SYDNEY BOYS HIGH SCHOOL

2020

YEAR 10 STAGE 5.3
TERM 4
YEARLY ASSESSMENT

Mathematics Stage 5.3

General Instructions

- All questions may be attempted.
- Writing using black or blue pen.
- Marks may be deducted for messy or badly arranged work. This can include insufficient working.
- All working and answers must be written in this booklet.
- NESA approved calculators may be used.
- Clearly indicate your class by placing an **X** next to it.
- For multiple choice, circle the letter of your chosen answer on the answer sheet provided.
- Leave your answers in the simplest exact form, unless otherwise stated.
- If you wish to rewrite an answer, cross out your faulty answer and write "**See back**" at the original question. Rewrite your new answer on the "**Spare working pages**" at the back of the exam booklet.

Time Allowed:

- Reading time: 5 minutes
- Working time: 120 minutes

Examiner: RW

Name: _____

Class	Teacher	
10Ma**A**	Mr A Wang	
10Ma**B**	Mr J James	
10Ma**C**	Mr AM Gainford	
10Ma**P**	Mr A Fuller	
10Ma**L**	Ms J Chan & Ms Z Zeng	
10Ma**U**	Ms B Kilmore	
10Ma**S**	Ms J Millar	

	Marks
Section I	/9
Section II	
Part A	/19
Part B	/17
Part C	/17
Part D	/18
Part E	/15
Total	/95

Section I **9 marks**

1. If two lines are perpendicular, what can be said about their gradients m_1 and m_2?

   A. $\frac{m_1}{m_2} = 1$

   B. $\frac{m_1}{m_2} = -1$

   C. $m_1 m_2 = 1$

   D. $m_1 m_2 = -1$

2. Consider the two way table below of 30 students.
   What is the probability of choosing a boy given that the student is a junior?

	Boy	Girl
Junior	7	5
Senior	8	10

   A. $\frac{7}{12}$

   B. $\frac{7}{15}$

   C. $\frac{12}{30}$

   D. $\frac{15}{30}$

3. Which of the following equations is a polynomial?

   A. $y = 3x$

   B. $y = \frac{3}{x}$

   C. $y = 3^x$

   D. $y = \sqrt[3]{x}$

4. Given that $\sin\theta > 0$ and $\cos\theta < 0$, which quadrant does θ lie in?

   A. Quadrant 1

   B. Quadrant 2

   C. Quadrant 3

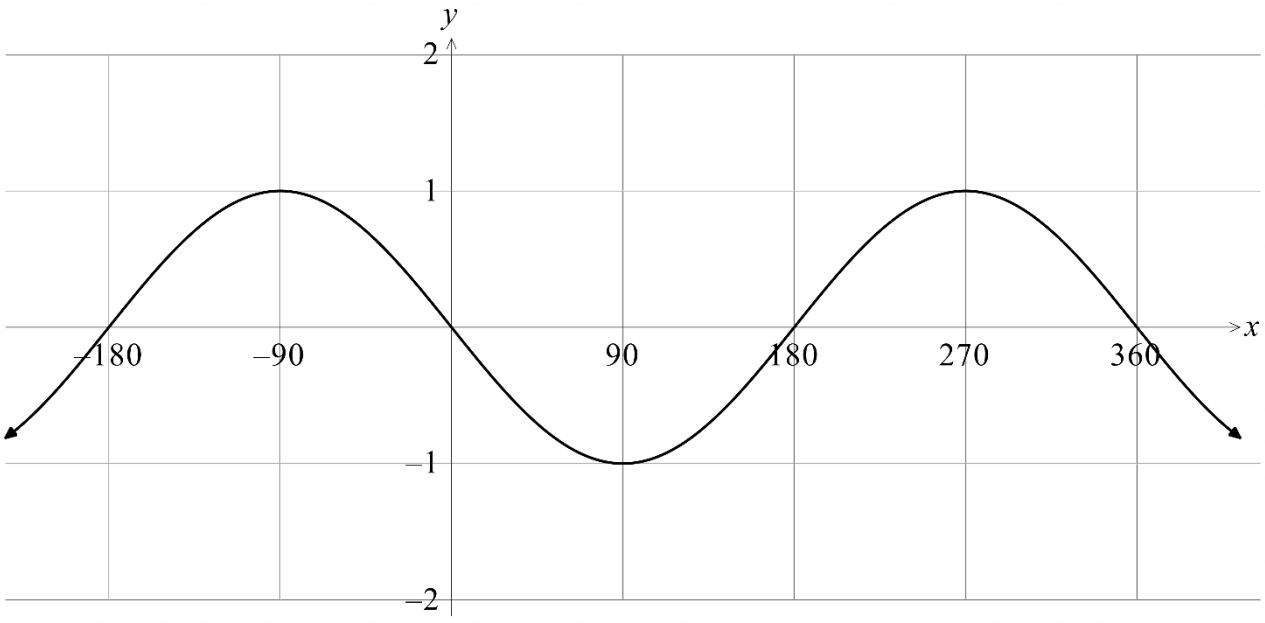
   D. Quadrant 4

5. Which of the following statements is true for a many to one function $f(x)$?
   A. $f(x)$ will pass both the vertical and horizontal line test
   B. $f(x)$ will fail both the vertical and horizontal line test
   C. $f(x)$ will pass the vertical line test and fail the horizontal line test
   D. $f(x)$ will fail the vertical line test and pass the horizontal line test

6. What is the domain of $f(x) = \frac{1}{\sqrt{-x}}$?
   A. $x \geq 0$
   B. $x > 0$
   C. $x \leq 0$
   D. $x < 0$

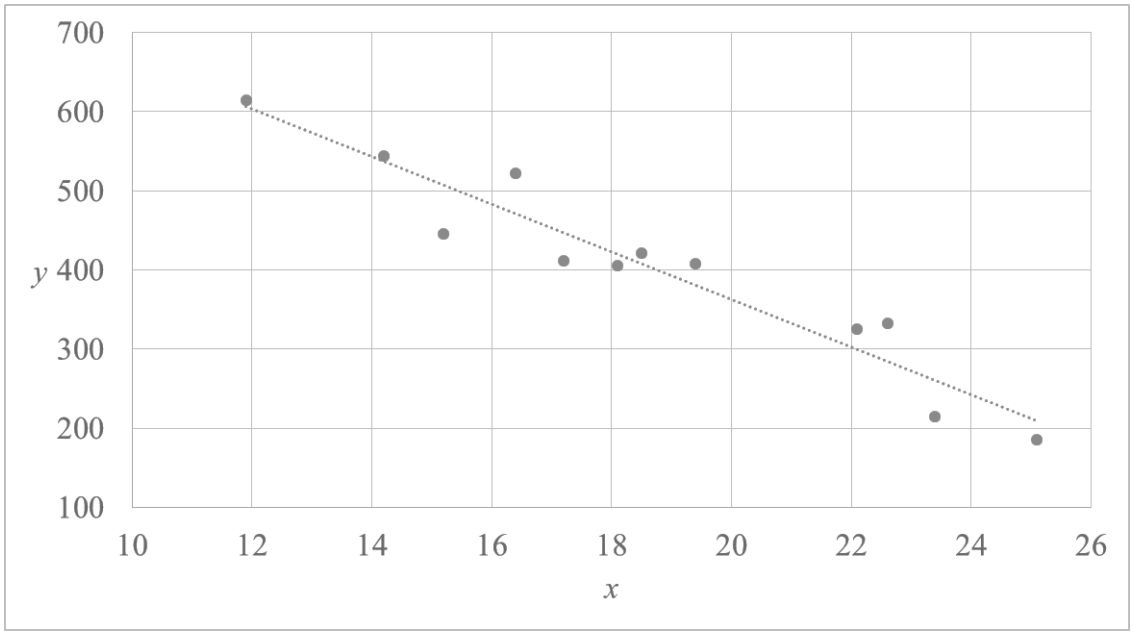
7. Which of the following expressions is equivalent to $\log_{10} x + \log_{10} y$?
   A. $\log_x y$
   B. $\log_{10}(x^y)$
   C. $\log_{10}(x+y)$
   D. $\log_{10}(xy)$

8. What is the equation of the graph below?



   A. $y = \sin x°$
   B. $y = \sin(-x)°$
   C. $y = \cos x°$
   D. $y = -\cos x°$

9. Which of the following best describes the correlation between x and y for the scatterplot below?



A. Strongly positive
B. Weakly positive
C. Strongly negative
D. Weakly negative

End of Section I

Section II, Part A **19 marks**

Question 11

Round $\left(\log_{10} \pi\right)^3$ to 3 significant figures. **1**

Question 12

Solve $3x^2 + x - 2 = 0$. **2**

Question 13

Rearrange the equation $a + bx = cx + d$ so that x is the subject of the equation. **1**

Question 14

Find the angle of inclination that the line $y = 4x - 5$ makes with the positive direction of the x axis. **1**

Correct your answer to the nearest minute.

Question 15

Find the equation of a circle centred at (1, –2) with a radius of 3. **1**

Question 16

If a standard coin is flipped twice, calculate the probability of:

a. Getting 2 Heads **1**

b. Getting at least 1 Tail **1**

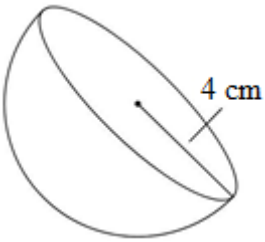
Question 17

Express $\sin 240°$ in simplest exact form. **1**

Question 18

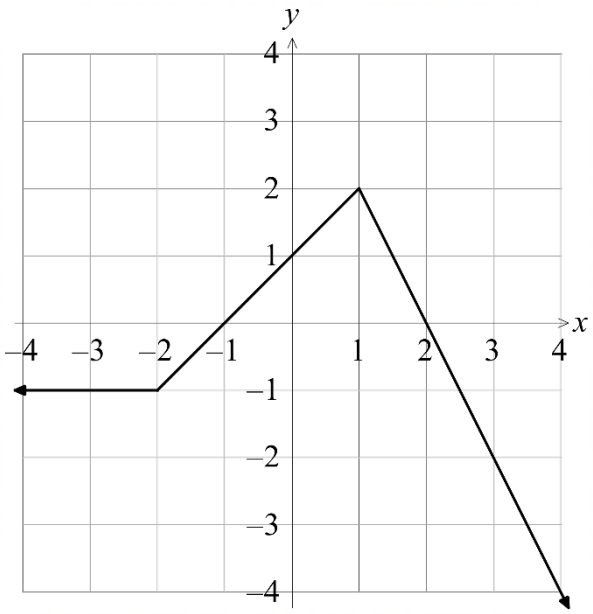
Calculate the total surface area of the closed hemisphere below. **2**

Express your answer in simplest exact form.



