SYDNEY BOYS HIGH SCHOOL

2023

YEAR 10 STAGE 5.3
TERM 4
YEARLY ASSESSMENT

Mathematics

Time Allowed: 90 minutes

Reading Time: 5 minutes

General Instructions

- All questions may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using black pen. Diagrams should be in ink too.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- All answers, unless otherwise stated, should be left in simplest exact form.
- Only NESA approved calculators may be used.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: PSP

Name: _____

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor & W Beare	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Part	Marks
A	/15
B	/15
C	/15
D	/15
E	/13
F	/12
Total	**/85**

Part A (15 marks)

1 Solve $(x + 7)(8 - 2x) = 0$. **2**

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2 Factorise $3m^3 + 24$. **2**

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3 Given that a, b, and c are integers, express $3x^2 + 12x + 19$ in the form $a(x + b)^2 + c$. **2**

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4 Given that $\left(\sqrt[3]{\frac{1}{x}}\right)^4 = x^n$, find the value of n. **1**

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5 Expand and simplify $n(n - 4)(n + 4)$. **2**

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6 Express $\frac{3}{x} - \frac{x-2}{2x} + \frac{1}{4}$ as a single fraction in its simplest form. **2**

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7 Simplify $\dfrac{2m^4n^2\times\left(3mn^{-3}\right)^2}{\left(5n^{-3}m^{-1}\right)^2}$ **3**

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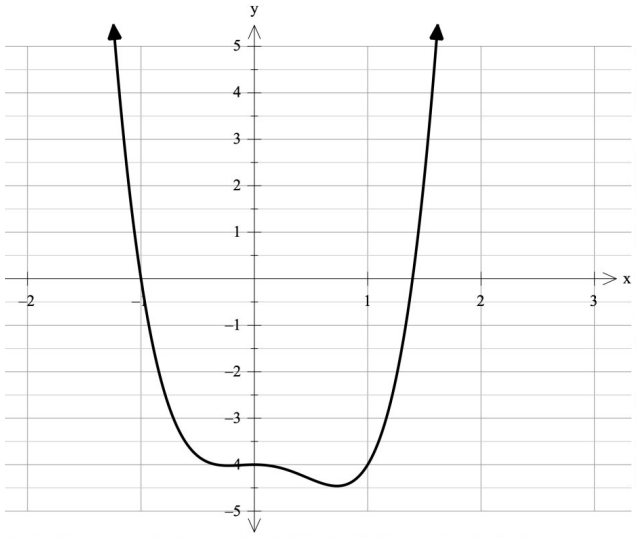
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8 State if the graph below is that of a function, giving a reason for your statement. **1**



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End of Section A

Extra Working Page for Part A only



Part B

Name: _____

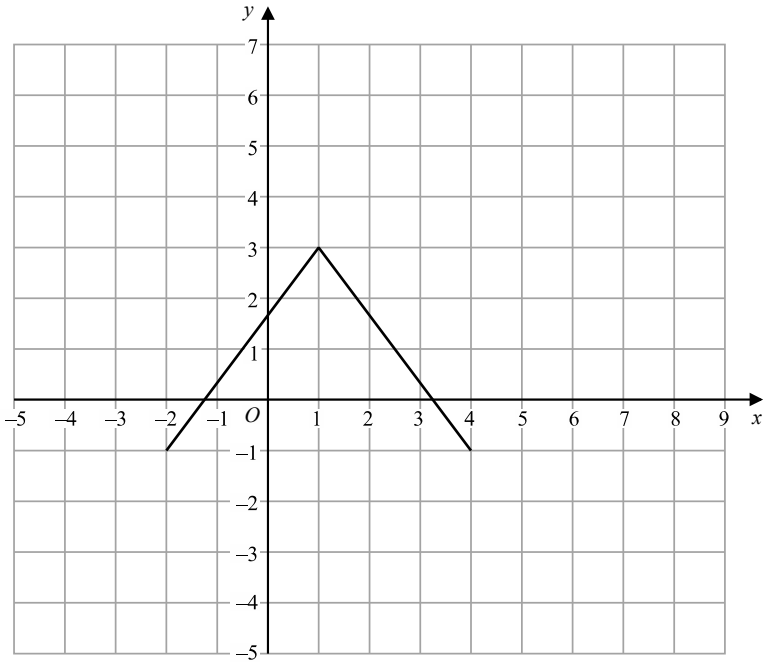
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10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	

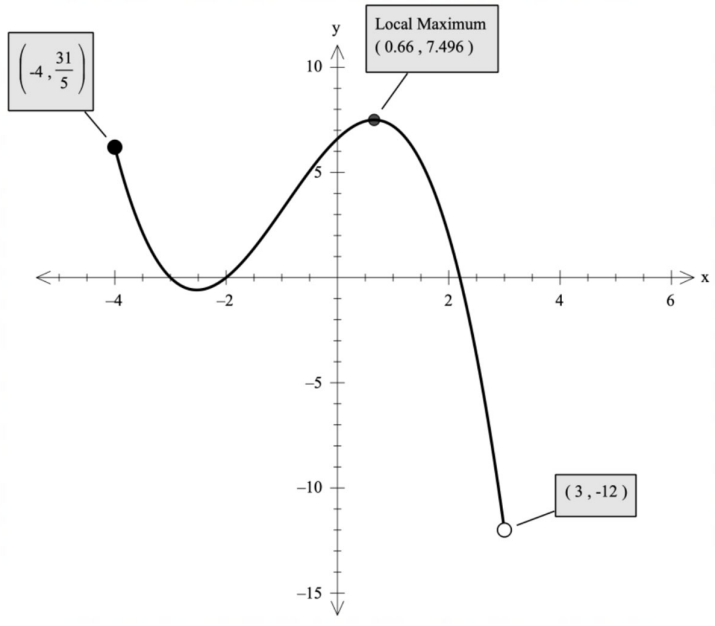
Part B (15 marks)

1 Below is the graph of $y = f(x)$

On the grid, draw the graph of $y = 2 - f(x)$. **2**



2 State the domain and range of the following function. **2**



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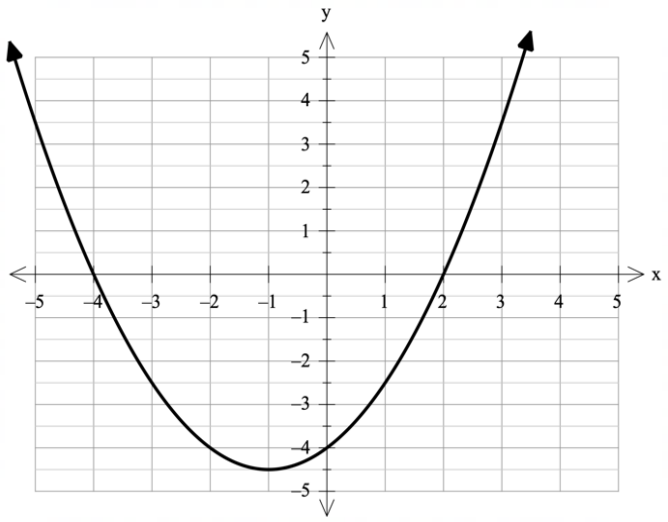
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3 Write down the equations of the following functions.

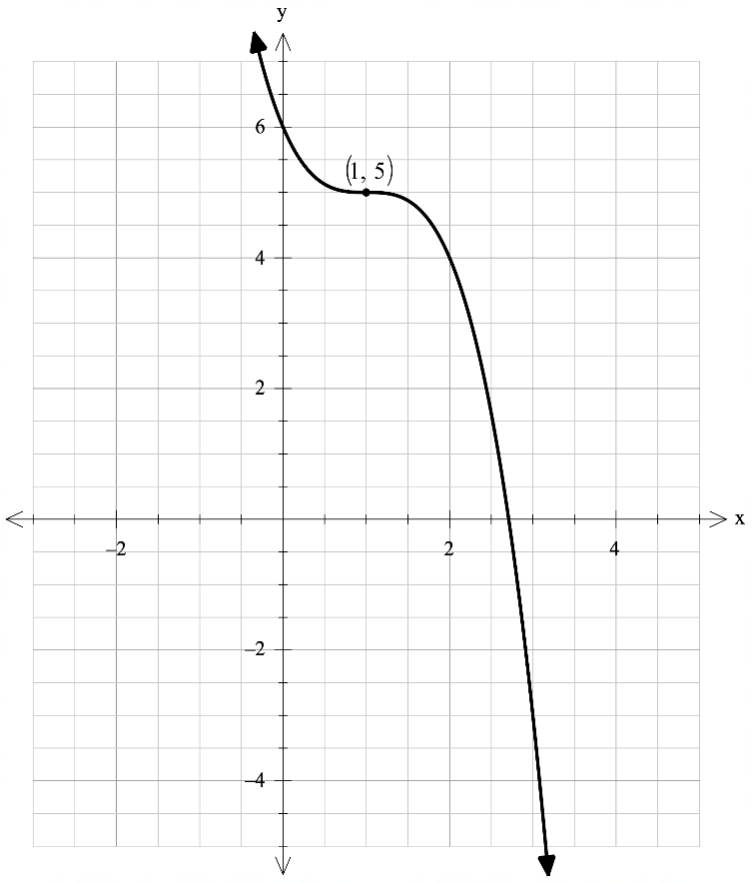
(a) **2**



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(b) **1**

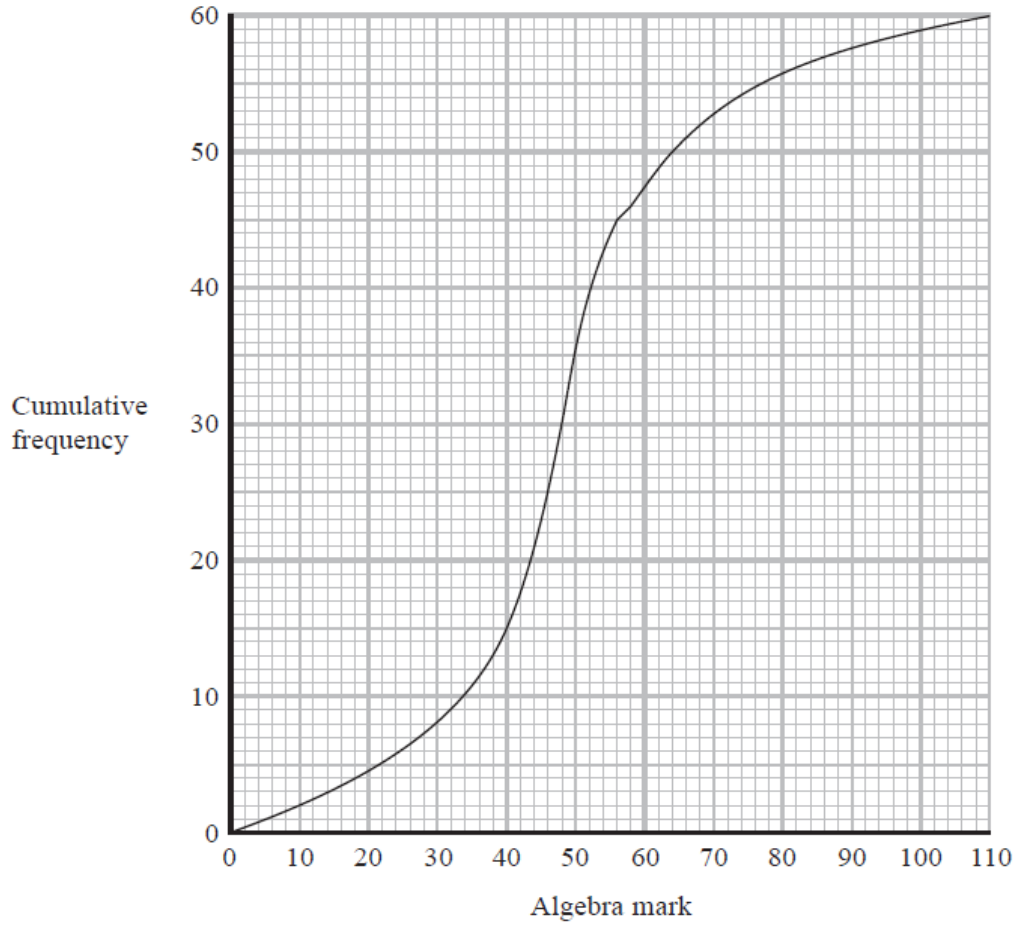


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4 A group of 60 students each sat an algebra test and a geometry test.
Each test was marked out of 110.

