SYDNEY
BOYS
HIGH
SCHOOL

2019

YEAR 10 YEARLY
STAGE 5.3 MATHEMATICS

General Instructions:

- All questions may be attempted.
- Write using black pen.
- Marks may be deducted for careless or badly arranged work.
- If you wish to rewrite an answer, draw a line through your faulty answer, and rewrite your answer on pages 25 and 26.
  Indicate you are doing this.
  Show the number and part of the answer being rewritten.
- All working and answers are to be written in this test booklet.
- NESA approved calculators may be used.
- Clearly indicate your class by placing an **X** next to your class.

Time Allowed:
120 minutes

Reading Time:
5 minutes

Examiner: PSP

Name: _____

Class	Teacher	
10**A**	BK	
10**B**	SG	
10**C**	JM	
10**P**	JJ	
10**L**	HC	
10**U**	RW	
10**S**	AW	

Section	Marks
A	/12
B	/12
C	/12
D	/14
E	/15
F	/15
G	/15
Total	/95

Section A (12 marks)

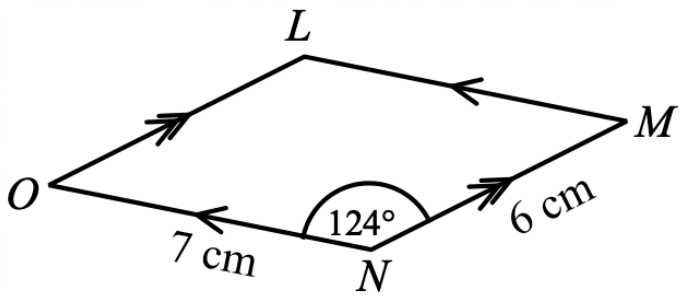
1. A function $f(x)$ is defined by $f(x) = x^3 - 2x^2 - 15x$.

   (a) Evaluate $f(-3)$. **1**

   (b) What can be deduced about $x + 3$ from the answer in part (i)? **1**

2. Solve the quadratic equation: $3x^2 - 7x - 10 = 0$ **2**

3. What is the area of quadrilateral *LMNO* to the nearest square centimetre? **2**



4. Write down the equation of a circle with centre (0, 0) and radius 3 units. **1**

5. The price of a fridge including 10% GST is $2 365. **1**
   What is the amount of GST charged of this fridge?

6. In a game, Iva Bett is to roll two dice, what is the probability that at least one of the dice shows a six? **2**

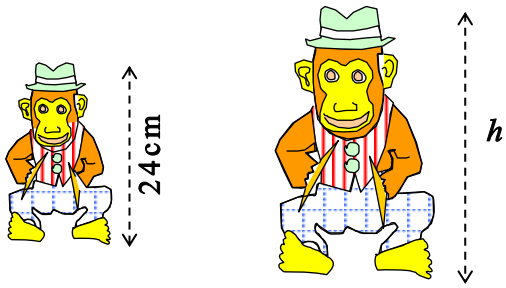
7. The polynomial $q(x) = x^4 - kx^3 - 2x + 33$ has $x - 3$ as a factor. **2**
What is the value of k?

Section B (12 marks)

1. Find the standard deviation of the set of scores correct to 2 decimal places. **1**

$$16\,,\ 10\,,\ 18\,,\ 12\,,\ 14\,,\ 20.$$

2. The diagram shows two similar monkey toys with volumes 3 600cm^3 and 12 150cm^3. **2**

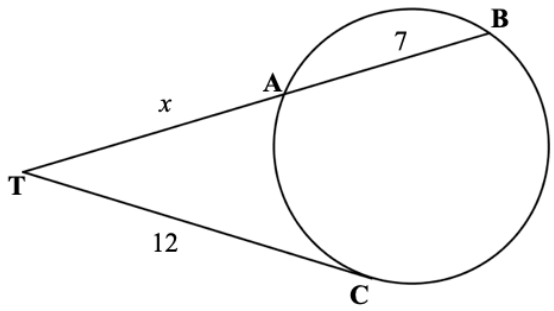


NOT TO SCALE

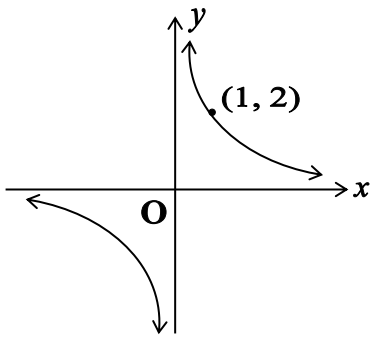
What is the height of the larger of the two monkey toys?

