasons are not needed, but working is essential.

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8) Given that $\log_a b = 1.5$ and $\log_a c = 0.3$, find the value of

i) $\log_a \frac{b}{c}$. **1**

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ii) $\log_a (cb)^5$ **2**

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9) A student tosses a fair coin three times.

i) What is the probability of three heads? **1**

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ii) What is the probability that the student observes exactly one head? **1**

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iii) Given that the student has observed *at least* one head, what is the probability that the student observes at least two heads? **2**

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End of Section A

Extra Working Page for Part A only

Part B

Name: _____

Class	Teacher	
10Ma**A**	A Wang	
10Ma**B**	M Zheng	
10Ma**C**	J Tao	
10Ma**P**	W Beare	
10Ma**L**	R Wang	
10Ma**U**	A Ward & J James	
10Ma**S**	J Chan	

Mark	**/15**

1) Let $g(x) = 13 - 5x^2$. If a is a constant, which of the following is $g(1-3a)$? **1**

A) $8 - 45a^2$

B) $8 + 45a^2$

C) $8 - 30a + 45a^2$

D) $8 + 30a - 45a^2$

2) Make y the subject for $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$ **2**

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3) The current population of planet Earth is 8.2 billion in 2024. If the projected population increase is 0.87% p.a., in which year will the population first reach 10 billion? **2**

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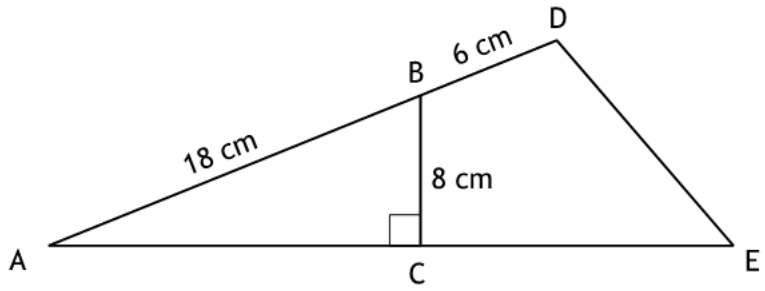
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4) In the diagram below, $AB = 18$ cm, $BD = 6$ cm, $BC = 8$ cm, and the area of triangle $ADE = 160$ cm^2. **3**



Calculate the length of AE, to 2 decimal places.

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5) Janice and Paul did a study on feelings of stress and life satisfaction. Participants completed a measure on how stressed they were feeling (on a 1 to 30 scale) and a measure of how satisfied they felt with their lives (measured on a 1 to 10 scale). The table below indicates the participants' scores.

Participant #	Stress score (X)	Life Satisfaction (Y)
1	11	7
2	25	1
3	19	4
4	7	9
5	23	2
6	6	8
7	11	8
8	22	3
9	25	3
10	10	6

Using this data, answer the following questions:

i) Calculate the correlation (r) between Stress and Life Satisfaction. **1**

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ii) The least-squares regression line can be written in the form $Y = mX + b$. **1**
What is the value of m?

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iii) Write a brief interpretation of this correlation, including the strength, direction, and an explanation of the effect. **1**

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iv) Can you say that being more stressed causes a lower level of life satisfaction? **1**
Why or why not?

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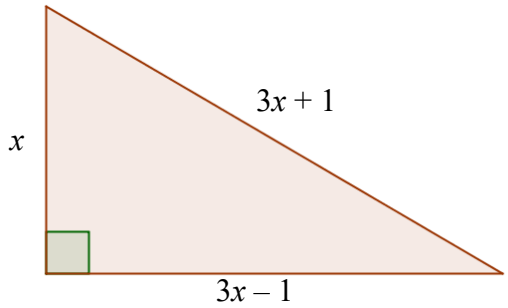
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6) A right-angled triangle is drawn so that the hypotenuse is 3 times the shortest side plus 1 cm, and the other side is 3 times the shortest side minus 1 cm. **3**

If the shortest side has length x, find all valid solutions for x, giving explicit mathematical reasoning.



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End of Section B

Extra Working Page for Part B only

Part C

Name: _____

Class	Teacher	
10Ma**A**	A Wang	
10Ma**B**	M Zheng	
10Ma**C**	J Tao	
10Ma**P**	W Beare	
10Ma**L**	R Wang	
10Ma**U**	A Ward & J James	
10Ma**S**	J Chan	

Mark	**/15**

1) By considering the discriminant, which of the following has 2 distinct real and rational x-intercepts? **1**

A) $y = -x^2 - 3x + 5$

B) $y = 4x^2 - x - 5$

C) $y = -x^2 - 4$

D) $y = 2x^2$

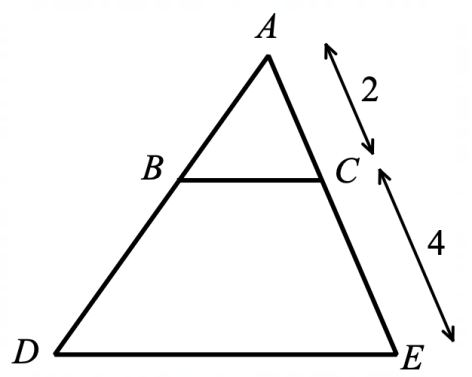
2) The table below shows the scores (in marks) and the corresponding z-scores of 3 students in an examination. **1**

Score (marks)	46	x	86
z-score	–3	1	2

Find x.

A) 64
B) 66
C) 70
D) 78

3) In the diagram below, $BC \parallel DE$, $AC = 2$, $CE = 4$, and $BC = 6$. **1**



Which of the following is true?

A) $DE = 12$

B) $BC : DE = 1 : 2$

C) Area ΔABC : Area $\Delta ADE = 1 : 9$

D) Perimeter ΔABC : Perimeter $\Delta ADE = 1 : 2$

4) Factorise $3x^3 - \frac{24}{125y^6}$ **3**

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5) A ball contains 5000 cm^3 of air. **3**
More air is pumped into the ball at a constant rate of 60 cm^3/sec for 1 minute.
The ball can maintain its spherical shape while it is being pumped up.



After the minute, the pumping stops.

What is the surface area of the pumped up ball, to the nearest square centimetre?

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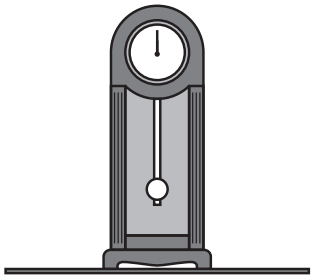
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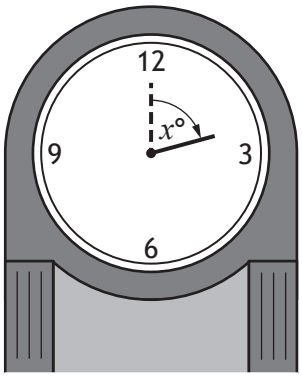
6) Elsa has a grandfather clock in her house. **3**



The height of the tip of the hour hand above the floor, in centimetres, is given by

$$h = 20\cos x° + 147,$$

where $x°$ is the angle the hour hand has rotated through since 12 o'clock.