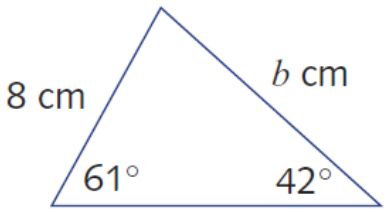
Question 19

Given the graph of $y = f(x)$ below, sketch $y = f(x+1)$ on the same number plane. **1**



Question 20

Given the diagram below, find the following, rounding your answers to 2 significant figures:

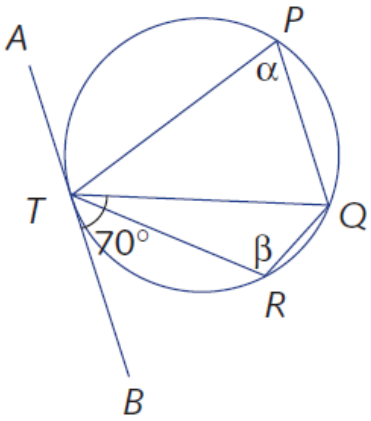


a. The value of b **1**

b. The area of the triangle **2**

Question 21

Consider the diagram below.



Find the values of the following pronumerals, justifying your answers geometrically.

a. α **2**

b. β **2**

End of Part A

Section II, Part B **17 marks**

Question 22

Factorise $x^3 + 8$. 1

Question 23

Below is a record of RW's Yearly Exam marks in Science out of 100 from Grade 8 to 10:

Year	Grade	Student Mark	Cohort Mean	Cohort Standard Deviation
2003	8	84	60	16
2004	9		70	8
2005	10	88	75	10

The cohort marks are known to be normally distributed.

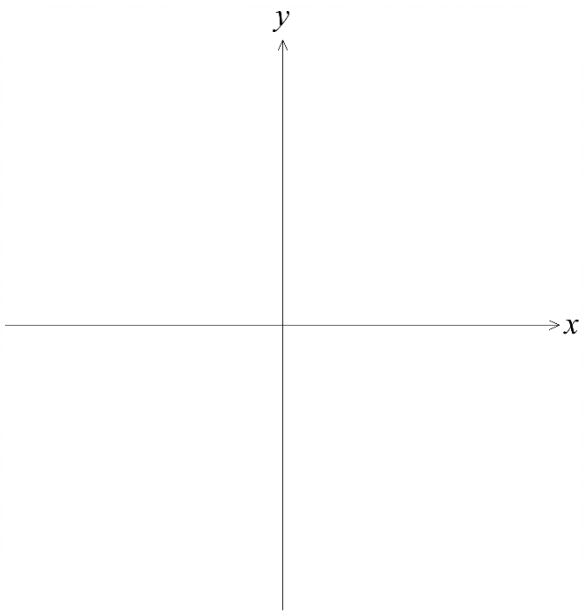
a. Explain mathematically whether RW performed better in 2003 or 2005. **2**

b. In 2004, RW was medically exempt from sitting the Yearly Exam. **1**
Use z scores and RW's results from 2003 to estimate her mark in 2004.

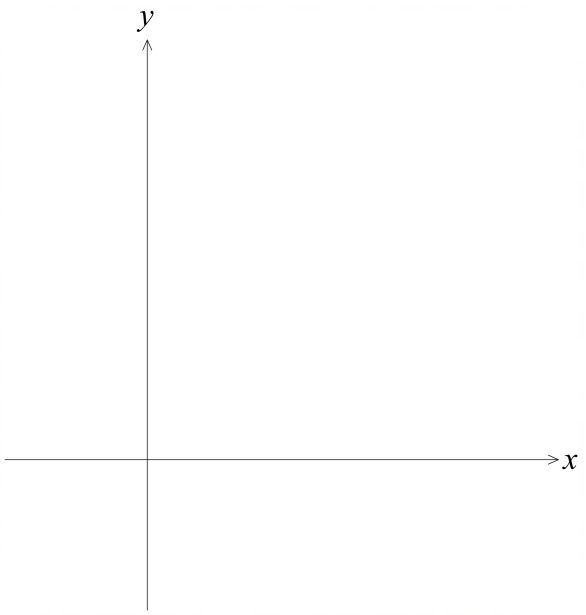
Question 24

Sketch the graphs of the following equations, including the coordinates of any intercepts and vertices and the equation of any asymptotes:

a. $y = -(x-1)^2 + 1$ **2**

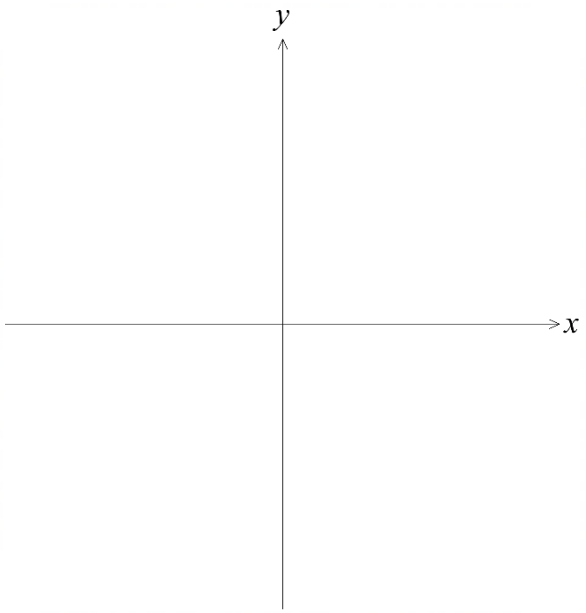


b. $y = \frac{6}{x-3} + 2$ **2**



Question 24 continues on the next page

c. $y = 8 - (x+1)^3$ **2**



Question 25

The ratio of the lengths of matching sides of two similar violins is 2 : 5. **2**

Find the volume of the smaller violin if the volume of the larger one is 37 500 mm^3.

Question 26

Consider the polynomial $P(x) = x^3 - 3x^2 - 10x + 24$.

a. Show that when $P(x)$ is divided by $x - 1$, the remainder is 12. **1**

b. Show that $x - 2$ is a factor of $P(x)$. **1**

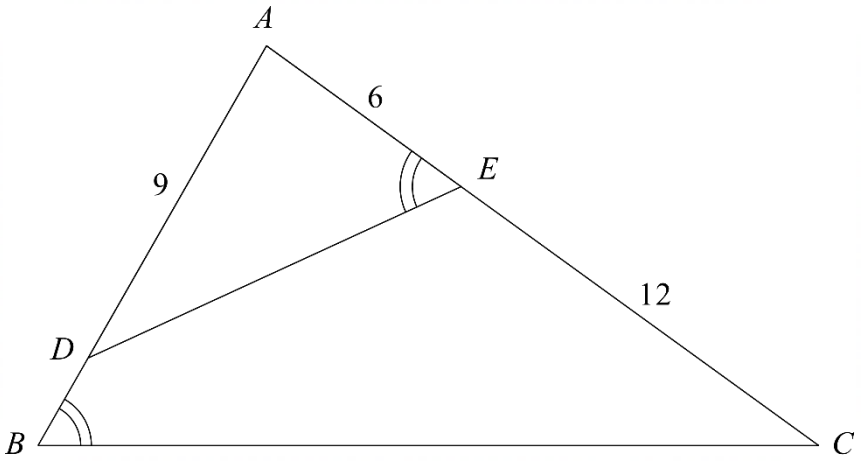
c. Hence, express $P(x)$ as a product of 3 linear factors. **3**

End of Part B

Section II, Part C **17 marks**

Question 27

In the diagram below, $AD = 9$, $AE = 6$, $EC = 12$ and $\angle ABC = \angle AED$.



a. State why $\Delta ABC \,|||\, \Delta AED$. **1**

b. Hence, find the length of DB. **2**

Question 28

Solve $\sin(180° - \theta) = \sin(90° - \theta)$ for θ where $0° \le \theta \le 360°$. **2**

Question 29

Solve $6^x = 8$, rounding your answer to 2 decimal places. **2**

Question 30

Below is the frequency distribution table of the class test results of one of RW's classes:

x	1	2	3	4	5	6	7	8	9	10
f	**1**	**0**	**2**	**3**	**10**	**5**	**6**	**0**	**2**	**1**
c.f.	1	1	3	6	16	21	27	27	29	30

a. State the mode. **1**

b. Calculate the mean and standard deviation, rounding your answers to 2 decimal places. **2**

c. If a student's mark is a low outlier, RW calls the student's parents. **3**
How many phone calls did RW make for this class test?

Question 31

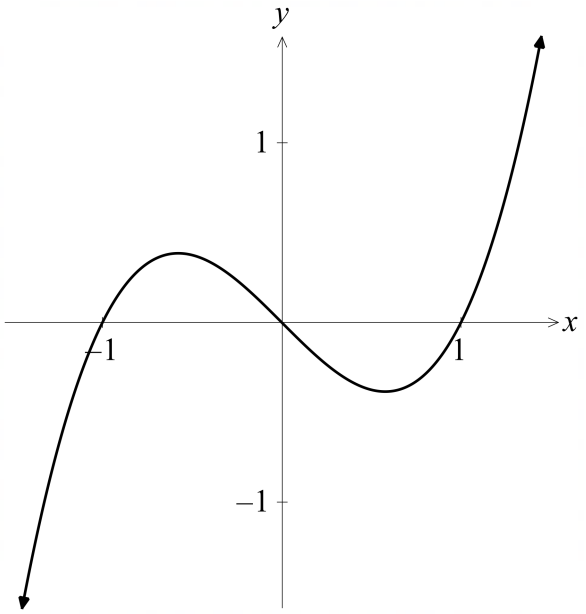
If $f(x)$ and $g(x)$ are inverse functions of each other, then $f\left(g\left(x\right)\right)=x$ and $g\left(f\left(x\right)\right)=x$. **2**

Let $f(x)=2x+3$ and $g(x)=\dfrac{x-3}{2}$.

Show that $f\left(g\left(x\right)\right)=x$.

Question 32

a. Given the graph of $f(x)=x^3-x$ below, sketch its inverse relation on the same number plane. **1**



b. Hence, locate all points of intersection between $f(x)$ and its inverse relation. **2**

End of Part C

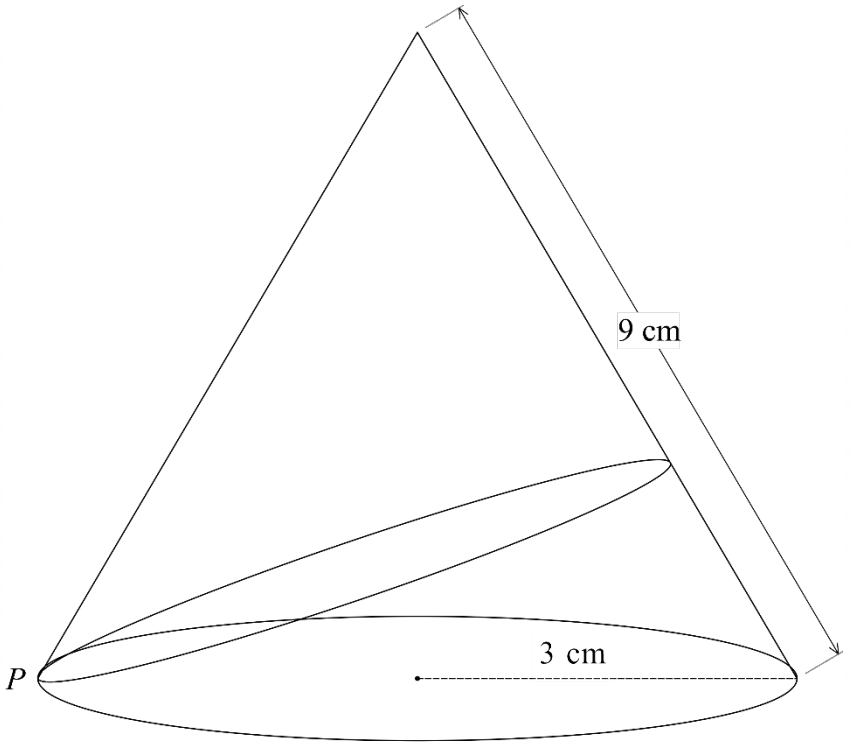
Section II, Part D **18 marks**

Question 33

In the diagram below, an ant starts at point P on the base of a conical pile of dirt. **3**

To search for food, the ant travels around the curved surface of the cone before returning to P.