The ogive graph gives information about the marks gained by the 60 students in the algebra test.



(a) Use the graph to find an estimate for the median mark in the algebra test. **1**

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(b) Use the graph to find the number of students who got a mark greater than 64. **1**

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(c) Use the graph to find an estimate for the interquartile range of the marks gained in the algebra test. **2**

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This question is continued on page 9

4 (continued)

The interquartile range of the marks gained in the geometry test is 9.
Jimmy Tree says
"The students' marks are more spread out in the algebra test than in the geometry test."

(d) Is Jimmy correct, giving a reason for your answer. **1**

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To be awarded a grade A in the algebra test, a student had to gain a mark greater than 64.
Two students are to be selected at random from the 60 students in the group.

(e) By using the answer to part (b), find the probability that both of these students were awarded a grade A in the algebra test. **2**

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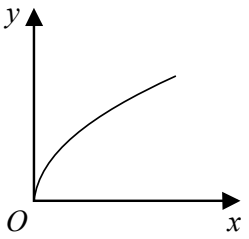
Turn over for Question 5

5 The graphs below of y against x represent four different types of proportionality: **1**

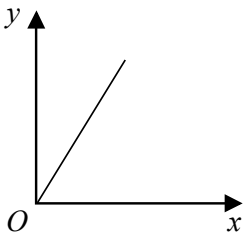
$$y \propto \frac{1}{x},\ y \propto x^2,\ y \propto \sqrt{x},\ \text{and}\ y \propto x.$$

Which graph shows $y \propto \sqrt{x}$? Circle the correct letter.

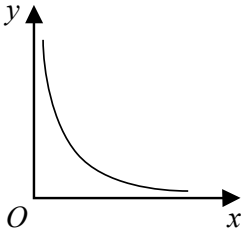
Graph A



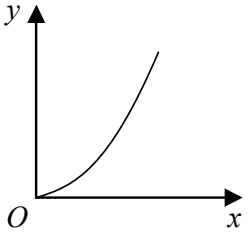
Graph B



Graph C



Graph D



End of Part B

Extra Working Page for Part B only

Part C

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10Ma**S**	A Fuller	

Mark	

Part C (15 marks)

1 **3**



An approximate equation for the lower arch of the Sydney Harbour Bridge is

$$y = -0.002(x - 250.5)^2 + 125.5,$$

where y is the height of the arch from water level and x is the horizontal distance from one end. Both x and y are in metres.

The roadway passes through the arch at two points, each 63.75 m above water level. Find the length of the road between these two points on the arch to the nearest metre.

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2 Make t the subject of $n^2 = \dfrac{4d + t^3}{t^3}$. **3**

In your answer, indicate what value(s) of n are not valid.

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3 The graph of $y = f(x)$ passes through the point $A(-6, -4)$.

The function $f(x)$ is transformed so that A goes to A'.

What are the coordinates of A'?

(a) $y = f(x) + 5$? **1**

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(b) $y = 2f(-3x)$? **3**

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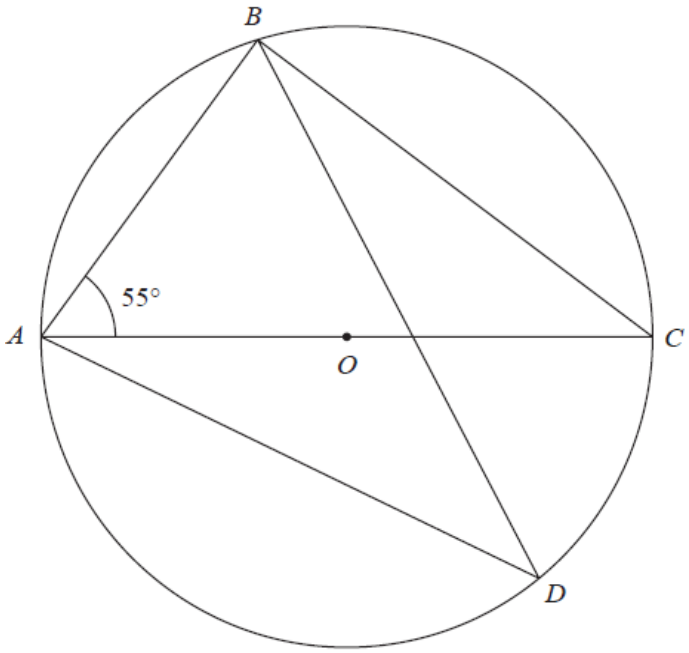
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4 In the diagram below, A, B, C, and D are points on a circle, centre O. **3**
AOC is a diameter of the circle.
$\angle BAC = 55°$.

Find $\angle ADB$, giving full reasons.



NOT TO SCALE

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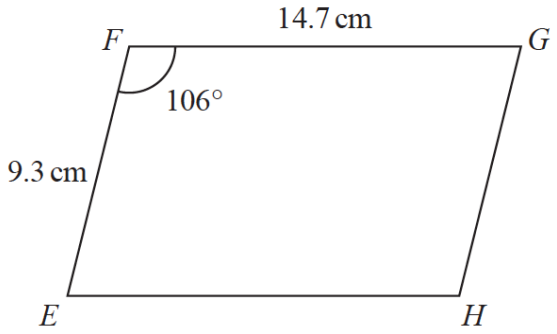
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5 The diagram shows the parallelogram *EFGH*. **2**



NOT TO SCALE

$EF = 9.3$ cm.
$FG = 14.7$ cm.
$\angle EFG = 106°$.

Calculate the area of the parallelogram, correct to 3 significant figures.

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End of Section C

Extra Working Page for Part C only

Part D

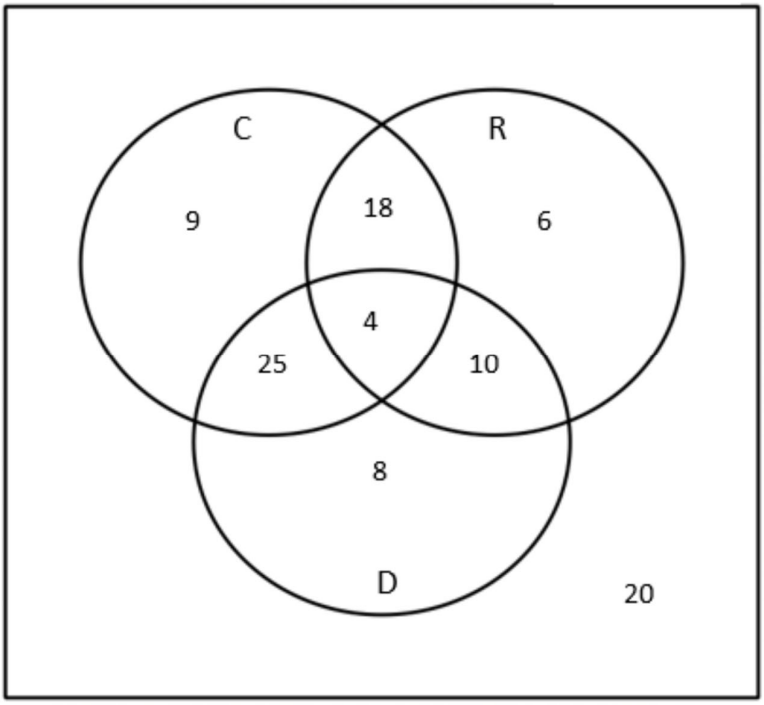
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10Ma**S**	A Fuller	

Mark	

Part D (15 marks)

1 Annee Mull's Vet Hospital did a survey of their 100 pet owners. **2**
The information they gathered about the pets they owned was placed in the Venn diagram below with *D* for dogs, *C* for cats, and *R* for rabbits.



What is the probability that a pet owner owns a rabbit, given that they own a dog?

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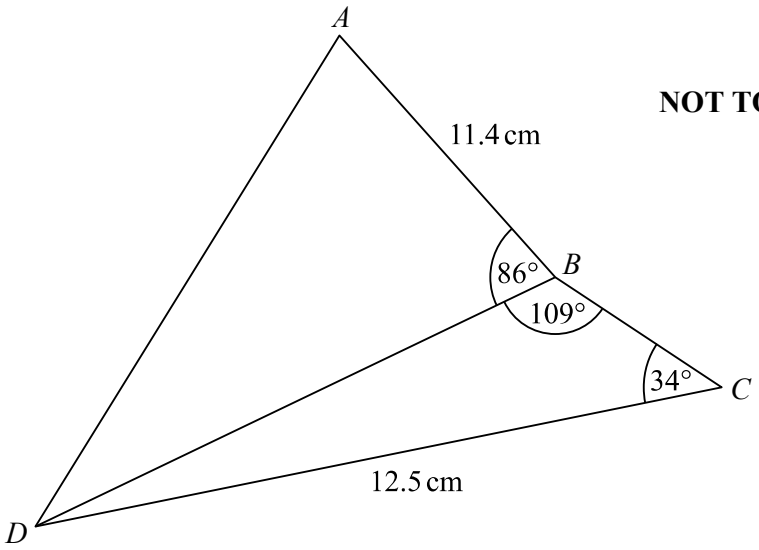
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2 Calculate the length of AD in the diagram below. **3**
Leave your answer correct to 3 significant figures.



NOT TO SCALE

3 Ikan Zave invests $6000 in a savings account for 3 years. **3**
The account pays compound interest at a rate of 1.5% p.a. for the first 2 years.

The compound interest rate changes for the third year.
At the end of three years, there is a total of $6311.16 in Ikan's account.

Calculate, to one decimal place, the compound interest rate for the third year.

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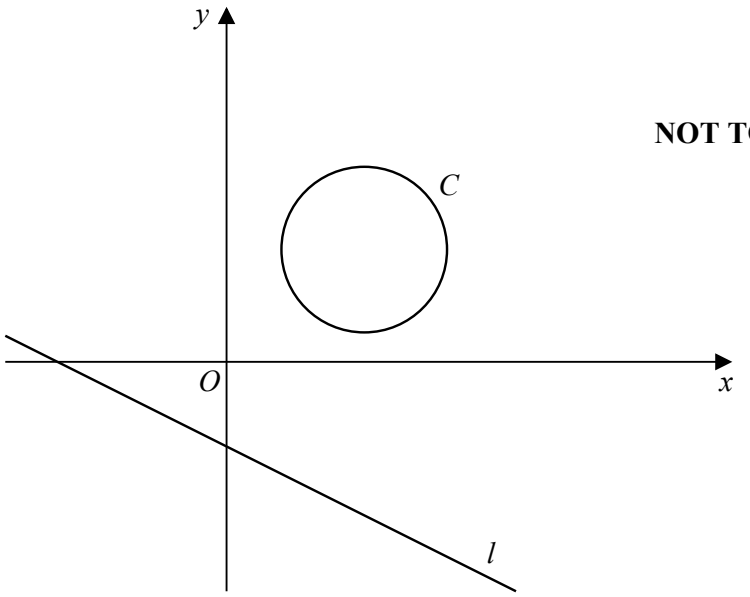
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NOT TO SCALE

The diagram above shows the circle C has equation $x^2 + y^2 - 10x - 8y + 32 = 0$, and the line l with equation $2y + x + 6 = 0$.

Find the exact value of the shortest distance between C and l.

You may use the following formula: $d = \dfrac{\left|ax_1 + by_1 + c\right|}{\sqrt{a^2 + b^2}}$

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5 Solve for x **3**

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\sqrt{x} = -11.$$

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End of Section D

Extra Working Page for Part D only

Part E

Name: _____

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor & W Beare	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	

Part E (13 marks)

1 Two functions f and g are defined as **3**

$$f(x) = 1 + \frac{1}{x} \text{ and } g(x) = \frac{x+1}{2}.$$

The function h is defined as $h(x) = f(g(x))$.

Find the inverse function h^{-1}.

2 If $\sin x = -\cos x$ and $x \in [0°, 360°]$, determine the value(s) of x. **2**

In this question you must show all stages of your working.

3 The ratio of the ages of Lynne and her father Jeff is 3 : 7. **3**
Twelve years later the ratio of their ages is 5 : 9.
Find their initial ages.

4 The mass, A kg, of algae in a small pond, is modelled by the equation **2**

$$A = pq^t,$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by

$$\log_{10} A = 0.03t + 0.5.$$

Find the values of p and q to four significant figures.

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5 Consider the polynomial **3**

$$P(x) = x^3 + ax^2 + bx - 12,$$

where a and b are constants.