Calculate the first 2 values of x for which the tip of the hour hand is 150 cm above the floor.

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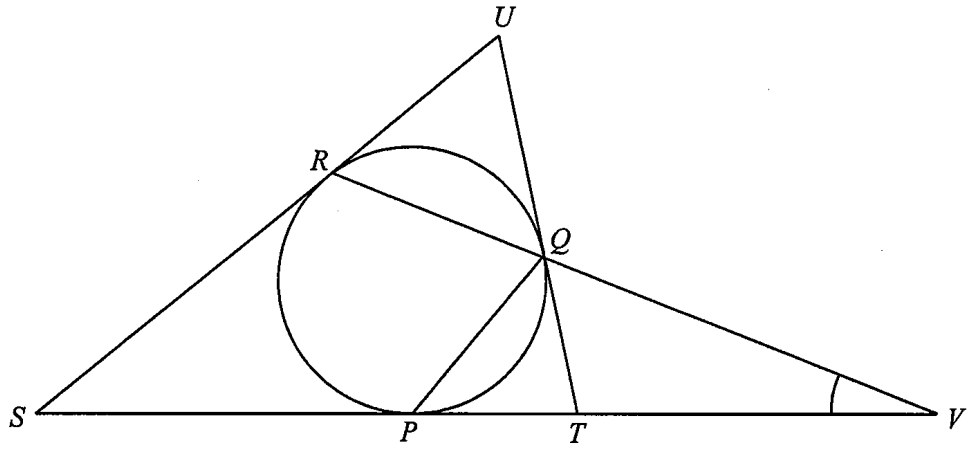
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7) In the figure below, P, Q and R are points lying on a circle. **3**

Lines ST, TU and SU are the tangents to the circle at P, Q, and R respectively.
RQ produced and ST produced meet at the point V.

If $\angle PSR = 34°$ and $\angle QPT = 46°$, then what is the value of $\angle PVQ$?

Geometrical reasons are needed.



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Extra Working Page for Part C only

Part D

Name: _____

Class	Teacher	
10Ma**A**	A Wang	
10Ma**B**	M Zheng	
10Ma**C**	J Tao	
10Ma**P**	W Beare	
10Ma**L**	R Wang	
10Ma**U**	A Ward & J James	
10Ma**S**	J Chan	

Mark	**/15**

1) Find any point(s) of intersection between the parabola $y = x^2 + x + 1$ and the line $y = 5x - 3$. **3**

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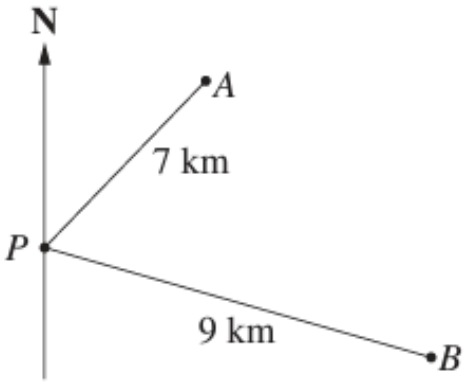
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2) Ms Azure and Mr Blue were camping at the site located at *P*. **3**

Ms Azure walked on a bearing of 035°T for 7 km to location *A*.

Mr Blue walked on a bearing of 100°T for 9 km to location *B*.



NOT TO SCALE

Find the distance between Ms Azure and Mr Blue to the nearest hundred metres.

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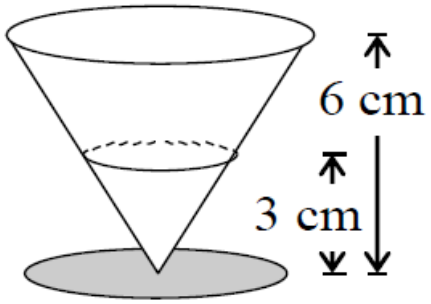
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3)



A medicine glass in the shape of a cone has a height of 6 cm. **3**

Initially, 3 mL of liquid fills the cone to a height of 3 cm.

How many more millilitres of liquid are required to fill the cone to a height of 6 cm?

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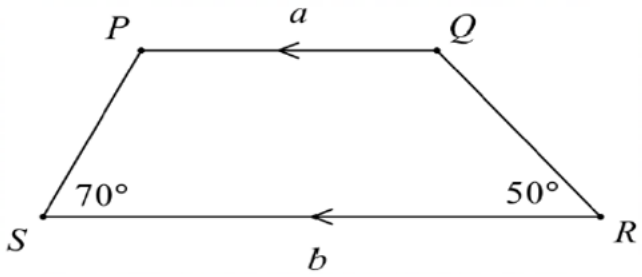
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4) In the figure below, $PQ \parallel RS$. **3**

$PQ = a$ and $RS = b$.

Show that the length of QR is $\dfrac{(b-a)\sin 70°}{\sin 60°}$.



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5) Solve the equation $\log_{10} x + \log_{10}(x+2) = 3\log_{10} 2$. **3**

End of Section D

Extra Working Page for Part D only

Part E

Name: _____

Class	Teacher	
10Ma**A**	A Wang	
10Ma**B**	M Zheng	
10Ma**C**	J Tao	
10Ma**P**	W Beare	
10Ma**L**	R Wang	
10Ma**U**	A Ward & J James	
10Ma**S**	J Chan	

Mark	**/12**

1) Let $f(x)=\dfrac{2x-1}{x-1}$.

i) Show that $\dfrac{2x-1}{x-1}=2+\dfrac{1}{x-1}$. **1**

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ii) What is the domain and range of $f(x)$? **2**

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iii) a) Find $f^{-1}(x)$. **2**

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b) Find $f(f(x))$. **2**

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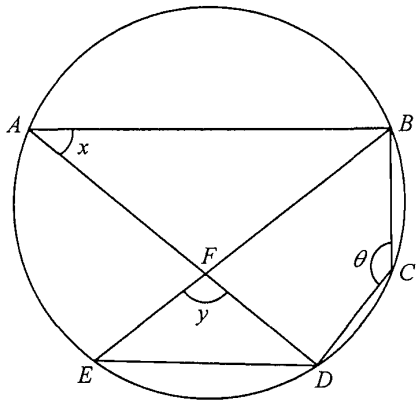
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2) In the diagram below, $ABCDE$ is a circle. It is given that $AB \parallel ED$.

AD and BE intersect at the point F.



i) Show that $2x + y = 180$, giving reasons. **3**

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ii) Express x in terms of θ. **1**

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iii) Express y in terms of θ. **1**

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Extra Working Page for Part E only

Part F

Name: _____

Class	Teacher	
10Ma**A**	A Wang	
10Ma**B**	M Zheng	
10Ma**C**	J Tao	
10Ma**P**	W Beare	
10Ma**L**	R Wang	
10Ma**U**	A Ward & J James	
10Ma**S**	J Chan	

Mark	**/10**

1) Let $f(x) = 3x^2 + 18mx + 22m^2$, where m is a real constant. **1**

Which of the following statements about the graph of $y = -f(3x)$ must be true?

I. The x-coordinate of the vertex of the graph is m.
II. The y-coordinate of the vertex of the graph is $5m^2$.
III. The equation of the axis of symmetry of the graph is $x + m = 0$.

