SYDNEY BOYS HIGH SCHOOL

2021

YEAR 10 STAGE 5.3
TERM 4
YEARLY ASSESSMENT

Mathematics

General Instructions:

Time Allowed: 120 minutes
Reading Time: 10 minutes

- All questions may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using black pen. Diagrams should be in ink too.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- NESA approved calculators may be used.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: PSP

Name: _____

Class	Teacher	
10Ma**A**	B Kilmore	
10Ma**B**	T Broockmann	
10Ma**C**	Z Zeng	
10Ma**P**	S Gurjar	
10Ma**L**	R Wang	
10Ma**U**	AM Gainford	
10Ma**S**	J James	

Part	Marks
A	/10
B	/20
C	/20
D	/10
E	/12
F	/12
G	/16
Total	**/100**

Part A

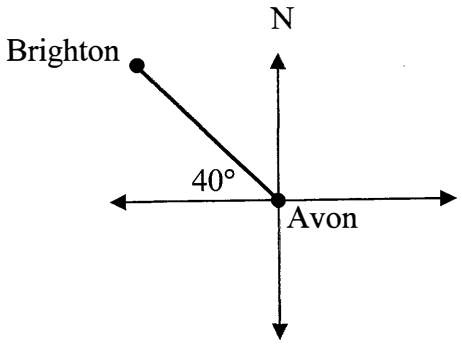
10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

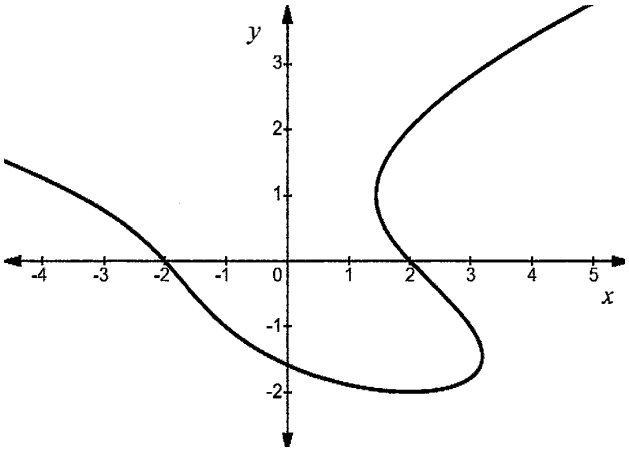
Use the multiple-choice answer sheet for Questions 1 – 10. Do NOT indicate answers below.

1 What is the compass bearing of Avon from Brighton in the diagram shown?



A. S50°E B. S40°W C. N50°E D. N50°W

2 With the graph below, which of the following best describes the relation?



A. one-to-one B. one-to-many

C. many-to-one D. many-to-many

3 Given that $v^2 = u^2 + 2as$. If u is always positive, which is the correct formula for u?

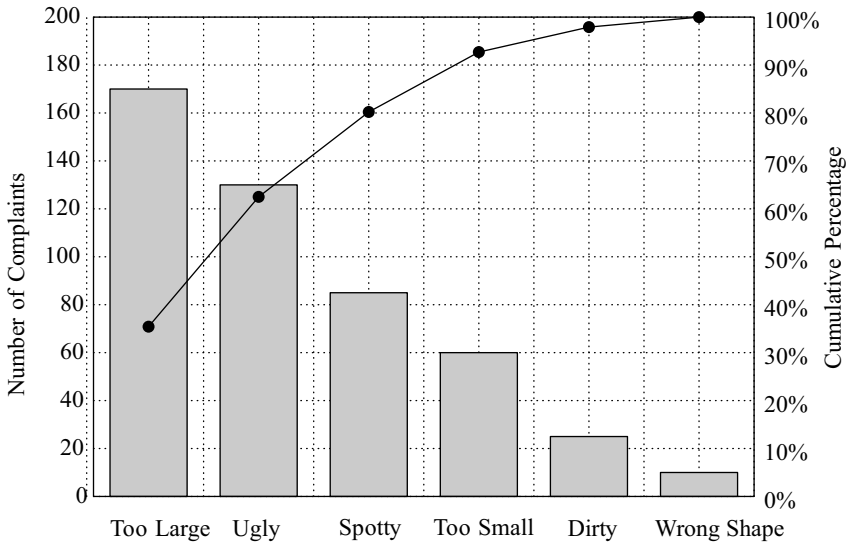
A. $u = v - 2as$

B. $u = \sqrt{2as - v^2}$

C. $u = \sqrt{v^2 + 2as}$

D. $u = \sqrt{v^2 - 2as}$

4 The number of complaints for the return of a particular item is shown in the Pareto chart below.



What percentage of the total number of complaints do the three largest complaints account for?

A. 60% B. 70% C. 80% D. 90%

5 Which of the following is the solution to $7^x = 2$?

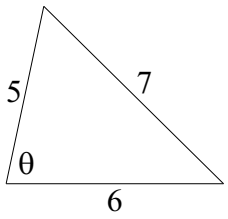
A. $x = \log_2 7$

B. $x = \log_{10} \frac{2}{7}$

C. $x = \frac{\log_{10} 2}{\log_{10} 7}$

D. $x = \frac{\log_{10} 7}{\log_{10} 2}$

6 In the diagram below, which of the following is a correct expression for $\cos\theta$?



A. $\dfrac{5^2+6^2-7^2}{2\times5\times6}$

B. $\dfrac{5^2+7^2-6^2}{2\times6\times7}$

C. $\dfrac{5^2+6^2+7^2}{2\times5\times6}$

D. $\dfrac{5^2+6^2-7^2}{2\times5\times7}$

7 Which inequality gives the domain of $y=\sqrt{2x-3}$?

A. $2x-3>0$

B. $2x-3\geq0$

C. $2x-3<0$

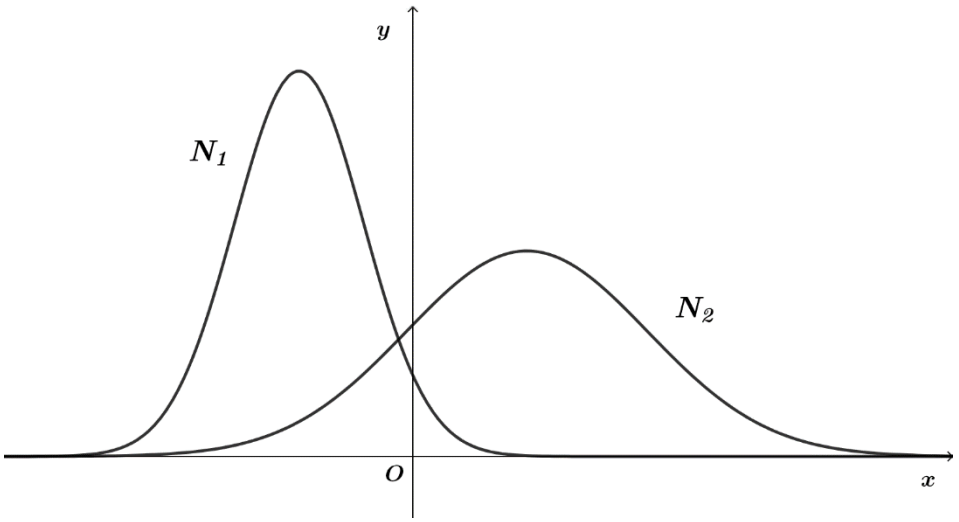
D. $2x-3\leq0$

8 The function $f(x)=x^3$ is transformed to $g(x)=(x-2)^3+5$ by a horizontal translation of 2 units followed by a vertical translation of 5 units.

Which row of the table shows the directions of the translations?

	Horizontal translation of 2 units	*Vertical translation of 5 units*
A.	Left	Up
B.	Right	Up
C.	Left	Down
D.	Right	Down

9 Consider the following graphs of two normal distributions, N_1 and N_2.



Assuming μ_1, σ_1 are the mean and standard deviation of N_1, and μ_2, σ_2 are the mean and standard deviation of N_2, which of the following statements is true?

A. $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$

B. $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$

C. $\mu_1 < \mu_2$ and $\sigma_1 > \sigma_2$

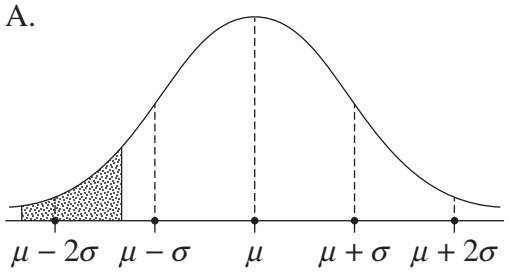
D. $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$

10 Suppose the weight of melons is normally distributed with a mean of μ and a standard deviation of σ.

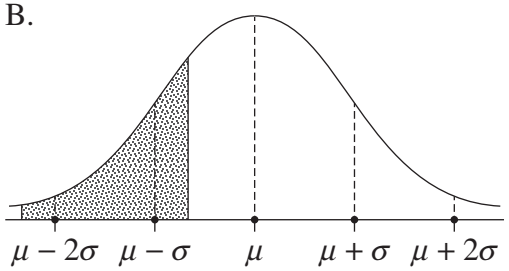
A melon has a weight below the lower quartile of the distribution but NOT in the bottom 10% of the distribution.

Which of the following most accurately represents the region in which the weight of this melon lies?

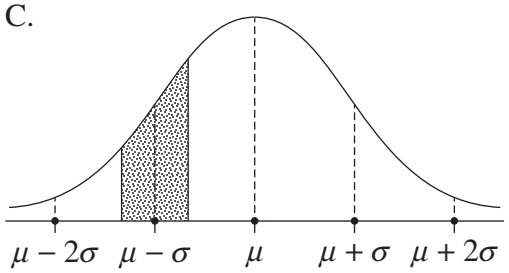
A.



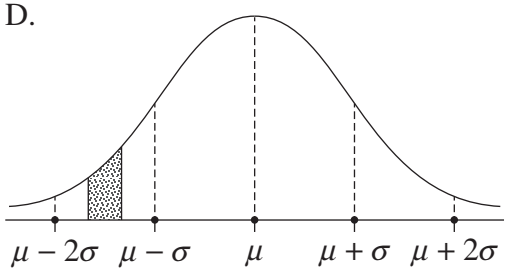
B.



C.



D.



Part B

Name: _____

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10Ma**L**	R Wang	
10Ma**U**	AM Gainford	
10Ma**S**	J James	

Mark	/20

(a) Solve $(x+7)(x-4)=0$ **1**

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(b) Simplify the expression $3y+2x-4y-x$. **1**

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(c) Write down the exact value of $\sin 120°$. **1**

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(d) Evaluate $\log_7 49$. **1**

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(e) Factorise $x^2-2x-15$. **1**

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(f) A cylinder is enlarged to give a similar second cylinder. **1**
If the ratio of their heights is 1 : 2, what is the ratio of their volumes?

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(g) State the vertex of the parabola with equation $y=(x-2)^2-1$. **1**

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(h) Consider the line with equation $12y+3x=4$. **1**
Write down its gradient.