It is known that $x + 1$ is a factor of $P(x)$ and that, when $P(x)$ divided by $x - 2$, the remainder is -18.

Find a and b.

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End of Section E

Extra Working Page for Part E only

Part F

Name: _____

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor & W Beare	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	

Part F (12 marks)

1 The relationship between two variables p and t is modelled by the regression line with equation

$$p = 22 - 1.1t.$$

The model is based on observations of the independent variable t, between 1 and 10.

(a) Describe the correlation between p and t implied by this model. **1**

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The variable p is measured in centimetres and the variable t is measured in days.

(b) State the units of the gradient of the regression line. **1**

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(c) Using the model, calculate the change in p over a 3-day period. **1**

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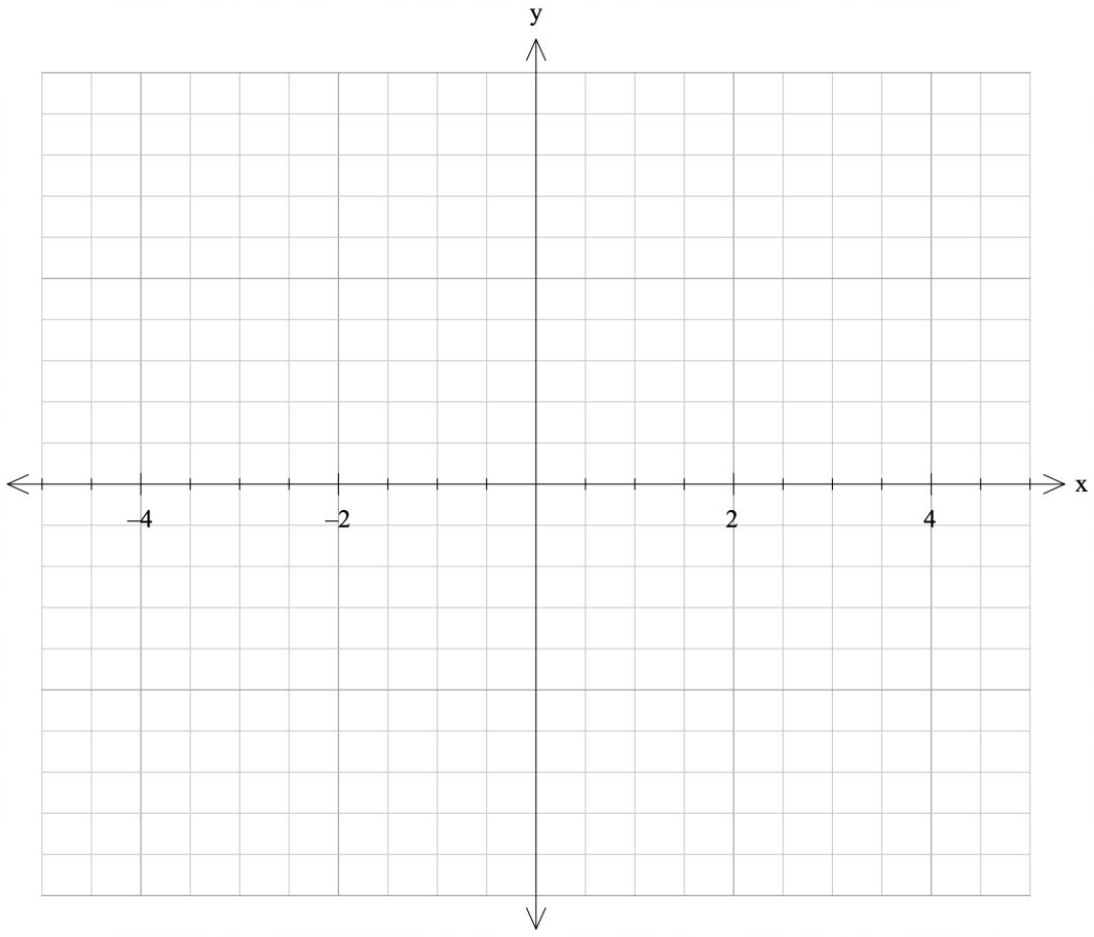
(d) Ray Grezion uses this model to estimate the value of p when $t = 19$. **1**
Comment, giving a reason, on the reliability of Ray's estimate.

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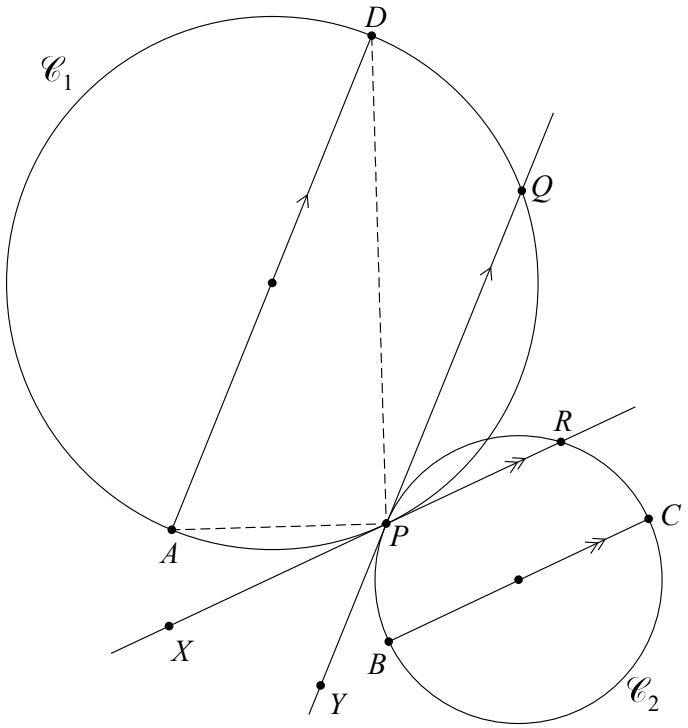
2 Sketch the function $y = 2(x - 1)^3(x + 2)^2$ on the graph below, clearly indicating all intercepts. Turning points are not required. **2**



3 The diagram shows two circles $\mathscr{C}_1$ and $\mathscr{C}_2$. The point P is one of their points of intersection. The tangent to $\mathscr{C}_2$ at P meets $\mathscr{C}_1$ at Q, and the tangent to $\mathscr{C}_1$ at P meets $\mathscr{C}_2$ at R.

The points A and D are chosen on $\mathscr{C}_1$ so that AD is a diameter of $\mathscr{C}_1$ and parallel to PQ. Likewise, points B and C are chosen on $\mathscr{C}_2$ so that BC is a diameter of $\mathscr{C}_2$ and parallel to PR.

The points X and Y lie on the tangents PR and PQ, respectively, as shown in the diagram.



(a) Show that $\angle APX = \angle DPQ$. **2**

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Question 4 continues over the page

4 (continued)

(b) Show that A, P, and C are collinear. **3**

(c) Show that $ABCD$ is a cyclic quadrilateral. **1**

End of paper

Extra Working Page for Part F only

Unused Questions

a) Without dividing, show that $(x+2)$ is a factor of $x^3 - 5x^2 - 2x + 24$

Hence, solve $x^3 + 24 = 5x^2 + 2x$ for x

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SYDNEY BOYS HIGH SCHOOL

2023

YEAR 10 STAGE 5.3
YEARLY

Sample Solutions

NOTE: This process of checking your mark is about reading the solutions and the comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.

Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen, and/or whiteout, may not be able to appeal.

Part A (15 marks)

1 Solve $(x + 7)(8 - 2x) = 0$. **2**

$8-2x=0 \rightarrow 8=2x$

$x=-7$ or $x=4$

2 Factorise $3m^3 + 24$. **2**

$=3(m^3+8)$

$=3(m^3+2^3)$

$=3(m+2)(m^2-2m+4)$

SOAP:

$(x^3+y^3)=(x+y)(x^2-xy+y^2)$

3 Given that a, b, and c are integers, express $3x^2 + 12x + 19$ in the form $a(x + b)^2 + c$. **2**

$=3(x^2+4x)+19$ completing squares.

$=3(x^2+4x+4)-4\times3+19$

$=3(x+2)^2+7$ where $a=3, b=2, c=7$

4 Given that $\left(\sqrt[3]{\frac{1}{x}}\right)^4 = x^n$, find the value of n. **1**

make x the subject on LHS: $\left(\frac{1}{x}\right)^{\frac{4}{3}} = x^{-\frac{4}{3}}$

$n=-\frac{4}{3}$

5 Expand and simplify $n(n-4)(n+4)$. Diff. of 2 cubes **2**

$=n(n^2-16)$

$=n^3-16n$

6 Express $\frac{3}{x} - \frac{x-2}{2x} + \frac{1}{4}$ as a single fraction in its simplest form. **2**

Method ①: LCD $= 4x$

$\frac{3}{x}\times\frac{4}{4}$, $\frac{x-2}{2x}\times\frac{2}{2}$, $\frac{1}{4}\times\frac{x}{x}$

$\frac{12-2(x-2)+x}{4x} = \frac{12-2x+4+x}{4x}$

Please see back for method ② $= \frac{16-x}{4x}$

7 Simplify $\dfrac{2m^4n^2\times(3mn^{-3})^2}{(5n^{-3}m^{-1})^2}$ 3

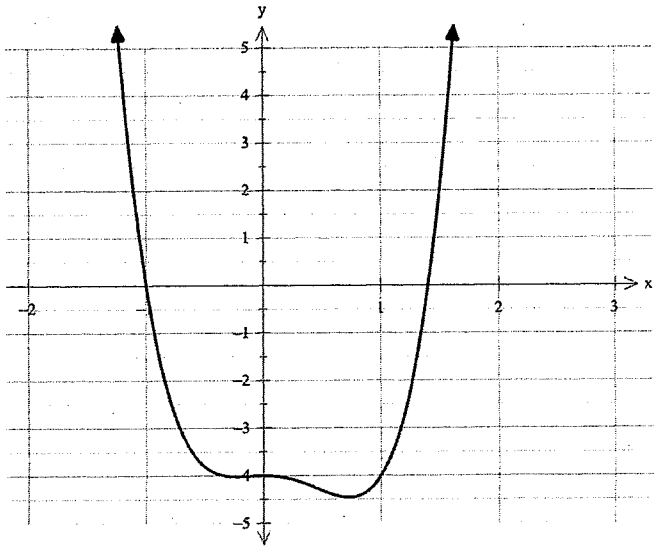
$= \dfrac{2m^4n^2 \times 9m^2n^{-6}}{25n^{-6}m^{-2}}$

$= \dfrac{18m^4n^2 \times m^2 \times n^6 \times m^2}{25n^6}$

$= \dfrac{18m^8n^2}{25}$

Comment: ① the 2 square terms were expanded out badly
② the negative powers were not placed correctly. Easy to move from num ↔ den. and remove (-)

8 State if the graph below is that of a function, giving a reason for your statement. 1



Yes. It is a function.
It passes the vertical line test (VLT)
which proves it is a one-to-one or many-to-one function.

End of Section A

Extra Working Page for Part A only

(6) Method ②: the right way to do the non-LCD way:

$$\frac{3}{x} - \frac{x-2}{2x} + \frac{1}{4}$$

$$= \frac{(3\times 4\times 2x) - [4x(x-2)] + [2x(x)]}{x \times 2x \times 4}$$

$$= \frac{24x - 4x^2 + 8x + 2x^2}{8x^2}$$

$$= \frac{32x - 2x^2}{8x^2}$$

$$= \frac{2x(16-x)}{2x(4x)}$$

$$= \frac{16-x}{4x}$$

Comment: students made lots of errors using this method for the following reasons:

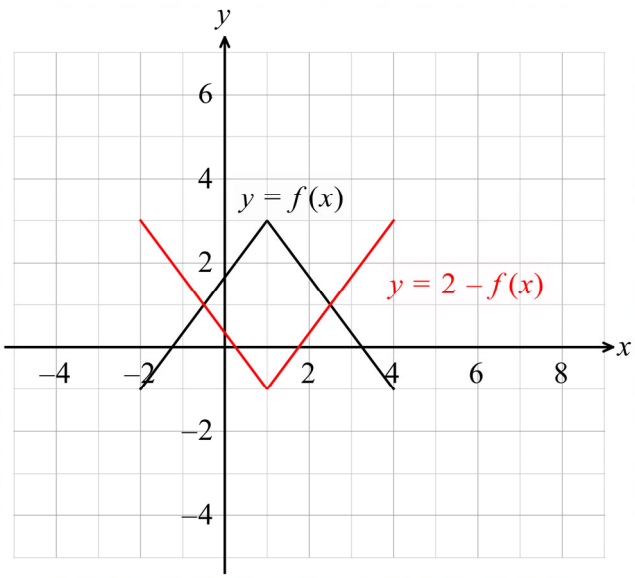
① not flipping the −2 in the 2nd fraction to +8(x) because a (−) × (−) should = (+)

② not factorising the second last line to cancel properly.

