SYDNEY BOYS HIGH SCHOOL

2022

YEAR 10 STAGE 5.3
TERM 4
YEARLY ASSESSMENT

Mathematics

Time Allowed: 90 minutes
Reading Time: 5 minutes

General Instructions

- All questions may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using black pen. Diagrams should be in ink too.
- Leave your answers in the simplest exact form, unless otherwise stated.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- NESA approved calculators may be used.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: PSP

Name: _____

Class	Teacher	
10Ma**A**	A. Wang	
10Ma**B**	A. Ward	
10Ma**C**	E. Choy	
10Ma**P**	R. Wang	
10Ma**L**	A. Fuller	
10Ma**U**	J. James	
10Ma**S**	AM Gainford	

Part	Marks
A	/17
B	/14
C	/15
D	/12
E	/10
F	/10
Total	**/78**

Part A

1 Given that $f(x) = 5x^3$, evaluate $f(-2)$. **1**

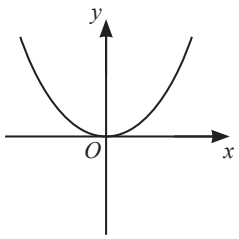
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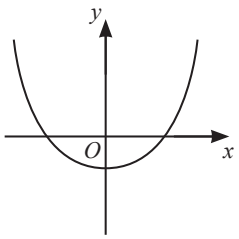
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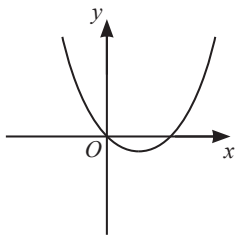
2 Here are four equations **3**

$$y = x^2 - 2x \qquad y = x^2 - 2 \qquad y = x^2 + 2x \qquad y = x^2$$

The graphs of three of these equations are sketched below.
Write the correct equation below each graph.







…… …… ……

3 If $h(t) = \sqrt{t^2 + 9}$ for all real values of t, which of the following is not in the range of $h(t)$? Circle the correct response. **1**

A. 1 B. 3 C. 9 D. 10

4 Given $f(x) = \{(0, -2), (1, 3), (5, -1), (-2, 7)\}$ and $g(x) = \{(0, 1), (1, 5), (5, 3), (3,0)\}$ evaluate $f\big(g(0)\big)$. **1**

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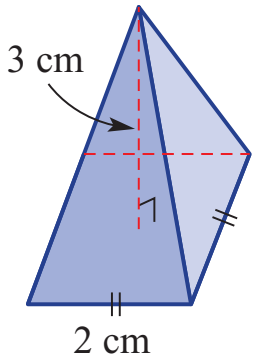
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5 A charity distributed 80 000 emergency packages during 2022. **1**
This number is expected to increase by 15% each year.
Calculate, to the nearest whole number, how many emergency packages the charity expects to distribute by the end of 2025.

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6 Using $V = \frac{1}{3}Ah$, what is the volume of the pyramid below? **1**



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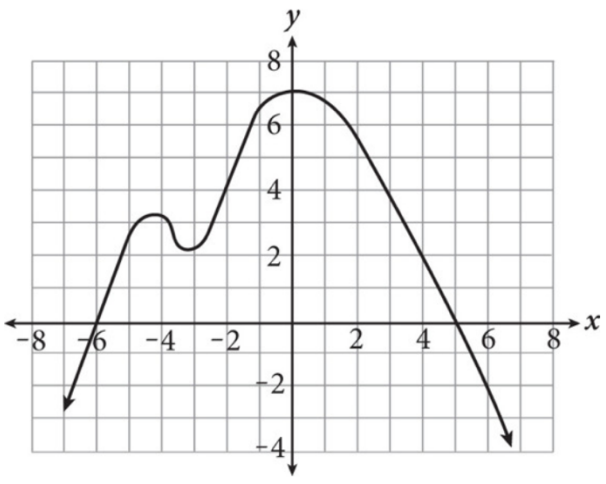
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7 Simplify $\left(\frac{x^6}{25y^4}\right)^{-\frac{1}{2}}$, giving your answer in positive indices. **2**

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8 **1**



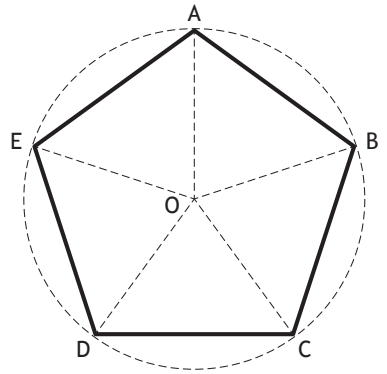
The figure above shows the graph of $p(x) - 4$. What is the value of $p(0)$?
Circle the correct response.

A. 3 B. 4 C. 7. D. 11

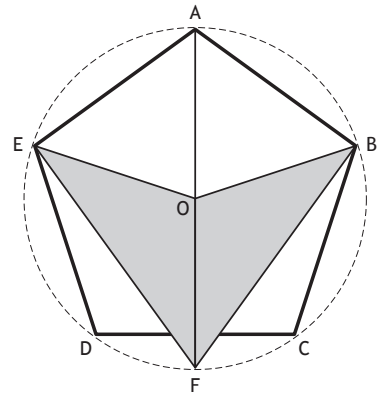
9 Ikan Deesine is creating a logo. **2**

Ikan starts by drawing a regular pentagon *ABCDE*.

The vertices of the pentagon lie on the circumference of a circle with centre *O*.



Ikan then adds to this as shown the diagram below.



AF is a diameter.

Find the value of angle *OBF*, reasons are not needed.

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10 By considering inverses, or otherwise, if $f(x) = 4x + 6$ and $g(x) = \dfrac{x-6}{4}$, **1**

find $f(g(-2))$.

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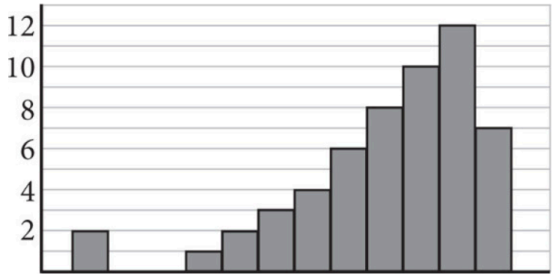
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11 Which of the following best describes the data represented by the figure shown below? Circle the correct response. **1**



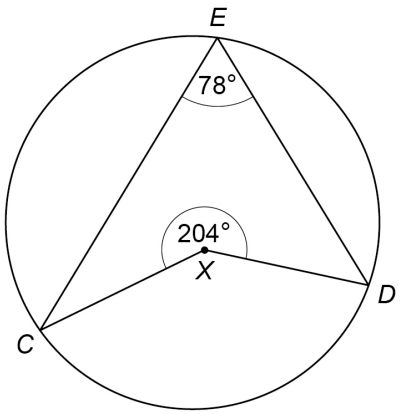
A. Skewed to the left with two outliers

B. Skewed to the left with an outlier of 2

C. Skewed to the right with two outliers

D. Skewed to the right with an outlier of 2

12 *C*, *D*, and *E* are points on a circle **2**



NOT DRAWN ACCURATELY

Justify whether *X* is the centre of the circle or not.

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Extra Working Page for Part A only

Part B

Name: _____

Class	Teacher	
10Ma**A**	A. Wang	
10Ma**B**	A. Ward	
10Ma**C**	E. Choy	
10Ma**P**	R. Wang	
10Ma**L**	A. Fuller	
10Ma**U**	J. James	
10Ma**S**	AM Gainford	

Mark	/14

1 Find the value of each of the following, giving your answer as an integer.

(a) $\log_5 5$ **1**

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(b) $\log_6 24 + \log_6 9$ **1**

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(c) $\log_{12} 2160 - \log_{12} 15$ **1**

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(d) $\frac{1}{3}\log_{11} 1331$ **1**

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2 A and B are two geometrically similar objects such that **2**

$$\frac{\text{surface area of } A}{\text{surface area of } B} = \frac{y}{y+7} \quad \text{and} \quad \frac{\text{volume of } A}{\text{volume of } B} = \frac{1}{8}.$$

Find the value of y.

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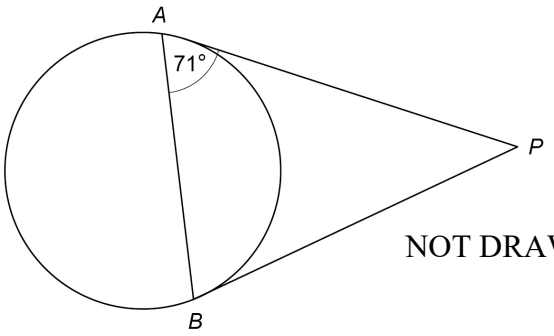
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3 *A* and *B* are points on a circle. *PA* and *PB* are tangents. **1**
Write down the value of angle *APB*.



NOT DRAWN TO SCALE

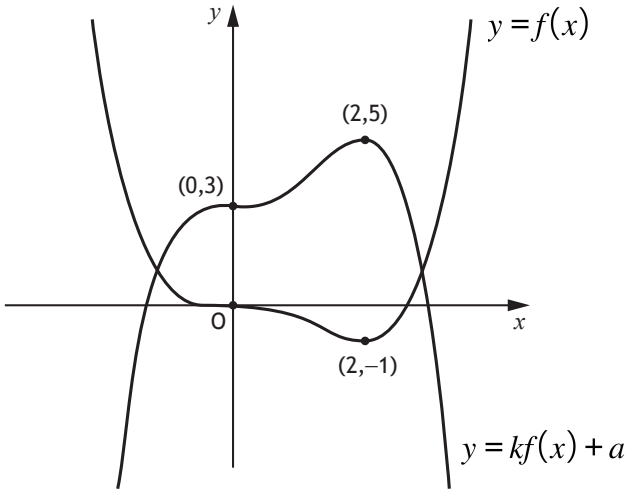
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4 The diagram shows the graphs with equations $y = f(x)$ and $y = kf(x) + a$.