A) I only

B) II only

C) I and III only

D) II and III only

2) A sequence, u_1, u_2, u_3, u_4, …, is defined by $u_{n+1} = \frac{4}{2 - u_n}$, $u_1 = 1$.

i) Write down the values of u_2, u_3, and u_4. **3**

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ii) What is the sum of the first 500 terms? **2**

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3) Each of 4 students have a fair coin.

They play a game in which each student tosses their coin and when the result of their toss is a head then that student is eliminated from the game.

The game continues with the remaining students tossing their coin again.
As before, any student who tosses a head is eliminated.

The game continues until all the students have been eliminated or there is a single winner.

Calculate the probability that:

i) All students are eliminated after their first toss of the coin. **1**

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ii) One student is eliminated in the first round, one student in the second round, and one student in the third round, leaving one clear winner by elimination. **3**

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End of paper

Extra Working Page for Part F only

SYDNEY BOYS HIGH SCHOOL

2024

YEAR 10
STAGE 5 TERM 4
YEARLY

Sample Solutions

NOTE: This process of checking your mark is about reading the solutions and the comments.

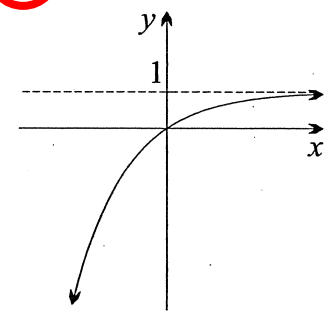
Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.
Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

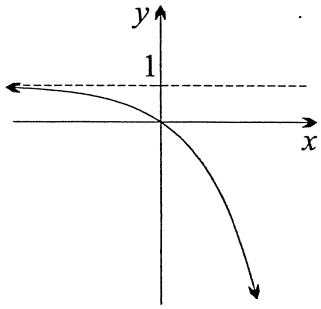
Section A Solutions & Comments

1) Which of the following graphs could have the equation $f(x) = 1 - 2^{-x}$? **1**

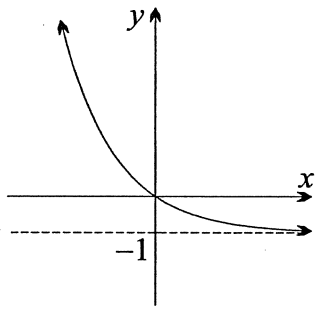
A)



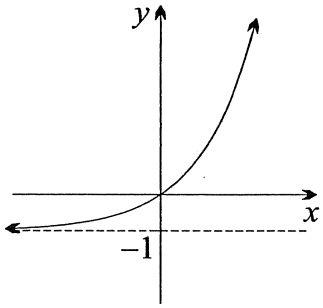
B)



C)



D)



To check: For large values of x, $f(x) \approx 1$ e.g. $x = 100$

Generally well done by most students.

2) Solve $x(3 - x) = 0$ **1**

Ans: $x = 0$ or $3 - x = 0$
$\therefore x = 0, 3$

It was surprising that this was not done well by all students.
This content was done in Term 1.
Think twice about Extension 1 Maths if you cannot just write down the answer.

A fair few only left one solution.
Students should not be expanding and using the quadratic formula.

3) Evaluate $\log_3 81$. **1**

Ans: $81 = 3^4$ and so $\log_3 81 = 4$.

Students should just use the definition for a logarithm, rather than using any of the rules.

4) Solve $\sin\theta = -0.5$ for $0° \le \theta \le 360°$. **2**

θ is in the 3rd & 4th quadrants.

Ans: The related angle (1st quadrant) is $\sin^{-1}(0.5) = 30°$.

$\therefore\ \theta = 180° + 30°, 360° - 30° = 210°, 330°$

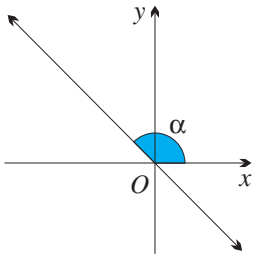
Students were not penalised this time for not leaving their answers in degrees.

Students are using notation that they don't understand: $\sin^{-1}(-0.5) = -30°$.
There is only a single value for $\sin^{-1}(\theta)$.
In fact, $-90° \le \sin^{-1}(\theta) \le 90°$.

5) What is the angle that the line $y = -x$ makes with the positive direction of the x-axis? **1**

Ans: $180° - \tan^{-1}(-1) = 135°$

Most students had forgotten the definition, and were trying to work it out from scratch.
For negative gradients:



6) Let M denote a length in metres
N denote a length in inches
F denote the length in feet

The conversion from metres to feet is given by $F = 3.28084M$.
The conversion from feet to inches is given by $N = 12F$
Let $f(x) = 3.28084x$ and $g(x) = 12x$.

i) Explain what $f^{-1}(x)$ represents for the units above. **1**

Ans: f: metres → feet so f^{-1}: feet → metres
So $f^{-1}(x)$ represents converting feet to metres

ii) Explain what $g(f(x))$ represents for the units above. **1**

Ans: $g(f(x))$ represents converting metres to inches.

I did accept metres to feet to inches for part (ii).
It was surprising that many got part (ii) right, but part (i) wrong.

Many students took the questions to mean 'Find the functions $f^{-1}(x)$ and $g(f(x))$', unfortunately they didn't receive any marks.

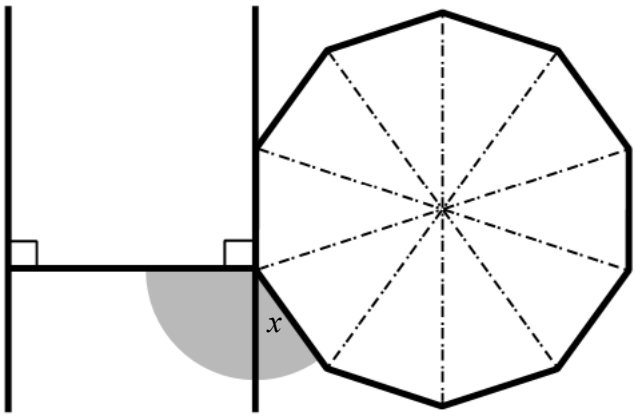
iii) Find the length in inches of a rope of 3.5 m. **1**
Give your answer correct to 5 significant figures.

Ans: $g(f(3.5)) = 137.80$ to 5 significant figures
$\therefore 3.5 \text{ m} \approx 137.80$ (5 sig fig)

Students had to get the rounding right to score full marks.
Rounding wasn't a concern if students had the wrong number.

7) A logo consists of an H shape and a regular decagon. **2**

The diagram below shows the logo.



State the size of the shaded angle. Geometric reasons are not needed, but working is essential.

Ans: The shaded angle is $90° + x$.

x is the exterior angle to the decagon i.e. $x = \dfrac{360°}{10} = 36°$.

$\therefore$ the shaded angle is 126°.

It was surprising that some students didn't know how many sides there are to a decagon.
Some had the same number of sides as an enneagon.

Some students were mixing up the formulae for internal and exterior angles.

That said, there were many students who were determined to do this in the longest way possible.