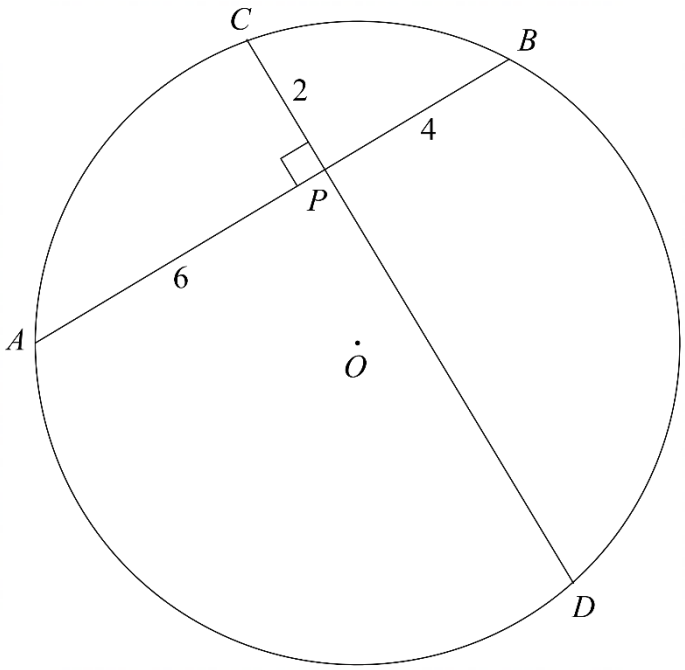
Show that the length of the shortest path that the ant can take is $9\sqrt{3}$ cm.



Question 34

In the diagram below, AB and CD are perpendicular chords that intersect at P on a circle centred at O. **4**

Given that $AP = 6$, $BP = 4$ and $CP = 2$, find the radius of the circle in simplest exact form.



Question 35

Solve $9^x + 15^x = 25^x$, rounding your answer to 2 decimal places. **3**

Question 36

Solve $(x^2 - 2x)^{x^2-3x} = 1$. **3**

Hint: There are 4 solutions.

Question 37

Ryu and Ken are two rivals among 128 competitors participating in a martial arts tournament.

The tournament format is single elimination, where only the winner advances to the next round.

The competitors for each round are randomly paired from the pool of those who remain.

The probability of either competitor winning in a matchup is 50%.

Find the probability of Ryu and Ken facing each other in:

a. The first round. **1**

b. The second round. **2**

c. The final round. **2**
Hint: For them to meet in the final, they must be in opposite halves of the draw.

End of Part D

Section II, Part E **15 marks**

Question 38

a. Show that $\frac{a+b}{2} \geq \sqrt{ab}$ for all real positive numbers a and b. **2**

Hint: Consider $\left(\sqrt{a}-\sqrt{b}\right)^2$.

b. Hence, find the range of $y = 54x - 48 + \frac{24}{x}$ for $x > 0$. **2**

Question 39

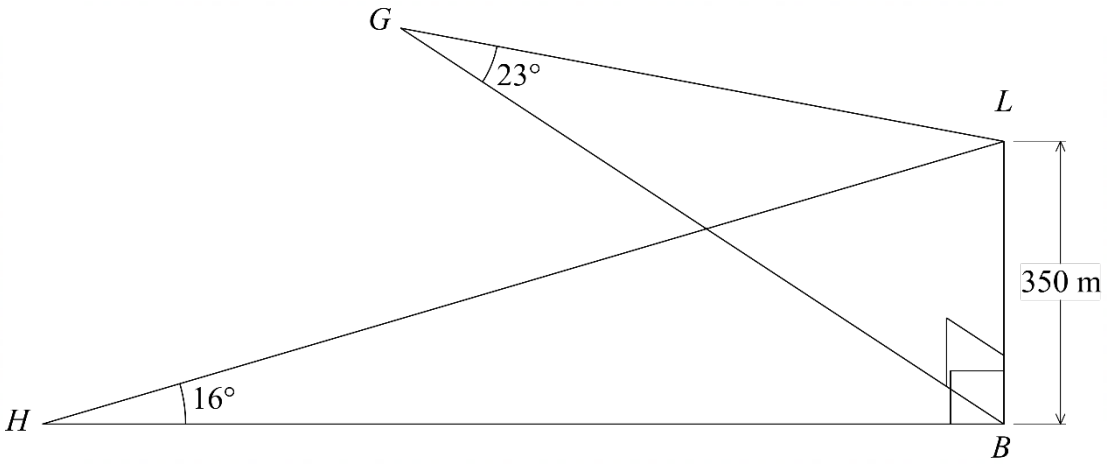
In the diagram below, Lijiang Tower, represented by BL, is 350 m in length. **4**

Hanzo stands at point H some distance away from the base of the tower B.

From H, the tower has a bearing of $109°T$ and the angle of elevation to the top L is $16°$.

Genji stands at some other point G away from the base of the tower.

From G, the tower has a bearing of $139°T$ and the angle of elevation to the top is $23°$.



By using two applications of the cosine rule, find the true bearing of H from G.

Round your answer to the nearest degree.

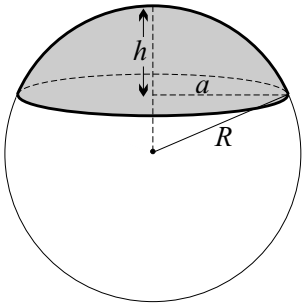
Hint: Find the distance GH.

Question 40

A spherical cap is the section of a sphere cut off by a plane as shown in the diagram below.

The volume of a spherical cap with base radius a and height h is given by $V_{CAP} = \frac{\pi h}{6}\left(3a^2 + h^2\right)$.

a. In a sphere of radius R, show that $V_{CAP} = \frac{\pi h^2}{3}\left(3R - h\right)$. **2**



Question 40 continues on the next page

b. Darth Parker is constructing a Maff Star by intersecting a sphere of radius 100 km with a sphere of radius 40 km such that their centres are 120 km apart.
The smaller sphere is then discarded, leaving a curved dent on the surface of the larger sphere, as shown in Diagram 1 below.

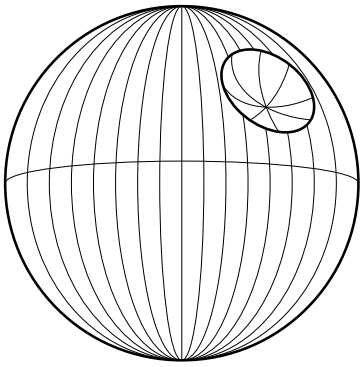


Diagram 1

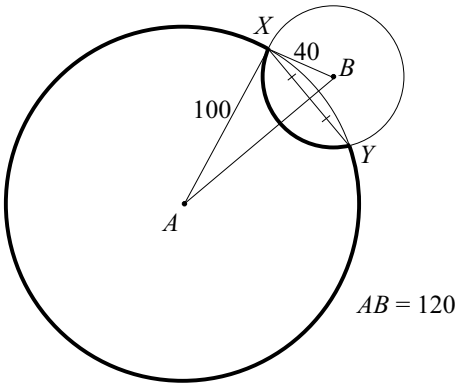


Diagram 2

Diagram 2 above shows a cross section of the intersecting spheres, where A and B are the centres of the larger and smaller sphere respectively and X and Y mark where the spheres intersect on the cross section.

I. Given that AB bisects XY, show that $AB \perp XY$. **1**

Question 40 continues on the next page

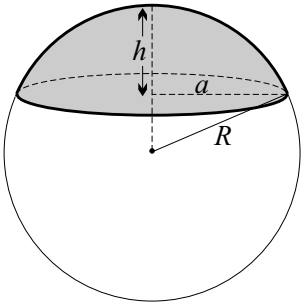
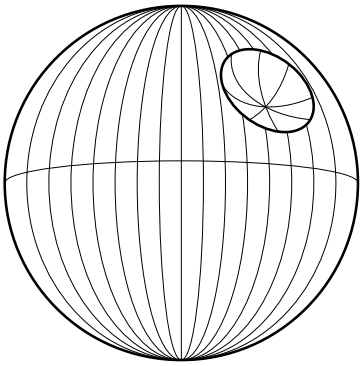


Diagram 1



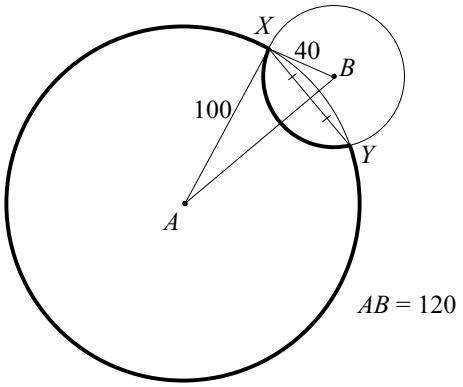


Diagram 2

II. Hence, find the exact volume of the Maff Star. **4**

End of paper

Spare Working Page 1

Spare Working Page 2

Spare Working Page 3

Blank page

SYDNEY BOYS HIGH SCHOOL

2020

YEAR 10
TERM 4
YEARLY ASSESSMENT TASK

Mathematics Stage 5.3

Sample Solutions

NOTE: **Some of you may be disappointed with your mark.**
This process of checking your mark is NOT the opportunity to improve your marks.
Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown 'working' does not justify that your solution is worth any marks.

If your solutions are in pencil or you have used white out/liquid paper you **CAN'T** appeal.

Multiple Choice Answers:

1. D
2. A
3. A
4. B
5. C
6. D
7. D
8. B
9. C

2020 Y10 Yearly Multiple Choice Solutions

Mean (out of 9): 7.53

1. $m_1 m_2 = -1$ (D)

A	9
B	11
C	8
D	180

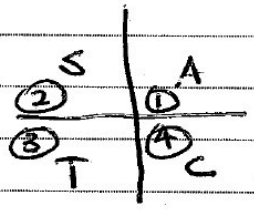
2. $P(\text{Boy} \mid \text{Junior}) = \frac{7}{12}$ (A)

A	196
B	9
C	3
D	0

3. $y = 3x$ (A)

A	154
B	5
C	21
D	27

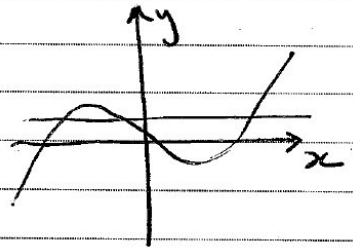
4.