(a) State the value of *a*. **1**

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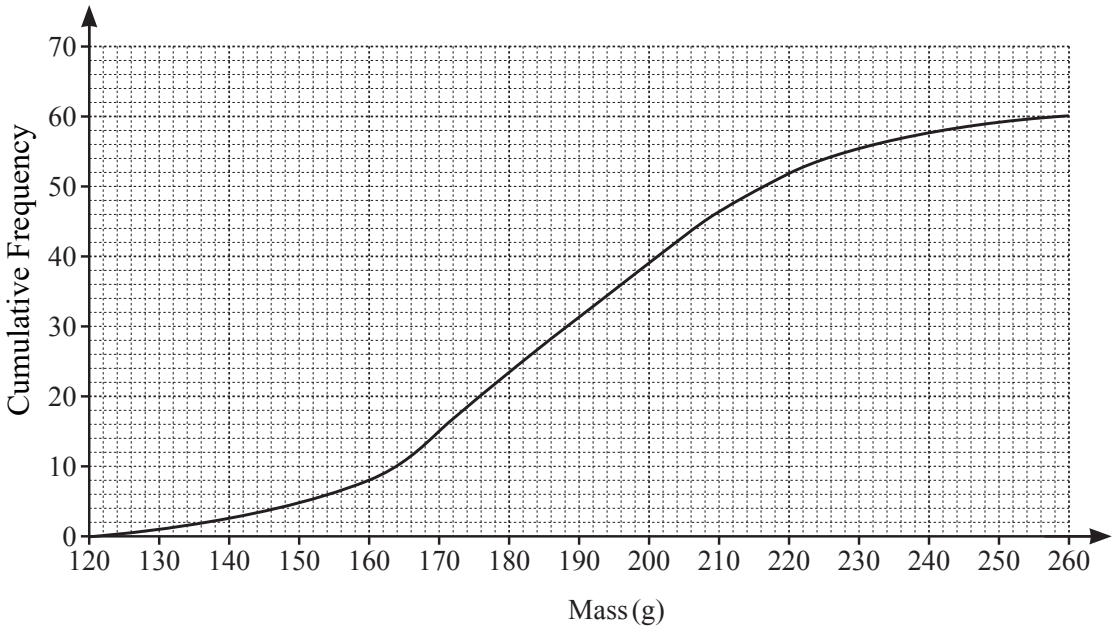
(b) State the value of *k*. **1**

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5 The cumulative frequency diagram shows the masses, in grams, of 60 potatoes of variety A.



(a) Use the diagram to estimate the interquartile range of variety A. **1**

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(b) The masses of 60 potatoes of variety B are also measured. **1**
For variety B, the median is 175 g and the interquartile range is 30 g.
Spud wants to buy potatoes that are more consistent in mass.
Should he choose variety A or variety B? Explain how you decide.

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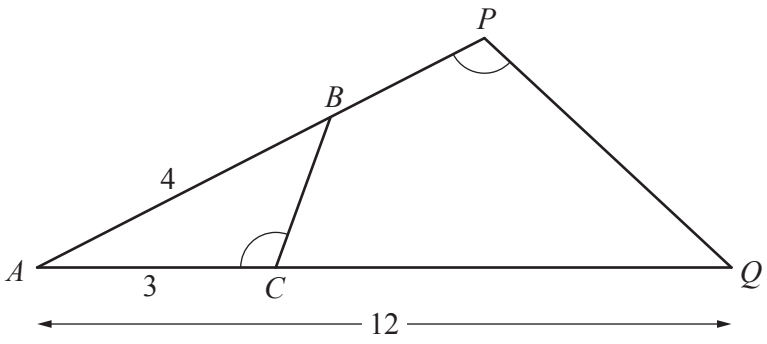
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NOT TO SCALE

In the diagram above, ABP and ACQ are straight lines. $\angle ACB = \angle APQ$.

(a) State why $\Delta ABC \,|||\, \Delta AQP$. Use AAA, SSS, RHS, or SAS. **1**

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(b) $AB = 4$ cm, $AC = 3$ cm, and $AQ = 12$ cm. **2**
Show that $AP = 9$ cm.

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Extra Working Page for Part B only

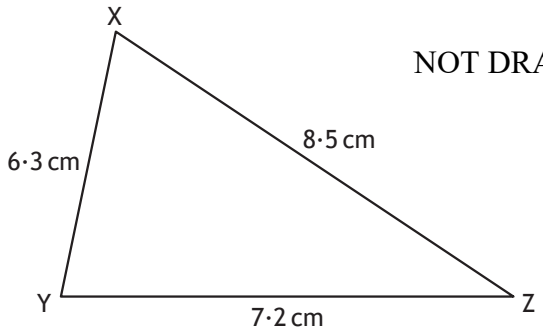
Part C

Name: _____

Class	Teacher	
10Ma**A**	A. Wang	
10Ma**B**	A. Ward	
10Ma**C**	E. Choy	
10Ma**P**	R. Wang	
10Ma**L**	A. Fuller	
10Ma**U**	J. James	
10Ma**S**	AM Gainford	

Mark	/15

1 Triangle XYZ is shown below. **2**



Calculate the size of the smallest angle in ΔXYZ, to the nearest degree.

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2 Circle C_1 has equation $x^2 + y^2 - 6x - 2y - 26 = 0$. **3**
Circle C_2 has a centre which is the reflection of the centre of C_1 in the x-axis.
The radius of C_2 is equal to the radius of C_1.
Find the equation of C_2.

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3 $\frac{1}{x}+\frac{3}{x}=\frac{1}{7}$. **1**

The equation shown above represents the following scenario:
A chemical laboratory uses two air purifiers to clean the air of contaminants emitted while working with hazardous materials. One is an older model, and the other is a new model that is considerably more energy efficient. The new model can clear the air of contaminants three times as quickly as the older model. Working together, the two air purifiers can clean the air in the lab in 7 hours.
Which of the following describes what the term $\frac{1}{x}$ in the equation can represent?

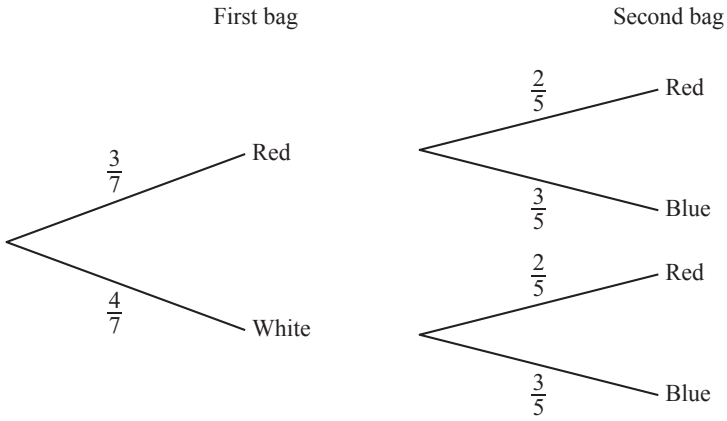
A. The portion of the air the new model can clean in 1 hour.

B. The portion of the air the older model can clean in 1 hour.

C. The time it takes the older model to clean the air by itself.

D. The time it takes the older model to clean $\frac{1}{7}$ the air by itself.

4 Two bags contain beads. The first bag contains 7 beads, of which 3 are red and 4 are white. The second bag contains 5 beads, of which 2 are red and 3 are blue.
One bead is taken, at random from each bag. The tree diagram is shown below.



Find the probability that

(a) both beads are red, **1**

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(b) exactly one bead is red. **2**

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5 (a) Solve $5p^2 - 11p + 2 = 0$. **2**

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(b) The curve with equation $y = 5\left(2^{2x}\right) - 6\left(2^{x}\right)$ intersects the curve with equation $y = 5\left(2^{x}\right) - 2$ at two points. **4**

Find the coordinates of each of these two points, giving your answers to three significant figures where appropriate.

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Extra Working Page for Part C only

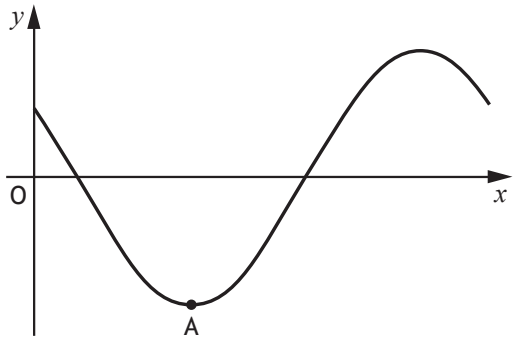
Part D

Name: _____

Class	Teacher	
10Ma**A**	A. Wang	
10Ma**B**	A. Ward	
10Ma**C**	E. Choy	
10Ma**P**	R. Wang	
10Ma**L**	A. Fuller	
10Ma**U**	J. James	
10Ma**S**	AM Gainford	

Mark	/12

1 Part of the graph of $y = 3\cos(x + 45°)$ is shown below. **2**



The graph has a minimum value at A.
State the coordinates of A.

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2 Functions p and q are defined by

$$p(x) = -\frac{1}{\sqrt{x-2}},\ x > 2 \text{ and } q(x) = 6 + x, \text{ for all real } x.$$

(a) Determine an expression for $p(q(x))$. **1**

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(b) For what value(s) of x is $p(q(x))$ undefined? **1**

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3 (a) Factorise completely $(y-2)^3 - 4y + 8$. **2**

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(b) Hence, find the minimum value of $(y-2)^3 - 4y + 8$ when $y \geq 4$. **2**

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4 Solve $\cos x = -\cos 225°$ for $0° \leq x \leq 360°$. **2**

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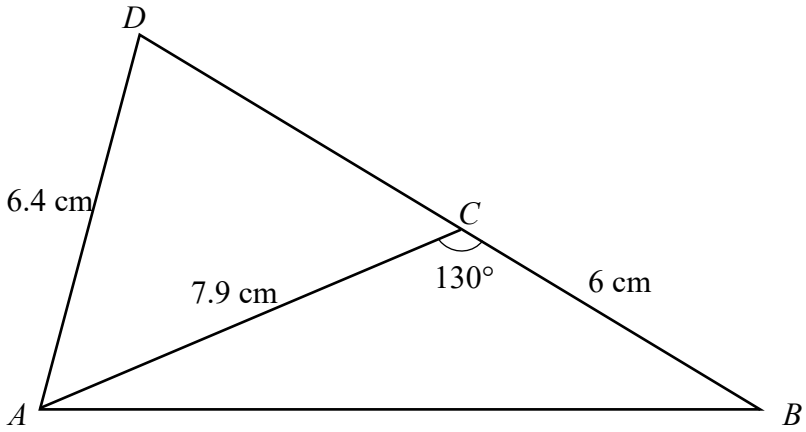
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4 **2**



In the diagram above, *BCD* is a straight line.