3. What is the angle of inclination to the nearest minute, of the line $3x + 2y = 7$ with the positive direction of the x-axis? **2**

4. In the diagram below, the line TB is a secant intersecting the circle ABC at the points A and B. TC is a tangent at the point C. $AT = x$. What is the value of x? **2**

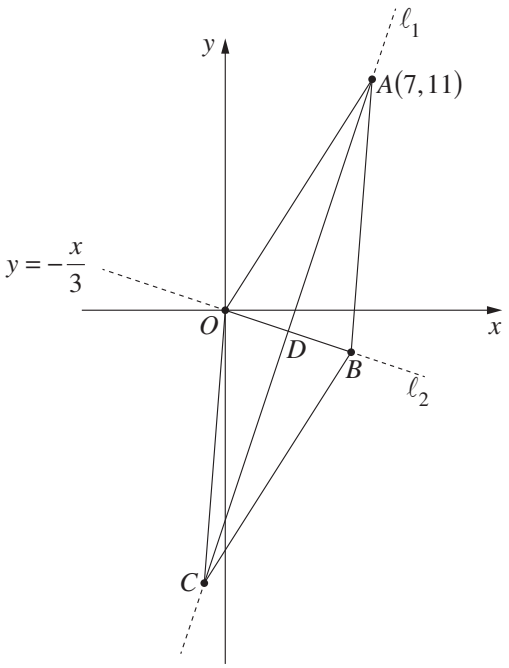


5. What is a possible equation for the graph below? **1**



6. The diagram shows the rhombus $OABC$.
The diagonal from the point $A(7, 11)$ to the point C lies on the line ℓ_1.

The other diagonal, from the origin O to the point B, lies on the line ℓ_2 which has equation $y = -\frac{x}{3}$.



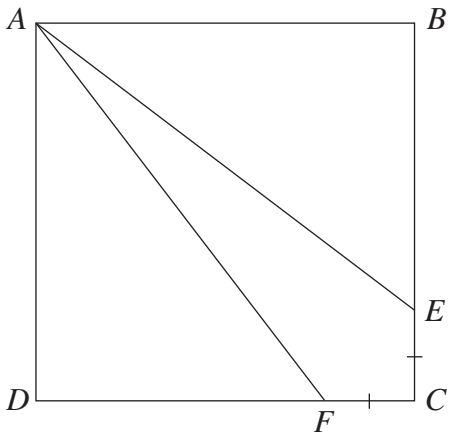
NOT TO SCALE

(a) Show that the equation of the line ℓ_1 is $y = 3x - 10$. **2**

(b) The lines ℓ_1 and ℓ_2 intersect at the point D. **2**
Find the coordinates of D.

Section C (12 marks)

1. The diagram shows the square $ABCD$. The point E is chosen on BC and the point F is chosen on CD so that $EC = FC$.



(a) Gio Meta wrote below his proof of $\Delta ADF \equiv \Delta ABE$. Unfortunately, parts of his proof were obscured. Fill in the blanks. **2**

AIM: $\Delta ADF \equiv \Delta ABE$

In ΔADF and ΔABE

$AD = AB$ ( )