Quadrant 2 (B)

A	4
B	190
C	10
D	4

5.



Passes vertical line test

Does not pass horizontal line test (C)

A	16
B	7
C	139
D	45

6. $\sqrt{\ } \Rightarrow -x \geq 0$

$\therefore x \leq 0$

Division $\Rightarrow x \neq 0$

$\therefore$ Domain is $x < 0$ (D)

A	10
B	19
C	29
D	150

7. $\log_{10} x + \log_{10} y = \log_{10}(xy)$ (D)

A	2
B	0
C	12
D	194

8. Zeroer suggest a curve related to $\sin x^\circ$

$\sin 90^\circ = 1$

$f(90^\circ) = -1$

$\therefore$ Curve is $y = -\sin x^\circ$

$= \sin(-x^\circ)$

B

A	12
B	190
C	4
D	2

9. The line of best fit has negative gradient.
Therefore correlation is negative.
Points are quite close to the line
$\therefore$ Strongly negative

(The correlation is close to -0.96)

C

A	11
B	2
C	173
D	22

Section II, Part A **19 marks**

Question 11

Round $(\log_{10}\pi)^3$ to 3 significant figures. **1**

0.123

- no half marks
- generally well done.
- some students counted 0 as sig, common error was rounding to 0.12

Question 12

Solve $3x^2+x-2=0$. **2**

Most common method:

$(3x-2)(x+1)=0$

$\therefore x=\frac{2}{3}, -1$

Using quadratic formulae:

$x=\frac{-1\pm\sqrt{1+24}}{6}$

$x=-1,\frac{2}{3}$

Question 13

Rearrange the equation $a+bx=cx+d$ so that x is the subject of the equation. **1**

$a-d=cx-bx$

$a-d=x(c-b)$

$x=\frac{a-d}{c-b}$

OR

$d-a=bx-cx$

$x(b-c)=d-a$

$x=\frac{d-a}{b-c}$

- generally well done.

Question 14

Find the angle of inclination that the line $y=4x-5$ makes with the positive direction of the x axis. **1**

Correct your answer to the nearest minute.

$\tan\theta=4$

$\theta=\tan^{-1}(4)=75.96^\circ\ldots$

$=75^\circ 58'$

- $\frac{1}{2}$ mark if not converted to degrees and minutes
- common errors; rounding to $75^\circ 57'$, or solving $\tan\theta=\frac{1}{4}$

Question 15

Find the equation of a circle centred at (1, –2) with a radius of 3. **1**

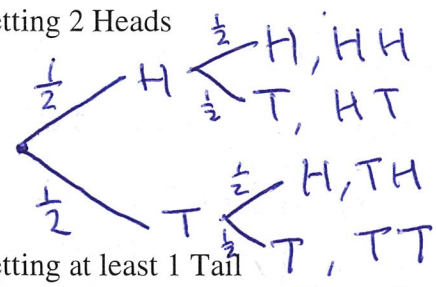
$(x-1)^2+(y+2)^2=9$

- most common errors: writing $(x^2-1)+(y^2+2)$, or $(x+1)^2+(y-2)^2$

Question 16

If a standard coin is flipped twice, calculate the probability of:

a. Getting 2 Heads **1**



$|S| = 4$

$P(HH) = \frac{1}{4}$

- generally very well done.
- no half marks.

b. Getting at least 1 Tail **1**

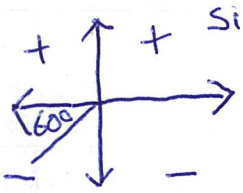
$P(\text{at least 1 tail}) = 1 - P(\text{no tails})$ OR $P(HT, \text{or } TH \text{ or } TT)$

$= 1 - \frac{1}{4}$ $= \frac{3}{4}$

$= \frac{3}{4}$

Question 17

Express $\sin 240°$ in simplest exact form. **1**



$\sin 240^\circ = -\sin 60^\circ$

$= -\frac{\sqrt{3}}{2}$

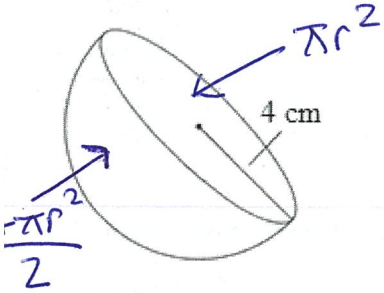


- $\frac{1}{2}$ mark for $+\frac{\sqrt{3}}{2}$
- $\frac{1}{2}$ for $-\sqrt{\frac{3}{4}}$

Question 18

Calculate the total surface area of the closed hemisphere below. **2**

Express your answer in simplest exact form.



$S/Area = \underbrace{\pi r^2}_{\text{circle}} + \underbrace{\frac{1}{2}(4\pi r^2)}_{\text{semi-sphere.}}$

$= \pi r^2 + 2\pi r^2$

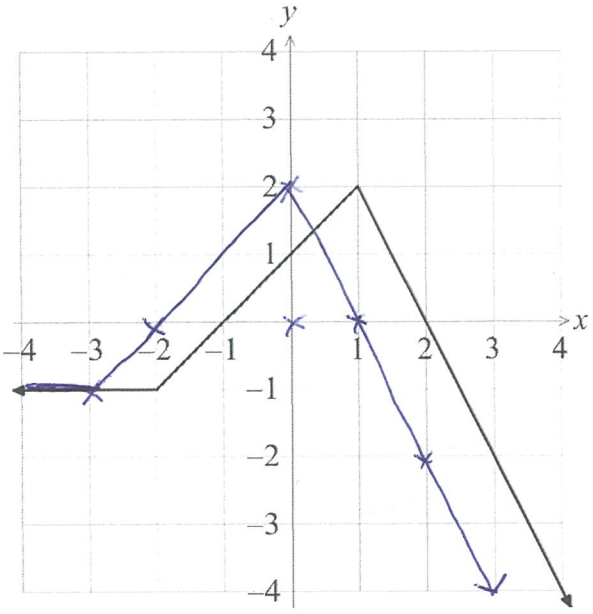
$= 3\pi r^2$

$= 48\pi \text{ cm}^2$

- common error: using the volume of a sphere formulae, or using "$\frac{4\pi r^2}{3}$"

Question 19

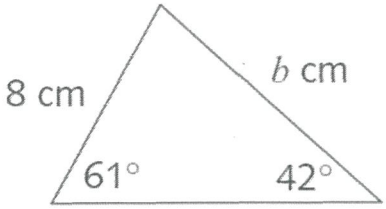
Given the graph of $y = f(x)$ below, sketch $y = f(x+1)$ on the same number plane. **1**



- generally well done. common errors were shifting the graph upward or right by 1 unit

Question 20

Given the diagram below, find the following, rounding your answers to 2 significant figures:



a. The value of b **1**

$$\frac{b}{\sin 61} = \frac{8}{\sin 42}$$

$$b = \frac{8 \sin 61}{\sin 42} = 10\text{cm}.$$

- common error: for both (a) and (b), using cos instead of sin.

b. The area of the triangle **2**

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 8 \left(\frac{8\sin 61}{\sin 42}\right) \sin 77.$$

$$= 41\text{cm}^2$$

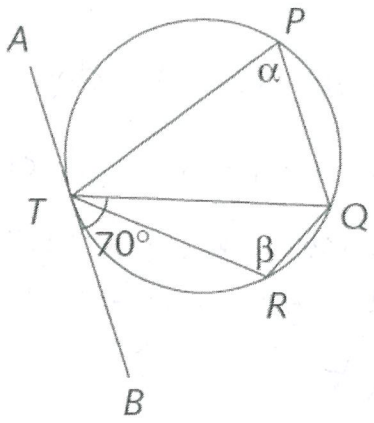
- common errors: using '61' or '42' as included angle

Solution using rounded answer from (a):

$A = \frac{1}{2} \times 8 \times 10 \times \sin 77^\circ = 39\text{cm}^2$

Question 21

Consider the diagram below.



Find the values of the following pronumerals, justifying your answers geometrically.

a. α $\alpha = 70^\circ$ (angle in alternate segment) **2**

b. β $\beta = 110^\circ$ (opposite angles of a cyclic quad are supplementary **2**

End of Part A

Part B

22. $x^3 + 8 = x^3 + 2^3$

$= (x + 2)(x^2 - 2x + 4)$ [1]

Comments:
Very poorly done

23.(a) $z\text{-score}(2003) = \frac{84 - 60}{16}$

$= \frac{24}{16}$

$= 1.5$

$z\text{-score}(2005) = \frac{88 - 75}{10}$

$= \frac{13}{10}$

$= 1.3$

Comments:
Some students didn't state show mathematically the difference if the z-scores.
Generally done reasonable well.

$\therefore$ RW performed better in 2003 as $1.5 > 1.3$. [2]

(b) Estimate 2004 mark:

$1.5 = \frac{x - 70}{8}$

$12 = x - 70$

$\therefore x = 82$ [1]

Comments:
Generally done reasonable well.
Many students used the both the 2003 and 2005, z-scores instead reading the question and just using the 2003.

24.

(a) $y = -(x-1)^2 + 1$

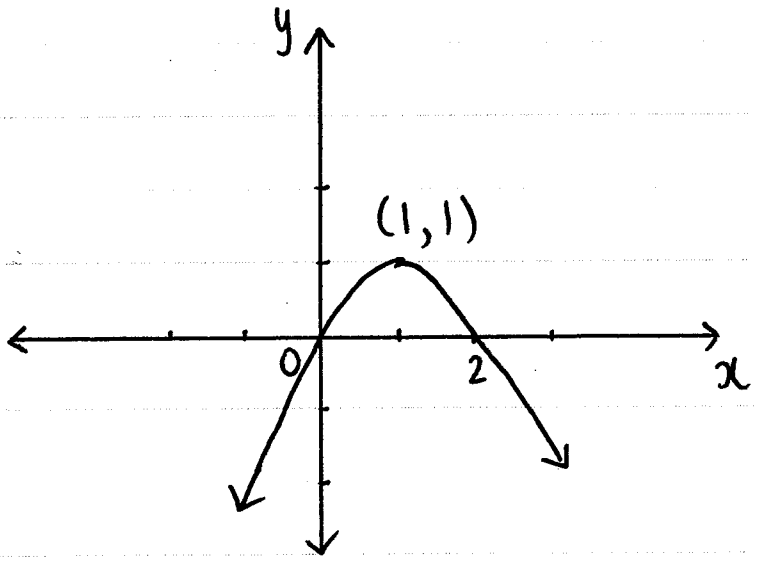
- x-int ($y = 0$):

$$0 = -(x-1)^2 + 1$$
$$(x-1)^2 = 1$$
$$x - 1 = \pm 1$$
$$x = 0, 2.$$

- y-int ($x = 0$):

$$y = -(0-1)^2 + 1$$
$$= 0$$

- vertex $(1, 1)$



[2]

Comments:
Generally done reasonable ok.
Many students didn't show any key points.