AC = 7.9 cm, *AD* = 6.4 cm and *BC* = 6 cm.

Angle *ACB* = 130°.

Calculate the acute angle *ADC*, to the nearest degree. Geometrical reasoning not needed.

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Extra Working Page for Part D only

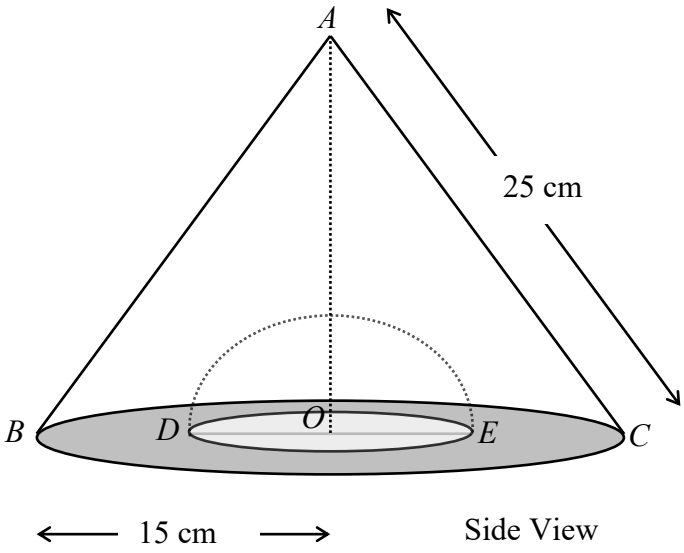
Part E

Name: _____

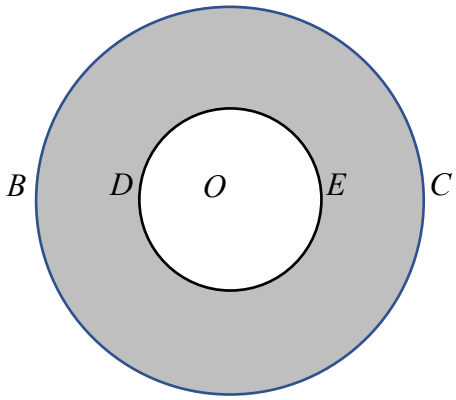
Class	Teacher	
10Ma**A**	A. Wang	
10Ma**B**	A. Ward	
10Ma**C**	E. Choy	
10Ma**P**	R. Wang	
10Ma**L**	A. Fuller	
10Ma**U**	J. James	
10Ma**S**	AM Gainford	

Mark	/10

1 The diagram below shows the side and base views of a solid party hat. **3**
It is made up of a cone and a hemisphere cut out from the bottom of the cone.



Side View



Base View

Given that the slant height, AC, and radius, OB of the cone are 25 cm and 15 cm respectively. The shaded area of the base is 176π cm^2.

Find the exact volume of the party hat, in terms of π.

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2 Students from three classes *S*, *U*, and *M* took a Mathematics examination and their mean marks and standard deviation were recorded in the table.

Class	Number of students	Mean mark	Standard deviation
S	10	85.4	0.313
U	10	86.5	2.128
M	9	75	0

(a) State how it may be deduced from the data that the mark scored by each student in class *M* was 75 marks. **1**

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(b) Compare the performance of the classes. **1**

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(c) A new student Stew Dent joined class *M*. **2**
The standard deviation of the marks of class *M* changed to 2.3.
Stew's *z*-score in class *M* was 1.5
What was Stew's mark?

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3 A function f, is given by $f(x) = \sqrt[3]{x} + 8$.
The domain of f is $1 \leq x \leq 1000$.
The inverse function, f^{-1}, exists.

(a) Find $f^{-1}(x)$. **2**

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(b) State the domain of f^{-1}. **1**

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Extra Working Page for Part E only

Part F

Name: _____

Class	Teacher	
10Ma**A**	A. Wang	
10Ma**B**	A. Ward	
10Ma**C**	E. Choy	
10Ma**P**	R. Wang	
10Ma**L**	A. Fuller	
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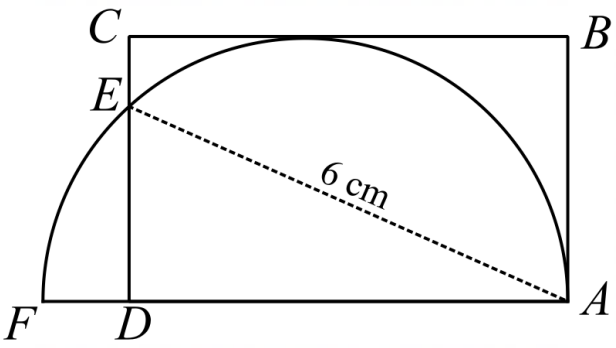
Mark	/10

1 Solve $\log_2(x+2)+\log_2(x-1)=2$. **3**

2 The remainder, when $f(x) = ax^4 + bx^3 + 15x^2 + 9x + 2$ is divided by $(x - 2)$, is 216. A factor of $f(x)$ is $(x + 1)$. **3**

Find the values of a and b.

3 In the diagram below, $ABCD$ is a rectangle and AF is a diameter of a semi-circle with tangent BC. If $AE = 6$ cm, find the area of $ABCD$. **4**



End of paper

Extra Working Page for Part F only

SYDNEY BOYS HIGH SCHOOL

2022

YEAR 10
STAGE 5.3 TERM 4
YEARLY EXAMINATION

Sample Solutions

NOTE: Some of you may be disappointed with your mark.
This process of checking your mark is NOT the opportunity to improve your marks. Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.

Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

Part A

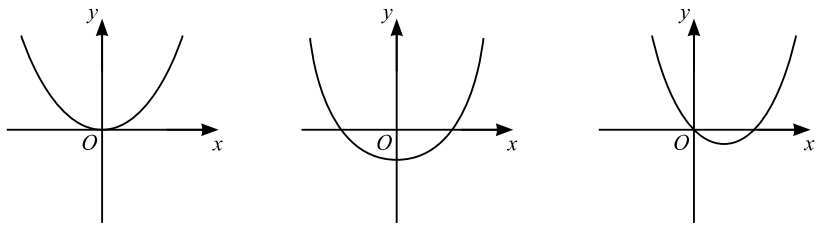
1 Given that $f(x) = 5x^3$, evaluate $f(-2)$. **1**

$f(-2) = 5 \times (-2)^3 = -40$ (1 — 12)
well done

2 Here are four equations **3**

$$y = x^2 - 2x \qquad y = x^2 - 2 \qquad y = x^2 + 2x \qquad y = x^2$$

The graphs of three of these equations are sketched below.
Write the correct equation below each graph.



going from L → R
$y = x^2$, $y = x^2 - 2$, $y = x^2 - 2x$.
well done with few chosen $y = x^2 + 2x$

3 If $h(t) = \sqrt{t^2 + 9}$ for all real values of t, which of the following is not in the range of $h(t)$? Circle the correct response. **1**

A. 1 B. 3 C. 9 D. 10

$\sqrt{t^2+9}$ $\because t^2 \geq 0$
$\therefore \sqrt{t^2+9} \geq 3$.
$\therefore$ Not in the range of $(h)(t) = 1$
$\therefore$ [A] well done

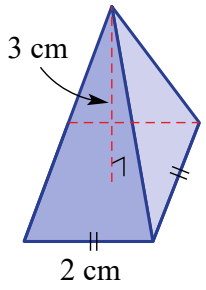
4 Given $f(x) = \{(0, -2), (1, 3), (5, -1), (-2, 7)\}$ and $g(x) = \{(0, 1), (1, 5), (5, 3), (3,0)\}$ evaluate $f\big(g(0)\big)$. **1**

$f[g(0)] = 3$
$\because g(0) = 1$ $\therefore f[1] = 3 \Rightarrow f[g(0)] = 3$
well done all except all scored 3.

5 A charity distributed 80 000 emergency packages during 2022. **1**
This number is expected to increase by 15% each year.
Calculate, to the nearest whole number, how many emergency packages the charity expects to distribute by the end of 2025.

$P = 80000$, $R = 1 + \frac{r}{100} = 1 + 0.15 = 1.15$
$n = 3$
well done $\therefore A_3 = 80000(1.15)^3 = 121676$ TO PR
Let A_n is the value of P after raised to 'n'

6 Using $V = \frac{1}{3}Ah$, what is the volume of the pyramid below? **1**

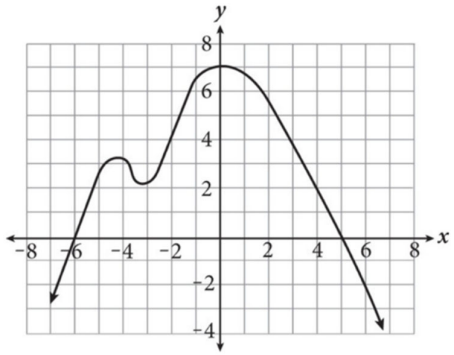


$V = \frac{1}{3}Ah$ $A = 4$, $h = 3$
$V = \frac{1}{3} \times 4 \times 3 = 4\text{ cm}^3$ Only a few got it wrong

7 Simplify $\left(\frac{x^6}{25y^4}\right)^{-\frac{1}{2}}$, giving your answer in positive indices. **2**

$\left(\frac{x^6}{25y^4}\right)^{-1/2} = \frac{x^{-3}}{5^{-1}y^{-2}} = \frac{5y^2}{x^3}$
well done some leave $(x^6)^{1/2}$

8 **1**



$P(0) + 4 = 7 + 4 = 11$
Quite a few chosen (A)

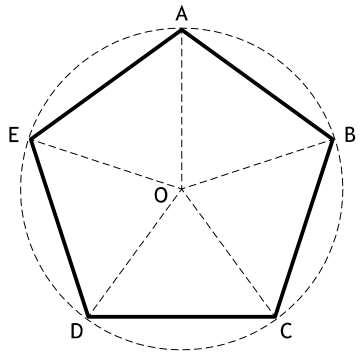
The figure above shows the graph of $p(x) - 4$. What is the value of $p(0)$?
Circle the correct response.