2023 YR10 Yearly Exam Part B Solutions and Comments

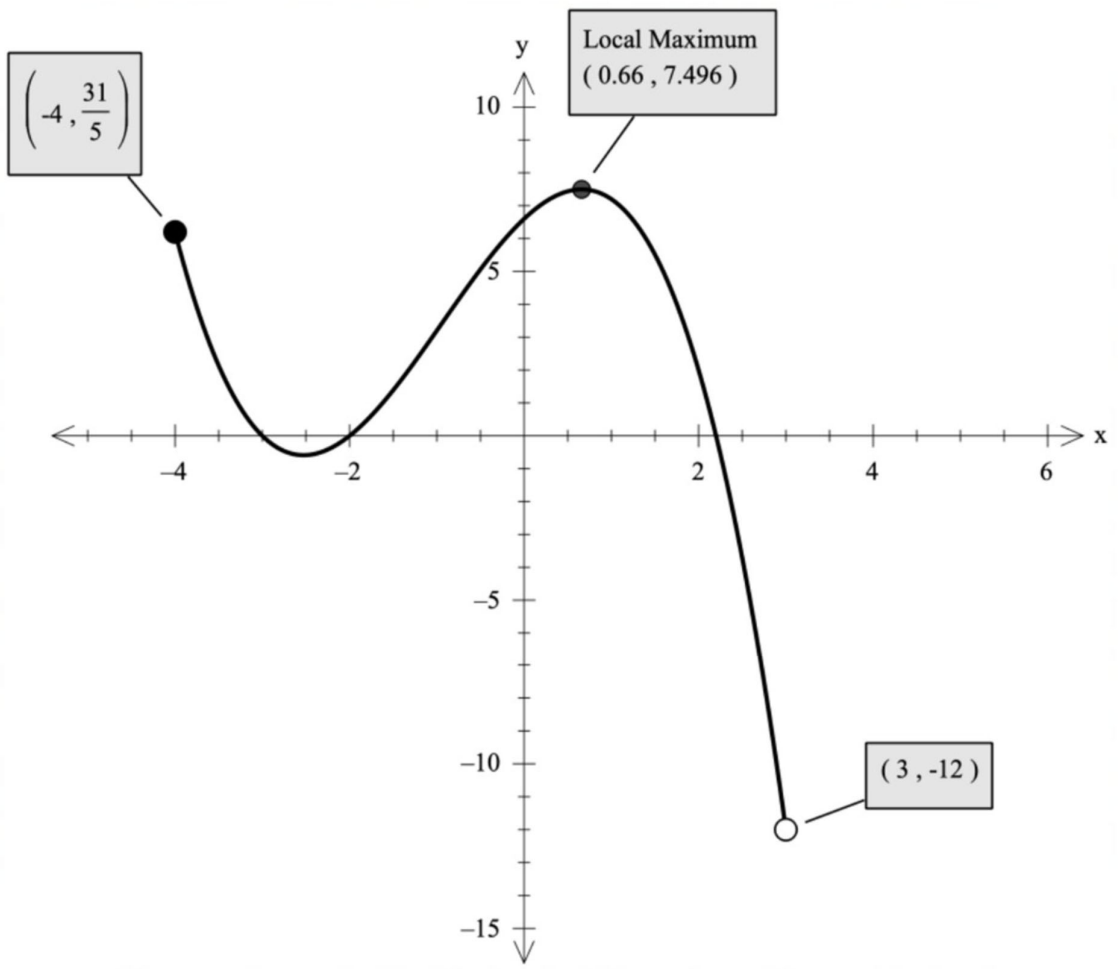
1. Below is the graph of $y = f(x)$. 2

On the grid, draw the graph of $y = 2 - f(x)$.



Solution	Comment(s)
Consider $g(x)$ and $h(x)$ such that: $g(x) = -f(x)$ $h(x) = g(x) + 2$ $= -f(x) + 2$ $= 2 - f(x)$ Hence, the graph of $y = h(x)$ is a reflection of $f(x)$ about the x axis, followed by a translation 2 units up. The resultant graph is shown above in red.	No half marks. 1 mark for the correct $y = -f(x)$ **Common error(s):** • Switching the order of transformation around. • Not using a ruler • Incorrect shape of the graph (horizontal dilation)

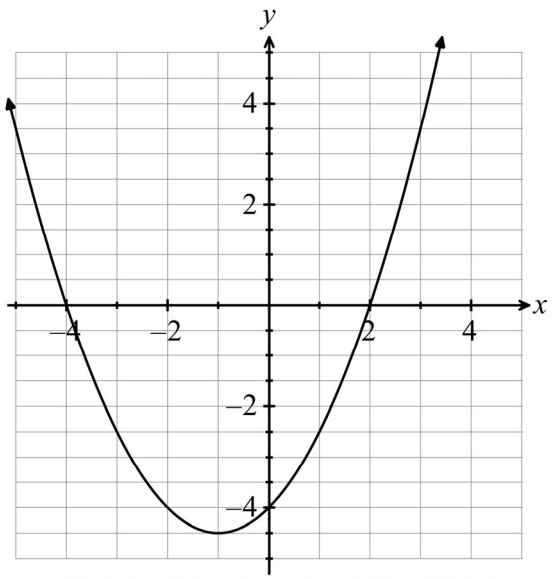
2. State the domain and range of the following function. 2



Solution	Comment(s)
The open circle at $(3, -12)$ means that the graph approaches it, but never reaches it. Hence, the domain of the function is $-4 \leq x < 3$. For the range: $\frac{31}{5} = 6.5$ < 7.496 Hence, the range is $-12 < y \leq 7.496$.	Alternatively, using interval notation: • The domain is $x \in [-4, 3)$. • The range is $y \in (-12, 7.496]$. ~ **Common error(s):** • Not addressing the fact that $(3, -12)$ is an open circle. • Not checking that $\frac{31}{5} < 7.496$. 0.5 per correct boundary value with inequality symbol.

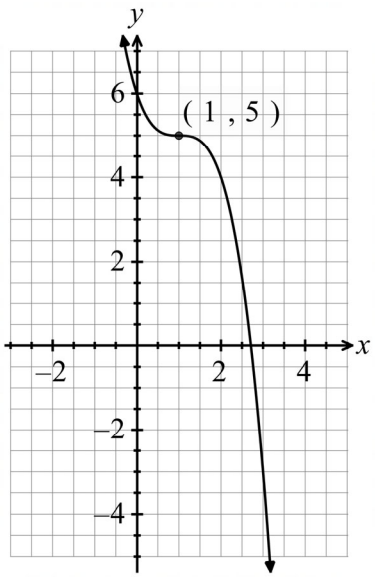
3. Write down the equations of the following functions.

A. 2



Solution	Comment(s)
The graph is a concave up parabola with x intercepts at 2 and –4, so for some positive number a, the equation is $y = a(x-2)(x+4)$. Substituting the vertex $(-1,-4.5)$ to solve for a: $-4.5 = a(-1-2)(-1+4)$ $= -9a$ $a = \frac{1}{2}$ Hence, the equation is $y = \frac{1}{2}(x-2)(x+4)$. Or, $y = \frac{1}{2}(x+1)^2 - \frac{9}{2}$	Alternatively, the graph is a concave up parabola with a vertex $(-1,-4.5)$, so for some positive number a, the equation is $y = a(x+1)^2 - 4.5$. Substituting the x intercept $(2,0)$ to solve for a: $0 = a(2+1)^2 - 4.5$ $= 9a - 4.5$ $a = \frac{1}{2}$ Hence, the equation is $y = \frac{1}{2}(x+1)^2 - 4.5$. ~ Award 1 mark for $y = (x-2)(x+4)$ or $y = (x+1)^2 - \frac{9}{2}$, leaving $\frac{1}{2}$ out. Deduct $\frac{1}{2}$ mark for incorrect concavity. **Common error(s):** • Not recognising that $y = (x-2)(x+4)$ will not give the correct vertex shown in the graph. • Students MUST remember any equations will have the left- and right-hand side. No marks deducted this time for only writing an expression.

B. 1

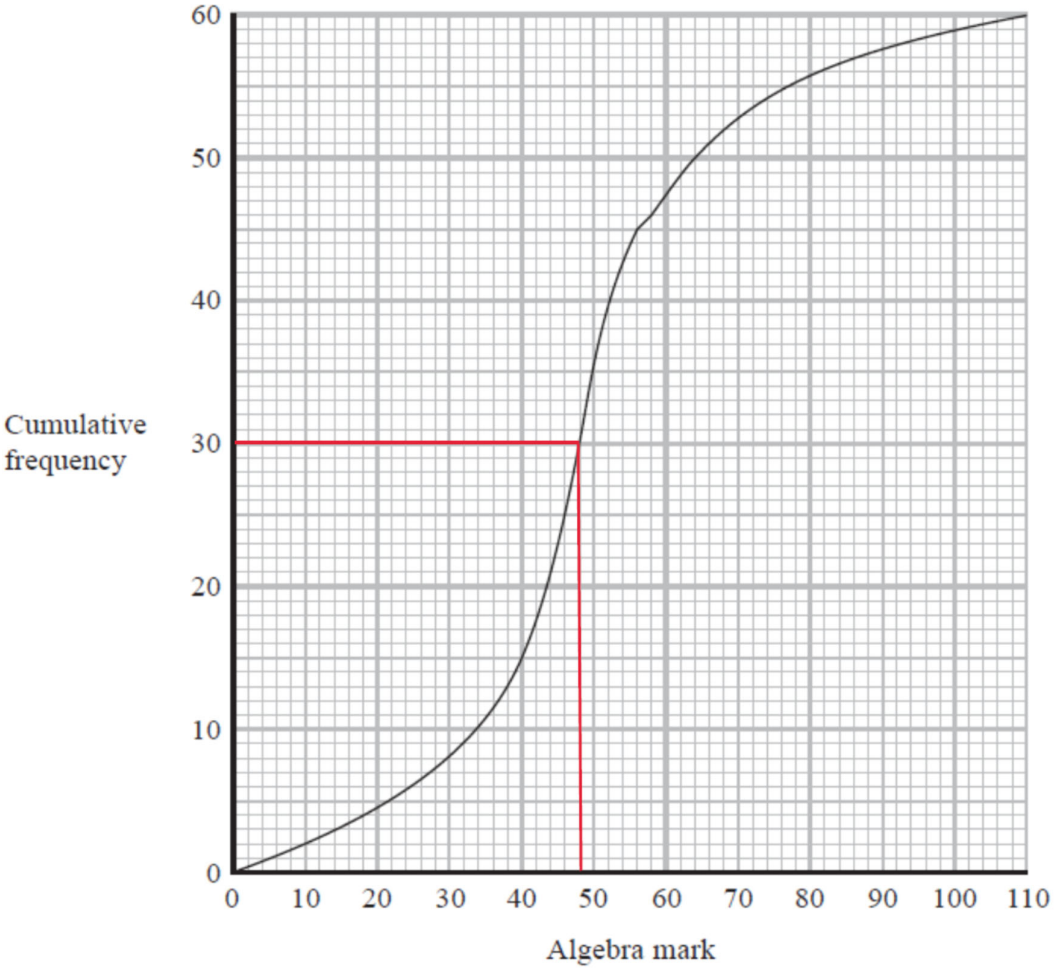


Solution	Comment(s)
The graph is a cubic with an inflection point at $(1, 5)$. Furthermore: • $f(x) \to \infty$ as $x \to -\infty$. • $f(x) \to -\infty$ as $x \to \infty$. This means the equation is $y = -a(x-1)^3 + 5$ for some positive number a. Substituting the y intercept $(0, 6)$ to solve for a: $6 = -a(0-1)^3 + 5$ $= a + 5$ $a = 1$ Hence, the equation is $y = -(x-1)^3 + 5$.	Award $\frac{1}{2}$ mark ONLY for not recognising negative cubic. **Common Error(s):** Students need to understand $y = -(x-1)^3 + 5$ is not the same as $y = -(x^3 - 1) - 5$ and $y = (-x-1)^3 + 5$. No marks awarded for these equations.