(b) $y = \frac{6}{x-3} + 2$

- asymptotes:

$x - 3 \neq 0$

$x \neq 3$

$y \neq 2.$

- x-int ($y = 0$):

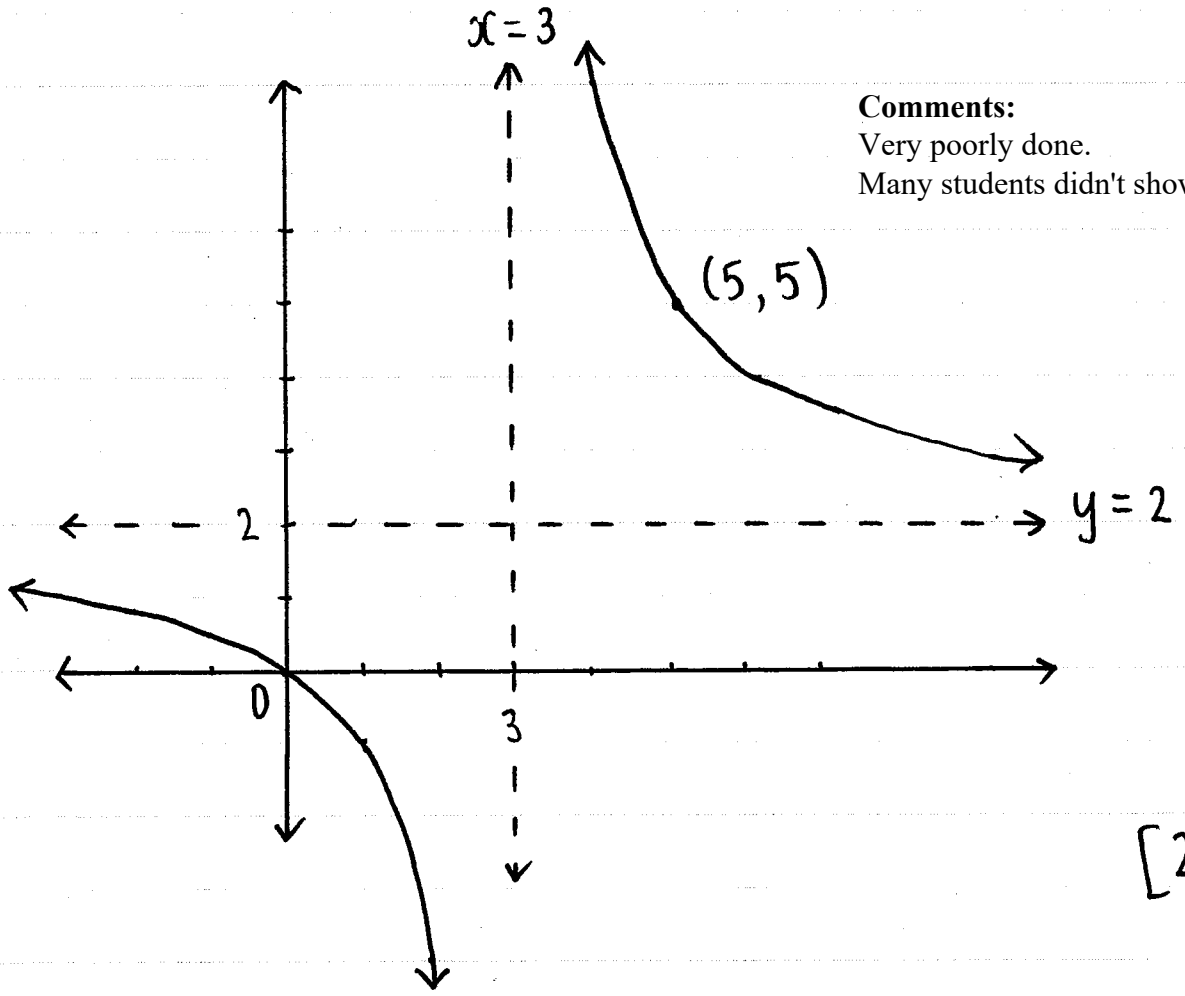
$0 = \frac{6}{x-3} + 2$

$\frac{6}{x-3} = -2$

$6 = -2x + 6$

$2x = 0$

$x = 0$ $\quad (0, 0).$



Comments:
Very poorly done.
Many students didn't show any key points.

[2]

(c) $y = 8 - (x+1)^3$

- x-int ($y=0$):

$0 = 8 - (x+1)^3$

$(x+1)^3 = 8$

$x + 1 = 2$

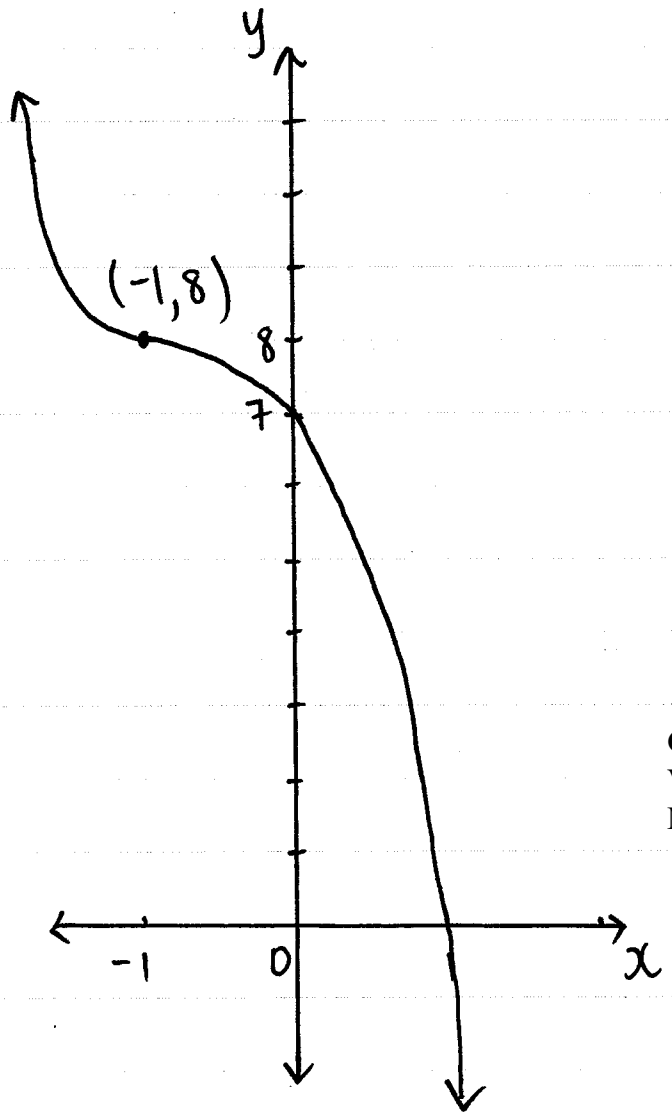
$x = 1$ $\quad (1, 0)$

- y-int ($x = 0$):

$y = 8 - (0+1)^3$

$= 7$ $\quad (0, 7)$

- turning point $(-1, 8)$.



Comments:
Very poorly done.
Many students didn't show any key points.

[2]

25. Ratio of sides $\Rightarrow$ $2:5$

$\therefore$ Ratio of volumes $\Rightarrow$ $2^3 : 5^3$

$= 8 : 125$

$\therefore \quad \frac{x}{37\,500} = \frac{8}{125}$

$x = 2400 \text{ mm}^3$ [2]

Comments:
Very poorly done.
Many students incorrect used the ratio of the lengths instead of cubing them for the volume..

26. $P(x) = x^3 - 3x^2 - 10x + 24$

(a) $P(1) = (1)^3 - 3(1)^2 - 10(1) + 24$

$= 12$

$\therefore$ The remainder is 12.

(b) $P(2) = (2)^3 - 3(2)^2 - 10(2) + 24$

$= 8 - 12 - 20 + 24$

$= 0$

$\therefore P(x)$ has a factor of $(x - 2)$.

(c)

$$
\begin{array}{r|l}
x^2 - x - 12 & \\
x - 2 \overline{)\ x^3 - 3x^2 - 10x + 24} & \\
-(x^3 - 2x^2) & \frac{x^3}{x} = x^2 \\
\quad -1x^2 - 10x & \\
\quad -(-x^2 + 2x) & \frac{-x^2}{x} = -x \\
\quad\quad -12x + 24 & \\
\quad\quad -(-12x + 24) & \frac{-12x}{x} = -12 \\
\quad\quad\quad 0 &
\end{array}
$$

$\therefore P(x) = x^3 - 3x^2 - 10x + 24$

$= (x - 2)(x^2 - x - 12)$

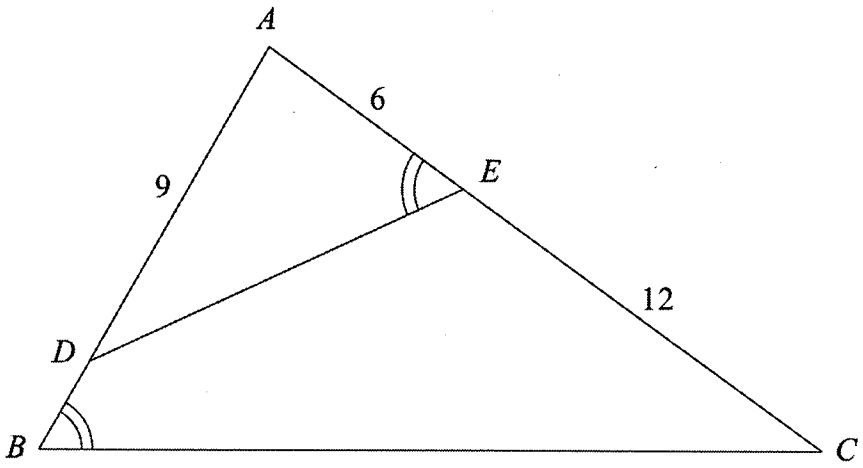
$= (x - 2)(x + 3)(x - 4)$

Comments:
Generally done very well. Some students factorised the quotient incorrectly

Section II, Part C **17 marks**

Question 27

In the diagram below, $AD = 9$, $AE = 6$, $EC = 12$ and $\angle ABC = \angle AED$.



a. State why $\Delta ABC \,|||\, \Delta AED$. **1**

In ΔABC and ΔAED: $\angle A$ is common

$\angle ABC = AED$ (given) (1)

$\therefore \Delta ABC \,|||\, \Delta AED$ (AA)

b. Hence, find the length of DB. **2**

$\frac{AE}{AB} = \frac{AD}{AC}$ (corresponding sides in same ratio in similar Δs)

$\frac{6}{9+BD} = \frac{9}{18}$ (1)

$9 + BD = 12 \qquad BD = 3$ (1)

Question 28

Solve $\sin(180° - \theta) = \sin(90° - \theta)$ for θ where $0° \le \theta \le 360°$. **2**

$\frac{1}{2}$ mark for 45° without working

1 mark for 45° with working.