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(i) Rationalise the denominator $\frac{3}{\sqrt{2}}$. **1**

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(j) Solve $2^x = 64$. **1**

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(k) Liz is asked to solve the equation $3x^2 + 8 = 83$ **1**

Here is her working.

$$3x^2 + 8 = 83$$
$$3x^2 = 75$$
$$x^2 = 25$$
$$x = 5$$

Explain what is wrong with Liz's answer

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(l) Simplify $5u^2w^4 \times 7uw^3$. **1**

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(m) The surface gravity of a planet can be worked out using the formula **1**

$$g = \frac{6.67 \times 10^{-11} m}{r^2},$$

where m kilograms is the mass of the planet and r metres is the radius of the planet.
For the Earth and Jupiter the values of m and r and listed below:

Earth	Jupiter
$m = 5.98 \times 10^{24}$ $r = 6.378 \times 10^{6}$	$m = 1.90 \times 10^{27}$ $r = 7.149 \times 10^{7}$

Calculate the ratio of the surface gravity of Earth to the surface gravity of Jupiter

Write your answer in the form 1 : n, correct to two significant figures.

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(n) Ibrar bought a cabin for $145 000. **1**

The value of the cabin depreciated by 4% in the first year.
The value of the cabin depreciated by 2.5% in the second year.

His cousin, Rarbi says,

"4 + 2.5 = 6.5, so in two years the value of your cabin depreciated by 6.5%"

Is Rarbi right? If you think he is wrong, give the correct percentage depreciation.

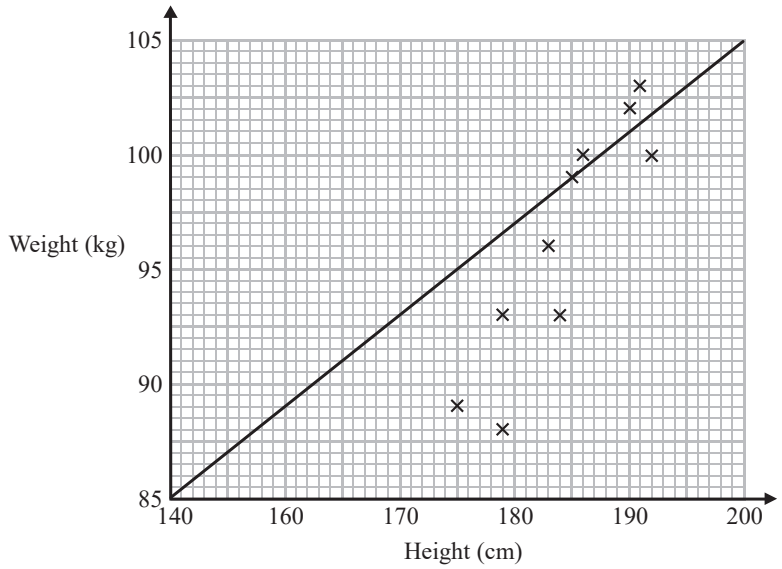
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(o) Sean has information about the height, in cm, and the weight, in kg, of each of ten rugby players. **2**
He is asked to draw a scatter graph and a line of best fit for this information.
Here is his answer



Sean has plotted the points accurately.
Write down two things that are wrong with his answer.

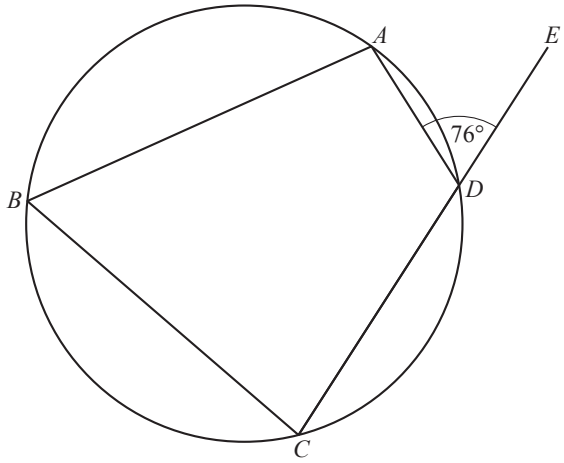
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(p)



A, B, C, and D are concyclic points. **1**

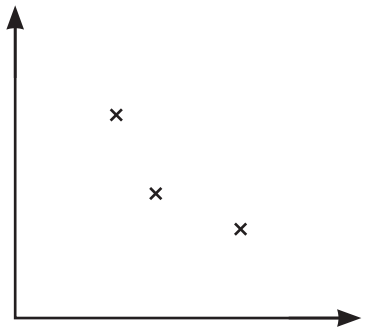
CDE is a straight line.

Write down the value of $\angle ABC$.

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(q) Henry drew the scatter diagram below. **1**



Put a circle around the most correct statement about this scatter diagram.

It shows no correlation.	It is not possible to tell if there is correlation as there are not enough points.	It shows negative correlation.	It shows positive correlation.

(r) Let $f(x) = 4\sin x°$ and $g(x) = 2x - 3$.

(i) Evaluate $f(23)$. **1**

Give your answer correct to three significant figures.

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(ii) Evaluate $f[g(24)]$. **1**

Give your answer correct to three significant figures.

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Extra Working Page for Part B only

Part C

Name: _____

Class	Teacher	
10Ma**A**	B Kilmore	
10Ma**B**	T Broockmann	
10Ma**C**	Z Zeng	
10Ma**P**	S Gurjar	
10Ma**L**	R Wang	
10Ma**U**	AM Gainford	
10Ma**S**	J James	

Mark	/20

(a) Re-arrange this formula to make x the subject **1**

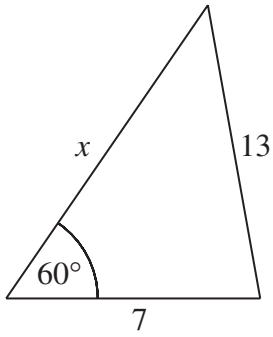
$$y = mx + c\,.$$

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(b) **4**



NOT TO SCALE

The diagram shows a triangle with sides 7 cm, 13 cm and x cm, and an angle of 60° as marked.

Use the cosine rule to show that $x^2 - 7x = 120$, and hence find the exact value of x.

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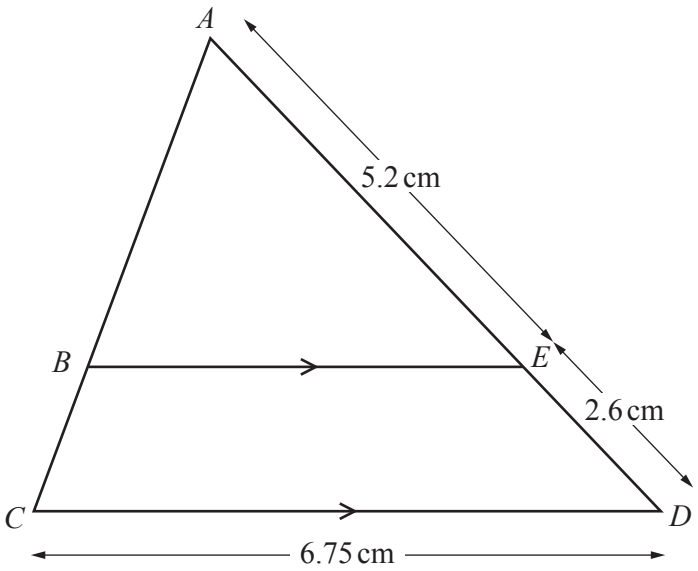
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(c) Divide $4x^4 - 36x^2 + 9$ by $\left(2x^2 - 1\right)$. **2**

Write your answer in the form $4x^4 - 36x^2 + 9 = \left(2x^2 - 1\right)Q\left(x\right) + R\left(x\right)$.

(d) In ΔACD, B lies on AC and E lies on AD such that $BE \parallel CD$.
$AE = 5.2$ cm and $ED = 2.6$ cm



NOT TO SCALE

(i) State why $\Delta ABE \;|||\; \Delta ACD$. **1**

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(ii) Calculate BE, with clear reasoning being shown. **2**

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(e) Write $\left(x+3\right) \div \dfrac{x^2+5x+6}{x-2}$ as a single fraction in its simplest form. **2**

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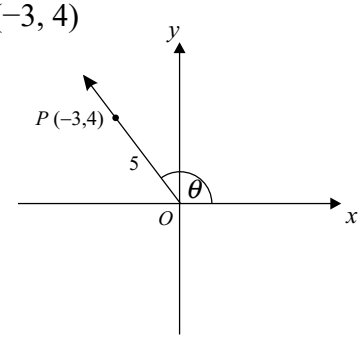
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(f) In the diagram above, the ray OP passes through the point $P(-3, 4)$ and makes the angle θ with the positive x-axis.



Given $OP = 5$ units, write down the values of $\sin\theta$ and $\tan\theta$. **2**

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(g) If $3^{2x-1} = \dfrac{1}{9^x} \times 3^{2y-x}$, find a simple expression for y in terms of x. **2**

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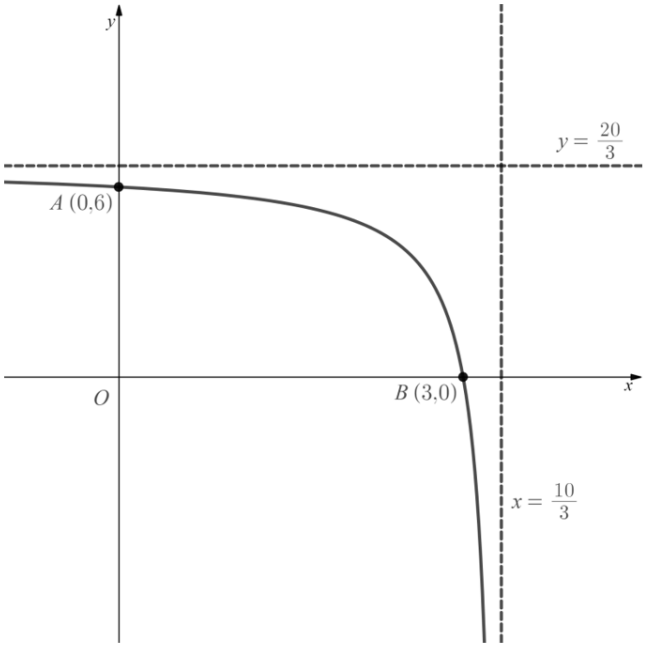
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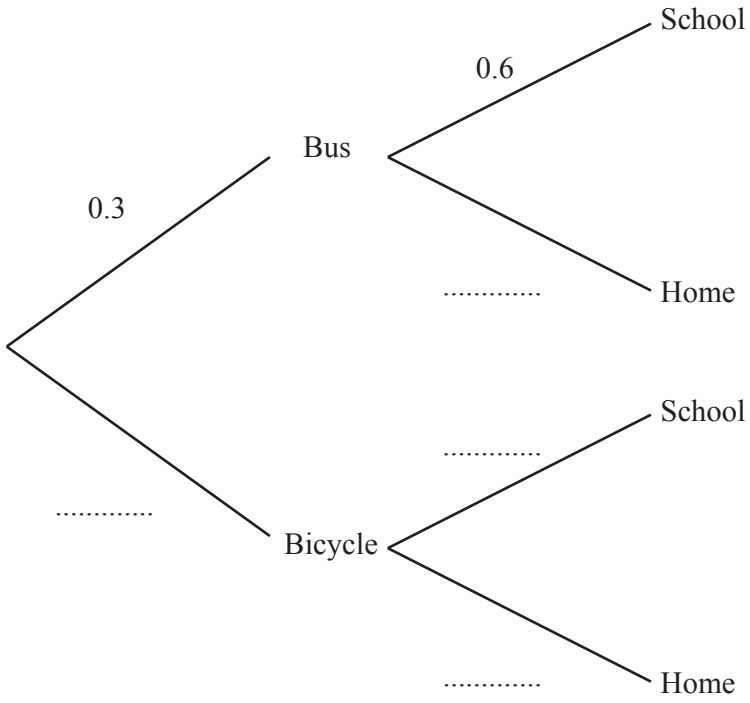
(h) The diagram below shows the graph of $y = f(x)$. The graph intersects the axes at points $A(0, 6)$ and $B(3, 0)$. The graph has two asymptotes as shown. **2**

Sketch the graph of $y = f(x) - 6$, showing clearly where A and B get transformed to. Also the location of the asymptotes after the transformation should be indicated.