4. A group of 60 students each say an algebra test and a geometry test.

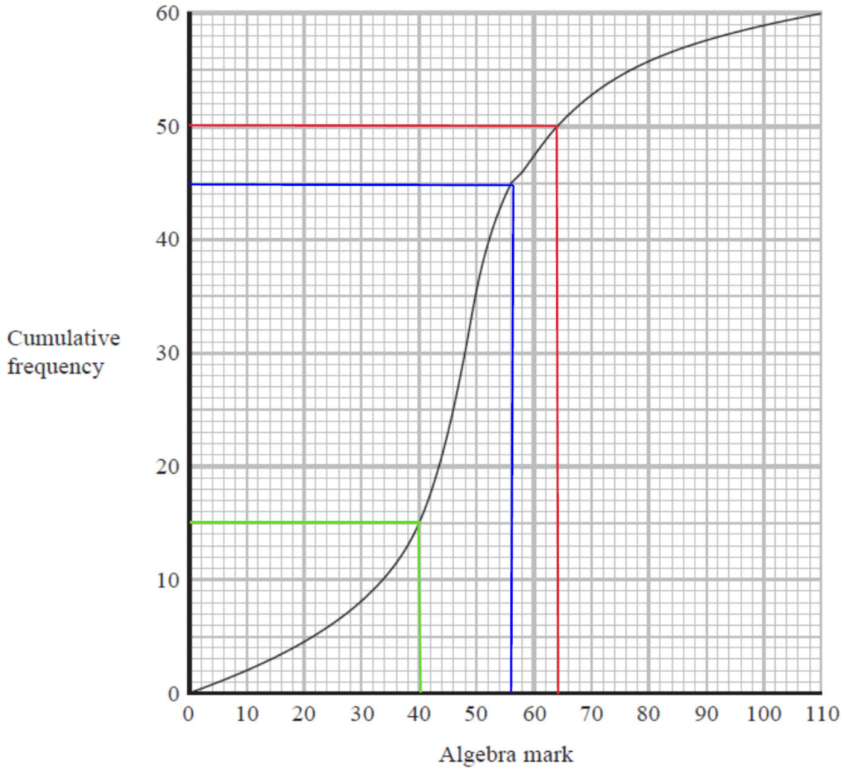
Each test was marked out of 110.

The ogive graph gives information about the marks gained by the 60 students in the algebra test.



A. Use the graph to find an estimate for the median mark in the algebra test. 1

Solution	Comment(s)
There are 60 students, so the median mark occurs at $c.f. = 30$. From the graph, the mark is 48.	No half marks. Students were not penalised for taking the average of the scores at $c.f. = 30$ and $c.f. = 31$ to find the median score, although this isn't necessary for an ogive. **Common error(s):** • Not noticing that each increment on the horizontal axis represents 2 units.

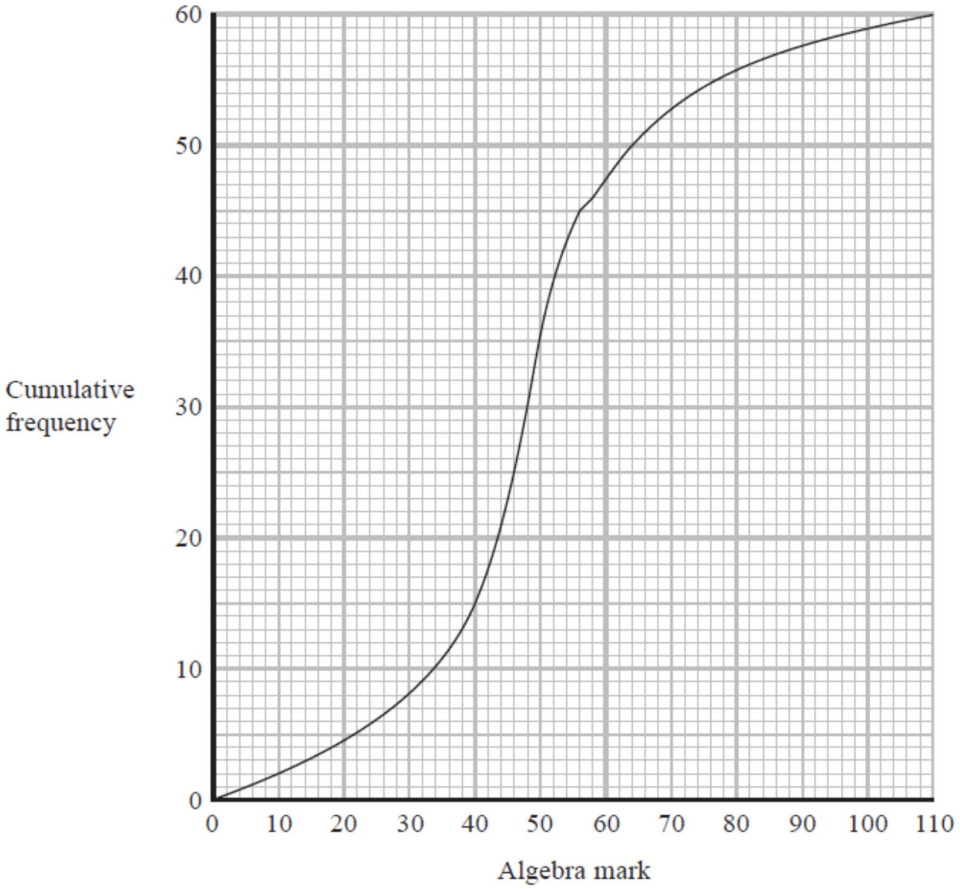


B. Use the graph to find the number of students who got a mark greater than 64. 1

Solution	Comment(s)
From the red line on the graph, a mark of 64 occurs at $c.f. = 50$. Hence, 10 students got a mark greater than 64.	No half marks. Allowed for ±1 error from estimation from the graph. **Common error(s):** • Giving an answer of 50, which corresponds to the number of students who got 64 or lower.

C. Use the graph to find an estimate for the interquartile range of the marks gained in the algebra test. 2

Solution	Comment(s)
There are 60 students, so: • Q_1 occurs at $c.f. = 15$, represented by the green line, giving $Q_1 = 40$. • Q_3 occurs at $c.f. = 45$, represented by the blue line, giving $Q_3 = 56$. Hence, the IQR is $56 - 40 = 16$.	Again, taking the average of the scores at $c.f. = 15$ and $c.f. = 16$ to find Q_1 wasn't necessary and similarly for Q_3. Allowed for ±1 error from estimation from the graph. Award 1 mark for recognising 40 or 56 or a bald answer of 16 without working. **Common error(s):** • Not noticing that each increment on the horizontal axis represents 2 units.



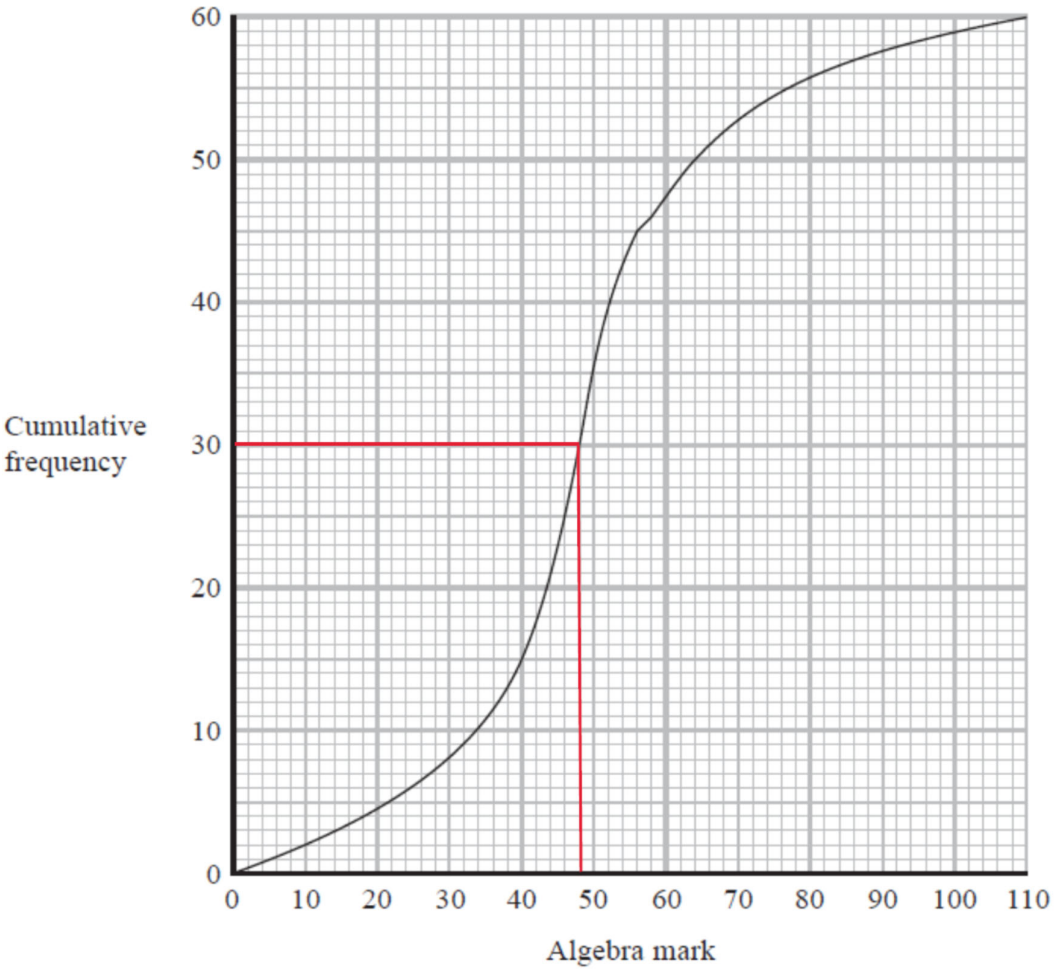
The interquartile range of the marks gained in the geometry test is 9.

Jimmy Tree says:

"The students' marks are more spread out in the algebra test than in the geometry test."

D. Is Jimmy correct? Give a reason for your answer. 1

Solution	Comment(s)
Yes, as the IQR is a measure of spread.	No half marks. Students who gave no reason or invalid reason without mentioned IQR could not score any marks irrespective of their choice of yes or no. Students seems to have difficulty understanding spread and IQR, or they did not understand the statement of "more spread out in the algebra test than in the geometry test".



To be awarded a grade A in the algebra test, a student had to gain a mark greater than 64.

Two students are to be selected at random from the 60 students in the group.

E. By using the answer to Part B, find the probability that both of these students were awarded a grade A in the algebra test. 2

Solution	Comment(s)
There are 10 out of 60 students who got a mark greater than 64. Hence, the probability of both students getting A is: $P(AA) = \frac{10}{60} \times \frac{9}{59}$ $= \frac{3}{118}$	Award $\frac{1}{2}$ mark for $\frac{n}{60} \times \frac{n-1}{60}$ or $\frac{10}{n} \times \frac{10}{n-1}$ Award 1 mark for $\frac{a}{b} \times \frac{a-1}{b-1}$. Full mark awarded for consistent mistake (CM) from Part (b) with the correct logic. **Common error(s):** • Not reducing the second fraction by 1 due to the reduced sample space.

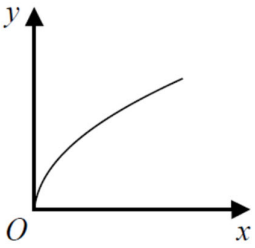
5. The graph below of y against x represent 4 different types of proportionality. 1

$$y \propto \frac{1}{x},\ y \propto x^2,\ y \propto \sqrt{x} \text{ and } y \propto x$$

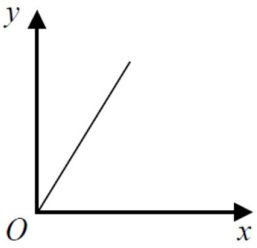
Which graph shows $y \propto \sqrt{x}$?

Circle the correct letter.