A. 3 B. 4 C. 7. D. 11

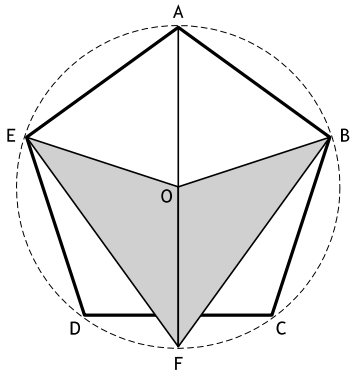
9 Ikan Deesine is creating a logo. **2**

Ikan starts by drawing a regular pentagon *ABCDE*.

The vertices of the pentagon lie on the circumference of a circle with centre *O*.



Ikan then adds to this as shown the diagram below.



AF is a diameter.

Find the value of angle *OBF*, reasons are not needed.

Each Interior Angles

$= \frac{(2n-4)\times 90}{5}$

$= \frac{540}{5} = 108 \Rightarrow \angle EAB = 108^\circ$

$\therefore \angle EOB \text{ (reflex)} = 216^\circ$

$\triangle EOF = \angle BOF = 216 \Rightarrow \angle FOB = 108$

$\therefore \triangle OFB$ is isosceles $\therefore \angle OBF = 36^\circ$

10 By considering inverses, or otherwise, if $f(x) = 4x + 6$ and $g(x) = \dfrac{x-6}{4}$, find $f(g(-2))$. **1**

$f(x)$ and $g(x)$ are inverses and so $f(g(x)) = x$

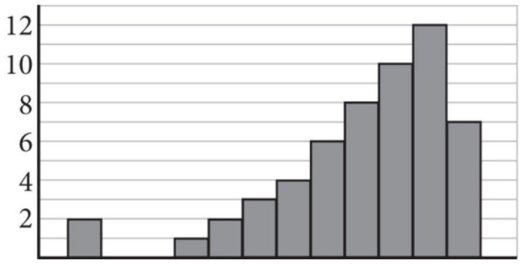
$\therefore f(g(-2)) = -2$

OR

$g(-2) = \dfrac{-2-6}{4} = -2$, $f(-2) = -8+6 = -2$

$\therefore f[g(-2)] = -2$.

11 Which of the following best describes the data represented by the figure shown below? Circle the correct response. **1**



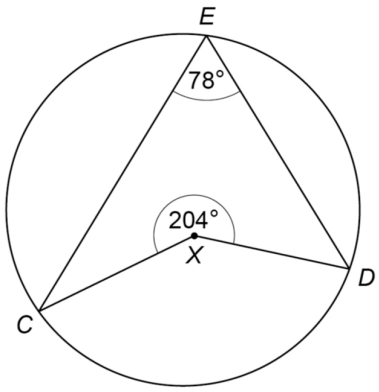
A. Skewed to the left with two outliers

B. Skewed to the left with an outlier of 2

C. Skewed to the right with two outliers

D. Skewed to the right with an outlier of 2

12 *C*, *D*, and *E* are points on a circle **2**



NOT DRAWN
ACCURATELY

Justify whether *X* is the centre of the circle or not.

Angle at the centre = 2 × ∠'s at the circumference standing on the same arc.

Now $\angle CXD + 204 = 360$ (revolution)

$\therefore \angle CXD = 360 - 204$

$= 156$

$= 2 \times 78^\circ = 2 \times \angle CED$

$\therefore$ X is the centre.



Part B

Comments for Part B

Q2. was not done well. Students needed to equate common lengths, areas or volumes in order to get an equation linking the unknown pronumeral y with a value.

Q3. No reasons were required.

Q4. Very little working was shown by students so it was hard to award any part marks.

Q5. There is no reason why it shouldn't be shown how the diagram is used to calculate the interquartile range. The diagram allows us to be precise. An IQR = 40g only warranted half marks.

Q6. Part (b) is worth 2 marks.
If students didn't mention that similar triangles is why corresponding sides were in the same ratio they could not gain full marks.
Remember it is a show that question

1 Find the value of each of the following, giving your answer as an integer.

(a) $\log_5 5$ **1**

$= 1$

(b) $\log_6 24 + \log_6 9$ **1**

$= \log_6 216$

$= \log_6 6^3 \quad = 3$

(c) $\log_{12} 2160 - \log_{12} 15$ **1**

$= \log_{12}\left(\frac{2160}{15}\right) \quad = \log_{12} 12^2$

$= \log_{12} 144 \quad = 2$

(d) $\frac{1}{3}\log_{11} 1331$ **1**

$= \frac{1}{3}\log_{11} 11^3$

$= \log_{11} 11 \quad = 1$

2 A and B are two geometrically similar objects such that **2**

$$\frac{\text{surface area of } A}{\text{surface area of } B} = \frac{y}{y+7} \quad \text{and} \quad \frac{\text{volume of } A}{\text{volume of } B} = \frac{1}{8}.$$

Find the value of y.

$V_A : V_B = 1 : 8$

$L_A : L_B = \sqrt[3]{1} : \sqrt[3]{8}$

$= 1 : 2$

$A_A : A_B = 1^2 : 2^2$

$= 1 : 4$

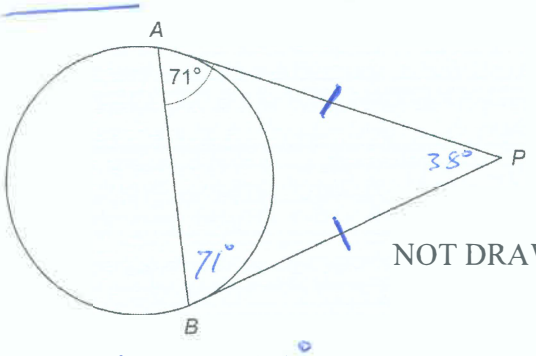
$\frac{y}{y+7} = \frac{1}{4}$

$4y = y + 7$

$3y = 7$

$y = \frac{7}{3}$

3 A and B are points on a circle. PA and PB are tangents. **1**
Write down the value of angle APB.

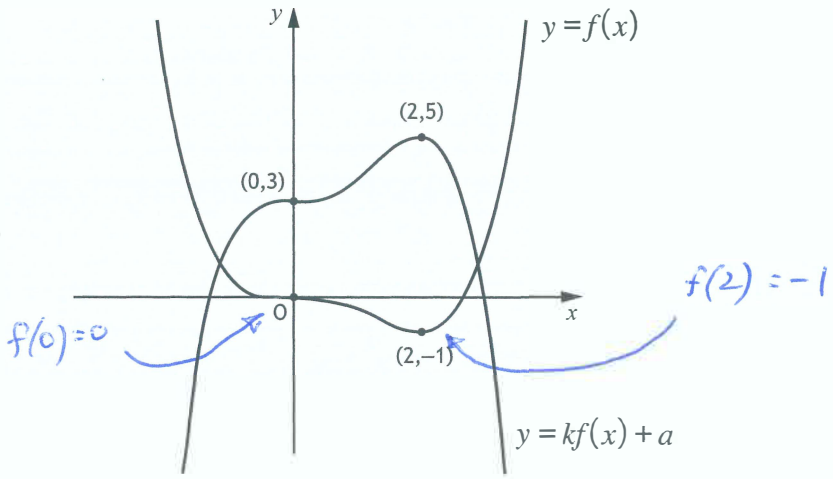


NOT DRAWN TO SCALE

$\angle PBA = 71°$

$\angle APB = 38°$

4 The diagram shows the graphs with equations $y = f(x)$ and $y = kf(x) + a$.



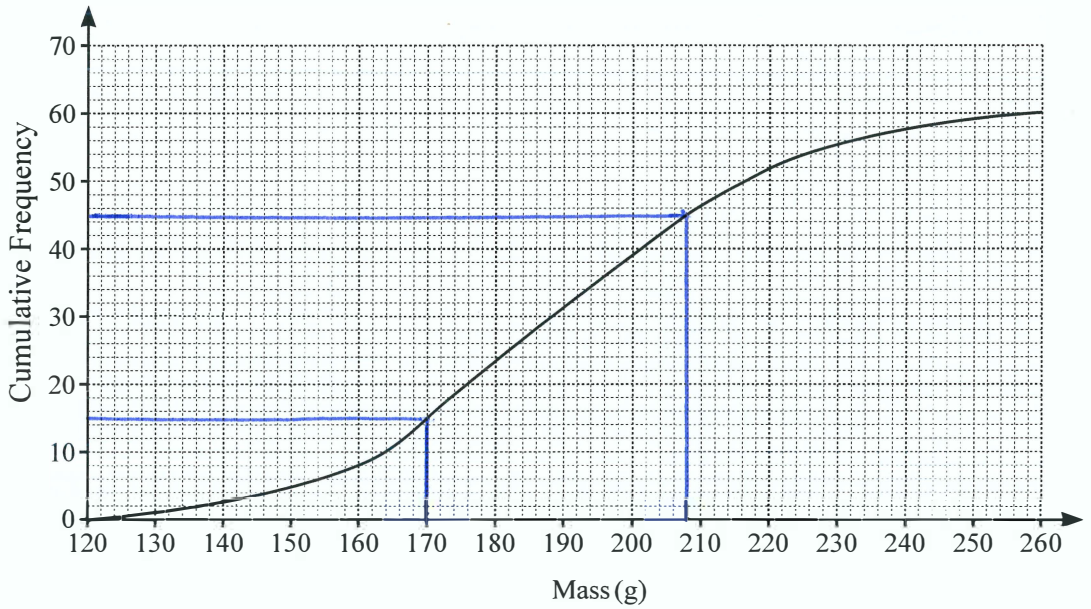
(a) State the value of a. **1**

$y = kf(x) + a$
sub in $(0, 3)$
$3 = kf(0) + a$
$3 = k(0) + a$
$a = 3$

(b) State the value of k. **1**

$y = kf(x) + 3$
sub in $(2, 5)$
$5 = kf(2) + 3$
$5 = k(-1) + 3$
$-k = 2$
$k = -2$

5 The cumulative frequency diagram shows the masses, in grams, of 60 potatoes of variety A.



(a) Use the diagram to estimate the interquartile range of variety A. **1**

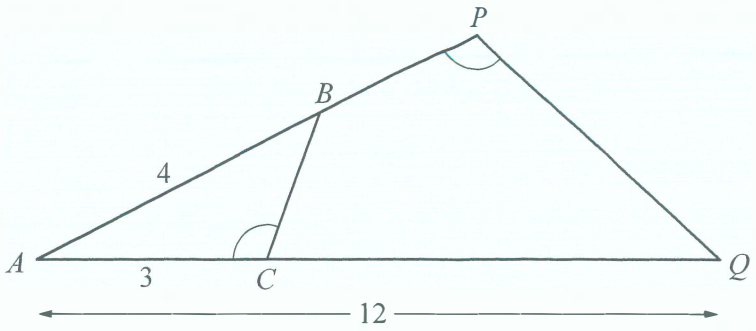
IQR = 208 - 170

= 38 g

(b) The masses of 60 potatoes of variety B are also measured. **1**
For variety B, the median is 175 g and the interquartile range is 30 g.
Spud wants to buy potatoes that are more consistent in mass.
Should he choose variety A or variety B? Explain how you decide.