8) Given that $\log_a b = 1.5$ and $\log_a c = 0.3$, find the value of

i) $\log_a \dfrac{b}{c}$. **1**

Ans: $\log_a \dfrac{b}{c} = \log_a b - \log_a c = 1.5 - 0.3 = 1.2$

ii) $\log_a (cb)^5$ **2**

Ans: $\log_a (cb)^5 = 5\left(\log_a b + \log_a c\right) = 5(1.5 + 0.3) = 9$

Many students don't know their logarithm rules.
This topic is in Mathematics Advanced.

Students are advised to use their calculators.

9) A student tosses a fair coin three times.

i) What is the probability of three heads? **1**

Ans: $P(HHH) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

No half marks.

ii) What is the probability that the student observes exactly one head? **1**

Ans: Exactly one head = P(HTT) + P(THT) + P(TTH) $= 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

No half marks
Many students confused 'exactly' one head with 'at least one' head.

iii) Given that the student has observed *at least* one head, what is the probability that the student observes at least two heads? **2**

Ans: A simple diagram is useful.
Given the condition, the sample space is reduced to at least one head.

HHH
HHT
HTH
THH

TTH
THT
HTT

So the probability of at least two heads in this sample space is $\frac{4}{7}$.

Students are advised to use their calculators.

Avoid a formula if you don't understand how the formula works.

This question also caused a problem with the language used.
This also manifest itself with a few students reading this as:
"*What is the probability of at least one head, given that the student observes at least two heads ?*"

End of Section A solutions

Part B Solutions and Comments

1) Let $g(x) = 13 - 5x^2$. If a is a constant, which of the following is $g(1-3a)$? **1**

A) $8 - 45a^2$

B) $8 + 45a^2$

C) $8 - 30a + 45a^2$

D) $8 + 30a - 45a^2$

$g(1-3a) = 13 - 5(1-3a)^2$

$= 13 - 5(1 - 6a + 9a^2)$

$= 13 - 5 + 30a - 45a^2$

$= 8 + 30a - 45a^2$

Comment:

Even though it is multiple choice, working should be done

2) Make y the subject for $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$ **2**

$\frac{1}{y} = \frac{1}{x} - \frac{1}{z}$

$\frac{1}{y} = \frac{z - x}{xz}$

$y = \frac{xz}{z - x}$

Comment:

Many students struggled with the algebra. Since we are solving the literal equation for y, it makes sense to keep y in the one place (then take reciprocals)

Part B (continued)

3) The current population of planet Earth is 8.2 billion in 2024. If the projected population increase is 0.87% p.a., in which year will the population first reach 10 billion? **2**

$A = P(1+r)^n$

$A = 8.2(1.0087)^n$

$10 = 8.2(1.0087)^n$

$1.0087^n = \frac{10}{8.2}$

$\log(1.0087)^n = \log\left(\frac{10}{8.2}\right)$

$n\log(1.0087) = \log\left(\frac{10}{8.2}\right)$

$n = \frac{\log\left(\frac{10}{8.2}\right)}{\log(1.0087)}$

$n = 22.909534944$

If we assume the population is 8.2 billion, January 1st 2024
then Answer = 2024 + 22 = 2046
If we assume the population is 8.2 billion, December 2nd 2024

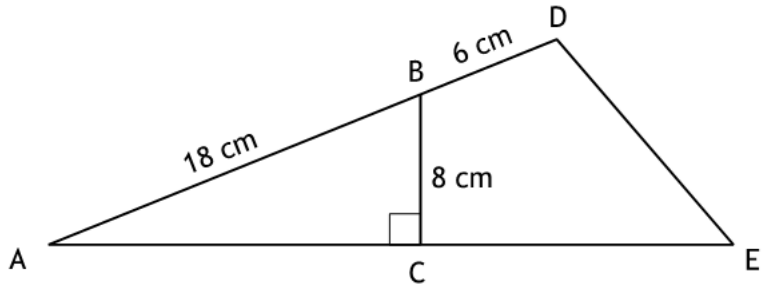
then answer = 2047

Comment:

It was not necessary to use 10000000000 & 8200000000.

Part B (continued)

4) In the diagram below, $AB = 18$ cm, $BD = 6$ cm, $BC = 8$ cm, and the area of triangle $ADE = 160$ cm^2. **3**



Calculate the length of AE, to 2 decimal places.

$\sin A = \frac{8}{18}$

$= \frac{4}{9}$

Area $= \frac{1}{2} \times AD \times AE \times \sin A$

$160 = \frac{1}{2} \times 24 \times AE \times \frac{4}{9}$

$AE = 30$ cm (ie 30.00 cm)

Comment:

Many students overcomplicated the question. Since we are finding the length of AE and are given the Area of $\triangle ADE$

$A = \frac{1}{2} \times AE \times$ (Altitude of D from AE)

OR

$A = \frac{1}{2} \times AD \times AE \times \sin A$

which tells us we first need to find the altitude of D from AE or $\sin A$.

5) Janice and Paul did a study on feelings of stress and life satisfaction.
Participants completed a measure on how stressed they were feeling (on a 1 to 30 scale) and a measure of how satisfied they felt with their lives (measured on a 1 to 10 scale).
The table below indicates the participants' scores.

Participant #	Stress score (X)	Life Satisfaction (Y)
1	11	7
2	25	1
3	19	4
4	7	9
5	23	2
6	6	8
7	11	8
8	22	3
9	25	3
10	10	6

Using this data, answer the following questions:

i) Calculate the correlation (r) between Stress and Life Satisfaction. **1**

$r = -0.957273896 \approx -0.96$

ii) The least-squares regression line can be written in the form $Y = mX + b$. **1**
What is the value of m?

$m = -0.357429718 \approx -0.36$

iii) Write a brief interpretation of this correlation, including the strength, direction, and an explanation of the effect. **1**

There is a strong, negative correlation. This means as stress increases, life satisfaction decreases and vice-versa

Turn over for the rest of Question 5 solutions

Part B (continued)

5) (continued)

iv) Can you say that being more stressed causes a lower level of life satisfaction? Why or why not? **1**

Although the correlation coefficient is close to -1, it should not be used to say anything about cause and effect relationships. We can conclude that the two variables are related, but it does not tell us that being more stressed is the cause of the change in life satisfaction. CORRELATION DOES NOT IMPLY CAUSATION

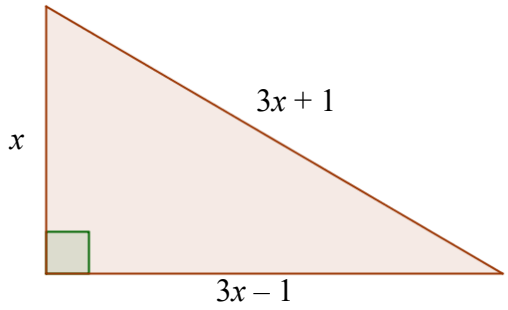
Comment:

i)ii) It appears as though many students do not know how to use their calculator.

iv) Not many students gained the mark. The key word is cause. There is a strong relationship, we can't use that to prove causation.