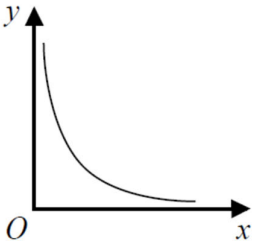
Graph A



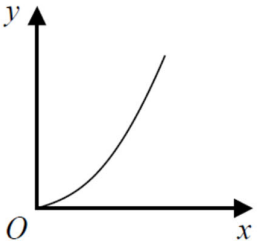
Graph B



Graph C



Graph D



Solution	Comment(s)
If $y \propto \sqrt{x}$, then $y = k\sqrt{x}$ for some constant k. This is the equation of half of a sideways parabola with domain $x \geq 0$ and range $y \geq 0$. The graph that best represents this is Graph A.	No half marks. Students should be able to recognise the graphs of $y = \frac{k}{x}$, $y = kx^2$ and $y = kx$. As such, by process of elimination, students should be able to answer this question correctly irrespective of whether they know the graph of $y = k\sqrt{x}$.

Part C Solutions — 15 marks

1 **3**



An approximate equation for the lower arch of the Sydney Harbour Bridge is

$$y = -0.002(x - 250.5)^2 + 125.5,$$

where y is the height of the arch from water level and x is the horizontal distance from one end. Both x and y are in metres.

The roadway passes through the arch at two points, each 63.75 m above water level. Find the length of the road between these two points on the arch to the nearest metre.

$63.75 = -0.002(x-250.5)^2 + 125.5$

$0.002(x-250.5)^2 = 61.75$

$(x-250.5)^2 = 30875$

$x - 250.5 = \pm 5\sqrt{1235}$

$x = 250.5 \pm 5\sqrt{1235}$

$d = 250.5 + 5\sqrt{1235} - \left(250.5 - 5\sqrt{1235}\right)$

$= 10\sqrt{1235}$

$\approx 351\ m$

Generally well done.

To get full marks, students need to:

- *substitute the correct y-value into the equation,*
- *solve the quadratic equation,*
- *find the length of the road.*

Common errors:

- *Only had* $250.5 + 5\sqrt{1235}$ *as the solution for the quadratic equation. Some students did not notice that* $250.5 - 5\sqrt{1235}$ *is positive.*
- *Many students did not read the question carefully. Thus, they only solved the quadratic equation and did not attempt to find the length of the road.*
- *Careless calculation mistakes happened because many students expanded the bracket and used the quadratic formula to solve the equation.*

2 Make t the subject of $n^2 = \dfrac{4d + t^3}{t^3}$. **3**

In your answer, indicate what value(s) of n are not valid.

$n^2 t^3 = 4d + t^3$

$n^2 t^3 - t^3 = 4d$

$\left(n^2 - 1\right) t^3 = 4d$

$\therefore t = \sqrt[3]{\dfrac{4d}{n^2 - 1}}$

$n^2 - 1 \neq 0$

$\therefore n \neq \pm 1$

Generally well done.

Common errors:

- *Students did not take the cube root, left their answers in the form of* t^3
- $t = \sqrt[3]{\dfrac{4d}{n-1}}$, $t = \sqrt[3]{\dfrac{4d + t^3}{n^2}}$
- *Only had one invalid value of* n : $n \neq 1$
- $n^2 - 1 > 0$

3 The graph of $y = f(x)$ passes through the point $A(-6, -4)$.

The function $f(x)$ is transformed so that A goes to A'.

What are the coordinates of A'?

(a) $y = f(x) + 5$? **1**

$y = f(x) + 5 = -4 + 5 = 1$

$\therefore A'(-6, 1)$

Well done.
Some students did not attempt this question.
No half marks.
Common errors: $(-6, -1), (-1, 1)$

(b) $y = 2f(-3x)$? **3**

x-coordinate:

$-3x = -6$

$\therefore x = 2$

y-coordinate:

$y = 2f(-3x)$

$x = 2: \Rightarrow y = 2f(-6) = 2 \times (-4) = -8$

$\therefore A'(2, -8)$

Poorly done.

To get full marks, students were required to demonstrate that:

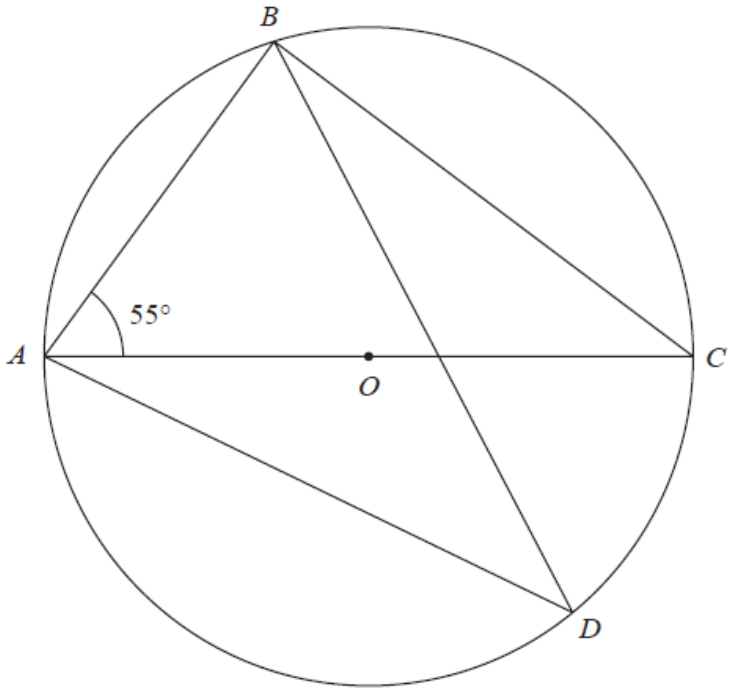
- *the sign of x-coordinate had changed,*
- *the x-value was one third of 6,*
- *the y-value had doubled.*

A partial correct answer, without working out, was not sufficient to demonstrate their understanding and so get the corresponding mark.

Common errors: $x = 18, y = -2, -24$

4 In the diagram below, A, B, C, and D are points on a circle, centre O. **3**
AOC is a diameter of the circle.
$\angle BAC = 55°$.

Find $\angle ADB$, giving full reasons.



NOT TO SCALE

$\angle ABC = 90°$ (right $\angle$ in semi-circle)

$\angle BCA = 180° - \angle BAC - LABC = 35°$ ($\angle$ sum of ΔABC)

$\angle ADB = \angle BCA = 35°$ ($\angle$s in the same seg.)

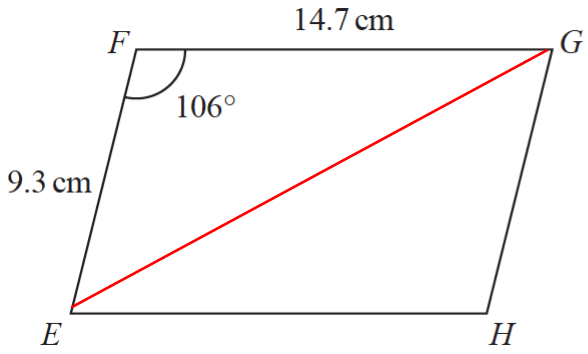
Well done.

Marks were deducted because some students did not provide clear and specific reasons.

Few students assumed that the intersection point of AC and BD is the centre O.

Marks were not awarded if students used the incorrect method to get the correct answer.

5 The diagram shows the parallelogram *EFGH*. **2**



NOT TO SCALE

$EF = 9.3$ cm.
$FG = 14.7$ cm.
$\angle EFG = 106°$.

Calculate the area of the parallelogram, correct to 3 significant figures.

$$S_{EFGH} = 2 \times S_{\Delta EFG}$$
$$= 2 \times \frac{1}{2} \times EF \times FG \times \sin 106°$$
$$= 9.3 \times 14.7 \times \sin 106°$$
$$\approx 131 cm^2$$

Well done.

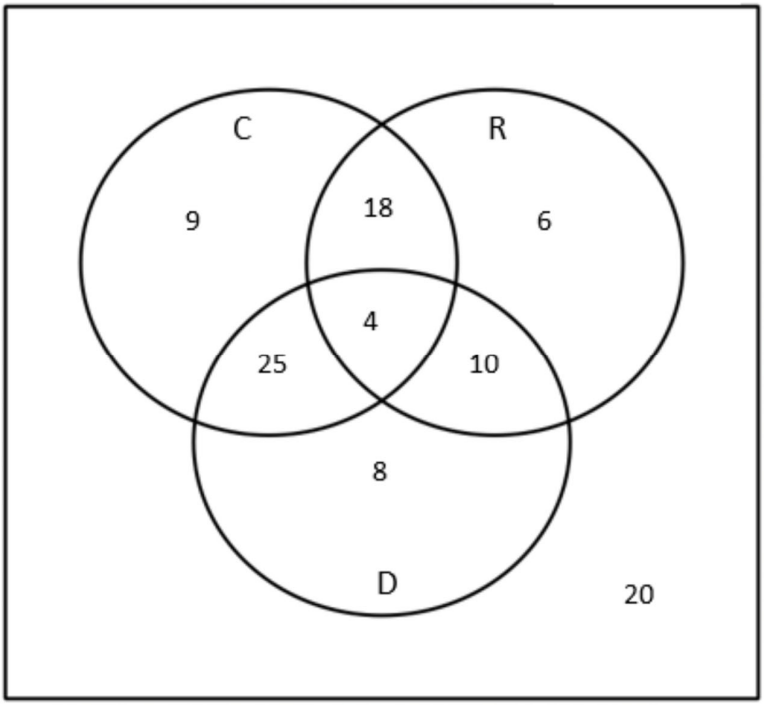
Various methods were accepted, but the quickest method was the area formula.

Some careless calculation mistakes.

End of Section C Solutions

Section D SOLUTIONS 15 Marks

1 Annee Mull's Vet Hospital did a survey of their 100 pet owners. **2**
The information they gathered about the pets they owned was placed in the Venn diagram below with *D* for dogs, *C* for cats, and *R* for rabbits.



What is the probability that a pet owner owns a rabbit, given that they own a dog?

The probability of "owns a rabbit, given that they own a dog" can be expressed as:

$$\begin{aligned} \mathrm{P}(R \mid D) &= \frac{\mathrm{P}(R \cap D)}{P(D)} \\ &= \frac{10+4}{10+4+25+8} \\ &= \frac{14}{47} \end{aligned}$$

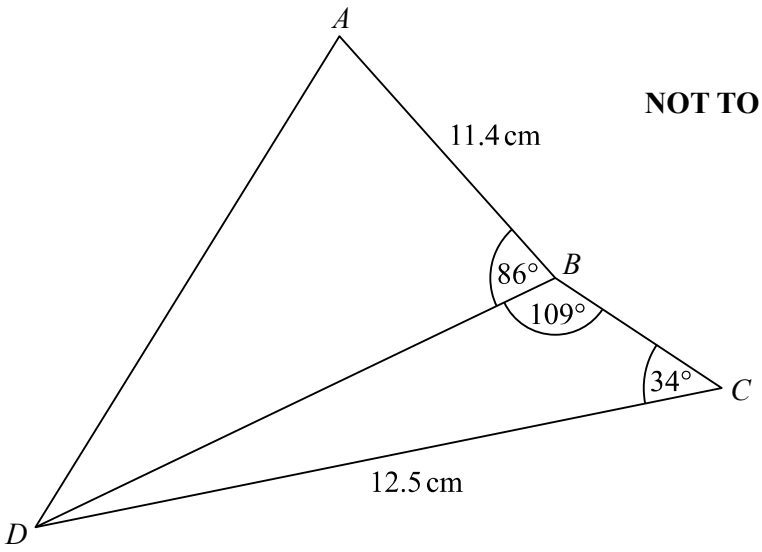
Common errors:

- Ans: 10/47, from not including the intersection of all three sets as part of P($R \cap D$).
- Ans: 14/100, from not recognising that "given that they own a dog" shrinks the sample space to be "owns a dog".

Generally done well.

Students are reminded to read the question and diagram carefully.

2 Calculate the length of AD in the diagram below. **3**
Leave your answer correct to 3 significant figures.



To solve this problem, students needed to apply the sine rule to find BD, then the cosine rule to find AD, and finally present their solution to 3 significant figures.

Sine Rule: $\dfrac{BD}{\sin 34°} = \dfrac{12.5}{\sin 109°}$

$$BD = \frac{12.5 \sin 34°}{\sin 109°}$$

$$\approx 7.393$$

Cosine Rule: $AD^2 = AB^2 + BD^2 - 2.AB.BD.\cos(86°)$

$$AD^2 \approx 11.4^2 + 7.39^2 - 2 \times 11.4 \times 7.39 \times \cos(86^\circ)$$
$$\approx 172.85$$

$$AD \approx \sqrt{172.85}$$
$$\approx 13.1472...$$
$$= 13.1 \qquad \text{(3 sig. fig.)}$$

Common errors:

- Ans: 13.2, due to rounding too early (always use exact values where possible).