(i) Claude travels to school either by bus or on a bicycle. **2**
The probability that he goes by bus is 0.3.
He can have lunch at home or at school.
If he catches the bus, the probability that he has lunch at school is 0.6
If he rides his bicycle, the probability that he has lunch at home is 0.6.

By using the diagram or otherwise, find the probability that Claude goes by bus given that he has lunch at school.



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Extra Working Page for Part C only

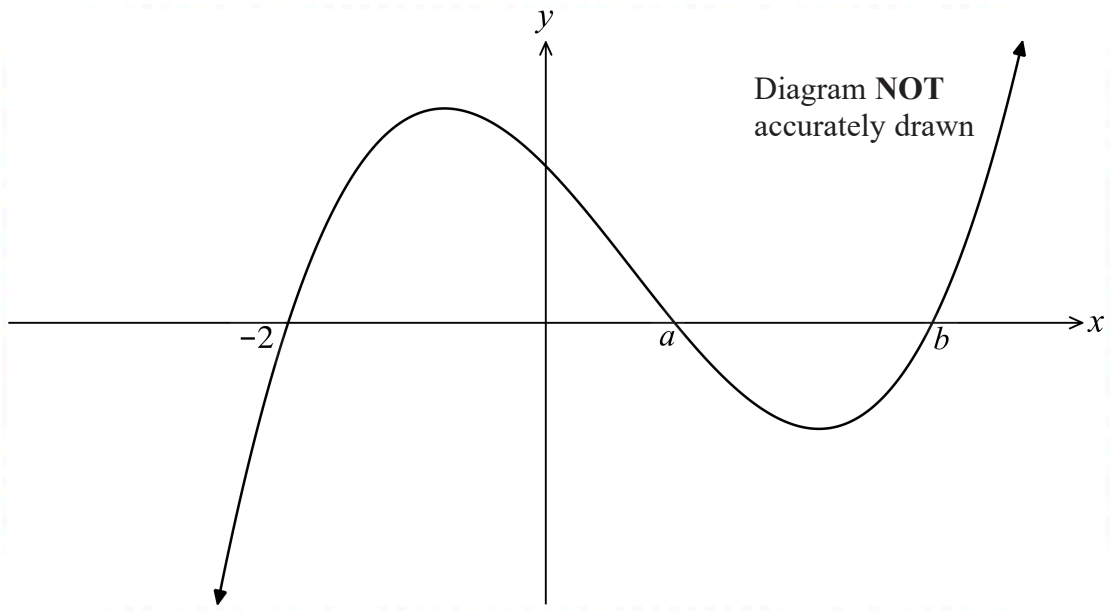
Part D

Name: _____

Class	Teacher	
10Ma**A**	B Kilmore	
10Ma**B**	T Broockmann	
10Ma**C**	Z Zeng	
10Ma**P**	S Gurjar	
10Ma**L**	R Wang	
10Ma**U**	AM Gainford	
10Ma**S**	J James	

Mark	/10

(a) The graph shows the polynomial with equation $y = x^3 - 2x^2 - 5x + 6$. **3**



Find the values of a and b, with clear reasoning being shown.

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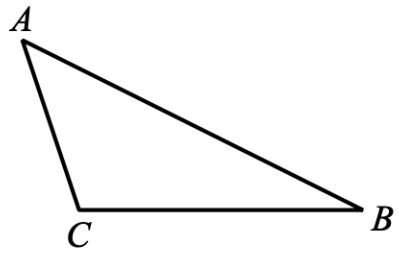
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(b) In ΔABC, $AB = 5$ cm, $AC = 3$ cm, $\angle B = 25°$, and $\angle C$ is obtuse.

(i) Find, to the nearest degree, the size of $\angle C$. **2**



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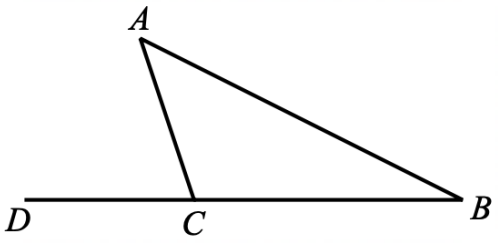
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(ii) The point D lies on BC produced and $AD = 3$ cm. **2**

Find, to three significant figures, the length of CD.



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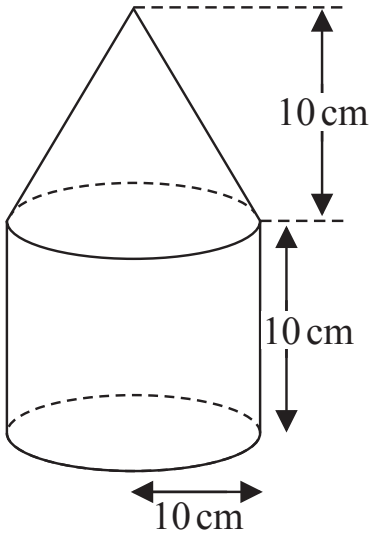
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(c) The diagram shows a solid shape made from a cone on top of a cylinder. **3**

The cone has a radius of 10 cm and a height of 10 cm.

The right cylinder has a radius of 10 cm and a height of 10 cm.



Show that the total surface area is $\left(300+100\sqrt{2}\right)\pi$ cm^2.

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Extra Working Page for Part D only

Part E

Name: _____

Class	Teacher	
10Ma**A**	B Kilmore	
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10Ma**P**	S Gurjar	
10Ma**L**	R Wang	
10Ma**U**	AM Gainford	
10Ma**S**	J James	

Mark	/12

(a) **3**

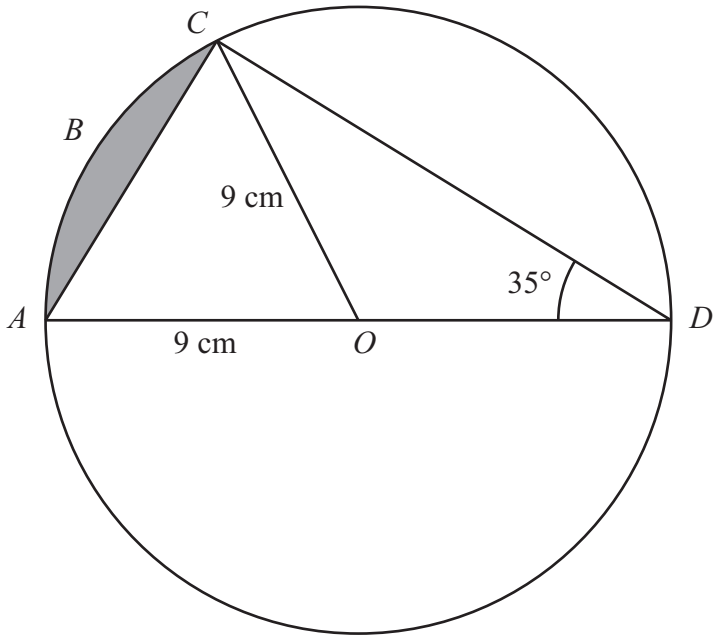


Diagram **NOT** accurately drawn

AOD is a diameter of a circle, with centre *O* and radius 9 cm.
ABC is an arc of the circle.
AC is a chord.
Angle *ADC* = 35°

Calculate the area of the shaded segment.
Give your answer correct to 3 significant figures.

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(b) Evaluate $\log_6 72 - \log_6 2$. **1**

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(c) If $\log_a 2 = p$ and $\log_a 6 = q$, express $\log_a 48$ in terms of p and q. **1**

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(d) Evaluate $49^{\log_7 6}$. **1**

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(e) Consider the function f defined by $f(x) = \dfrac{x}{3x+1}$.

(i) Find $f(f(1))$. **1**

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(ii) Show that $\dfrac{x}{3x+1} = \dfrac{1}{3} - \dfrac{1}{3(3x+1)}$. **1**

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(iii) Hence, explain why f^{-1} exists without finding $f^{-1}(x)$. **1**

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(iv) Find where $f(x) = f^{-1}(x)$. **2**

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(v) Write down the domain of f^{-1}. **1**

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Extra Working Page for Part E only

Part F

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10Ma**S**	J James	

Mark	/12

(a) Solve $\log_{\sqrt{3}}(\sin x) - \log_{\sqrt{3}}(\cos x) = 1$ for $0° < x < 90°$. **2**

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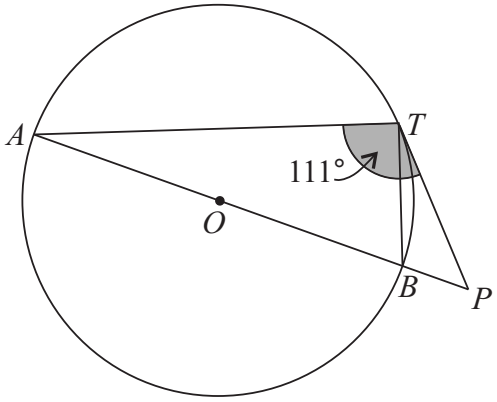
(b) The point $P(-1, 4)$ lies on the curve with equation $y = f(x)$. **1**

State the coordinates of the image of point P on the curves with equation $y = -f(3x)$.

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(c) In the diagram AB is a diameter of the circle, TP is a tangent at point T, O is the centre of the circle and $\angle ATP = 111°$.



Write down the value of $\angle BAT$, giving clear reasoning. **2**

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(d) Given that $(x-4)$ is a factor of $f(x)=x^3-2x^2+ax+b$ and that division of $f(x)$ by $(x+2)$ leaves a remainder of 18. Find the values of a and b. **3**

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(e) Students at a Sydney high school took an examination. The examination scores are normally distributed with a mean of 72 and standard deviation of 8.