6) A right-angled triangle is drawn so that the hypotenuse is 3 times the shortest side plus 1 cm, and the other side is 3 times the shortest side minus 1 cm. **3**

If the shortest side has length x, find all valid solutions for x, giving explicit mathematical reasoning.



since x, $3x-1$ and $3x+1$ are all lengths in a triangle.

$x > 0$ and $3x-1 > 0$ and $3x+1 > 0$

$\boxed{x > \frac{1}{3}}$ $x > -\frac{1}{3}$

We are also told that x is the shortest side

$x < 3x-1$

$1 < 2x$

$\boxed{x > \frac{1}{2}}$

The triangle inequality must also hold

$x + 3x - 1 > 3x + 1$

$\boxed{x > 2}$

$x^2 + (3x-1)^2 = (3x+1)^2$ Pythagoras' Theorem

$x^2 + 9x^2 - 6x + 1 = 9x^2 + 6x + 1$

$x^2 - 12x = 0$

$x(x-12) = 0$

$x = 0, \; x = 12$

$\therefore$ $x = 12$ is the only valid solution.

Turn over for the rest of Question 6 solutions

6) (continued)

Comment:

It should have been answered better. Some students were able to eliminate 0 as a possibility because x is a length and greater than zero. Very few students found stronger restrictions on x.

End of Section B Solutions

Part C Solutions & Comments

1) By considering the discriminant, which of the following has 2 distinct real and rational x-intercepts? **1**

A) $y = -x^2 - 3x + 5$

B) $y = 4x^2 - x - 5$

C) $y = -x^2 - 4$

D) $y = 2x^2$

A. $\Delta = (-3)^2 - 4 \times 5 \times (-1) = 29 \neq$ perfect square

$\therefore$ the x-intercepts are not rational, but they are distinct.

B. $\Delta = (-1)^2 - 4 \times 4 \times (-5) = 121 > 0$

$\therefore$ the x-intercepts are rational and distinct

C. The graph is completely below the y-axis, so no real roots.

OR

$\Delta = (0)^2 - 4 \times (-1) \times (-4) = -16 < 0$

D. Only 1 x-intercept.

Generally well done.

The most common error is A.

2) The table below shows the scores (in marks) and the corresponding z-scores of 3 students in an examination. **1**

Score (marks)	46	x	86
z-score	–3	1	2

Find x.

A) 64

B) 66

C) 70

D) 78

$2 - (-3) = 5$ standard deviations $\equiv 86 - 46 = 40$ marks

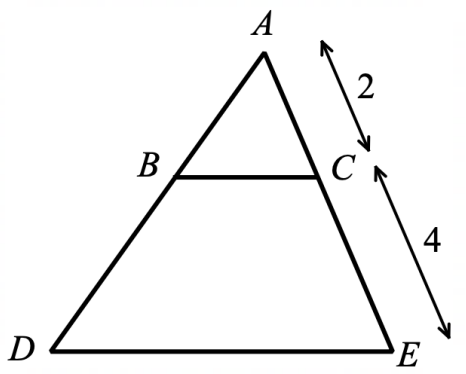
$\therefore$ one standard deviation is 8.

$\therefore x = 86 - 8 = 78$

Generally well done.

Common error: C

3) In the diagram below, $BC \parallel DE$, $AC = 2$, $CE = 4$, and $BC = 6$. **1**



Which of the following is true?

A) $DE = 12$

B) $BC : DE = 1 : 2$

C) Area ΔABC : Area $\Delta ADE = 1 : 9$

D) Perimeter ΔABC : Perimeter $\Delta ADE = 1 : 2$

A. $BC : DE = 1 : 3 \Rightarrow DE = 18$

B. See above

C. $\Delta ABC \,|||\, \Delta ADE$

$AC : AE = 1 : 3$

$\therefore$ Area ΔABC : Area $ADE = 1 : 9$

D. As $\Delta ABC \,|||\, \Delta ADE$ and $AC : AE = 1 : 3 \Rightarrow$ Perimeter ΔABC : Perimeter $\Delta ADE = 1 : 3$

Well done.

Common error: B

4) Factorise $3x^3 - \dfrac{24}{125y^6}$ **3**

Solution 1:

$$3x^3 - \frac{24}{125y^6} = \frac{3}{125y^6}\left(125x^3y^6 - 8\right)$$
$$= \frac{3\left(\left(5xy^2\right)^3 - 2^3\right)}{125y^6}$$
$$= \frac{3\left(5xy^2 - 2\right)\left(25x^2y^4 + 10xy^2 + 4\right)}{125y^6}$$

Solution 2:

$$3x^3 - \frac{24}{125y^6} = 3\left(x^3 - \frac{8}{125y^6}\right)$$
$$= 3\left(x^3 - \left(\frac{2}{5y^2}\right)^3\right)$$
$$= 3\left(x - \frac{2}{5y^2}\right)\left(x^2 + \frac{2x}{5y^2} + \frac{4}{25y^4}\right)$$

Solution 2 is NOT great as the final expression is only partially factorised, but no mark was deducted this time.

Most students who got full marks used this method.

In general:

- *Poorly done.*
- *Some students did not attempt this question.*
- *Some students did not understand the meaning of factorise, so they tried to solve for x.*
- *Many students did not take the common factor 3 out.*
- *Many students were not able to correctly apply the formula for the difference of two cubes.*
- *Many careless mistakes.*

5) A ball contains 5000 cm^3 of air. **3**

More air is pumped into the ball at a constant rate of 60 cm^3/sec for 1 minute.
The ball can maintain its spherical shape while it is being pumped up.



After the minute, the pumping stops.

What is the surface area of the pumped up ball, to the nearest square centimetre?

$$V = \frac{4}{3}\pi r^3 = 5000 + 60 \times 60 = 8600 \text{ cm}^3$$

$$\therefore r = \left(\frac{8600 \times 3}{4\pi}\right)^{\frac{1}{3}} = \left(\frac{6450}{\pi}\right)^{\frac{1}{3}} \text{ cm}^2$$

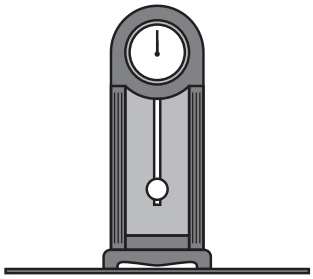
$$\therefore SA = 4\pi r^2 = 4\pi\left(\frac{6450}{\pi}\right)^{\frac{2}{3}} = 2030 \text{ cm}^2$$

Well done.

Common errors:

- *Incorrect volume. e.g. 5000, 3600, 5360 etc.*
- *Incorrect volume formula:* $V = \frac{4}{3}\pi r^2, \frac{3}{4}\pi r^3, 4\pi r^3$ *etc.*
- *Incorrect surface area formula:* $SA = \frac{4}{3}\pi r^2$
- *Incorrect answer:* $2070\ cm^2$.
  This error occurred because students rounded the value of r *to 12.7 cm, and then used this rounded value.*
  This affected the accuracy of their final result.
  Students should use the exact value (left on calculator) and avoid rounding too early.