2 marks for 45° and 225° with working.

$\sin\theta = \sin(180° - \theta)$

$\sin\theta = \sin(90° - \theta)$

$\theta = 90° - \theta$

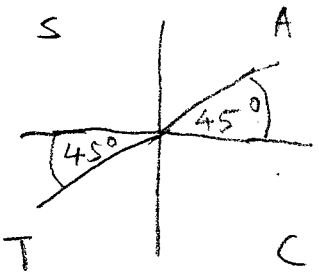
$\sin\theta = \cos\theta$

Since $\sin\theta = \cos\theta$

$\tan\theta = 1$

$\theta = 45°, 225°$ (1)

(1)



Question 29

Solve $6^x = 8$, rounding your answer to 2 decimal places. **2**

$\ln 6^x = \ln 8$

$x \ln 6 = \ln 8$

$x = \frac{\ln 8}{\ln 6}$ (1)

$= 1.16$ (1)

Question 30

Below is the frequency distribution table of the class test results of one of RW's classes:

x	1	2	3	4	5	6	7	8	9	10
f	1	0	2	3	10	5	6	0	2	1
c.f.	1	1	3	6	16	21	27	27	29	30

a. State the mode. **1**

5 (1)

b. Calculate the mean and standard deviation, rounding your answers to 2 decimal places. **2**

mean = 5.63 (1)

$6x = 1.83$ (1)

c. If a student's mark is a low outlier, RW calls the student's parents. How many phone calls did RW make for this class test? **3**

Q1 = 5 (8th score) $\frac{1}{2}$

Q3 = 7 (23rd score) $\frac{1}{2}$

IQR = 7 − 5

= 2 $\frac{1}{2}$

$1.5 \times IQR = 3$ $\frac{1}{2}$

$Q1 - 3 = 2$ $\frac{1}{2}$

lower outlier = 1 $\frac{1}{2}$

1 phone call made.

everyone gets 3 marks by default.

Question 31

If $f(x)$ and $g(x)$ are inverse functions of each other, then $f(g(x))=x$ and $g(f(x))=x$. **2**

Let $f(x)=2x+3$ and $g(x)=\frac{x-3}{2}$.

Show that $f(g(x))=x$.

$f(g(x)) = f\left(\frac{x-3}{2}\right)$ (1)

$= 2\times\frac{x-3}{2}+3$

$= x-3+3$

$= x$ (1)

OR

Find $f^{-1}(x)$; since $f^{-1}(x)=g(x)$, $f(g(x))=x$ (1)

$y=2x+3$

$x=2y+3$

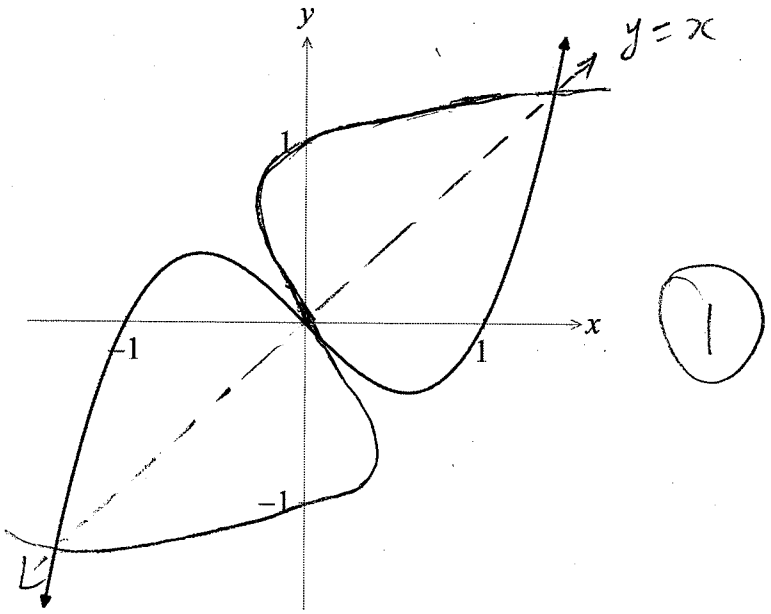
$2y=x-3$ (1)

$y=\frac{x-3}{2}$

$f^{-1}(x)=g(x)$

Question 32

a. Given the graph of $f(x)=x^3-x$ below, sketch its inverse relation on the same number plane. **1**



(1)

b. Hence, locate all points of intersection between $f(x)$ and its inverse relation. **2**

All intersections are on $y=x$ and $y=x^3-x$

Solve $y=x^3-x$, $y=x$

$x^3-x=x$

$x^3-2x=0$

$x(x^2-2)=0$

$x=0, \quad x=\pm\sqrt{2}$

$(0,0), (\sqrt{2},\sqrt{2}), (-\sqrt{2},-\sqrt{2})$

(2)

$(0,0)$ $\frac{1}{2}$ mark only

$x=0, x=\pm\sqrt{2}$ $1\frac{1}{2}$ marks

End of Part C

Section II, Part D
18 marks

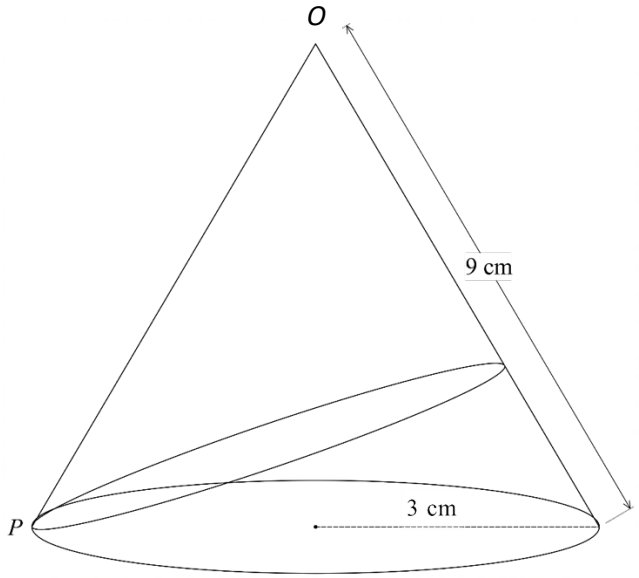
Question 33

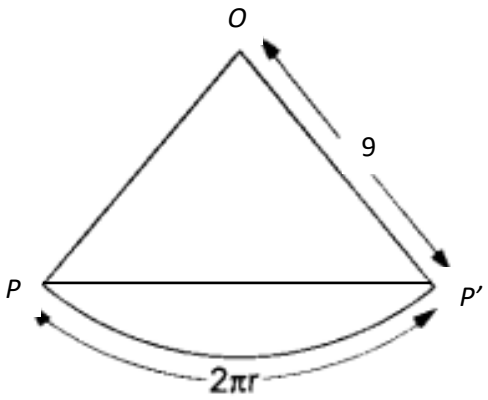
In the diagram below, an ant starts at point *P* on the base of a conical pile of dirt.

3

To search for food, the ant travels around the curved surface of the cone before returning to *P*.

Show that the length of the shortest path that the ant can take is $9\sqrt{3}$ cm.





Cut curved surface along *OP*, and around the base.
Let *l* be the circumference of the base. So $l = 2\pi r$
$= 6\pi$

Let $\angle POP' = \theta$

By proportion $\frac{\theta}{360°} = \frac{6\pi}{2\pi \times 9}$

$\theta = 120°$

Now in $\Delta POP'$, by cosine rule

$(PP')^2 = 9^2 + 9^2 - 2\times9\times9\times\cos 120°$

$= 243$

$\therefore PP' = \sqrt{243}$

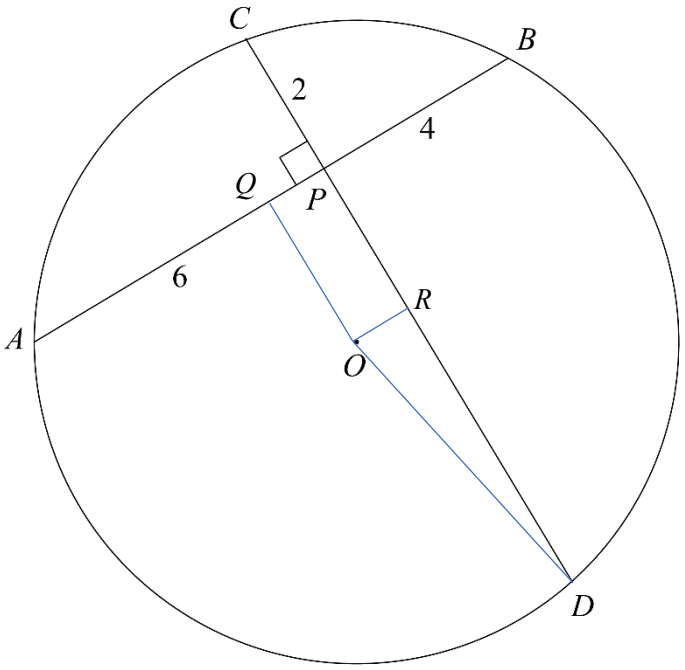
$= 9\sqrt{3}$

[*Most candidates failed to realise that the shortest path is a straight line on the net of the curved surface. Hence very few marks were awarded for working not leading to a solution. Many "back formed' the answer, also for no marks.*]

Question 34

In the diagram below, AB and CD are perpendicular chords that intersect at P on a circle centred at O. **4**

Given that $AP = 6$, $BP = 4$ and $CP = 2$, find the radius of the circle in simplest exact form.