(i) A student is chosen at random.

Calculate the probability that their examination score is:

1. less than 80; **1**

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2. higher than 56; **1**

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(ii) It is found that 6 students' scores are higher than 88. **2**
Estimate how many students took the test.

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Extra Working Page for Part F only

Part G

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10Ma**L**	R Wang	
10Ma**U**	AM Gainford	
10Ma**S**	J James	

Mark	/16

(a) (i) Solve the simultaneous equations. **3**

$$2x^2 - y^2 = 17$$
$$x + 2y = 1$$

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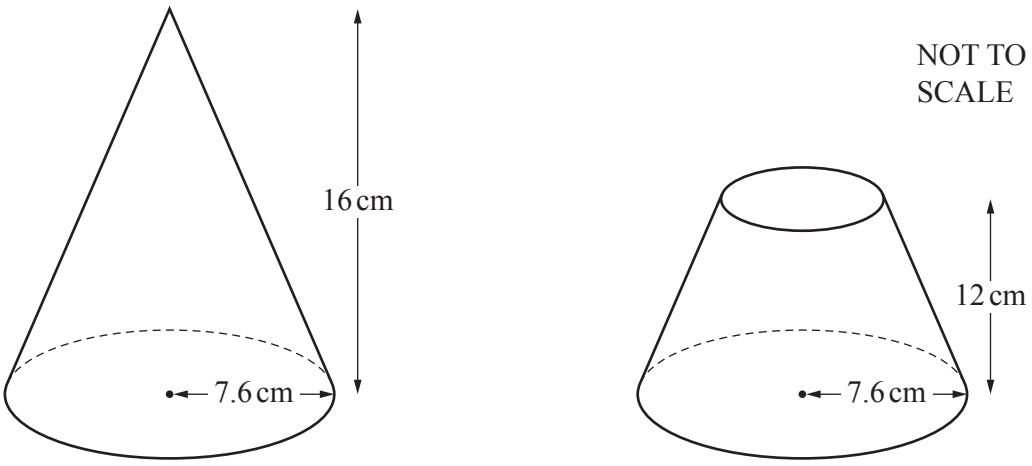
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(b) **3**



The diagram shows a solid cone with radius 7.6 cm and height 16 cm.
A cut is made parallel to the base of the cone and the top section is removed.
The remaining solid has height 12 cm, as shown in the diagram.

Calculate the volume of the remaining solid, correct to two decimal places.

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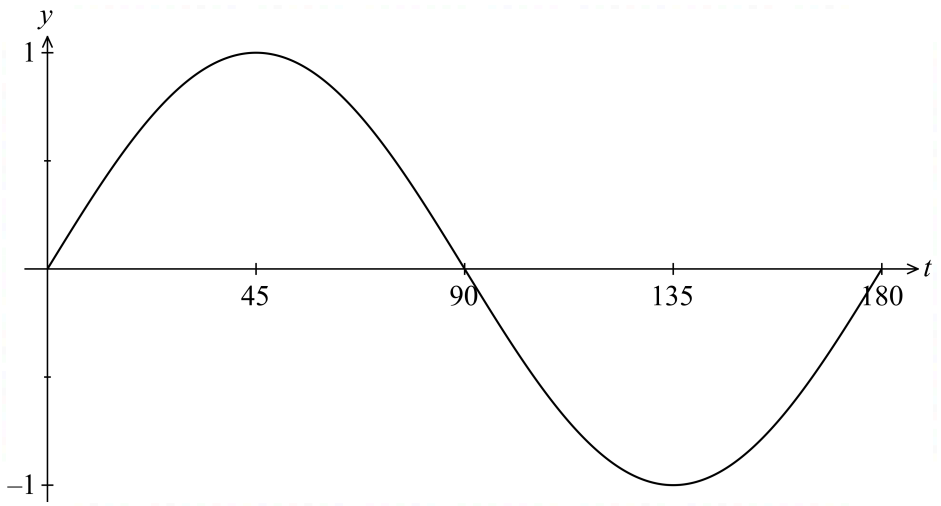
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(c) The diagram shows the graph of $y = f(t)$, where $f(t) = \sin 2t°$, $0 \le t \le 180$.



(i) Write down the range of y when $y = -5f(t+30)$. **1**

……

(ii) Write down the first positive value of t for when the minimum value of $y = -5f(t+30)$ occurs. **1**

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(d) When $p(x) = 2x^2 + 4x + c$ is divided by $(x - r)$, the remainder is 3. **3**
Given that c and r are constants, find the largest possible value for c.

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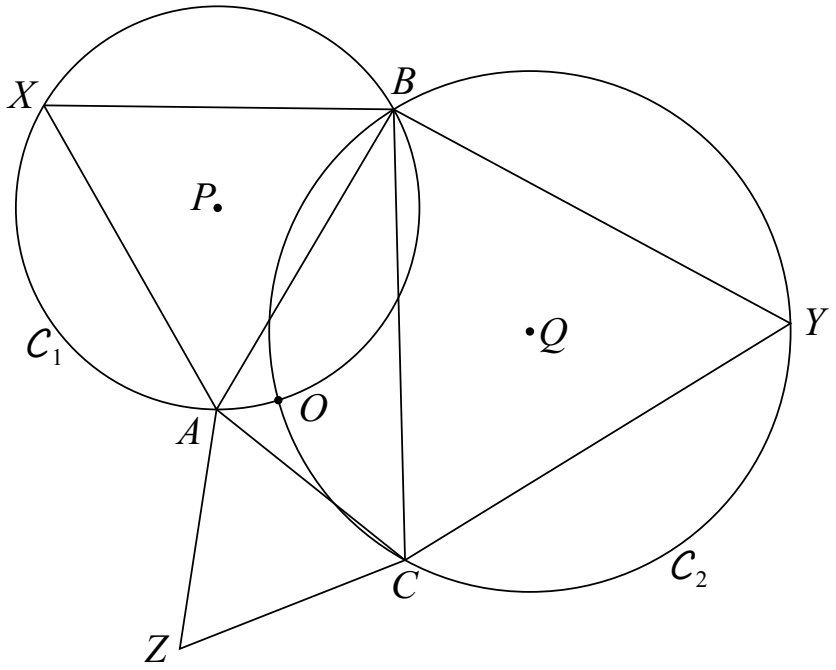
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(e)



A scalene triangle ABC has equilateral triangles ABX, BCY and ACZ constructed on its three sides. Circle $\mathcal{C}_1$ is drawn to pass through the vertices of $\triangle ABX$ and has centre P. Similarly circle $\mathcal{C}_2$ is drawn through the vertices of $\triangle BCY$ and has centre Q. These two circles meet at B and O as shown in the diagram above.

(i) Find the sizes of $\angle AOB$, $\angle BOC$, and $\angle AOC$. **1**

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(ii) Prove that $AOCZ$ is a cyclic quadrilateral. **1**

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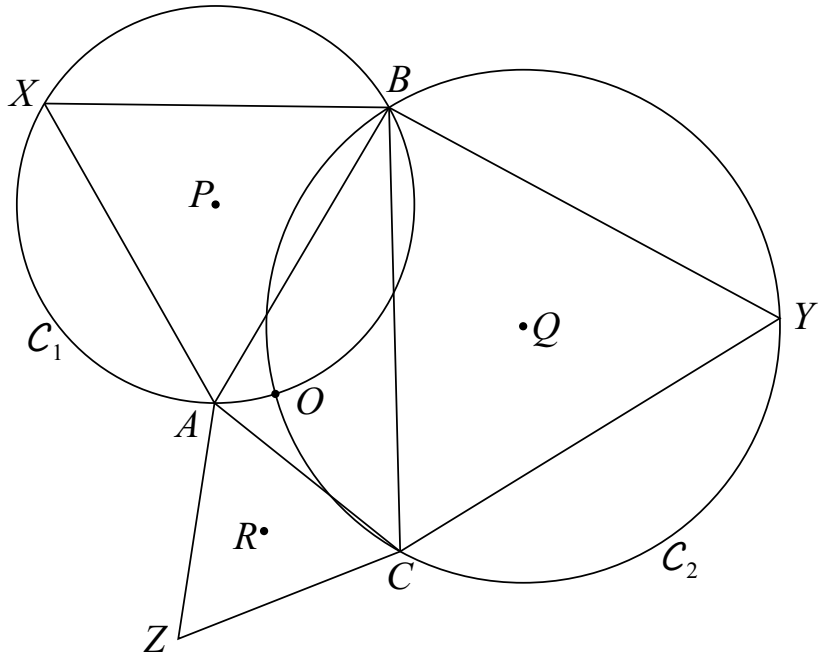
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This question continues on page 36

(e) (continued)

(iii) Taking R as the centre of circle $AOCZ$, prove that ΔPQR is equilateral. **3**



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End of paper

Extra Working Page for Part G only

SYDNEY
BOYS
HIGH
SCHOOL

2021

YEAR 10
TERM 4
YEARLY ASSESSMENT TASK

Mathematics Stage 5.3

Sample Solutions

NOTE: **Some of you may be disappointed with your mark.**
This process of checking your mark is NOT the opportunity to improve your marks.
Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown 'working' does not justify that your solution is worth any marks.

If your solutions are in **pencil** or you have used white out/liquid paper you **CAN'T** appeal.

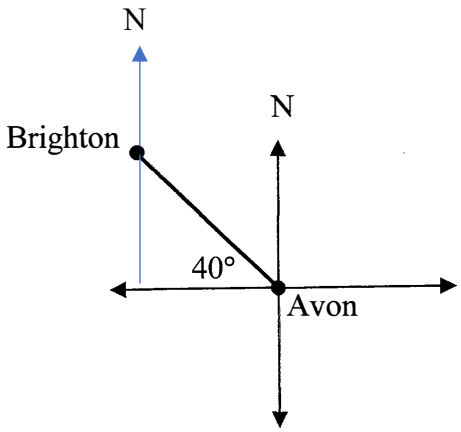
Multiple Choice Answers:

1. A
2. D
3. D
4. C
5. C
6. A
7. B
8. B
9. D
10. C

Part A Solutions

Question 1

Answer: **A**



The angle formed at Brighton from south towards east is 50° (co-interior angles on parallel lines). Therefore, the compass bearing of Avon from Brighton is S 50° E.

Question 2

Answer: **D**

The graph does not pass either the horizontal or vertical line tests, and so is many-to-many.