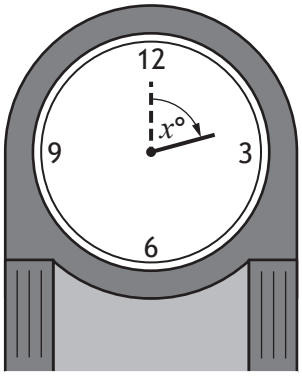
6) Elsa has a grandfather clock in her house. **3**



The height of the tip of the hour hand above the floor, in centimetres, is given by

$$h = 20\cos x° + 147,$$

where $x°$ is the angle the hour hand has rotated through since 12 o'clock.



Calculate the first 2 values of x for which the tip of the hour hand is 150 cm above the floor.

$h = 20\cos x + 147 = 150$

$\therefore x = \cos^{-1}\left(\frac{3}{20}\right) = 81.37°$

And $x = 360 - \cos^{-1}\left(\frac{3}{20}\right) = 278.63°$

Well done.

Common errors:

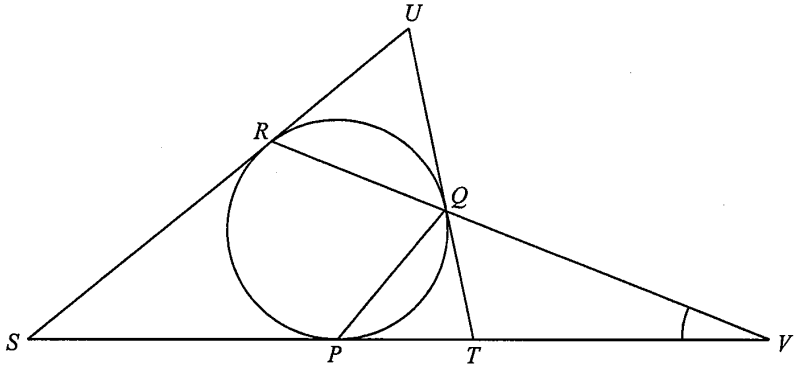
- *Few students only had 1 solution*
- *Incorrect second value of x, e.g.* 98.63°
- *The sum of the 2 values of x is not* 360°

7) In the figure below, P, Q and R are points lying on a circle. **3**

Lines ST, TU and SU are the tangents to the circle at P, Q, and R respectively.
RQ produced and ST produced meet at the point V.

If $\angle PSR = 34°$ and $\angle QPT = 46°$, then what is the value of $\angle PVQ$?

Geometrical reasons are needed.



Ans: $\angle PVQ = 27°$

READ FIRST

Generally well done.

To achieve full marks, students needed to:

- *Provide full geometrical reasons.*
  A maximum 1 mark was awarded if students did not provide any geometrical reasons.

- *Demonstrate their understanding of*
  - *The angle between a tangent and a chord equals the angle at the circumference in the alternate segment. e.g.* $\angle SRP = \angle RQS$ *or* $\angle PRQ = \angle QPT$

  - *Intercepts on tangents drawn from a point to a circle are equal.*
    e.g. $SR = SP$, $UR = UQ$ *or* $TQ = TP$

  - *Angle properties in triangle, including but not limited to angle sum of a triangle, exterior angle of a triangle, base angles of isosceles triangle.*

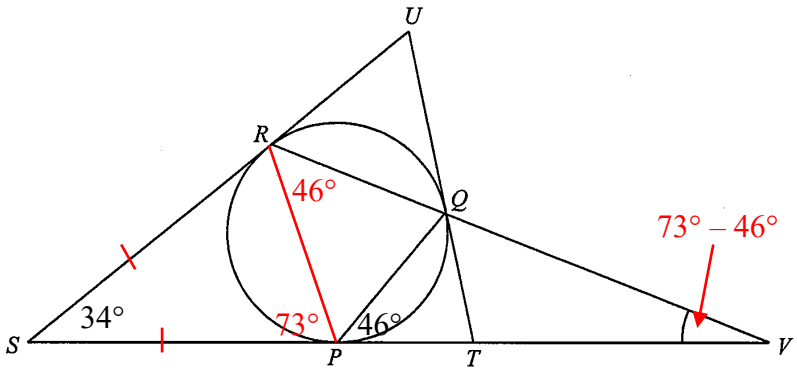
- *Not combine multiple geometrical reasons in one step.*

Common error: $\angle SRQ = \angle SPQ$

If you are struggling with the geometrical reasoning, please refer to the SBHS Reasoning in Geometry and Reasoning in Circle Geometry files.

Solution on next page

Question 7 (continued)



Construct RP

$\because SU$ and SP are tangents (given)

$\therefore SR = SP$ (tangents from external point)

$\angle SRP = \angle RQP$ (angle in alternate segment)

$\therefore \angle SRP = \angle SPR$ (angle opposite equal sides in a triangle)

Also $\because \angle PSR = 34°$ (given)

$\therefore \angle SRP = (180° - 34°) \div 2 = 73°$ (angle sum of triangle)

$\therefore \angle RQP = 73°$

$\because \angle RQP = \angle QPV + \angle PVQ$ (exterior angle of triangle)

Also $\because \angle QPT = 46°$ (given)

$\therefore \angle PVQ = 73° - 46° = 27°$

End of Part C solutions

Part D Solutions and Comments

1) Find any point(s) of intersection between the parabola $y = x^2 + x + 1$ and the line $y = 5x - 3$.

$x^2 + x + 1 = 5x - 3$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

$y = 5(2) - 3 = 7$

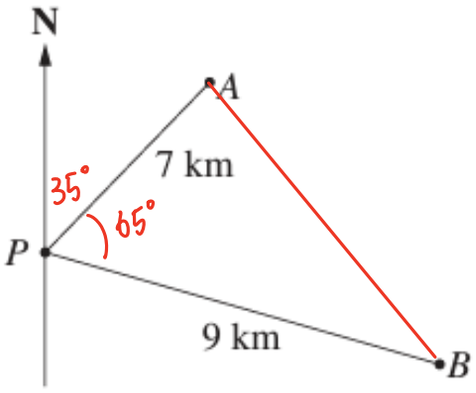
$\therefore (2, 7)$

- Generally well done.
- There were a few arithmetic errors e.g. $x^2 - 4x - 4 = 0$, $x^2 - 4x - 2 = 0$ etc.
- Question asked for point of intersection, so students were required to find 'y' and also rewrite as a coordinate pair. Students were not awarded full marks if they didn't do this.

2) Ms Azure and Mr Blue were camping at the site located at P. **3**

Ms Azure walked on a bearing of 035°T for 7 km to location A.

Mr Blue walked on a bearing of 100°T for 9 km to location B.



NOT TO SCALE

Find the distance between Ms Azure and Mr Blue to the nearest hundred metres.

$\angle APB = 65°$ (adjacent angles)

$AB = \sqrt{7^2 + 9^2 - 2(7)(9)\cos 65°}$

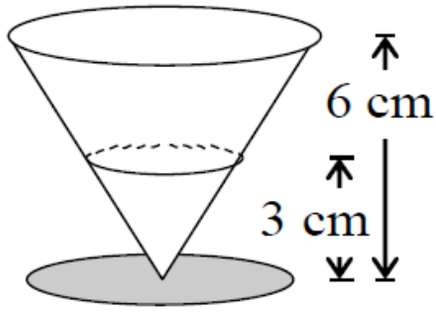
$= 8.760713385$

$= 8\,800$ m (nearest hundred metre)

or 8.8 km

- Generally well done.
- Some students should revise the cosine rule. The most common mistake was forgetting the 2: $c^2 = a^2 + b^2 - 2ab\cos C$
- Units and rounding were important in this question. Students could not receive full marks if they didn't round correctly.