$AP \times PB = CP \times PD$
(product of intersecting chords)
$6 \times 4 = 2 \times PD$
$\therefore PD = 12$

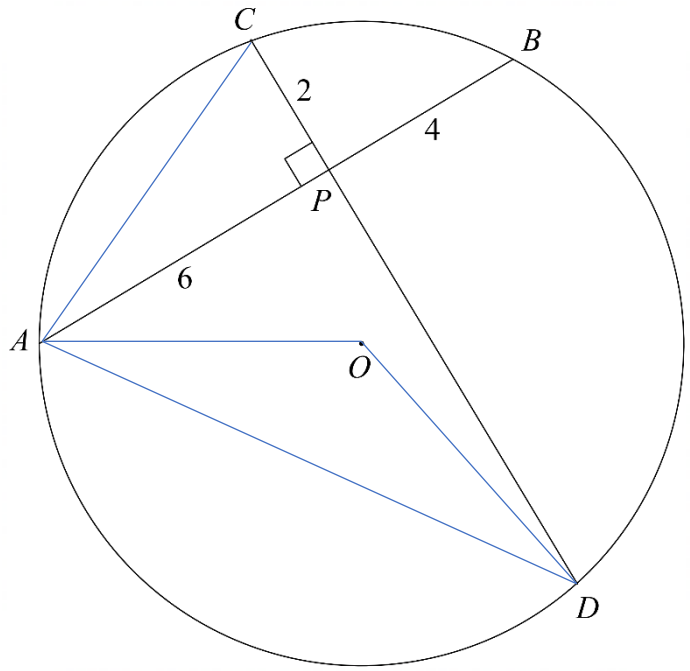
Join $OR \perp CD$
$CR = RD = 14/2 = 7$
(perpendicular thro' centre bisects chord)

Join $OQ \perp AB$
$AQ = QB = 10/2 = 5$
(perpendicular thro' centre bisects chord)

Join OD. $OR = QP = 6 - 5 = 1$

In ΔORD, by Pythagoras
$OD^2 = OR^2 + RD^2$
$= 1^2 + 7^2$
$= 50$
$\therefore$Radius $= \sqrt{50} = 5\sqrt{2}$ [*Many (most) only got as far as the length of PD, for 2 marks, provided they gave the reason.]*

Alternatively



Join AC, AO, DO, AD

$AP \times PB = CP \times PD$
(product of intersecting chords)
$6 \times 4 = 2 \times PD$
$\therefore PD = 12$

In ΔAPD
$AD^2 = 6^2 + 12^2$ (Pythagoras)
$\therefore AD = \sqrt{(180)}$

Let $\angle ACP = \theta \therefore$
$\tan \theta = 6/2 = 3$
$\therefore \theta = \tan^{-1} 3$

$\angle AOD = 2\theta$ (angle at the centre)
$= 2 \tan^{-1} 3$

In ΔAOD, using the cosine rule
$180 = r^2 + r^2 - 2 r^2 \cos(2 \tan^{-1} 3)$
$= 2 r^2 (1 - \cos(2 \tan^{-1} 3))$
$= 2 r^2 \times (9/5)$

Thus $r = \sqrt{50} = 5\sqrt{2}$ [*Several candidates used this approach, or something similar.*]

Question 35

Solve $9^x + 15^x = 25^x$, rounding your answer to 2 decimal places.

Dividing by 9^x:

$$1 + \left(\frac{15}{9}\right)^x - \left(\frac{25}{9}\right)^x = 0$$

$$1 + \left(\frac{5}{3}\right)^x - \left(\frac{5}{3}\right)^{2x} = 0$$

Let $u = \left(\frac{5}{3}\right)^x$, and so $u^2 - u - 1 = 0$

And $u = \frac{1 \pm \sqrt{5}}{2}$

Hence $\left(\frac{5}{3}\right)^x = \frac{1+\sqrt{5}}{2}$ (Negative root is extraneous)

So $x = \frac{\log \frac{1+\sqrt{5}}{2}}{\log \frac{5}{3}}$

$\div 0.94$

[*Very few candidates (about 1) realised the necessity to divide, and find an equation reducible to a quadratic.*]

Question 36

Solve $(x^2 - 2x)^{x^2-3x} = 1$. **3**

Hint: There are 4 solutions.

We consider the three ways in which a power may equal 1.

Case 1 $1^a = 1$

ie

$$x^2 - 2x = 1$$
$$x^2 - 2x - 1 = 0$$
$$x = \frac{2 \pm \sqrt{4+4}}{2}$$
$$= 1 \pm \sqrt{2}$$

Case 2 $a^0 = 1$

ie

$$x^2 - 3x = 0$$
$$x(x - 3) = 0$$
$$x = 0, 3$$

Substitute $x = 0$, so $x^2 - 2x = 0$

But 0^0 is undefined, so $x \neq 0$.

Substitute $x = 3$, so $x^2 - 2x = 3 \neq 0$

So $x = 3$ is a solution.

Case 3 $(-1)^a = 1$, where a is divisible by 2.

So $x^2 - 2x = -1$

$x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0$

$x = 1, 1$

Substitute $x = 1$, so $x^2 - 3x = -2$

Thus $(-1)^{-2} = 1$

So the solutions are $x = 1, 3, 1 + \sqrt{2}, 1 - \sqrt{2}$.

[*Whilst several candidates found 3 or 4 solutions, those who found 0 failed to realise it is not a valid solution, and very few found 1 to be a solution.*]

Question 37

Ryu and Ken are two rivals among 128 competitors participating in a martial arts tournament.

The tournament format is single elimination, where only the winner advances to the next round.

The competitors for each round are randomly paired from the pool of those who remain.

The probability of either competitor winning in a matchup is 50%.

Find the probability of Ryu and Ken facing each other in:

a. The first round. **1**

There are 128 places, and R may go in any one of them. $p = 1$.
K must go in the only place next to R, out\ of 127 remaining.
Total probability $p = 1 \times \frac{1}{127} = \frac{1}{127}$.

[*This was not well answered. Half a mark was awarded for showing some understanding.*]

b. The second round. **2**

R goes in any place. K must be in one of the two places in the match next to R.

R and K must both win.

$$\therefore p = \frac{1}{2} \times \frac{1}{2} \times \frac{2}{127} = \frac{1}{254}$$

[*This was not well answered at all. A mark was awarded for showing some understanding.*]

c. The final round. **2**
Hint: For them to meet in the final, they must be in opposite halves of the draw.

R goes in any position, and K goes in 64 of 127 (other side of the draw).
R and K must win 6 matches each.

$$\therefore p = \frac{1}{2^6} \times \frac{1}{2^6} \times \frac{64}{127} = \frac{1}{8128}$$

[*Whereas several candidates made arguments about winning 6 matches, most failed to deal with the hint. Very few part marks were awarded.*]

End of Part D

PART E Solution

Question 38

a. Show that $\frac{a+b}{2} \geq \sqrt{ab}$ for all real positive numbers a and b. **2**

Hint: Consider $\left(\sqrt{a}-\sqrt{b}\right)^2$.

Solution:

a) Consider $(\sqrt{a}-\sqrt{b})^2$

$(\sqrt{a}-\sqrt{b})^2 \geq 0$

$a-2\sqrt{ab}+b \geq 0$

$a+b \geq 2\sqrt{ab}$

$\frac{a+b}{2} \geq \sqrt{ab}$

Marking Scheme:

1 mark correctly expanding $(\sqrt{a}-\sqrt{b})^2$

1 mark simplifying the expression to get correct result.

Marker's Comments:

- Significant number of candidates started with the result to prove, which is not correct.
- Candidates were penalised if they didn't explicitly state that $(\sqrt{a}-\sqrt{b})^2 \geq 0$.

b. Hence, find the range of $y = 54x - 48 + \frac{24}{x}$ for $x > 0$. **2**

Solution:

b) From (a)

$\frac{54x + \frac{24}{x}}{2} \geq \sqrt{54x \times \frac{24}{x}}$ as $x > 0$

$\frac{54x + \frac{24}{x}}{2} \geq \sqrt{1296}$

$\frac{54x + \frac{24}{x}}{2} \geq 36$

$54x + \frac{24}{x} \geq 36 \times 2$

$54x + \frac{24}{x} \geq 72$

$54x + \frac{24}{x} \geq 48 + 24$

$54x + \frac{24}{x} - 48 \geq 24$

$\therefore y \geq 24$.

Marking Scheme:

1 mark for using the result in (a) correctly to obtain 36.

1 mark for correct answer.

Marker's Comments:

Since this question had the word *hence*, candidates must use previous result otherwise no marks were given for working.

This was poorly done by majority of the candidates.

Question 39

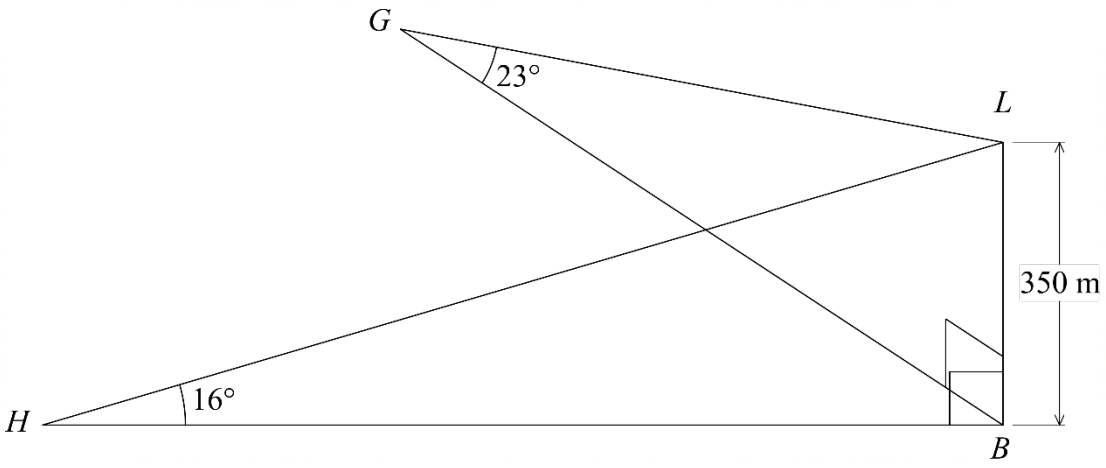
In the diagram below, Lijiang Tower, represented by BL, is 350 m in length. **4**

Hanzo stands at point H some distance away from the base of the tower B.

From H, the tower has a bearing of $109°T$ and the angle of elevation to the top L is $16°$.

Genji stands at some other point G away from the base of the tower.

From G, the tower has a bearing of $139°T$ and the angle of elevation to the top is $23°$.