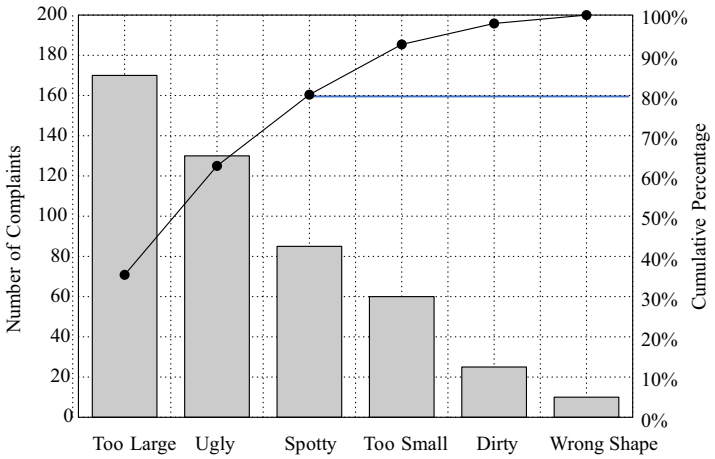
Question 3

Answer: **D**

$$v^2 = u^2 + 2as$$
$$u^2 = v^2 - 2as$$
$$u = \sqrt{v^2 - 2as}, u > 0$$

Question 4

Answer: **C**



Question 5

Answer: **C**

$$x = \log_7 2$$

$$x = \frac{\log_{10} 2}{\log_{10} 7}$$

Question 6

Answer: **A**

Question 7

Answer: **B**

The term inside the radical must be zero or positive, as we cannot take the square root of a negative number.

Question 8

Answer: **B**

If we start with the cubic $f(x) = x^3$, then $g(x) = (x-h)^3 + k$ translates the curve to the right by h units and upward by k units. Hence $g(x) = (x-2)^3 + 5$ translates the graph to the right by 2 and up by 5.

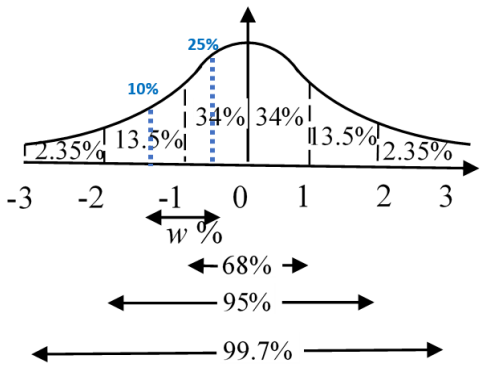
Question 9

Answer: **D**

N_1 is further to the left than N_2, and so therefore has a lower mean.
N_1 is also narrower than N_2, and therefore has lower standard deviation.

Question 10

Answer: **C**



From the Normal Distribution percentages.
The given weight of the melon is in the region $10\% < w < 25\%$.

Part B

(a) Solve $(x+7)(x-4)=0$ **1**

$x^2+3x-28$ $x=-7$ or $x=4$

(b) Simplify the expression $3y+2x-4y-x$. **1**

$-y+x$ or $x-y$

(c) Write down the exact value of $\sin 120°$. **1**

$\frac{\sqrt{3}}{2}$

(d) Evaluate $\log_7 49$. **1**

$7^2=49 \Rightarrow x=2$

(e) Factorise $x^2-2x-15$. **1**

$(x+3)(x-5)$

(f) A cylinder is enlarged to give a similar second cylinder. **1**
If the ratio of their heights is 1 : 2, what is the ratio of their volumes?

$1^3:2^3 = 1:8$

(g) State the vertex of the parabola with equation $y=(x-2)^2-1$. **1**

$(2,-1)$

(h) Consider the line with equation $12y+3x=4$. **1**
Write down its gradient.

$12y=-3x+4$

$y=\frac{-1}{4}x+\frac{1}{3}$

$m=-\frac{1}{4}$

8

(i) Rationalise the denominator $\frac{3}{\sqrt{2}}$. **1**

$\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

(j) Solve $2^x = 64$. **1**

$2^x = 2^6$

$\Rightarrow x = 6$

(k) Liz is asked to solve the equation $3x^2 + 8 = 83$ **1**

Here is her working.

$$3x^2 + 8 = 83$$
$$3x^2 = 75$$
$$x^2 = 25$$
$$x = 5$$

Explain what is wrong with Liz's answer

Last line – should be $x = \pm 5$

(l) Simplify $5u^2w^4 \times 7uw^3$. **1**

$35u^3w^7$

(m) The surface gravity of a planet can be worked out using the formula **1**

$$g = \frac{6.67 \times 10^{-11} m}{r^2},$$

where m kilograms is the mass of the planet and r metres is the radius of the planet.
For the Earth and Jupiter the values of m and r and listed below:

Earth	Jupiter
$m = 5.98 \times 10^{24}$ $r = 6.378 \times 10^6$	$m = 1.90 \times 10^{27}$ $r = 7.149 \times 10^7$

5

Calculate the ratio of the surface gravity of Earth to the surface gravity of Jupiter

Write your answer in the form 1 : n, correct to two significant figures.

$g_E = 9.805234578$ $= 1 : 2.5288$

$g_J = 24.79641183$

$= 1 : 2.5$ (2sf)

(n) Ibrar bought a cabin for $145 000. **1**

The value of the cabin depreciated by 4% in the first year.
The value of the cabin depreciated by 2.5% in the second year.

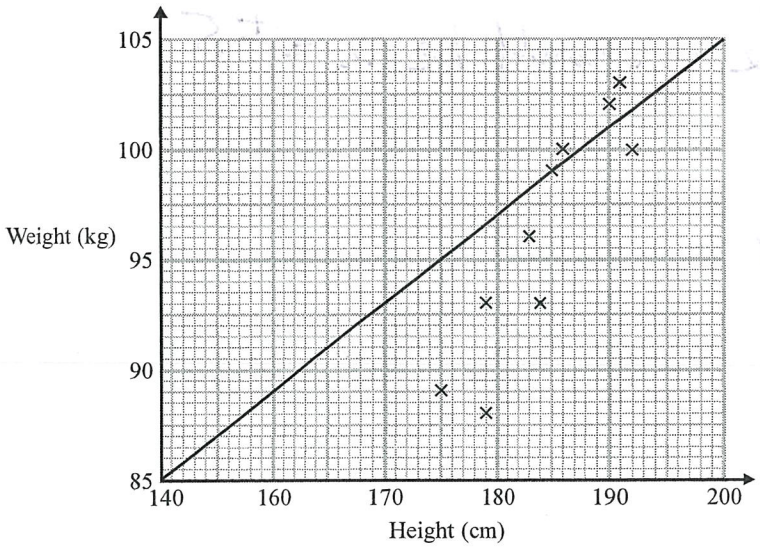
His cousin, Rarbi says,

"4 + 2.5 = 6.5, so in two years the value of your cabin depreciated by 6.5%"

Is Rarbi right? If you think he is wrong, give the correct percentage depreciation.

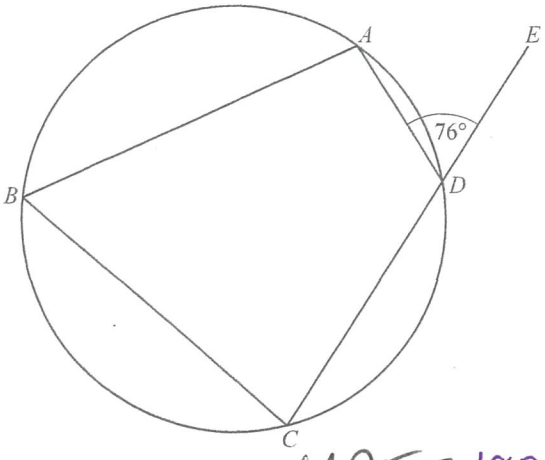
No — Amount $= A \times 0.96 \times 0.975$
$= A \times .936$
$\Rightarrow$ Depreciation $= 0.064$
$= 6.4\%$

(o) Sean has information about the height, in cm, and the weight, in kg, of each of ten rugby players. **2**
He is asked to draw a scatter graph and a line of best fit for this information.
Here is his answer



Sean has plotted the points accurately.
Write down two things that are wrong with his answer.

1. Line of best fit should have been steeper so that distances of points above line should approx equal distance of points below the line.
2. He has extrapolated well below the shortest recorded height of 175cm.
3. Scale on x-axis is not consistent.

(p) 

A, B, C, and D are concyclic points. **1**

CDE is a straight line.

Write down the value of $\angle ABC$.

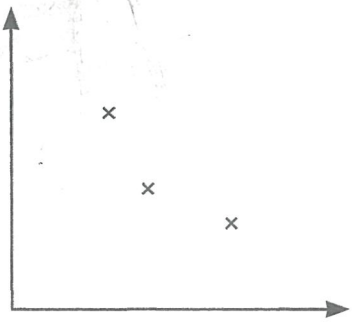
Note Reasons not required.

$\angle ADE = 180 - 76 = 104°$

$\Rightarrow \angle ABC = 76°$

OR $\angle ABC = 76°$ (opp angle = ext. angle of cyc. quad)

(q) Henry drew the scatter diagram below. **1**



Put a circle around the most correct statement about this scatter diagram.

It shows no correlation.	It is not possible to tell if there is correlation as there are not enough points.	It shows negative correlation.	It shows positive correlation.

(r) Let $f(x) = 4\sin x°$ and $g(x) = 2x - 3$.

(i) Evaluate $f(23)$. **1**

Give your answer correct to three significant figures.

1·56

(ii) Evaluate $f[g(24)]$. **1**

Give your answer correct to three significant figures.

$f(45) = 2.83$

Part C

(a) Re-arrange this formula to make x the subject **1**

$$y = mx + c.$$

$y = mx + c$

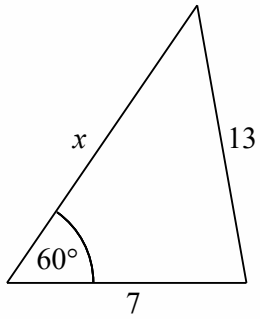
$y - c = mx$

$mx = y - c$

$$x = \frac{y - c}{m}$$

[Well answered by most candidates.]

(b) **4**



NOT TO SCALE

The diagram shows a triangle with sides 7 cm, 13 cm and x cm, and an angle of 60° as marked.

Use the cosine rule to show that $x^2 - 7x = 120$, and hence find the exact value of x.

$13^2 = x^2 + 7^2 - 2 \times x \times 7\cos 60°$

$169 = x^2 + 49 - 14x \times ½$

$120 = x^2 - 7x$

$x^2 - 7x = 120$ as required

$$x = \frac{7 \pm \sqrt{49 + 480}}{2}$$

$= 15$ or -8 but -8 is extraneous (length of a side)

So $x = 15$

[Marks were lost for using the sign rule, which does not lead to the quadratic required. Also marks were lost if the negative solution was not explained and discounted.]

(c) Divide $4x^4 - 36x^2 + 9$ by $\left(2x^2 - 1\right)$. **2**

Write your answer in the form $4x^4 - 36x^2 + 9 = \left(2x^2 - 1\right)Q\left(x\right) + R\left(x\right)$.

$$
\begin{array}{r|l}
2x^2 - 1 & 2x^2 \quad -17 \\
\hline
)4x^4 \quad -36x^2 \quad +9 & \\
\underline{4x^4 \quad -2x^2} & \\
-34x^2 \quad +9 & \\
\underline{34x^2 \quad +17} & \\
-8 &
\end{array}
$$

$\therefore 4x^4 - 36x^2 + 9 = (2x^2 - 1)(2x^2 - 17) - 8$

[Well answered in general, but many made a small arithmetic error. There were also several transcription errors with the divisor.]