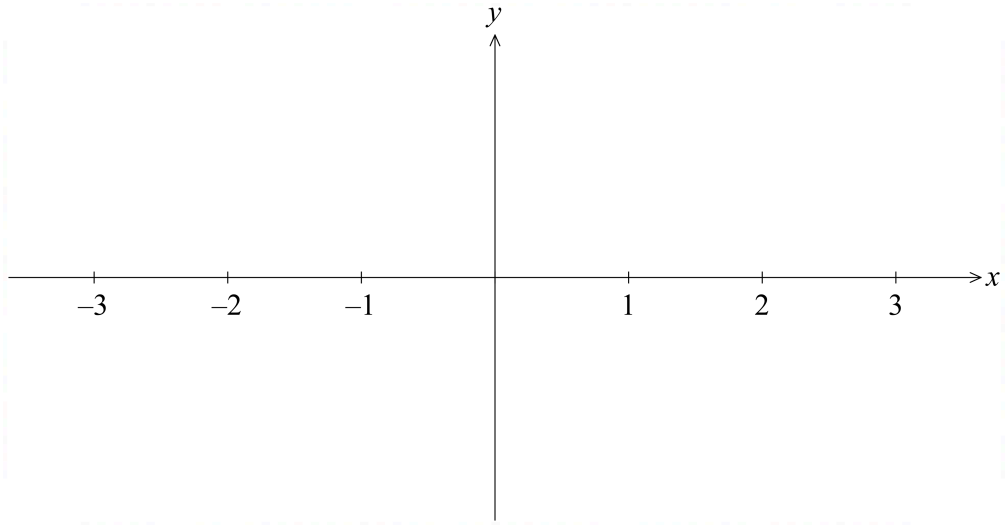
$\angle ADF = \angle ABE =$ ___ ( )

$DF = DC - FC$
$=$ _____ ( )
$= BE$

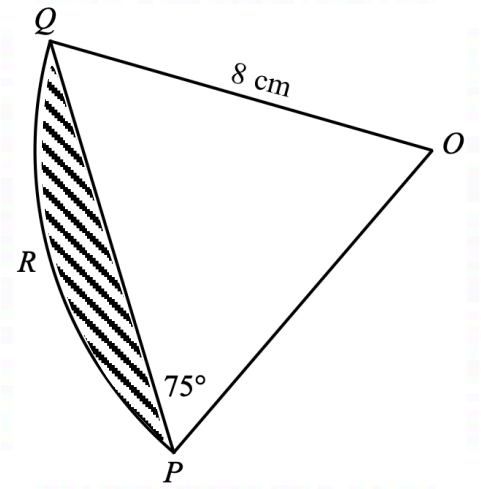
$\therefore \Delta ADF \equiv \Delta ABE$ ( )

(b) The side length of the square is 14 cm and EC has length 4 cm. **2**
Find the area of $AECF$.

2. On the number plane below sketch $y = x^2(x+1)(2-x)^2$. **2**



3. A sector OPQ of a circle is shown in the diagram. The circle has its centre at O and a radius of 8 cm. $\angle OPQ = 75°$



(a) Show that $PQ = \dfrac{4}{\sin 75°}$ cm. **2**

(b) Calculate the perimeter of the segment PQR, correct to 1 decimal place. **1**

4. At Inner City High School, the senior students were surveyed about wearing school uniform.

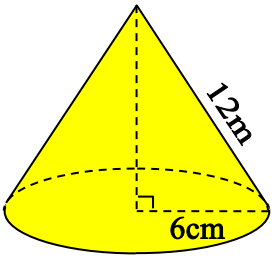
	Year 10	Year 11	Year 12	Total
In favour	50	55	25	130
Not in favour	70	35	25	130
Uncertain	20	10	10	40
Total	140	100	60	300