Variety B since the IQR (30 g.) is less than Variety A (38 g.) meaning there is less variation in the middle 50% of potatoes mass ie more consistent

6



NOT TO SCALE

In the diagram above, ABP and ACQ are straight lines. $\angle ACB = \angle APQ$.

(a) State why $\Delta ABC \,|||\, \Delta AQP$. Use AAA, SSS, RHS, or SAS. **1**

AAA

(b) $AB = 4$ cm, $AC = 3$ cm, and $AQ = 12$ cm. **2**
Show that $AP = 9$ cm.

$\frac{AP}{AC} = \frac{AQ}{AB}$ (corresponding sides in the same ratio, $\triangle ABC \,|||\, \triangle AQP$)

$\frac{AP}{3} = \frac{12}{4}$

$AP = 3 \times \frac{12}{4}$

$= 9$ cm

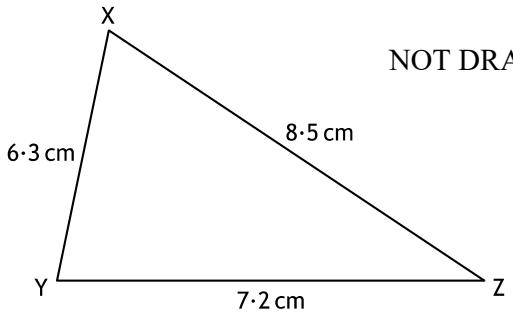


Part C

1 **NOTE:** Some pupils tried to use the sine rule to find the result, but this method could never be successful alone.

Using the calculator proved too much for some pupils and others found their result after using approximations in their calculations – which sometimes led to an incorrect response.

Triangle *XYZ* is shown below. **2**



Calculate the size of the smallest angle in ΔXYZ, to the nearest degree.

The smallest angle is opposite the smallest length.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Let C be the angle at vertex *Z*, $c = 6.3$, $a = 7.2$ and $b = 8.5$.

$$Z = \cos^{-1}\left(\frac{7.2^2 + 8.5^2 - 6.3^2}{2 \times 7.2 \times 8.5}\right)$$
$$\simeq 46.4060966°$$

Which is $46°$ to the nearest degree.

2 **NOTE:** Many pupils assumed that a reflection in the *x*-axis involves swapping the sign of the *x* value (the abscissa).
Some pupils put a negative sign in front of the ordinate terms which is not equivalent (in this case) to swapping the sign of the ordinate.
Many pupils who used the alternative method above got an incorrect square for the ordinate: $(y-2)^2-4$.
Others got 4 as the radius because the added the constant terms from completing the square.

Circle C_1 has equation $x^2 + y^2 - 6x - 2y - 26 = 0$. **3**
Circle C_2 has a centre which is the reflection of the centre of C_1 in the *x*-axis.
The radius of C_2 is equal to the radius of C_1.
Find the equation of C_2.

For $y = f(x)$ a reflection in the *x* axis may be described a $-y = f(x)$. That is, all one has to do is swap *y* for $-y$ for a reflection in the *x* axis. Since the radii for both circles is the same, we need only concern ourselves with the reflection.

$x^2 + (-y)^2 - 6x - 2(-y) - 26 = 0$ is the reflection in the *x* axis, which simplifies to

$$x^2 + y^2 - 6x + 2y - 26 = 0$$

Alternatively:

Find the centre of C_1 and reflect that in the *x* axis by making the *y* value (the ordinate) negative and create C_2 with the new centre and same radius.

$$x^2 - 6x + 9 - 9 + y^2 - 2y + 1 - 1 - 26 = 0$$
$$(x-3)^2 + (y-1)^2 = 36$$

C_1 has centre (3, 1) and radius 6. Then C_2 has centre (3, –1) with radius 6:

$$(x-3)^2 + (y+1)^2 = 36 \text{ or } x^2 + y^2 - 6x + 2y - 26 = 0$$

3 $\frac{1}{x}+\frac{3}{x}=\frac{1}{7}$. **1**

The equation shown above represents the following scenario:
A chemical laboratory uses two air purifiers to clean the air of contaminants emitted while working with hazardous materials. One is an older model, and the other is a new model that is considerably more energy efficient. The new model can clear the air of contaminants three times as quickly as the older model. Working together, the two air purifiers can clean the air in the lab in 7 hours.

Which of the following describes what the term $\frac{1}{x}$ in the equation can represent?

A. The portion of the air the new model can clean in 1 hour.

B. The portion of the air the older model can clean in 1 hour.

C. The time it takes the older model to clean the air by itself.

D. The time it takes the older model to clean $\frac{1}{7}$ the air by itself.

The air is completely cleaned every 7 hours (given) and hence a seventh of the air is cleaned in one hour.

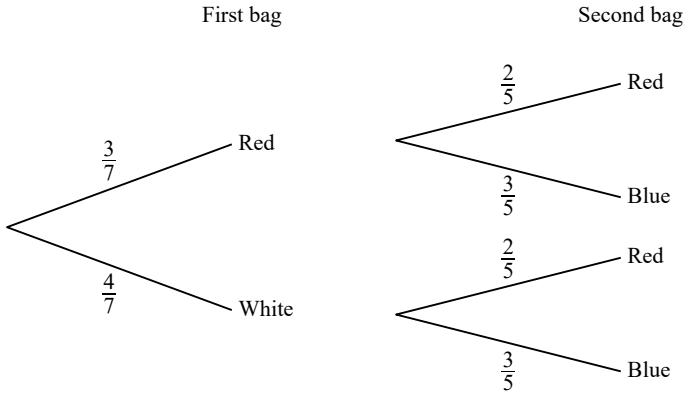
Only a portion of the air is cleaned every hour: $\frac{4}{28}$.

The newer machine is more efficient in that it does three times more work over the same time period.

Hence the answer is B.

4 **NOTE:** Most pupils did this well, however often the multiplication of the numerator was done as an addition.
This led to 4×2 being equated to 6 and other similar things.
This error was ignored **if** the correct working was laid out beforehand. **BUT** this is an unacceptable error and those making it should practice manipulating fractions from a year 7 textbook.

Two bags contain beads. The first bag contains 7 beads, of which 3 are red and 4 are white.
The second bag contains 5 beads, of which 2 are red and 3 are blue.
One bead is taken, at random from each bag. The tree diagram is shown below.



Find the probability that

(a) both beads are red, **1**

$$P(\text{R from 1}^{\text{st}}\text{ bag \& R from 2}^{\text{nd}}\text{ bag}) = P(\text{R from 1}^{\text{st}}\text{ bag})\times P(\text{R from 2}^{\text{nd}}\text{ bag})$$
$$= \frac{3}{7}\times\frac{2}{5}$$
$$= \frac{6}{35}$$

(b) exactly one bead is red. **2**

$$P(\text{exactly 1 R}) = P(\text{R from 1}^{\text{st}}\text{ \& B from 2}^{\text{nd}}) + P(\text{B from 1}^{\text{st}}\text{ \& R from 2}^{\text{nd}})$$

$$= \frac{3}{7}\times\frac{3}{5}+\frac{4}{7}\times\frac{2}{5}$$
$$= \frac{9}{35}+\frac{8}{35}$$
$$= \frac{17}{35}$$

5 **NOTE:** Many pupils failed to see the relationship between the two parts.
When their equation differed from part (a), they failed to realise they had made an error.

Many pupils could not reconcile 2^{2x} as $\left(2^x\right)^2$.

Most pupils who got the correct abscissa (x-value), did not go on to find the coordinates (which was what the question asked to find) and hence could not get full marks.

(a) Solve $5p^2 - 11p + 2 = 0$. **2**

The factorisation is $(5p-1)(p-2)=0$.

Hence the solutions for p are $\frac{1}{5}$ and 2.

(b) The curve with equation $y = 5\left(2^{2x}\right) - 6\left(2^x\right)$ intersects the curve with equation $y = 5\left(2^x\right) - 2$ at two points. **4**

Find the coordinates of each of these two points, giving your answers to three significant figures where appropriate.

Let $p = 2^x$, then $2^{2x} = \left(2^x\right)^2 = p^2$

(1) $y = 5p^2 - 6p$
(2) $y = 5p - 2$

$(1) = (2) \Rightarrow 5p^2 - 6p = 5p - 2$

$\Rightarrow 5p^2 - 11p + 2 = 0$

Then 2^x is $\frac{1}{5}$ and 2 (from part a)

Use (2) to find the ordinate.