- Not having the calculator in degrees mode (Ans: 11.8 for grad, Ans: 16.3 for rad).

Done very well.

Most students recognised the necessary steps and followed through.

A reminder to students that writing down the relevant formulas for a question is a way to demonstrate understanding and is marked accordingly.

3 Ikan Zave invests $6000 in a savings account for 3 years. **3**
The account pays compound interest at a rate of 1.5% p.a. for the first 2 years.

The compound interest rate changes for the third year.
At the end of three years, there is a total of $6311.16 in Ikan's account.

Calculate, to one decimal place, the compound interest rate for the third year.

To find the compound interest for the third year, students needed to determine the account balance at the end of the second year, and then use the interest earned in the third year to determine the rate.

$$A_2 = 6000(1+0.015)^2$$
$$= 6181.35$$

$A_3 = A_2(1+r)$ (where r is the new interest rate)

$$r = \frac{A_3}{A_2} - 1$$
$$= \frac{6311.16}{6181.35} - 1$$
$$= 0.021000267...$$
$$= 2.1\% \quad (1 \text{ d.p.})$$

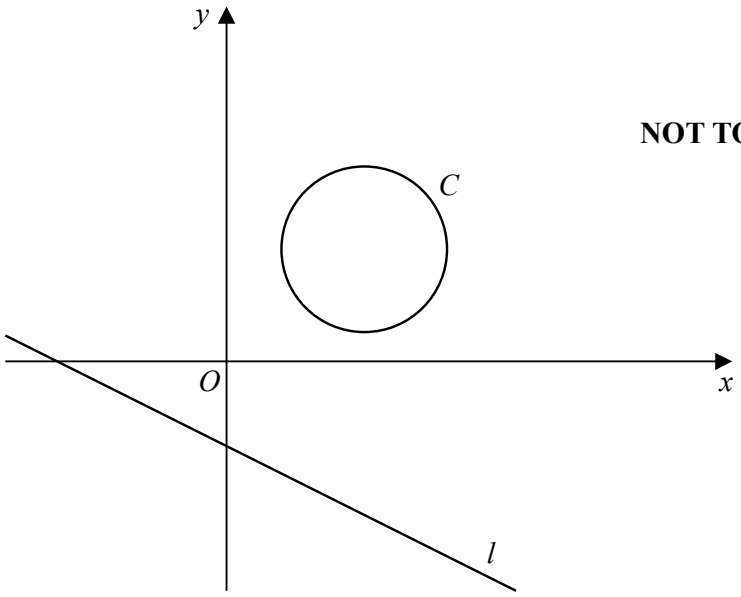
Common errors:

- Ans: 2.2%, by dividing the interest earned by 6000 instead of 6181.35.
- Swapping the numerator and denominator when finding 'r' (while this method also gives 2.1% as the final answer, it is an incorrect approach and was marked as such).
- Showing that 2.1% satisfied the equation for A_3 instead of actually solving for 'r' is not complete working (students were asked to "calculate", not "show").

Generally done very well.

Some students opted to calculate A_1 and A_2 individually using simple interest, which was fine.

4



NOT TO SCALE

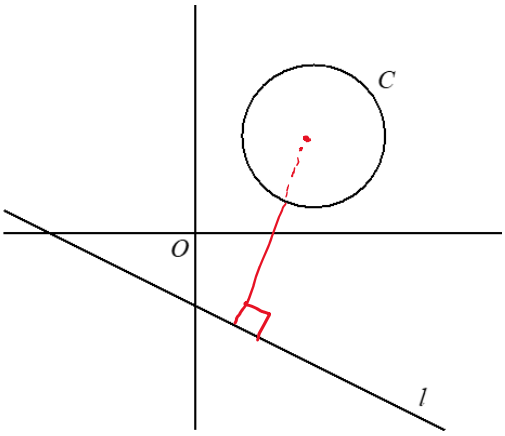
The diagram above shows the circle C has equation $x^2 + y^2 - 10x - 8y + 32 = 0$, and the line l with equation $2y + x + 6 = 0$.

Find the exact value of the shortest distance between C and l.

You may use the following formula: $d = \dfrac{\left|ax_1 + by_1 + c\right|}{\sqrt{a^2 + b^2}}$

Solution over the page

4. To find the exact value required, students needed to recognise that the shortest distance was the perpendicular distance from the centre of the circle to the line, minus the radius.



To determine the perpendicular distance, students needed to find the coordinates (x_1, y_1) for the centre of the circle and the coefficients a, b, and c of the line, and substitute those values into the given formula.

$$x^2 + y^2 - 10x - 8y + 32 = 0$$
$$x^2 - 10x + 25 + y^2 - 8y + 16 = 25 + 16 - 32$$
$$(x-5)^2 + (y-4)^2 = 9$$

Therefore, the circle has a centre of (5, 4) and a radius of 3 units.

This gives us: $a = 1, b = 2, c = 6, x_1 = 5, y_1 = 4$.

Substituting these values into the distance formula gives:

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$
$$= \frac{|1 \times 5 + 2 \times 4 + 6|}{\sqrt{1^2 + 2^2}}$$
$$= \frac{19}{\sqrt{5}}$$

Therefore, the shortest distance from the circle to the line is $\frac{19}{\sqrt{5}} - 3$ units.

Common errors:

- Incorrectly matching the coefficients of the line to values ‘a’ and ‘b’.
- Not subtracting the radius from the perpendicular distance to get a shortest distance.
- Using the centre coordinates for ‘a’ and ‘b’.
- Incorrectly assuming that the lowest point on the circle (5, 1) was closest to the line.

Students generally did very well to find the centre and radius of the circle, and most students recognised that the shortest distance involved taking a perpendicular distance.

Revision of how the perpendicular distance formula is applied is recommended.

5 Solve for x **3**

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\sqrt{x} = -11.$$

To solve for x, students needed to apply the logarithm laws to combine the left-hand side into a single expression and then convert into exponential form and simplify. The order in which the rules were applied did not affect the final result.

$$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11$$

$$\log_3\left(\frac{x^2}{81}\right)+\log_3\left(\sqrt{x}^3\right)=-11 \qquad \text{(multiplication law)}$$

$$\log_3\left(\frac{x^2}{81}\times\sqrt{x}^3\right)=-11 \qquad \text{(addition law)}$$

$$\log_3\left(\frac{x^{\frac{7}{2}}}{81}\right)=-11$$

$$\frac{x^{\frac{7}{2}}}{81}=3^{-11} \qquad \text{(converting logarithms to exponentials)}$$

$$x^{\frac{7}{2}}=3^{-11}.81$$

$$=3^{-11}.3^4$$

$$=3^{-7}$$

$$x=\left(3^{-7}\right)^{\frac{2}{7}}$$

$$=3^{-2}$$

$$=\frac{1}{9},0.\dot{1}$$

Common errors:

- Incorrectly applying one or more of the logarithm laws.
- Simplifying $\sqrt{x}^3$ as x
- Incorrectly adding the exponents 2 and $\frac{3}{2}$ together to get $\frac{5}{2}$.
- Multiplying the exponents 2 and $\frac{3}{2}$, instead of adding them.
- Algebraic errors when multiplying/dividing both sides of the equation by some term.

Not done particularly well.

Many students made multiple algebraic errors (either applying the logarithm laws incorrectly or making a silly mistake).

Higher performing students tended to use the alternate method shown below which used the inverse logarithm laws to simply the left-hand side.

Alternate method:

$$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11$$

$$2\log_3(x)-2\log_3(9)+3\log_3\left(\sqrt{x}\right)=-11 \qquad \text{(inverse subtraction law)}$$

$$2\log_3(x)-2\log_3\left(3^2\right)+3\log_3\left(x^{\frac{1}{2}}\right)=-11$$

$$2\log_3(x)-4\log_3(3)+\frac{3}{2}\log_3(x)=-11 \qquad \text{(inverse multiplication law)}$$

$$\frac{7}{2}\log_3(x)-4=-11 \qquad (\log_a a=1)$$

$$\frac{7}{2}\log_3(x)=-7$$

$$\log_3(x)=-2$$

$$x=3^{-2} \qquad \text{(converting logarithms to exponentials)}$$

$$x=\frac{1}{9}, 0.\bar{1}$$

Part E Solutions — 13 Marks

In each of these questions you may see 'ecf.' written on your script.
This means your error in a previous part of the question has been disregarded in subsequent parts. 'ECF' is only applied if it is clear the erroneous result has been used.

1 Two functions f and g are defined as **3**

$$f(x) = 1 + \frac{1}{x} \text{ and } g(x) = \frac{x+1}{2}.$$

The function h is defined as $h(x) = f(g(x))$.

Find the inverse function h^{-1}.

$h(x) = 1 + \frac{2}{x+1}$

Let $y = h(x) = 1 + \frac{2}{x+1}$

(interchange x and y) $x = 1 + \frac{2}{y+1}$

$x - 1 = \frac{2}{y+1}$

(make y the subject) $y + 1 = \frac{2}{x-1}$

$y = \frac{2}{x-1} - 1$ | $y = \frac{2 - x + 1}{x-1}$

$\therefore h^{-1}(x) = \frac{2}{x-1} - 1 \quad x \neq 1$ | $= \frac{3-x}{x-1} \quad x \neq 1$

This question was very well done.

Most students knew the process of finding the inverse although when rearranging some students made errors with the positive and negative signs.

There was more success for those students who made a single fraction at the outset.

2 If $\sin x = -\cos x$ and $x \in [0°, 360°]$, determine the value(s) of x. **2**

In this question you must show all stages of your working.

$\sin x = -\cos x$

$\frac{\sin x}{\cos x} = \frac{-\cos x}{\cos x}$ $\cos x \neq 0$

$\frac{\sin x}{\cos x} = -1$

$\tan x = -1$

$x = 180 - 45, 360 - 45$

$= 135°$ and $315°$

This question was very well done.

Most students heeded the instructions in the question, to show all working.

Most errors occurred in finding which quadrant the solutions were in, for which marks were deducted.

Some students gave only one answer.

3 The ratio of the ages of Lynne and her father Jeff is 3 : 7.
Twelve years later the ratio of their ages is 5 : 9.
Find their initial ages.

3

Let Lynne's initial age be $3x$

Let Jeff's initial age be $7x$

$3:7 \Rightarrow \frac{3x}{7x}$

$\frac{3x+12}{7x+12} = \frac{5}{9}$

$9(3x+12) = 5(7x+12)$

$27x + 108 = 35x + 60$

$48 = 8x$

$x = 6$

Lynne's initial age $3x = 18$

Jeff's initial age $7x = 42$

This question was well done.

A variety of methods were used with varying levels of success.

There was an issue with layout and neatness.

Some solutions were not readable or able to be followed.

In a few cases marks were deducted for "messy or badly arranged" work.

4 The mass, A kg, of algae in a small pond, is modelled by the equation **2**

$$A = pq^t,$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by

$$\log_{10} A = 0.03t + 0.5.$$

Find the values of p and q to four significant figures.

Method 1

$A = 10^{0.03t + 0.5}$

$pq^t = 10^{0.03t} \times 10^{0.5}$

$= (10^{0.03})^t \times \sqrt{10}$

$p = \sqrt{10} = 3.162$ $q = 10^{0.03} = 1.072$

This question was not done well.

Many students did not know how to start and/or left the question blank.

Those that changed the log into an exponent generally got the correct answer.

Most students who answered knew how to round to 4 significant figures.

Marks were not deducted for rounding the last digit.

Method 2

$A = pq^t$

when $t = 0$ $A = P$

If $\log_{10} A = 0.03t + 0.5$

then $\log_{10} P = 0.03(0) + 0.5$

$P = 10^{0.5}$

$= 3.162$ (4 sig fig)

$\log_{10} A = \log_{10} pq^t = 0.03t + 0.5$

$\log_{10} p + t\log q = 0.03t + 0.5$

when $t = 1$

$0.5 + (1)\log_{10} q = 0.03(1) + 0.5$

$\log_{10} q = 0.03$

$q = 10^{0.03}$

$q = 1.072$ (4 sig fig)

5 Consider the polynomial **3**

$$P(x) = x^3 + ax^2 + bx - 12,$$

where a and b are constants.