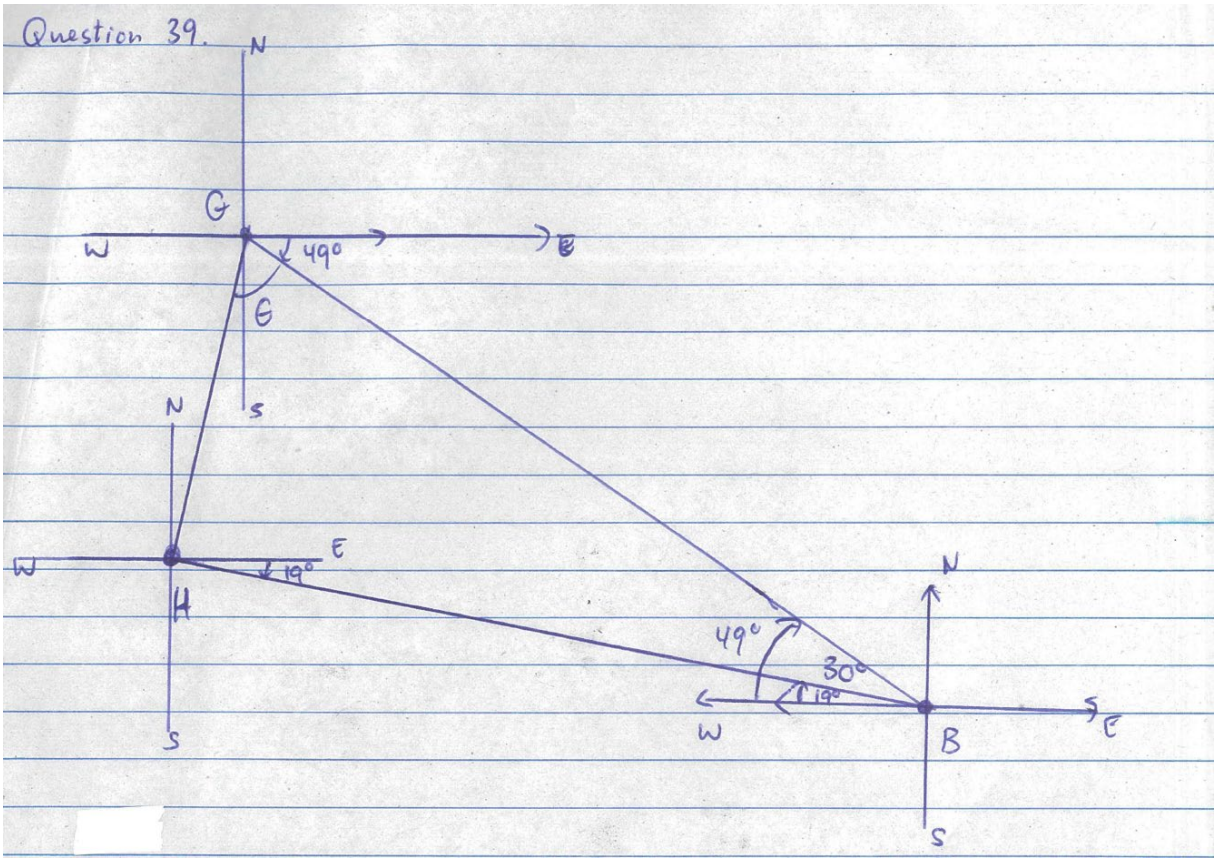


By using two applications of the cosine rule, find the true bearing of H from G.

Round your answer to the nearest degree.

Hint: Find the distance GH.

Solution



$\tan 23 = \frac{350}{GB} \rightarrow GB = \frac{350}{\tan 23}$

$\tan 16 = \frac{350}{HB} \rightarrow HB = \frac{350}{\tan 16}$

From the diagram, using the information and alternate angles on parallel lines,

$\angle GBH = 30°$

$\therefore GH^2 = GB^2 + BH^2 - 2 \times (GB) \times (BH) \times \cos 30$

$GH^2 = \left(\frac{350}{\tan 23}\right)^2 + \left(\frac{350}{\tan 16}\right)^2 - 2 \times \left(\frac{350}{\tan 23}\right)\left(\frac{350}{\tan 16}\right) \times \cos 30$

≈ 426527.6915

$\therefore GH \approx \sqrt{426527.6915}$

≈ 653.09 (2 d.p.)

let $\angle HGB = \theta$

$\therefore \cos\theta = \frac{(GH)^2 + (GB)^2 - (HB)^2}{2(GH)(GB)}$

$\cos\theta = \frac{653.09^2 + \left(\frac{350}{\tan 23}\right)^2 - \left(\frac{350}{\tan 11}\right)^2}{2\left(\frac{350}{\tan 23}\right)(653.09)}$

$\cos\theta \approx -0.356$ (3 d.p.)

$\therefore \theta \approx \cos^{-1}(-0.356)$

$\approx 111°$ (nearest degree)

$\therefore 90 + 49 + 111° = 250°$

$\therefore$ Bearing of H from G is 250°T

Marking Scheme:

1 mark for both working out correctly *GB* and *HB*.
1 mark for working out *GH* using cosine rule.
1 mark for working out $\angle HGB$ using cosine rule.
1 mark for correct answer with correct final step.

Marker's Comments

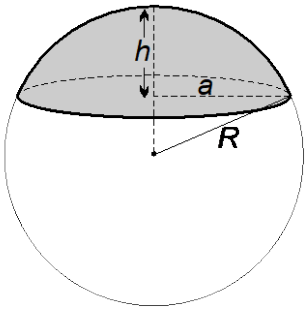
Many candidates chose to use sine rule to find *GB* and *HB* which is not necessary as ΔGLB and ΔHLB are both right angle.
No marks were given for finding *GL* and *HL*, which did not contribute to the correct answer.
Candidates whom did not use cosine rule the second time (READ THE QUESTION) were penalised.

Question 40

A spherical cap is the section of a sphere cut off by a plane as shown in the diagram below.

The volume of a spherical cap with base radius a and height h is given by $V_{CAP} = \frac{\pi h}{6}\left(3a^2 + h^2\right)$.

a. In a sphere of radius R, show that $V_{CAP} = \frac{\pi h^2}{3}(3R - h)$. **2**



Solution

$a^2 = R^2 - (R-h)^2$ (Pythagoras' theorem)

$= R^2 - (R^2 - 2Rh + h^2)$

$a^2 = 2Rh - h^2$ (1)

Sub (1) into $V_{cap} = \frac{\pi h}{6}(3a^2 + h^2)$

$= \frac{\pi h}{6}(3(2Rh - h^2) + h^2)$

$= \frac{\pi h}{6}(6Rh - 2h^2)$

$= \frac{\pi h}{6} \times 2h(3R - h)$

$V_{cap} = \frac{\pi h^2}{3}(3R - h)$

Marking Scheme:

1 mark for correctly working out a^2 (or a)

1 mark for correct substitution and working to obtain the result.

Marker's Comments:

Many candidates failed to find the relationship of a in terms of R and h.

Candidates who equate the two equations from the questions gained no marks.

b. Darth Parker is constructing a Maff Star by intersecting a sphere of radius 100 km with a sphere of radius 40 km such that their centres are 120 km apart.
The smaller sphere is then discarded, leaving a curved dent on the surface of the larger sphere, as shown in Diagram 1 below.

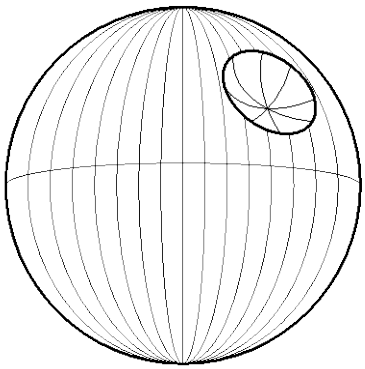


Diagram 1

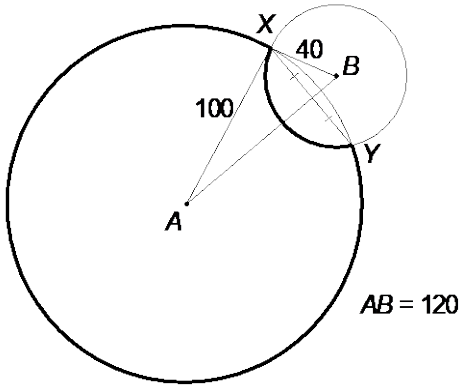


Diagram 2

Diagram 2 above shows a cross section of the intersecting spheres, where A and B are the centres of the larger and smaller sphere respectively and X and Y mark where the spheres intersect on the cross section.

I. Given that AB bisects XY, show that $AB \perp XY$. **1**

Solution

Since AB bisects XY (a chord)
The line from the centre (i.e A) of a circle to the midpoint of a chord (ie. XY) is perpendicular to the chord.
$\therefore AB \perp XY$.

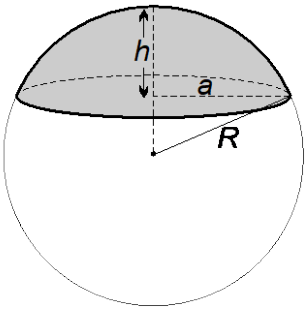
Marking Scheme

1 mark using the correct circle theorem.

Marker's comments

Candidates needs to be precise with language and not use the converse theorem otherwise full mark is not given.

Significant number of candidates should pay attention to this question being 1 mark, as many used congruent triangles which is not efficient.



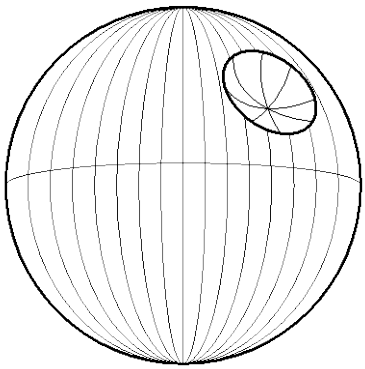


Diagram 1

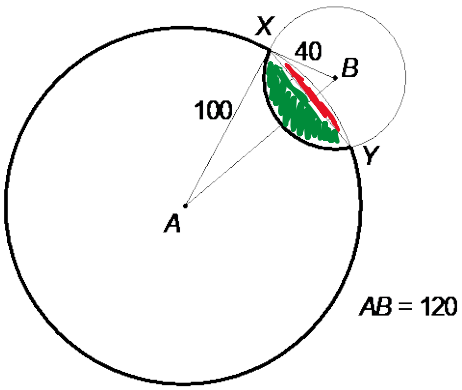
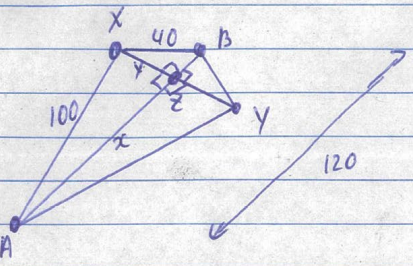


Diagram 2

I. Hence, find the exact volume of the Maff Star. **4**

Solution

Volume of Big Sphere $= \frac{4}{3}\pi \times (100)^3$



From diagram above, let AZ be x and XZ be y

$100^2 = x^2 + y^2 \rightarrow y^2 = 100^2 - x^2$ (1)

$40^2 = (120-x)^2 + y^2 \rightarrow y^2 = 40^2 - (120-x)^2$ (2)

Sub (1) into (2)

$100^2 - x^2 = 40^2 - (120-x)^2$

$100^2 - x^2 = 40^2 - (120^2 - 240x + x^2)$

$100^2 - x^2 = 40^2 - 120^2 + 240x - x^2$

$240x = 100^2 - 40^2 + 120^2$

$240x = 22800$

$x = \frac{22800}{240} = 95$

$\therefore$ AZ = 95km BZ = 120 − 95 = 25km

"green cap"

$\therefore$ Volume $= \frac{4}{3} \times \pi \times 100^3 - \frac{(40-25)^2}{3} \times \pi \times (3\times 40 - (40-25))$

$- \frac{(100-95)^2}{3} \times \pi \times (3\times 100 - (100-95))$ ← "red cap"

$= \frac{\pi}{3}\left[4 \times 100^3 - 15^2 \times 105 - 5^2 \times 295\right]$

$= 1323000\pi \text{ km}^3$

Marking Scheme:

1 mark for the volume of the big sphere.
1 mark for finding correctly the “green” cap (see diagram in the previous page).
1 mark for finding correctly the “red” cap (see diagram in the previous page).
1 mark for the correct answer in exact value.

Marker’s comments

- Poorly done by many candidates.