When $p = 2$, $x = 1$ and $y = 5 \times 2 - 2 = 8$ and the coordinate is (1, 8)

When $p = \frac{1}{5}$, $2^x = \frac{1}{5}$. Take the logarithm of both sides (base 2) to solve for x.

$$\log_2 2^x = \log_2 \frac{1}{5}$$
$$x = -\log_2 5$$
$$x = -\frac{\ln 5}{\ln 2}$$
$$x \approx -2.32$$

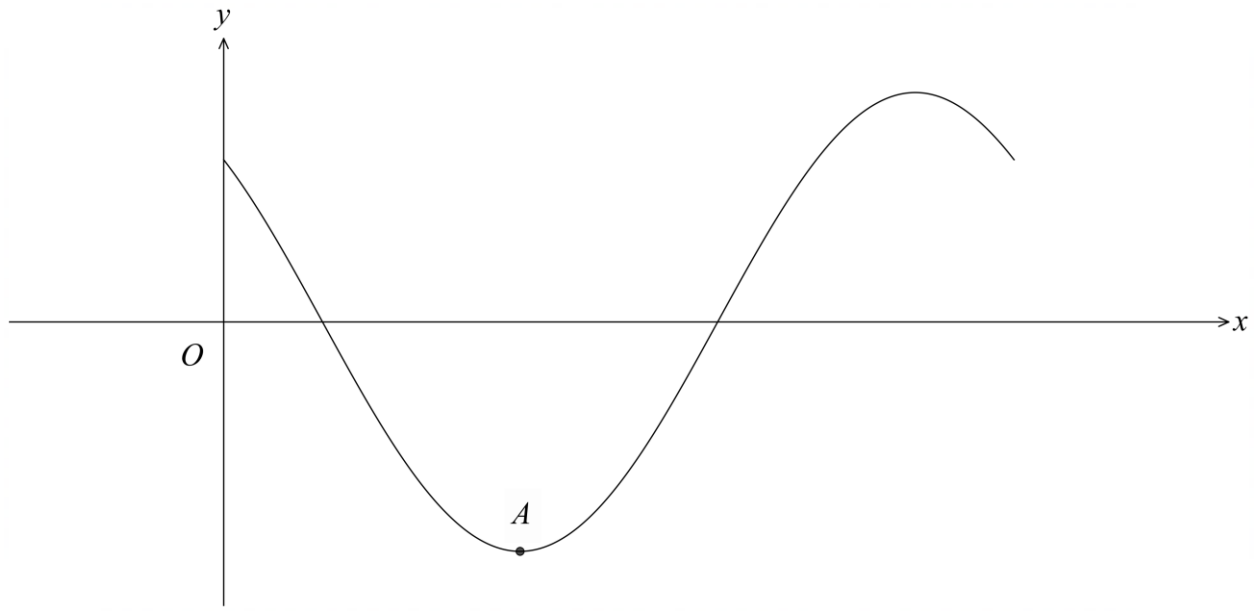
Then, $y = 5 \times \frac{1}{5} - 2 = -1$ and the coordinate is approximately (– 2.32, –1).



Part D

2022 YR10 Yearly Exam Part D Solutions and Comments

1. Part of the graph of $y = 3\cos(x + 45°)$ is shown below. 2



The graph has a minimum value at A.
State the coordinates of A.

Solution	Comment(s)
The minimum value of $y = \cos x$ is $y = -1$, so the minimum value of $y = 3\cos x$ is $y = -3$. The minimum value of $y = 3\cos x$ occurs at $x = 180°$, so the minimum value of $y = 3\cos(x + 45°)$ occurs at: $x + 45° = 180°$ $x = 135°$ Hence: $A = (135°, -3)$	No half marks. Some students lost marks due to providing an incorrect answer in radians when they potentially could have scored full marks if they worked in degrees instead. Furthermore, the question is written in degrees, so answer in radians is redundant. Common error(s): • Not stating both $x = 135°$ and $y = -3$ when A is a point.

2022 YR10 Yearly Exam Part D Solutions and Comments

2. Functions p and q are defined by:

$$p(x)=\frac{-1}{\sqrt{x-2}},\ x>2 \text{ and } q(x)=6+x, \text{ for all real } x$$

A. Determine an expression for $p(q(x))$. 1

Solution	Comment(s)
$p(q(x)) = p(6+x)$ $= \frac{-1}{\sqrt{6+x-2}}$ $= \frac{-1}{\sqrt{x+4}}$	Common error(s): • Forgetting the minus sign. • Not simplifying $\frac{-1}{\sqrt{6+x-2}}$.

B. For what value(s) of x is $p(q(x))$ undefined? 1

Solution	Comment(s)
$x+4 \leq 0$ $x \leq -4$	Common error(s): • Stating the values for which $p(q(x))$ is defined, rather than undefined. • Forgetting that $\sqrt{x+4} \neq 0$ as a denominator.

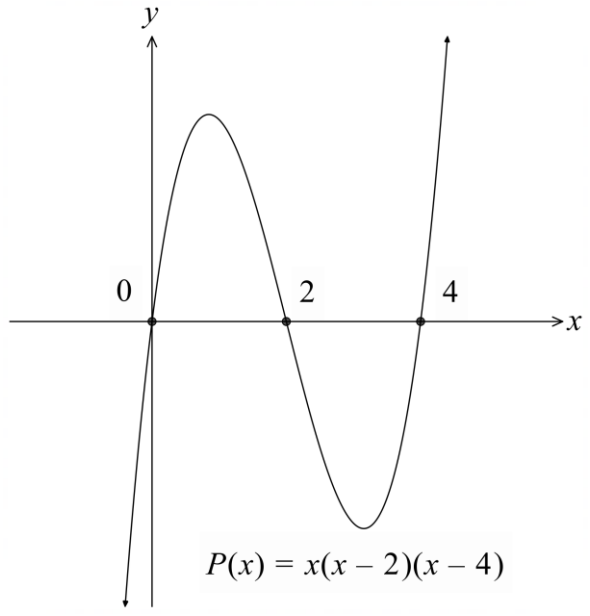
2022 YR10 Yearly Exam Part D Solutions and Comments

3.

A. Factorise $(y-2)^3-4y+8$. 2

Solution	Comment(s)
Method 1: $(y-2)^3-4y+8$ $=(y-2)^3-4(y-2)$ $=(y-2)((y-2)^2-4)$ $=(y-2)(y-2+2)(y-2-2)$ $=y(y-2)(y-4)$ Method 2: $(y-2)^3-4y+8$ $=y^3-(3\times 2y^2)+(3\times 4y)-8-4y+8$ $=y^3-6y^2+12y-4y$ $=y^3-6y^2+8y$ $=y(y^2-6y+8)$ $=y(y-2)(y-4)$	Most students failed to notice Method 1, which is a significantly easier approach. Due to the high number of variations in student responses, particularly in Method 2, carry through errors were penalised accordingly. Common error(s): • Incorrectly treating $(y-2)^3$ as a difference of cubes and factorising it into $(y-2)(y+2y+4)$. • Incorrectly factorising $-4y+8$ into $-4(y+2)$. • Forgetting the coefficients and/or the alternating signs when expanding $(a-b)^3$.

B. Hence, find the minimum value of $(y-2)^3-4y+8$ when $y\geq 4$. 2

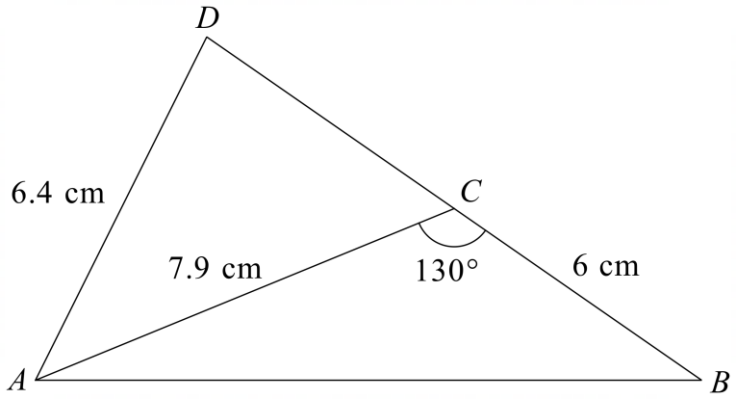
Solution	Comment(s)
Consider the graph of the polynomial: $P(x)=(x-2)^3-4y+8$ $=x(x-2)(x-4)$  From the graph, $P(x)\geq 0$ when $x\geq 4$. Hence, the minimum value of $(y-2)^3-4y+8$ is $y=0$.	IWO: Insufficient working out. Almost all students were penalised for failing to justify why the minimum occurs at $y=4$.

2022 YR10 Yearly Exam Part D Solutions and Comments

4. Solve $\cos x = -\cos 225°$ for $0° \leq x \leq 360°$. 2

Solution	Comment(s)
$\cos x = -\cos 225°$ $= \frac{1}{\sqrt{2}}$ Solving for the related angle: $x = \cos^{-1}\frac{1}{\sqrt{2}}$ $= 45°$ As $\cos x > 0$, x is in Quadrants I and IV. $\therefore x = 45°, 315°$	No half marks. Students who wrote $x = 135°, 225°$ as their answer could not score any marks. Common error(s): • Forgetting the $x = 315°$ solution.

5. In the diagram below, BCD is a straight line. 2



$AC = 7.9$ cm, $AD = 6.4$ cm and $BC = 6$ cm.
Angle $ACB = 130°$.
Calculate the acute angle ADC, to the nearest degree.
Geometrical reasoning not needed.

Solution	Comment(s)
As $\angle BCD$ is a straight angle, $\angle ACD = 50°$. Applying the sine rule to ΔACD: $\frac{\sin \angle ADC}{7.9} = \frac{\sin 50°}{6.4}$ $\sin \angle ADC = \frac{7.9 \sin 50°}{6.4}$ $\angle ADC = \sin^{-1}\frac{7.9 \sin 50°}{6.4}$ $= 71°$ (n. deg)	Students generally performed well on this question.



Part E

2022 Year 10 Yearly Part 1 Section E Comments

Q1

Relatively well done. The most common mistake was in the formulas for the volumes, generally these were carried though correctly. (Error Carried Forward e.c.f)

Q2

(a) This was extremely well done but some of the explanations were very poor. Marks were deducted for declaring that there was 'no' standard deviation when in fact there was a standard deviation, and it was zero.
(b) This was relatively well done. Mention should have been made to the means, but more importantly to the significance of the standard deviation and its effect on the spread of the data. The level of literacy was poor, as was the handwriting. Students need to learn to explain clearly, concisely, and legibly.
(c) This was very poorly done. Some used the z-score formula but did not state it. Most used the wrong mean being that of the initial set of scores (75), rather than $\frac{(75\times14)+x}{15}$.