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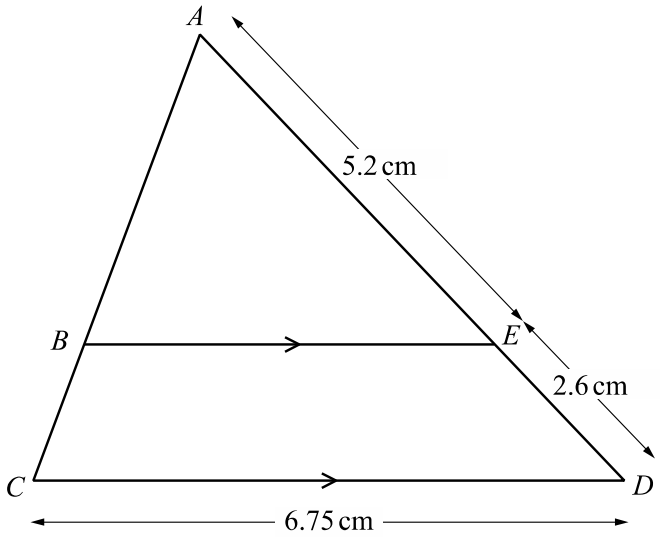
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(d) In ΔACD, B lies on AC and E lies on AD such that $BE \parallel CD$.
$AE = 5.2$ cm and $ED = 2.6$ cm



NOT TO SCALE

(i) State why $\Delta ABE \;|||\; \Delta ACD$. **1**

Equiangular ($\angle AEB = \angle EDC$ corresponding angles)

[Just the single word, or the abbreviation (AA), was required. Well answered.]

(ii) Calculate BE, with clear reasoning being shown. **2**

$\frac{BE}{CD} = \frac{AE}{AD}$ Corresponding sides in proportion

$BE = \frac{CD \times AE}{AD}$

$= (6.75 \times 5.2)/7.8$

$\therefore BE = 4.5$

[While most got the right result, many failed to show clear reasoning.]

(e) Write $\left(x+3\right) \div \frac{x^2+5x+6}{x-2}$ as a single fraction in its simplest form. **2**

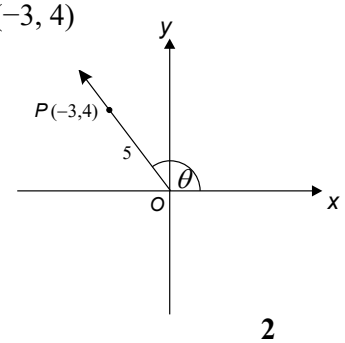
$Exp = \frac{(x+3)(x-2)}{x^2+5x+6}$

$= \frac{(x+3)(x-2)}{(x+3)(x+2)}$

$= \frac{x-2}{x+2}$

[Mostly well answered, but several gave the reciprocal, and some tried to divide, unsuccessfully.]

(f) In the diagram above, the ray OP passes through the point $P(-3, 4)$ and makes the angle θ with the positive x-axis.



Given $OP = 5$ units, write down the values of $\sin\theta$ and $\tan\theta$. **2**

$$\tan\theta = -\frac{4}{3} \qquad \sin\theta = \frac{4}{5}$$

*[Many over-complicated the problem by finding **θ**, then approximating sin**θ** and tan**θ**.]*

(g) If $3^{2x-1} = \frac{1}{9^x} \times 3^{2y-x}$, find a simple expression for y in terms of x. **2**

$3^{2x-1} = 3^{-2x} \times 3^{2y-x}$ $\qquad$ $2y = 5x - 1$

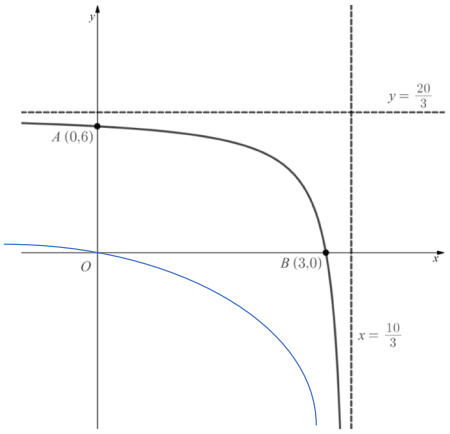
$3^{2x-1} = 3^{-2x+2y-x}$ $\qquad$ $y = ½ (5x - 1)$

$\therefore 2x - 1 = 2y - 3x$

[Well done by 60%. Those who tried a log approach generally failed.]

(h) The diagram below shows the graph of $y = f(x)$. The graph intersects the axes at points $A(0, 6)$ and $B(3, 0)$. The graph has two asymptotes as shown. **2**

Sketch the graph of $y = f(x) - 6$, showing clearly where A and B get transformed to. Also the location of the asymptotes after the transformation should be indicated.



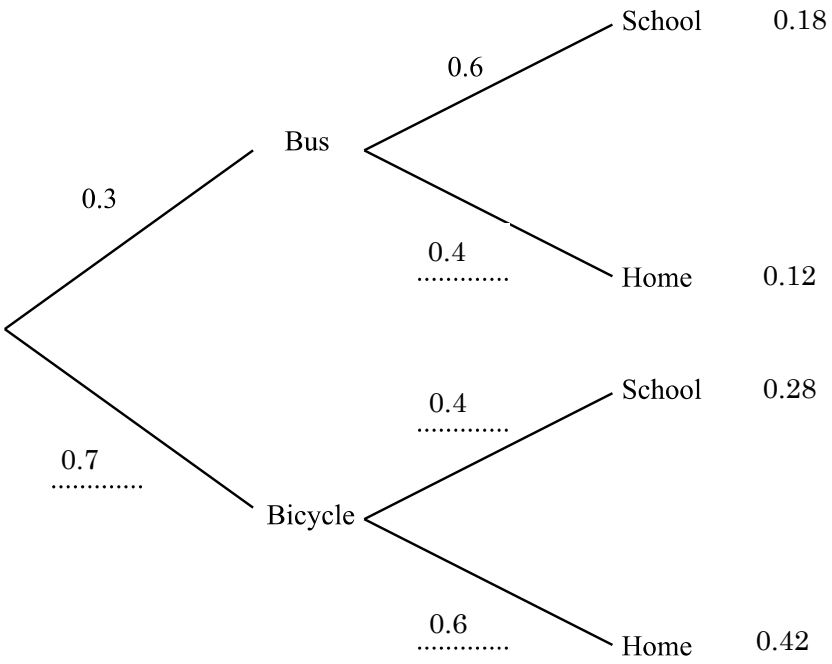
The image of A is the origin O. Horizontal asymptote $y = 2/3$.

The image of B is $B'(3, -6)$. Vertical asymptote $x = 10/3$.

[Usually well done. Some failed to show B' and new Horiz asymptote.]

(i) Claude travels to school either by bus or on a bicycle. **2**
The probability that he goes by bus is 0.3.
He can have lunch at home or at school.
If he catches the bus, the probability that he has lunch at school is 0.6
If he rides his bicycle, the probability that he has lunch at home is 0.6.

By using the diagram or otherwise, find the probability that Claude goes by bus given that he has lunch at school.



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= 0.18/0.46$$

$$= 9/23$$

$$\simeq 0.39$$

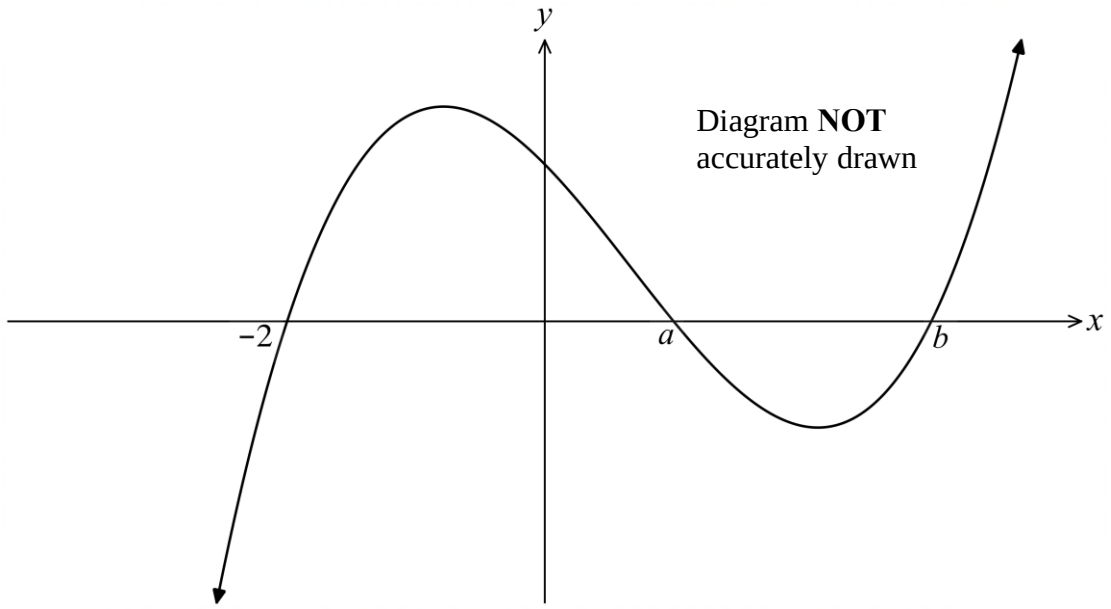
[Not very well answered. Most had no grasp of Bayes's Teorem.]

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2021 YR10 YE Part D Solutions and Comments

Answers in pencil **CANNOT** appeal for marks.

A. The graph shows the polynomial with equation $y = x^3 - 2x^2 - 5x + 6$. **3**



Find the values of a and b, with clear reasoning being shown.

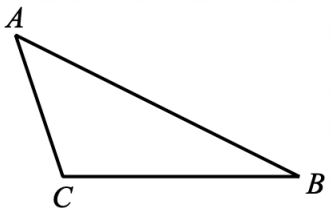
Solution	Comment(s)
Let $P(x) = y$. From the graph, there is a single root at $x = -2$. Hence, $x + 2$ is a factor of $P(x)$. By polynomial division: $$\begin{array}{r} x^2 - 4x + 3 \\ x+2 \overline{)\, x^3 - 2x^2 - 5x + 6} \\ \underline{x^3 + 2x^2} \\ -4x^2 - 5x \\ \underline{-4x^2 - 8x} \\ 3x + 6 \\ \underline{3x + 6} \\ 0 \end{array}$$ Hence: $$P(x) = (x+2)(x^2 - 4x + 3)$$ $$= (x+2)(x-1)(x-3)$$ Hence, $P(x)$ has three single roots at $x = -2, 1, 3$. From the graph, $-2 < a < b$, so $a = 1$ and $b = 3$.	Most students found another factor, e.g. $x - 1$, to use for polynomial division instead of using the given factor $x + 2$. Some examples of alternative methods that were used to solve this problem: • Synthetic division • Trial and error using the factor theorem • Expanding $(x + 2)(x - a)(x - b)$ and comparing coefficients However, many students who used these alternatives were penalised for failing to show sufficient relevant working out. Common error(s): • Attempting to solve the $P(x)$ using the quadratic formula

2021 YR10 YE Part D Solutions and Comments

Answers in pencil **CANNOT** appeal for marks.