3)



A medicine glass in the shape of a cone has a height of 6 cm. **3**

Initially, 3 mL of liquid fills the cone to a height of 3 cm.

How many more millilitres of liquid are required to fill the cone to a height of 6 cm?

$x_{liquid} : x_{glass} = 3:6 = 1:2$

$V_{liquid} : V_{glass} = 1:8$

$\frac{3}{V_{glass}} = \frac{1}{8}$

$V_{glass} = 3 \times 8 = 24\ cm^3 = 24mL$

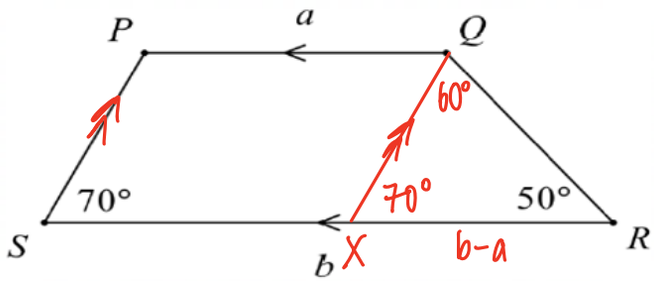
$\therefore 24 - 3 = 21\ mL$

- Poorly done.
- Many students thought the ratio was $3:9 = 1:3$.
- Some students did not read the question carefully and left their answer as 24 mL.

4) In the figure below, $PQ \parallel RS$. **3**

$PQ = a$ and $RS = b$.

Show that the length of QR is $\dfrac{(b-a)\sin 70°}{\sin 60°}$.



Construct line QX such that PS ∥ QX.

$\angle QXR = 70°$ (corresponding angles, QX ∥ PS).

$\angle XQR = 60°$ (angle sum of triangle).

∴ SXQR is a parallelogram (two pairs of opposite sides parallel)

∴ $SX = a$ (opposite sides in parallelogram equal).

$XR = SR - RX$

$= b - a$

$\dfrac{QR}{\sin 70°} = \dfrac{XR}{\sin 60°}$ (Sine Rule)

$QR = \dfrac{XR \sin 70°}{\sin 60°}$

$\therefore QR = \dfrac{(b-a)\sin 70°}{\sin 60°}$

- Poorly done.
- This is a SHOW question, so ALL steps must be shown clearly.
- Many students tried to reverse engineer.
- Many students did not explain how they got '70°, 60°, b-a' and so the students could not receive full marks.
- Some students 'cut out' the rectangle in the middle. This was an equally valid approach.

5) Solve the equation $\log_{10} x + \log_{10}(x+2) = 3\log_{10} 2$. **3**

$x > 0$, $x > -2$

$\therefore x > 0$.

$\log_{10} x(x+2) = \log_{10} 2^3$

$x(x+2) = 8$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2$

$\because x > 0$

$\therefore x = 2$ only

- Generally well done.
- Common mistake was $3\log_{10} 2 = \log_{10} 3^2 = \log_{10} 9$.
- Many students forgot to omit $x = -4$.
- Of the students who did omit $x = -4$, their explanation was mostly invalid. ($x > -2$). Students needed to find the intersection of $x > 0$ and $x > -2$, which is $x > 0$. Marks were not deducted this time.

End of Section D solutions

Part E Solutions & Comments

1) Let $f(x)=\dfrac{2x-1}{x-1}$.

i) Show that $\dfrac{2x-1}{x-1}=2+\dfrac{1}{x-1}$. **1**

LHS: $\dfrac{2x-1}{x-1} = \dfrac{2x-2+1}{x-1} = \dfrac{2(x-1)}{x-1} + \dfrac{1}{x-1} = 2+\dfrac{1}{x-1} = \text{RHS}$

OR

$$\begin{aligned}\text{RHS} &= 2+\frac{1}{x-1}\\ &= \frac{2(x-1)+1}{x-1}\\ &= \frac{2x-2+1}{x-1}\\ &= \frac{2x-1}{x-1}\\ &= \text{LHS}\end{aligned}$$

Comment:
Well done.

Although most students demonstrated all the concept to be tested.

Layout was very poor.

Few students declared LHS and RHS preferring just to show the algebraic working.

ii) What is the domain and range of $f(x)$? **2**

domain $\Rightarrow x \neq 1$

range $\Rightarrow y \neq 2$

Comment:
Reasonably well done.

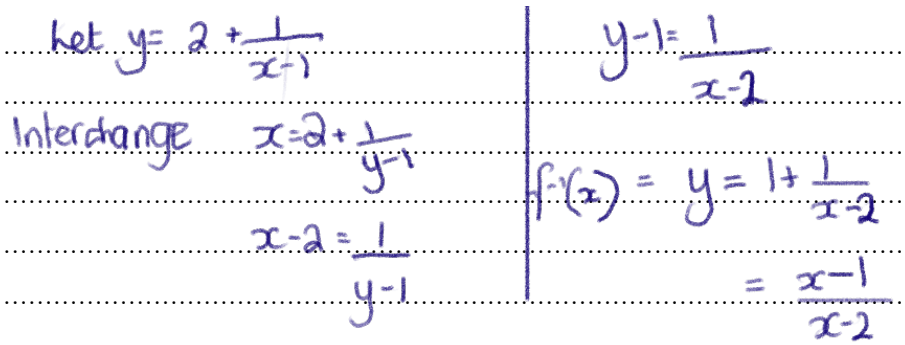
Most students got the domain correct but were unaware of how to work out the range.

A wide variety of values were given, even ranges of values.

Question 1 solutions continued over the page

1) (continued)

iii) a) Find $f^{-1}(x)$. **2**

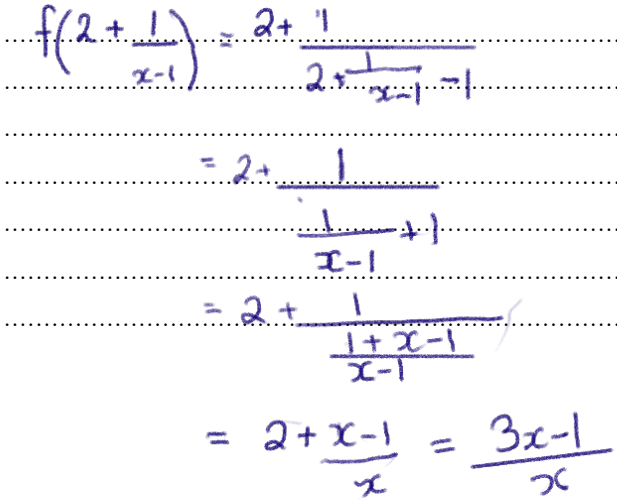


Comment:
Well done.