It is known that $x + 1$ is a factor of $P(x)$ and that, when $P(x)$ divided by $x - 2$, the remainder is -18.

Find a and b.

$P(x) = x^3 + ax^2 + bx - 12$

$P(-1) = -1 + a - b - 12 = 0$ (factor thm)

$a - b = 13$ —— ①

$P(2) = 8 + 4a + 2b - 12 = -18$ (remainder thm)

$4a + 2b = -14$ —— ②

Rearranging ①

$a = b + 13$ —— ③

sub ③ into ②

$4(b + 13) + 2b = -14$

$6b + 52 = -14$ $\quad 6b = -68$

$b = -11$

sub $b = -11$ into ①

$a - 11 = 13$

$a = 2$

The solutions are $a = 2$ $b = -11$

Reasonably well done.

Many students employed the right method but their arithmetic and algebra were the main cause of errors.

Part F Solutions — 12 marks

1 The relationship between two variables p and t is modelled by the regression line with equation

$$p = 22 - 1.1t.$$

The model is based on observations of the independent variable t, between 1 and 10.

(a) Describe the correlation between p and t implied by this model. **1**

negative correlation

Many students wrote about the relationship between p and t or described how they are proportional to each other, however what was required was to directly state something about the correlation.
Many students described how strong or weak the correlation was.
There is nothing in the question that could be used to determine the strength of the correlation.
Any student who did this clearly does not have a deep understanding of how the regression line is related to correlation.

The variable p is measured in centimetres and the variable t is measured in days.

(b) State the units of the gradient of the regression line. **1**

cm/day

(c) Using the model, calculate the change in p over a 3-day period. **1**

$p_3 - p_0 = 22 - 1.1 \times 3 - (22 - 1.1 \times 0)$

$= -3.3$

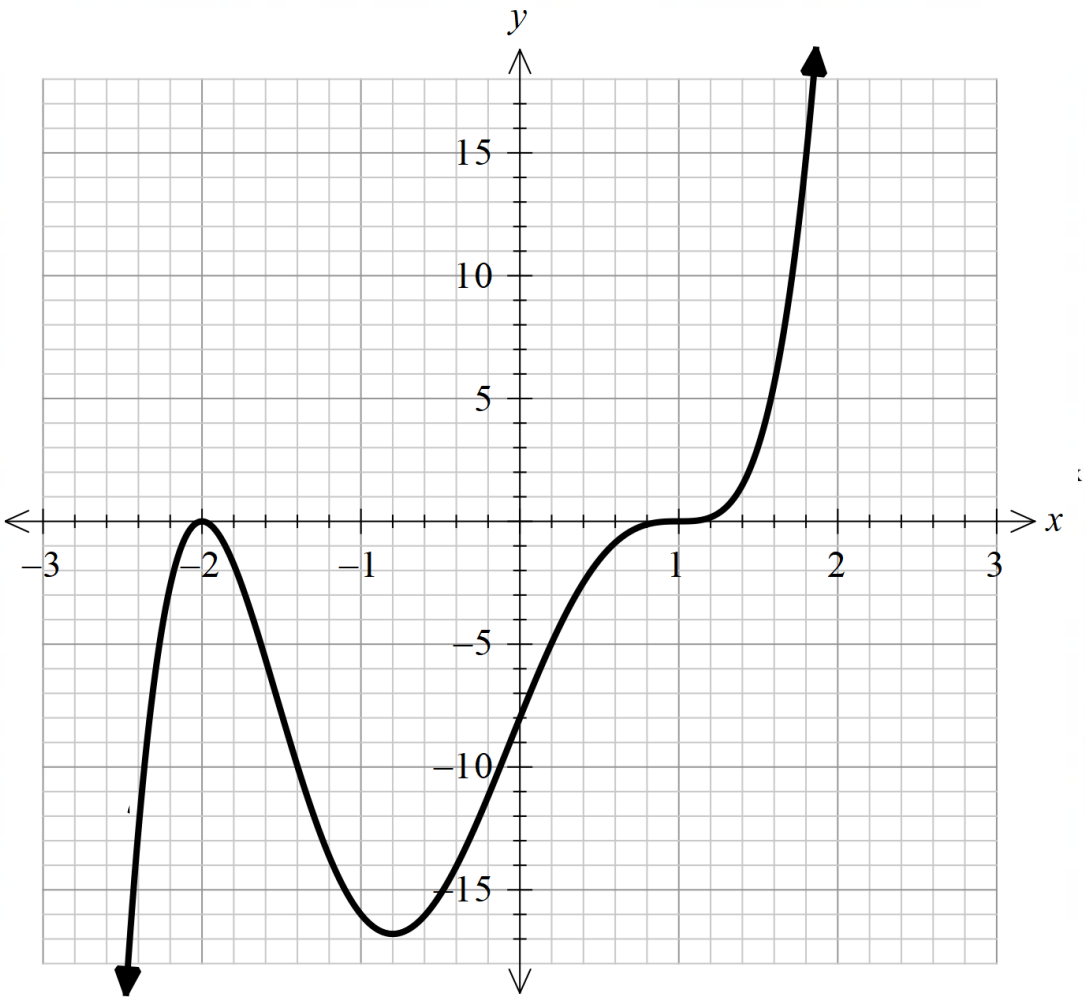
This was generally done poorly.
Many students used the values for $p = 1$ to $p = 3$ to determine the change over three days.
Others determined the exact value of p after three days, rather than find the change in the value of p.

(d) Ray Grezion uses this model to estimate the value of p when $t = 19$. **1**
Comment, giving a reason, on the reliability of Ray's estimate.

He is extrapolating. This is quite unreliable as the model may change outside of the known data set.

Students needed to allude to the fact that extrapolation was occurring and that the regression line cannot be relied upon to give a good estimate outside of the known dataset.
A good example of this is taking growth over time data for a human being from the ages of 0 to 14.
If one uses a regression line to extrapolate the height of a person at age 28, one would find an alarming result.
Beyond the age of 16 growth pattern changes for humans.
One never knows what might happen outside of the dataset given.

2 Sketch the function $y = 2(x - 1)^3(x + 2)^2$ on the graph below, clearly indicating all intercepts. Turning points are not required. **2**

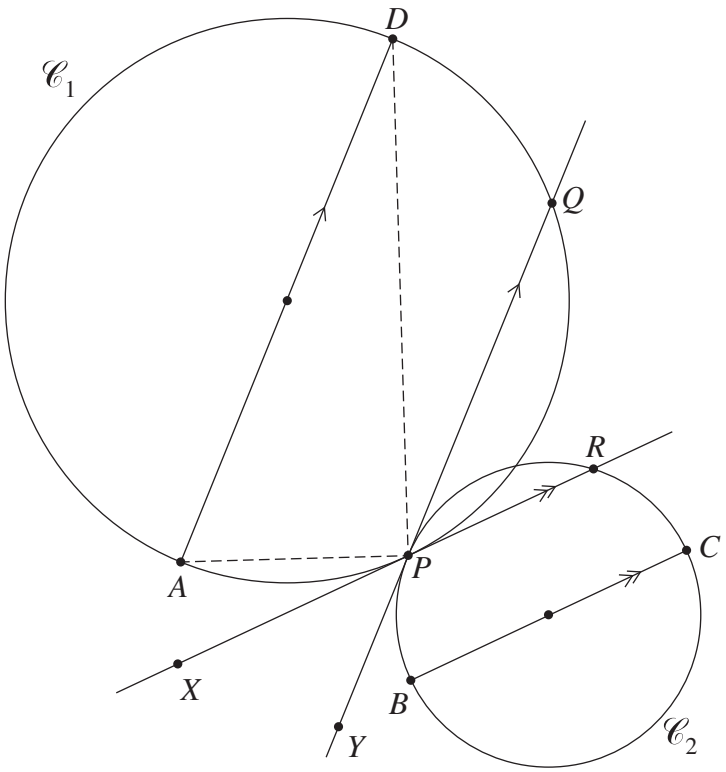


This was the question that was most often awarded full marks.

3 The diagram shows two circles $\mathscr{C}_1$ and $\mathscr{C}_2$. The point P is one of their points of intersection. The tangent to $\mathscr{C}_2$ at P meets $\mathscr{C}_1$ at Q, and the tangent to $\mathscr{C}_1$ at P meets $\mathscr{C}_2$ at R.

The points A and D are chosen on $\mathscr{C}_1$ so that AD is a diameter of $\mathscr{C}_1$ and parallel to PQ. Likewise, points B and C are chosen on $\mathscr{C}_2$ so that BC is a diameter of $\mathscr{C}_2$ and parallel to PR.

The points X and Y lie on the tangents PR and PQ, respectively, as shown in the diagram.



(a) Show that $\angle APX = \angle DPQ$. **2**

$\angle ADP = \angle APX$ (Alternate segment theorem)

$\angle DPQ = \angle APD$ (Alt. $\angle$ s, $AD \parallel PQ$)

This was done reasonably well, by most students.
Many students did the appropriate lines of reasoning but failed to link the two lines of reasoning in a final statement. Thus leaving it to the marker to infer the what the point of their work was.

Question 3 continues over the page

3 (continued)

(b) Show that A, P, and C are collinear. **3**

Consider a line through DP which cuts circle C_2 at point F.

$\angle APD = 90°$ ($\angle$ in a semi-circle)

$\angle EPF = \angle APD$ (vertically opposite $\angle$ s)

EF must be the diameter of C_2 (converse of $\angle$ in a semi-circle)

$\angle APX = \angle RPF$ (vertically opposite $\angle$ s) – (1)

From part (a), $\angle APX = \angle DPQ = \angle ADP$

$\angle DPQ = \angle YRE$ (vertically opposite $\angle$ s) – (2)

$\angle YPE = \angle EFP$ (Alternate segment theorem)

$\Rightarrow \angle EFP = \angle APX = \angle RPF$ (From (1) and (2))

$\therefore\ EF \parallel XR$ (Alt. $\angle$ s on parallel lines)

$\therefore E \leftrightarrow B$ and $F \leftrightarrow C$

$\therefore C$ lies on the line AP or A, P, and C are collinear

Only two students got this question correct, and only one of those got full marks.
Almost every single other student who attempted this assumed that which they were required to prove.
Essentially, they used their assumption that *A*, *P* and *C* are collinear to prove these very points were collinear.
Others chose to assume collinearity for *D*, *P* and *B*.
***D*, *P* and *B* are not drawn in the diagram as a line and it is not stated in the question that they belong on the same line.**
Assuming either of these things made proving the collinearity of *A*, *P* and *C* extremely easy.
The simplification meant that students who did this could not get any more than half the marks available.
Since all but a handful of students who attempted this question did this, most of the cohort was not awarded very many marks for this question.

3 (continued)

(b) Show that A, P, and C are collinear.

Alternative Solution

Using a similar argument to that in part (a), it can be shown that

$\angle BPY = \angle BCP = \angle CPR$

$\angle APD = 90°$ (Angle in a semi-circle)

Similarly $\angle BPC = 90°$

$\angle XPQ = \angle YPR$ (Vertically opposite $\angle$s)

$\therefore \angle APX + \angle APD + \angle DPQ = \angle BPY + \angle BPC + \angle CPR$

$\therefore 2.\angle APX + 90° = 90° + 2.\angle CPR$

$\therefore \angle APX = \angle CPR$

$\therefore APC$ is a straight line (equal vertically opposite $\angle$s)

$\therefore A, P, C$ are collinear.

(c) Show that $ABCD$ is a cyclic quadrilateral. **1**

From (b), $\angle PCB$ (or $\angle ACB$) equals $\angle ADP$ (or $\angle ADB$)

$\therefore ABCD$ is a cyclic quadrilateral

Why? An interval that subtends equal $\angle$ s at two points on the same side of the interval implies the 2 end points of the interval and the two points are a cyclic quadrilateral.
Simpler: converse of $\angle$ s in the same segment)

This should have been an easy question for most students.
The easiest way to prove this is described in the solutions and the reasoning provided in the solutions is clear.
The reasoning must either be "converse of angles on same arc" or wording that means the same thing - just using "angles on the same arc" was not enough for full marks.
Many students assumed that the angles of the quadrilateral at vertices A and C were ninety degrees. This is because AB appears to be tangential to circle 1 and DC appears to be tangential to circle 2. At this level students should not be assuming something is true based on appearance - proof by geometric reasoning is required to make such claims.

End of solutions