B. In ΔABC, $AB = 5\ cm$, $AC = 3\ cm$, $\angle ABC = 25°$ and $\angle ACB$ is obtuse.

I. Find, to the nearest degree, the size of $\angle ACB$. **2**

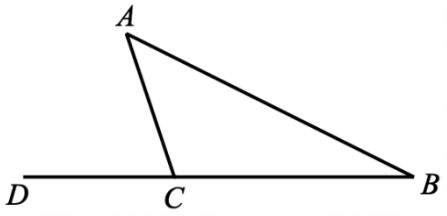


Solution	Comment(s)
Let $\angle ACB = \theta$ Using the sine rule: $\frac{\sin\theta}{5} = \frac{\sin 25°}{3}$ $\sin\theta = \frac{5\sin 25°}{3}$ $\theta = 45°\ (n.deg.)$ However, as the sine function is positive in Quadrants 1 and 2, θ can be either $45°$ or $135°$. Since θ is given to be obtuse: $\theta = 180° - 45°$ $= 135°$	Some students used the cosine rule when it wasn't necessary. Of those who used the cosine rule, most failed to use it correctly to solve the problem. Common error(s): • Forgetting to round to the nearest degree • Forgetting that $\angle ACB$ is obtuse

2021 YR10 YE Part D Solutions and Comments

Answers in pencil **CANNOT** appeal for marks.

II. The point D lies on BC produced and $AD = 3\ cm$. **2**
Find, to three significant figures, the length of CD.

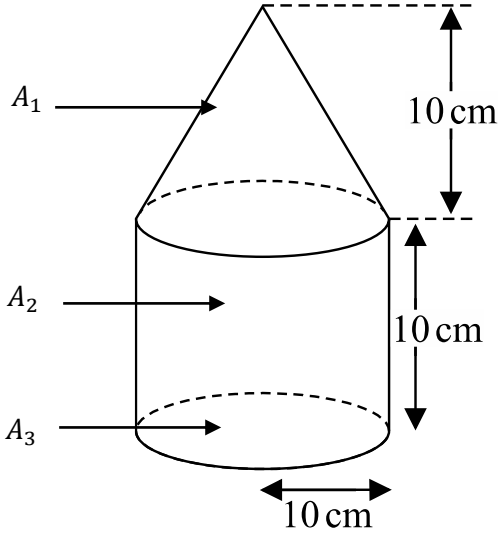


Solution	Comment(s)
From Part I, $\angle ACB = 135°$. Then, as BCD is a straight line: $\angle ACD = 180° - 135°$ $= 45°$ Since $AD = AC$, equal angles are opposite to equal sides. $\angle ADC = 45°$ By considering the angle sum of ΔADC: $\angle DAC = 180° - 45° - 45°$ $= 90°$ Using the sine rule: $\frac{CD}{\sin 90°} = \frac{3}{\sin 45°}$ $CD = \frac{3 \sin 90°}{\sin 45°}$ $= 4.24\ cm$	Again, some students used the cosine rule unnecessarily and most were incorrect in their attempt to solve the problem. Also, some students assumed that ΔADC is a right angle triangle instead of recognising it as an approximation. As such, they risked losing marks by using Pythag instead of the sine rule to find the length of CD. Furthermore, some students risked losing marks by failing to round their answer correctly to 3 significant figures. Most students were also penalised for failing to provide sufficient geometrical reasoning when deducing the size of the relevant angles.

2021 YR10 YE Part D Solutions and Comments

Answers in pencil **CANNOT** appeal for marks.

C. The diagram shows a solid shape made from a cone on top of a cylinder. **3**
The cone has a radius of $10\ cm$ and a height of $10\ cm$.
The right cylinder has a radius of $10\ cm$ and a height of $10\ cm$.



Show that the total surface area is $\left(300 + 100\sqrt{2}\right)\pi\ cm^2$.

Solution	Comment(s)
Let A_1, A_2 and A_3 be the curved surface of the cone, the curved surface of the cylinder and the base of the cylinder respectively. Then: $A_1 = \pi rl$ $= \pi \times 10 \times \sqrt{10^2 + 10^2}$ $= 100\pi\sqrt{2}\ cm^2$ Also: $A_2 = 2\pi rh$ $= 2\pi \times 10 \times 10$ $= 200\pi\ cm^2$ Finally: $A_3 = \pi r^2$ $= \pi \times 10^2$ $= 100\pi\ cm^2$ Hence: $A = A_1 + A_2 + A_3$ $= 100\pi\sqrt{2} + 200\pi + 100\pi$ $= 100\pi\sqrt{2} + 300\pi$ $= \left(300 + 100\sqrt{2}\right)\pi\ cm^2$	Some students risked losing marks by working with rounded values for areas instead of exact values. Otherwise, students generally performed well on this question.

PART E

(a) **3**

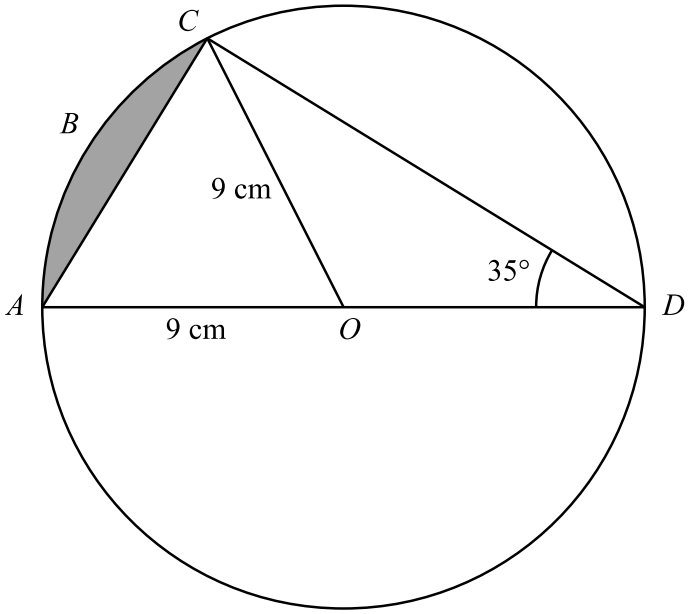


Diagram **NOT** accurately drawn

AOD is a diameter of a circle, with centre *O* and radius 9 cm.
ABC is an arc of the circle.
AC is a chord.
Angle *ADC* = 35°

Calculate the area of the shaded segment.
Give your answer correct to 3 significant figures.

$\angle AOC = 70°$ (angle at the centre is double the angle at the circumference)

The area of the shaded region is the area of triangle AOC subtracted from the area of sector AOC.

Area of Sector $\left(A_S\right) = \dfrac{\theta°}{360°}\pi r^2$, where $\theta = 70$ and $r = 9$.

Area of ΔACO $\left(A_T\right) = \dfrac{1}{2}bc\sin A$, where $b = c = 9$ and $A = 70$.

$A = A_S - A_T$

$A = \dfrac{70}{360}\pi 9^2 - \dfrac{1}{2}9^2 \sin 70$

= 11.4 cm^2 (to 3 significant figures)

(b) Evaluate $\log_6 72 - \log_6 2$. **1**

Expression equals $\log_6 \dfrac{72}{2} = \log_6 36$ which simplifies to $\log_6 6^2 = 2$

(c) If $\log_a 2 = p$ and $\log_a 6 = q$, express $\log_a 48$ in terms of p and q. **1**

$\log_a 48 = \log_a 8 + \log_a 6 \Rightarrow 3\log_a 2 + \log_a 6 = 3p + q$

(d) Evaluate $49^{\log_7 6}$. **1**

$\left(7^2\right)^{\log_7 6} = 7^{2\log_7 6} \Rightarrow 7^{\log_7 6^2} = 36$

(e) Consider the function f defined by $f(x)=\dfrac{x}{3x+1}$.

(i) Find $f(f(1))$. **1**

$$f(1)=\frac{1}{3\times 1+1}=\frac{1}{4}$$

$$f\left(\frac{1}{4}\right)=\frac{\frac{1}{4}}{3\times\frac{1}{4}+1}\Rightarrow\frac{\frac{1}{4}}{\frac{7}{4}}=\frac{1}{7}$$

(ii) Show that $\dfrac{x}{3x+1}=\dfrac{1}{3}-\dfrac{1}{3(3x+1)}$. **1**

$$RHS=\frac{1}{3}-\frac{1}{3(3x+1)}\Rightarrow\frac{3x+1}{3(3x+1)}-\frac{1}{3(3x+1)}=\frac{3x+1-1}{3(3x+1)}\Rightarrow\frac{x}{3x+1}=LHS$$

(iii) Hence, explain why f^{-1} exists without finding $f^{-1}(x)$. **1**

$\dfrac{1}{3}-\dfrac{1}{3(3x+1)}$ is a translation (two in fact) and dilation of the 'base' function $f(x)=\dfrac{1}{x}$.

Essentially, this is a hyperbola which is a one-to-one function (passes both the vertical and horizontal line tests) and therefore its inverse exists without the need to restrict $f(x)$.

Note: $f^{-1}(x)$ is defined as a function and hence it must pass the vertical line test. For this to be the case, $f(x)$ must pass the horizontal line test. Since $f(x)$ is already defined as a function, it must be one-to-one to pass both line tests.

(iv) Find where $f(x)=f^{-1}(x)$. **2**

Since the only values that satisfy this equation lie on the line $y = x$, we can solve the two equations: $y = x$ and $y=\dfrac{x}{3x+1}$ simultaneously by using substitution.

$$x=\frac{x}{3x+1}\Rightarrow 1=\frac{1}{3x+1}\Rightarrow 3x+1=1\Rightarrow x=0$$

(v) Write down the domain of f^{-1}. **1**

Since the range of $f(x)$ is all real y, except that $y\neq\dfrac{1}{3}$, the domain of $f^{-1}(x)$ is all real x, except that $x\neq\dfrac{1}{3}$.

Note: The section of the exam was done reasonably well. The questions on the inverse function were done less well and students generally chose approaches that made the working out more difficult for themselves.