Most students had a good grasp of what was required here.

The negatives did produce problems for some.

b) Find $f(f(x))$. **2**



Comment:
Not well done.

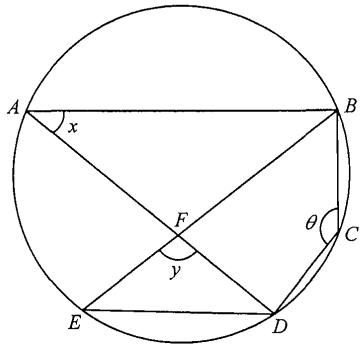
Most students had a good grasp of what was required here.

The negatives did produce a problem for some.

The manipulation of the algebraic fractions also produced some errors.

2) In the diagram below, $ABCDE$ is a circle. It is given that $AB \parallel ED$.

AD and BE intersect at the point F.



i) Show that $2x + y = 180$, giving reasons. **3**

In $\triangle FED$ and $\triangle FBA$
$\angle EFD = \angle BFA = y$ (vertically opposite $\angle$'s)
$\angle DEF = \angle BAF = x$ ($\angle$'s on the same arc BD)
$\angle DEF = \angle FBA = x$ (alternate $\angle$'s AB || ED)

In $\triangle ABF$ $\angle BFA + \angle BAF + \angle FBA = 180$ $\angle$ Sum $\triangle$
$x + x + y = 180$

Comment:
Relatively well done.

Most students got most of the required reasoning, although layout was sloppy and occasionally illogical in nature.

ii) Express x in terms of θ. **1**

$x = 180 - \theta$

(Opposite angles of cyclic quadrilateral $ABCD$)

Comment:
Very well done. Some students guessed incorrectly.

iii) Express y in terms of θ. **1**

$y + 2x = 180$
$y = 180 - 2(180 - \theta)$
$y = 2\theta - 180$

Comment:
Poorly done.

Although there was a fairly simple solution, many students over complicated their answer.

Part F Solutions & Comments

1) Let $f(x)=3x^2+18mx+22m^2$, where m is a real constant. **1**

Which of the following statements about the graph of $y=-f(3x)$ must be true?

I. The x-coordinate of the vertex of the graph is m.
II. The y-coordinate of the vertex of the graph is $5m^2$.
III. The equation of the axis of symmetry of the graph is $x+m=0$.

A) I only B) II only

C) I and III only (D) II and III only

(1) Explanation:

Given $f(x) = 3x^2 + 18mx + 22m^2$

$-f(3x) = -(3(3x)^2 + 18m(3x) + 22m^2)$

$y = -27x^2 - 54mx - 22m^2$

∴ Axis of symmetry: $x = \frac{-b}{2a}$

$= \frac{-(-54m)}{2(-27)}$

$x = -m \rightarrow x+m=0$

∴ $y = -27(-m)^2 - 54m(-m) - 22m^2$

$= -27m^2 + 54m^2 - 22m^2$

$= 5m^2$

∴ Vertex $= (-m, 5m^2)$

∴ I is false

II is true

III is true

∴ (D) is correct.

Marking scheme	Marker's comments
1 mark Correct answer	- Poorly done by many candidates. - Statistics of the candidates' responses are shown below. **Options** | A | B | C | D | No response **Candidates' choice** | 10.7% | 21.5% | 25.9% | 30.7% | 11.2% - Candidates should always attempt multiple-choice questions and avoid leaving them blank.

2) A sequence, $u_1, u_2, u_3, u_4, \ldots$, is defined by $u_{n+1} = \frac{4}{2 - u_n}$, $u_1 = 1$.

i) Write down the values of u_2, u_3, and u_4. **3**

$u_2 = \frac{4}{2-1} = 4$

$u_3 = \frac{4}{2-4} = -2$

$u_4 = \frac{4}{2-(-2)} = 1$

Marking scheme	Marker's comments
3 marks All 3 correct values. **2 marks** 2 correct values. . **1 mark** 1 correct value.	- Candidates were not awarded relevant marks if they wrote values incorrectly for the wrong corresponding u^{th} term or did not clearly indicate which values corresponded to each u^{th} term. - Common mistakes included failing to substitute the correct value for u_n or arithmetic errors. - Finding an undefined u^{th} term should prompt some candidates to recheck their solution.

ii) What is the sum of the first 500 terms? **2**

Pattern: 1, 4, -2, 1, 4, -2, ...

$500 \div 3 = 166$ remainder 2

∴ 166 lots of -2

167 lots of 1

and 167 lots of 4

∴ Sum $= 166 \times (-2) + 167 \times 1 + 167 \times 4 = 503$

Marking scheme	Marker's comments
2 marks Correct solution. . **1 mark** Substantial progress to correct solution.	- A significant number of candidates who identified the correct pattern in part (i) were able to find the correct answer successfully. - Common errors included incorrectly identifying the 499th or 500th term, arithmetic errors or using formulas that are not correct e.g. AP series.

3) Each of 4 students have a fair coin.

They play a game in which each student tosses their coin and when the result of their toss is a head then that student is eliminated from the game.

The game continues with the remaining students tossing their coin again.
As before, any student who tosses a head is eliminated.

The game continues until all the students have been eliminated or there is a single winner.

Calculate the probability that:

i) All students are eliminated after their first toss of the coin. **1**

$(\frac{1}{2})^4 = \frac{1}{16}$

Marking scheme	Marker's comments
1 mark Correct answer	- This question was done well by majority of the candidates.

Question 3 continues on the next page

3) (continued)

ii) One student is eliminated in the first round, one student in the second round, and one student in the third round, leaving one clear winner by elimination. **3**

4 options for the first student to be eliminated
3 options for the 2nd student to be eliminated
2 options for the 3rd student to be eliminated
1 option for the winner
$\therefore 4 \times \left(\frac{1}{2}\right)^4 \times 3\left(\frac{1}{2}\right)^5 \times 2\left(\frac{1}{2}\right)^2 \times 1$

$= 4 \times \frac{1}{16} \times \frac{3}{8} \times 2 \times \frac{1}{4}$

$= \frac{1}{4} \times \frac{3}{8} \times \frac{1}{2}$

$= \frac{3}{64}$

Marking scheme	Marker's comments
3 marks Correct solution. **2 marks** Substantial progress to correct solution e.g. show the probability is $\frac{1}{16} \times \frac{1}{8} \times \frac{1}{4} = \frac{1}{512}$ with some reasoning or show correct probabilities for 2 out of the 3 rounds being multiplied. **1 mark** Partial (non-trivial) progress towards correct solution e.g. show the probability is $\frac{1}{16} + \frac{1}{8} + \frac{1}{4} = \frac{7}{16}$ or show correct probability for only 1 out of the 3 rounds being multiplied.	- This question was not done well by many candidates. - Many candidates who made significant progress failed to consider about the number of students that can be eliminated in each round e.g. any of the 4 students in the first round, any of the 3 students in the second round and any of the 2 students in the last round. - Some candidates failed to understand that the product rule, rather than the addition rule, was required for calculating the probability at each stage in this multistage event. This was due to the 'and' relationship between rounds, meaning probabilities should be multiplied rather than added.

End of solutions