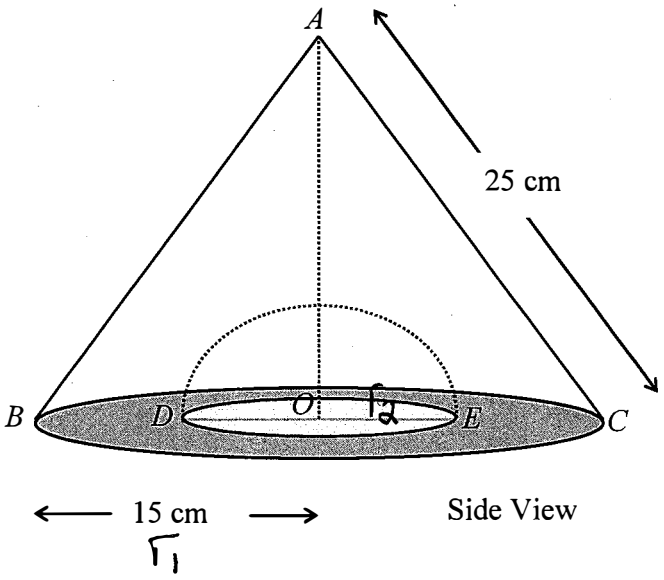
Q3

(a) This was well done. Most students knew to interchange *x* and *y* but the following algebra was poorly done in some cases.
(b) Very poorly done. Many did not attempt the question. Of those that did, many confused the domain and range.

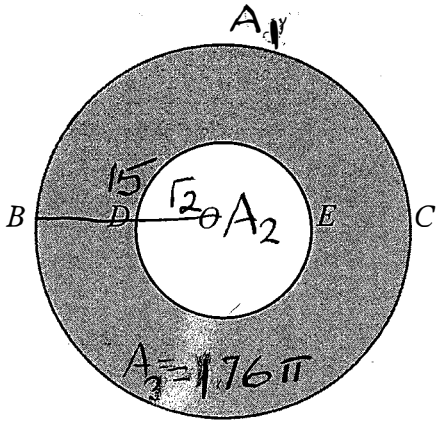
NOTES:

Overall students read and understood the questions well. In some cases, the layout was very poor with some being difficult to follow. In the written questions, the use of English was very poor, and handwriting atrocious.

1 The diagram below shows the side and base views of a solid party hat. **3**
It is made up of a cone and a hemisphere cut out from the bottom of the cone.



Side View



Base View

Given that the slant height, AC, and radius, OB of the cone are 25 cm and 15 cm respectively. The shaded area of the base is 176π cm^2.

Find the exact volume of the party hat, in terms of π.

V_1 = cone $= \frac{\pi r_1^2 h}{3}$

r_2

$A_3 = A_1 - A_2 = 176\pi$

$= \pi 15^2 - \pi r_2^2 = 176\pi$

$= \pi(15^2 - r_2^2) = 176\pi$

$15^2 - r_2^2 = 176$

$r^2 = 49$

$r_2 = 7$

$V_1 = \frac{\pi(15^2)(\sqrt{25^2 - 15^2})}{3}$

$= (225 \times \sqrt{400}\pi) \div 3$

$= 225 \times 20 \div 3$

$= 1500\pi$

$V_2 = \left(\frac{4}{3}\pi r_2^3\right) \times \frac{1}{2}$ = hemisphere

7^3

$V_2 = \frac{1}{2}\left(\frac{4}{3}\pi \times 343\right)$

$= \frac{686}{3}\pi$

$V = V_1 - V_2$

$= 1500\pi - \frac{686}{3}\pi$

$= \frac{3814}{3}\pi$

$= 1271\frac{1}{3}\pi$

2 Students from three classes *S*, *U*, and *M* took a Mathematics examination and their mean marks and standard deviation were recorded in the table.

Class	Number of students	Mean mark	Standard deviation
S	15	85.4	0.313
U	10	86.5	2.128
M	14	75	0

(a) State how it may be deduced from the data that the mark scored by each student in class *M* was 75 marks. **1**

By the fact that the standard deviation is zero

(b) Compare the performance of the classes. **1**

MEAN SD around the mean

S 2nd Close distribution of marks

U 1st Wide distribution of marks

M 3rd All students got the same mark

(c) A new student Stew Dent joined class *M*. **2**
The standard deviation of the marks of class *M* changed to 2.30.
Stew's *z*-score in class *M* was 1.5.
What was Stew's mark?

$SD(1) = 0$ $SD(1+Stew) = 2.30$ $\bar{x}(1) = 75$

$z(stew) = \frac{x - \bar{x}}{\sigma} = 1.5$

$\bar{x}(1) = \frac{\Sigma fx}{\Sigma f} = \frac{1050}{14} = 75$

$= \left[x - \left(70 + \frac{x}{15}\right)\right] \div 2.30 = 1.5$

$\bar{x}(1+stew) = \frac{1050 + x}{15}$

$= 70 + \frac{x}{15}$

$\left(\frac{14x}{15} - 70\right) \div 2.3 = 1.5$

$\frac{14x}{15} - 70 = 3.45$

$\frac{14x}{15} = 73.45$

$= 78.6964$

≈ 78.5 _ 79

3 A function f, is given by $f(x) = \sqrt[3]{x} + 8$.
The domain of f is $1 \leq x \leq 1000$.
The inverse function, f^{-1}, exists.

(a) Find $f^{-1}(x)$. **2**

let $y = \sqrt[3]{x} + 8$

Interchange x and y

$x = \sqrt[3]{y} + 8$

Make y the subject

$x - 8 = \sqrt[3]{y}$

$y = (x-8)^3$

$\therefore f^{-1}(x) = (x-8)^3 =$

(b) State the domain of f^{-1}. **1**

range $f(x)$ = domain $f^{-1}(x)$

$f(1) = \sqrt[3]{1} + 8 = 9$ $= 9 \leq x \leq 18$

$f(1000) = \sqrt[3]{1000} + 8 = 18$

as increasing



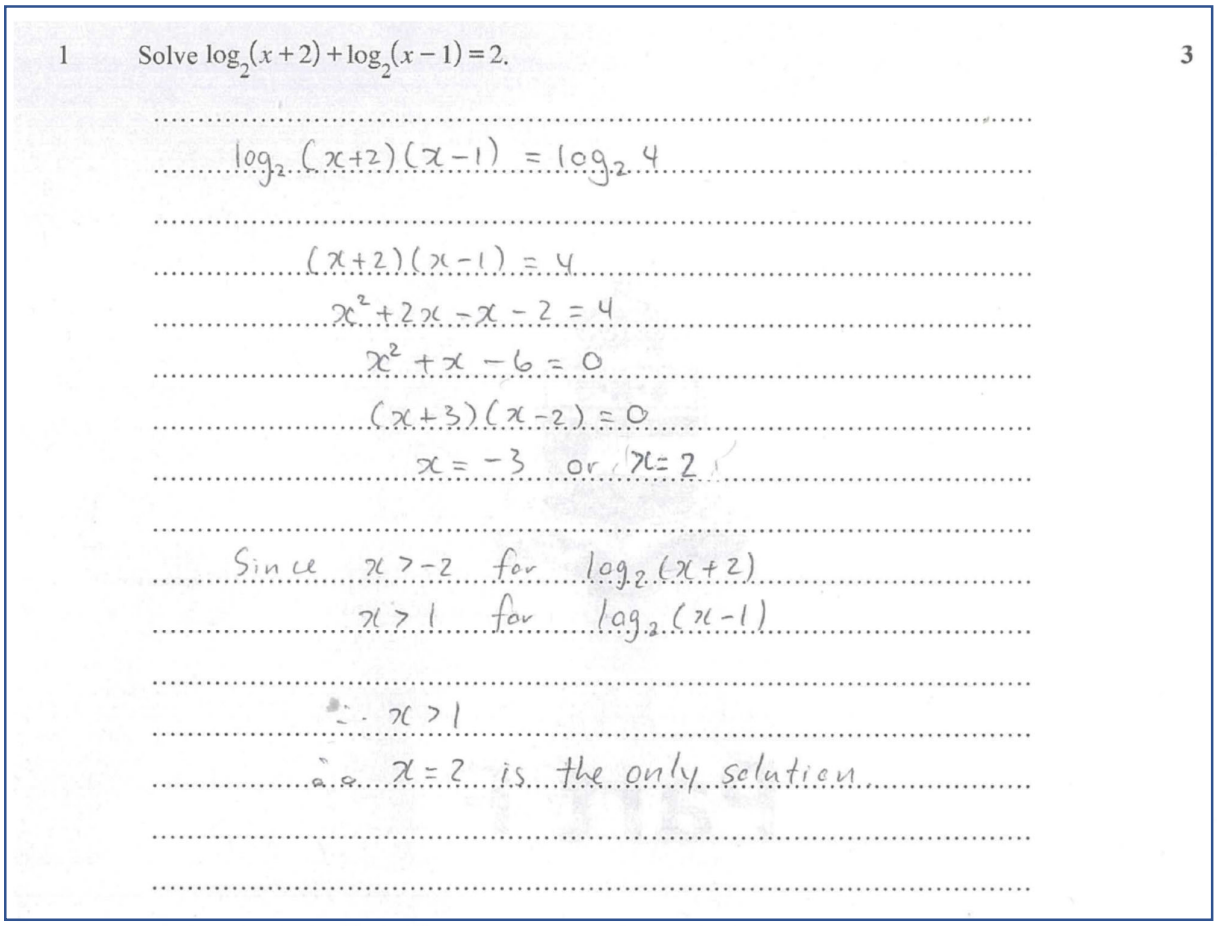
Part F

1 **Marker's comments:**

- Many candidates were not able to receive full marks due to not recognising the domain for logarithmic function i.e., that it can't be negative.

- Candidates should give a brief reason why one solution is not valid like as shown in the solution above.