Part F

(a) Solve $\log_{\sqrt{3}}(\sin x) - \log_{\sqrt{3}}(\cos x) = 1$ for $0° < x < 90°$. **2**

$\log_{\sqrt{3}}\left(\frac{\sin x}{\cos x}\right) = 1$ | $\tan x = \sqrt{3}$ → 1 mark

$x = 60°$

1 mark ← $\log_{\sqrt{3}}(\tan x) = 1$

(b) The point $P(-1, 4)$ lies on the curve with equation $y = f(x)$. **1**

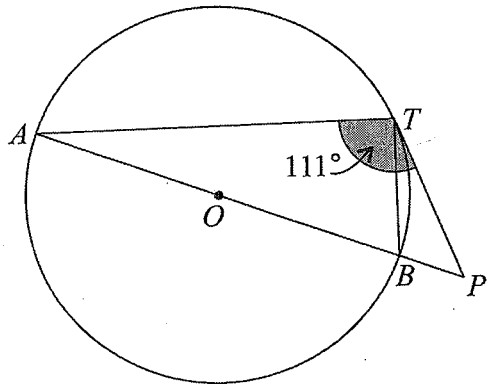
State the coordinates of the image of point P on the curves with equation $y = -f(3x)$.

$y = f(3x)$: $(-\frac{1}{3}, 4)$

$y = -f(3x)$: $(-\frac{1}{3}, -4)$

$\frac{1}{2}$ mark if either coordinate is correct.

(c) In the diagram AB is a diameter of the circle, TP is a tangent at point T, O is the centre of the circle and $\angle ATP = 111°$.



Write down the value of $\angle BAT$, giving clear reasoning. **2**

1 mark { $\angle ATB = 90°$ ($\angle$ in a semi-circle, AB is diameter)

$\angle BTP = 111° - 90°$

$= 21°$

1 mark { $\angle BTP = \angle BAT$

$= 21°$ ($\angle$ in alternate segment)

OR

1 mark { $OA = OA$ (radii of same circle)

$\angle OAT = \angle OTA$ ($\angle$s opposite equal sides)

1 mark { $\angle OTP = 90°$ (radius ⊥ tangent)

$\angle OTA = 21°$

$= OAT$

(d) Given that $(x-4)$ is a factor of $f(x)=x^3-2x^2+ax+b$ and that division of $f(x)$ by $(x+2)$ leaves a remainder of 18. Find the values of a and b. **3**

$P(4)=0$

$4^3-2\times4^2+4a+b=0$

$32+4a+b=0$ ①

$P(-2)=18$

$(-2)^3-2\times(-2)^2-2a+b=18$

$-16-2a+b=18$

$-34-2a+b=0$ ②

① − ② $66+6a=0$

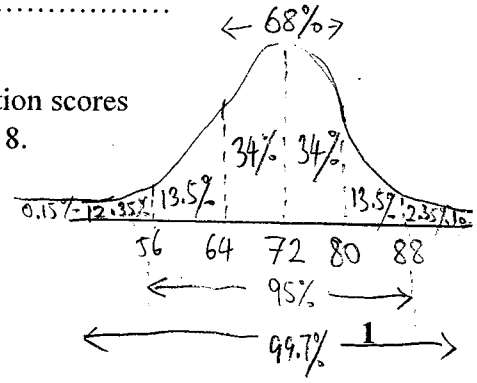
$a=-11$, $b=12$

2 marks

1 mark

1 mark if show correct understanding of remainder/factor theorem but made error with substitution

(e) Students at a Sydney high school took an examination. The examination scores are normally distributed with a mean of 72 and standard deviation of 8.



(i) A student is chosen at random.

Calculate the probability that their examination score is:

1. less than 80;

$50+34=84\%$ 1 mark

Students who wrote 83.85% in (a) and 97.35% in (b) got 1 mark as it was E.C.F

2. higher than 56; **1**

$50\%+34\%+13.5\%=97.5\%$ 1 mark

(ii) It is found that 6 students' scores are higher than 88. **2**
Estimate how many students took the test.

$1-97.5\%=2.5\%$

$2.5\% \times N=6$

$N=240$

- 1 mark if students recognise it's $2\sigma+\bar{x}$, even if answer is incorrect.
- Accept 255 as error carried forward.

Part G

(a)

① $2x^2 - y^2 = 17$

② $x + 2y = 1$

②* $x = 1 - 2y$

②* → ① $2(1-2y)^2 - y^2 = 17$

$2(1 - 4y + 4y^2) - y^2 = 17$

$7y^2 - 8y - 15 = 0$

$(7y - 15)(y + 1) = 0$

$7y - 15 = 0 \Rightarrow y = \frac{15}{7}$, or

$y + 1 = 0 \Rightarrow y = -1$

If $y = \frac{15}{7}$,

$x = 1 - 2 \times \frac{15}{7}$

$= \frac{-23}{7}$

If $y = -1$,

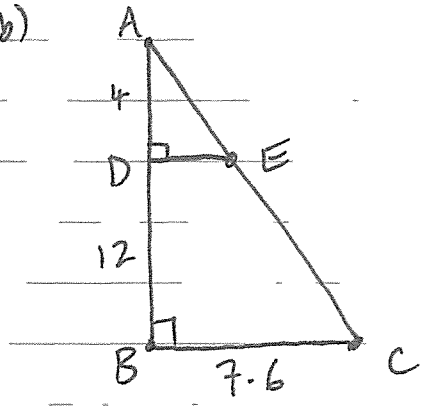
$x = 1 - 2 \times (-1)$

$= 3$

$\therefore \left(\frac{-23}{7}, \frac{15}{7}\right)$ or $(3, -1)$

* Done well by some students. Alternate method has quadratic in x^2.
* Common algebraic errors were expanding brackets incorrectly:
  eg. $(1-2y)^2 = 1 - 4y + y^2$, or $(1-2y)^2 = 1 + 4y^2$
* If working in x, errors with fractions:
  eg. $2x^2 - \left(\frac{1-x}{2}\right)^2 = 17 \rightarrow 2x^2 - (1-x)^2 = 68$ instead of $8x^2 - (1-x)^2 = 68$.
* If students didn't explicitly link pairs of x and y, they did not receive full marks.
* Some students, if working in y, did not find corresponding x-values, or only one.

(b)



By similar triangles (you do not need to prove similarity), DE (the upper radius of the fructum) is found as

$$\frac{DE}{7.6} = \frac{4}{16}$$

$$DE = 1.9$$

$$\text{Volume total} = \frac{1}{3} \times \pi \times 7.6^2 \times 16 - \frac{1}{3} \pi \times 1.9^2 \times 4$$

$$= 952.66 \text{ cm}^3$$

* Done well by many students. Most were correctly able to demonstrate the formula for volume of a pyramid.
* Various errors in finding the smaller pyramid, including using $r = 5.7$ or $h = 12$.

c) i) Since $\sin(2t)$ takes values between -1 and 1, $-5\sin(2t)$ takes values between -5 and 5.

i.e. $-5 \leq y \leq 5$

* Done well. Students who wrote $5 \geq y \geq -5$ were not penalised, but it is frowned upon.

ii) y takes a minimum value where $f(t+30)$ is a maximum. Since $f(45)$ is the maximum, $t + 30 = 45$.

$t = 15$

* Some students got this. Many had $t = 60$, but there was a range of incorrect answers.

d) By the remainder theorem, $p(r) = 3$

$$2r^2+4r+c=3$$

$$2r^2+4r+2+c=5$$

$$2(r+1)^2+c=5$$

since $(r+1)^2 \geq 0$,

c is at a max when $r=-1$,

where $c=5$

$$2r^2+4r+(c-3)=0$$

$$\Delta = b^2-4ac$$

$$= 4^2-4\times 2\times(c-3)$$

$$= 40-8c$$

Since $\Delta \geq 0$

$$40-8c \geq 0$$

$$c \leq 5$$

* Not many students managed this question well.
* Many students got 1 mark for correctly applying the remainder theorem.
* Some students got the right answer through trial-and-error, but did not receive full marks because they did not justify why $c=5$ is the largest.

e) i) In cyclic quadrilateral AXBO,

$\angle AOB + \angle AXB = 180°$ (opposite angles in cyclic quad are supplementary)

$\angle AXB = 60°$ (ΔAXB is equilateral)

$\therefore \angle AOB = 180-60 = 120°$

By similar logic in BTCO, $\angle BOC = 120°$

$\angle AOC + \angle AOB + \angle BOC = 360°$ (angles at a point)

$\therefore \angle AOC = 360-120-120 = 120°$

* Done well. Bald answers were accepted.

ii) In quadrilateral AOCZ,

$\angle AZC = 60°$ (angle in equilateral ΔAZC)

$\angle AOC + \angle AZC = 120+60 = 180°$

$\therefore$ AOCZ is cyclic (opposite angles are supplementary)

* Many answered this well. Remember that proofs must be clear.

(iii) Construct chord OB with midpoint M

In circle C_1, $\angle PMO = 90°$ (line from centre of circle that bisects a chord is perpendicular to the chord)

By the same logic in C_2 $\angle QMO = 90°$

Since $\angle PMO + \angle QMO = 180°$, PMQ is a straight line.

Construct chord OA with midpoint N

By the same logic as above, $\angle PNO = 90°$ and PNR is a straight line.

Consider quadrilateral PMON

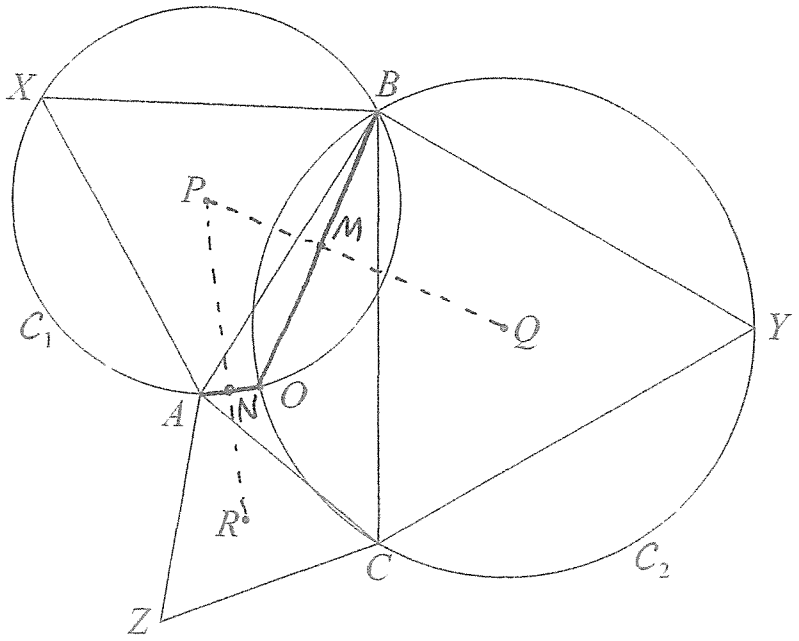
$\angle NPM + \angle PMO + \angle MON + \angle ONP = 360°$ (angle sum of quad)

$\angle NPM + 90° + 120° + 90° = 360°$ ($\angle MON$ found in (i))

$\therefore \angle NPM = \angle RPQ = 60°$

By the same logic, $\angle PQR = \angle PRQ = 60°$

$\therefore \triangle PQR$ is equilateral (all angles are 60°)



* Very few students completed this. All other approaches failed to make meaningful progress, and received no marks.