- Significant number of candidates will need to review the logarithmic laws as their working showed no understanding of the log laws.



1 Solve $\log_2(x+2)+\log_2(x-1)=2$. 3

$\log_2(x+2)(x-1) = \log_2 4$

$(x+2)(x-1) = 4$

$x^2+2x-x-2 = 4$

$x^2+x-6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ or $x = 2$

Since $x > -2$ for $\log_2(x+2)$

$x > 1$ for $\log_2(x-1)$

$\therefore x > 1$

i.e. $x = 2$ is the only solution

Marking Scheme:

1 mark: For the correct equation, $x^2 + x - 6 = 0$.

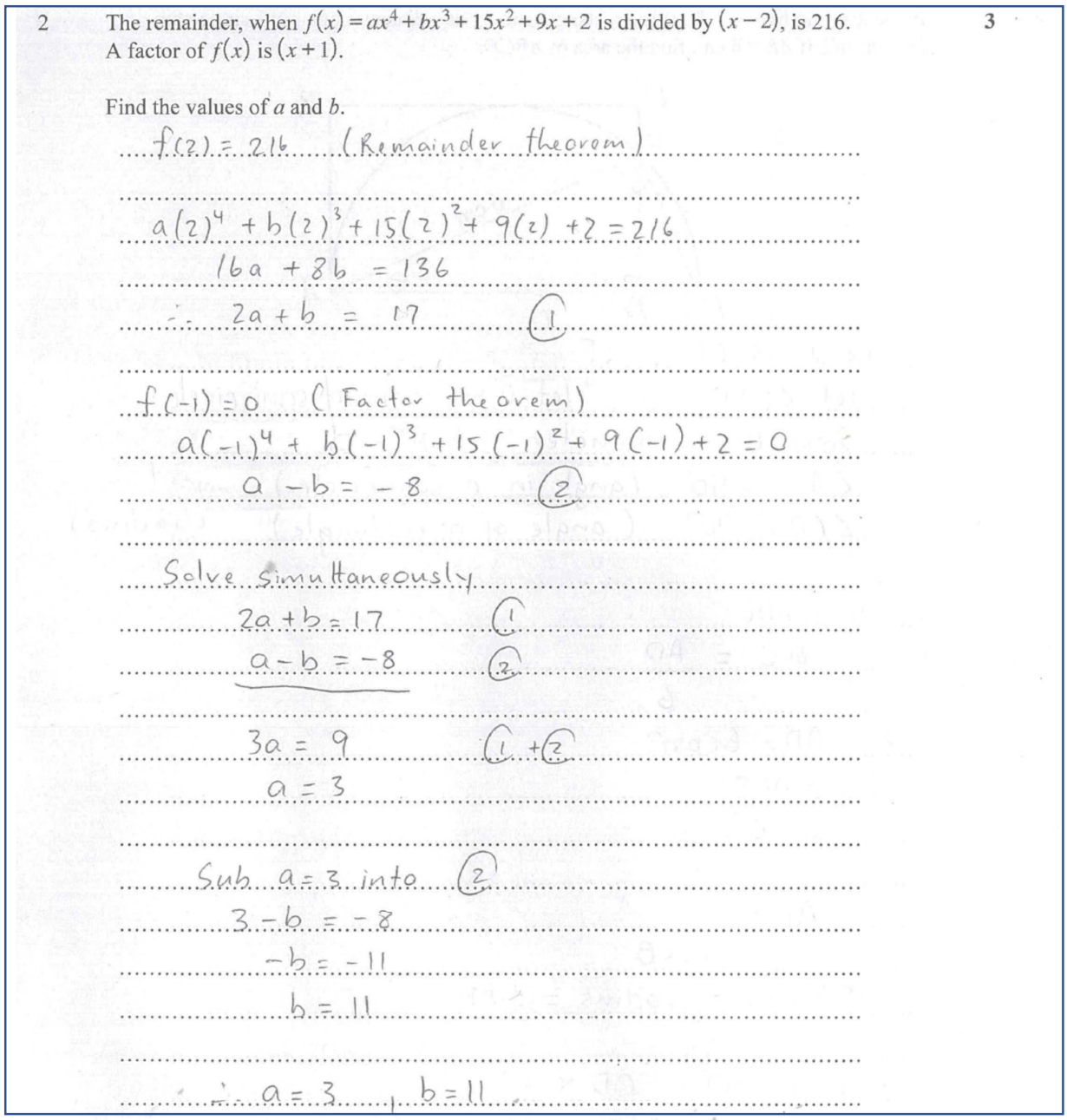
1 mark: Finding the correct two possible solutions i.e. $x = 2$ and $x = -3$.

1 mark: Justifying with reason why only 1 correct answer, i.e. $x = 2$.

2 **Marker's comments:**

- Many candidates were not able to find the two correct equations due to arithmetic errors from substitution and simplifying the equations. Candidates should take more care with silly errors.

- Candidates who did not use the remainder and factor theorem were not successful in answering this question.



2 The remainder, when $f(x) = ax^4 + bx^3 + 15x^2 + 9x + 2$ is divided by $(x-2)$, is 216. **3**
A factor of $f(x)$ is $(x+1)$.

Find the values of a and b.

$f(2) = 216$ (Remainder theorem)

$a(2)^4 + b(2)^3 + 15(2)^2 + 9(2) + 2 = 216$

$16a + 8b = 136$

$\therefore 2a + b = 17$ (1)

$f(-1) = 0$ (Factor theorem)

$a(-1)^4 + b(-1)^3 + 15(-1)^2 + 9(-1) + 2 = 0$

$a - b = -8$ (2)

Solve simultaneously

$2a + b = 17$ (1)

$a - b = -8$ (2)

$3a = 9$ (1) + (2)

$a = 3$

Sub $a = 3$ into (2)

$3 - b = -8$

$-b = -11$

$b = 11$

$\therefore a = 3, \quad b = 11$

Marking Scheme:

1 mark: Finding 2 correct equations to solve simultaneously.
1 mark: Correctly finding the value of one of the pronumerals with working.
1 mark: Correctly finding the value of the other pronumeral with working.

3 **Marker's comments:**

- Many candidates did not attempt this question.

- Majority of the candidates who attempted this question were not able to answer this question successfully.

- Many candidates were able to write some steps, but clearly got to a point where they can't find a numerical answer or gave a wrong numerical answer.
  Candidates were penalised for this.

- Candidates need to be aware that writing many steps that are not logical or does not aid in finding the correct answer will not be given any marks.

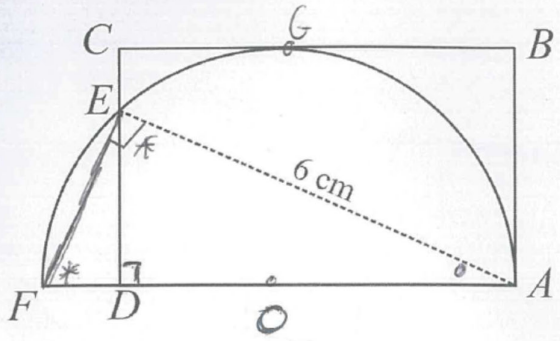
- Some candidates were lucky with to find the correct answer with minimal and incorrect working. Only 1 mark was awarded.

Marking Scheme:

2 marks: Correct logical working to achieve the correct answer.
1 mark: Correct reasoning to justify working out where required.
1 mark : Correct answer.

Method 1

3 In the diagram below, $ABCD$ is a rectangle and AF is a diameter of a semi-circle with tangent BC. If $AE = 6$ cm, find the area of $ABCD$. **4**



Construct : Join FE

$\angle FED = 90°$ (angle in a semi-circle)

let O be the centre of semi-circle

let G be a point on CB such that tangent BC meets the semi-circle

$GO \perp AD$ (tangent perpendicular to radius)

$GO = CD$ (From diagram)

Area of $ABCD = CD \times AD$

$\angle CDA = 90°$ (angle of a rectangle DABC)

In $\triangle FEA$ and $\triangle EDA$

$\angle EDA = \angle FEA = 90°$ (proved above)

$\angle FAE = \angle DAE$ (common angle A)

$\therefore \triangle FEA \parallel\parallel \triangle EDA$ (equiangular)

$\therefore \frac{DA}{EA} = \frac{EA}{FA}$ (corresponding sides of similiar $\triangle$ are proportional)

$\frac{DA}{6} = \frac{6}{FA}$

$DA \times FA = 36$

$OG = \frac{1}{2} FA$ (OG is radii of circle and FA is diameter of semi-circle

$\therefore CD = \frac{1}{2} FA$

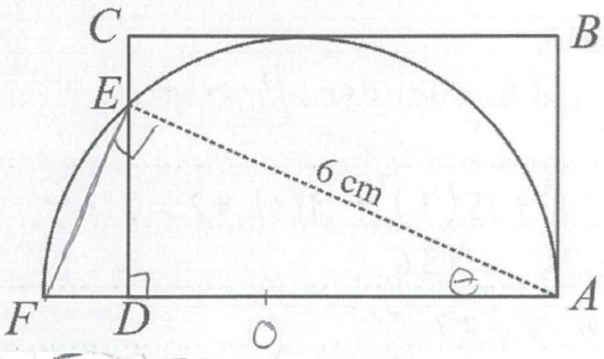
$\therefore$ Area of $ABCD = CD \times AD$

$= \frac{1}{2} FA \times AD$

$= \frac{1}{2} \times 36 = 18 \text{ cm}^2$.

End of paper

Method 2



Construct : Join EF

let O be the centre of semi-circle and E on CB such that $OE \perp CB$ ...

∴ OE is radius of semi-circle (tangent perpendicular to radius)

$\angle FED = 90°$ (Angle in a semi-circle)

In $\triangle ADE$

$\cos\theta = \frac{AD}{6}$

$AD = 6\cos\theta$

In $\triangle AEF$

$\cos\theta = \frac{6}{AF}$

$AF = \frac{6}{\cos\theta}$

$CD = BA = \text{radius} = \frac{1}{2}AF$

Area of ABCD = $CD \times AD$

$= \frac{1}{2}AF \times AD$

$= \frac{1}{2} \times \frac{6}{\cos\theta} \times 6\cos\theta$

$= 18\,cm^2$