What is the probability that a student chosen at random from this school

(a) is in Year 10 and in favour of the uniform? **1**

(b) is in Year 10, given that they are in favour of the uniform? **1**

5. Find the curved surface area of this cone shown in the diagram, correct to one decimal place. **1**

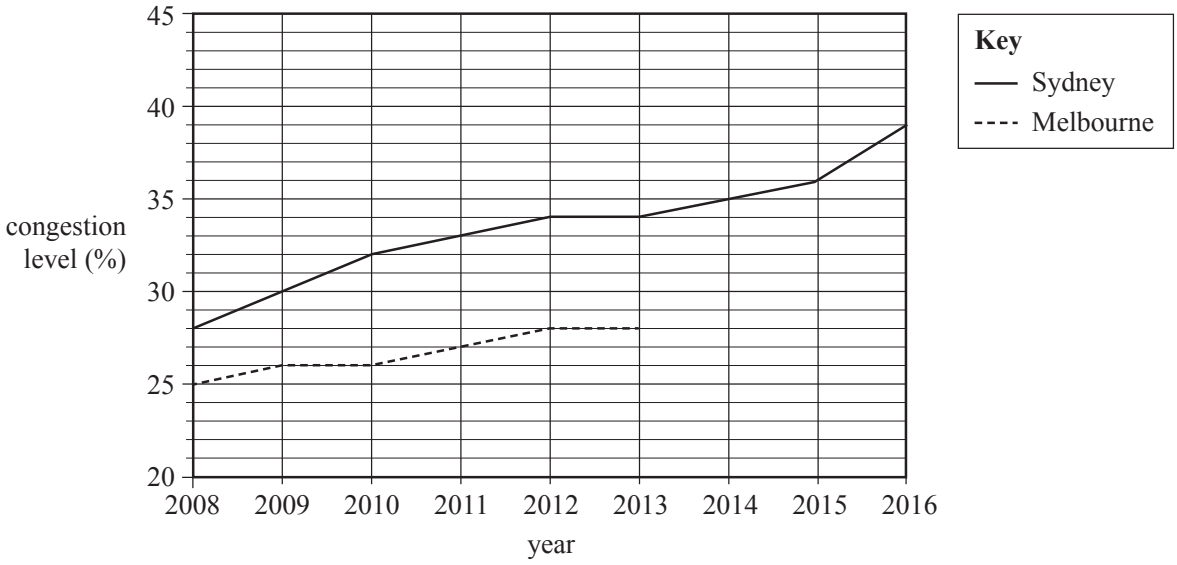


Section D (14 Marks)

1. The table below shows the yearly average traffic congestion levels in two cities, Melbourne and Sydney, during the period 2008 to 2016.

   Also shown is a time series plot of the same data. The time series plot for Melbourne is incomplete.

	Congestion level (%)								
Year	2008	2009	2010	2011	2012	2013	2014	2015	2016
Melbourne	25	26	26	27	28	28	28	29	33
Sydney	28	30	32	33	34	34	35	36	39



Data: TomTom International BV, <www.tomtom.com/en_gb/trafficindex>

(a) Use the data in the table to complete the **time series plot above** for Melbourne. **1**

(b) A line of best fit is used to model the trend line in the time series plot for Sydney.

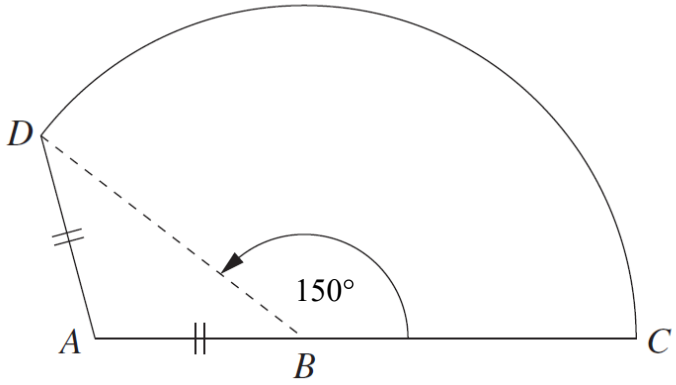
The equation is $congestion\ level = -2280 + 1.15 \times year$

(i) Draw this line on the time series plot above. **1**

(ii) Determine the average rate of increase in percentage congestion level for the period 2008 to 2016 inclusive. **1**

(iii) Use the equation to predict when the percentage congestion level in Sydney will be 43%. **1**

2. In the diagram below $ABCD$ represents a garden.
The sector BCD has centre B and $\angle DBC = 150°$.
The points A, B and C lie on a straight line and $AB = AD = 3$ metres.

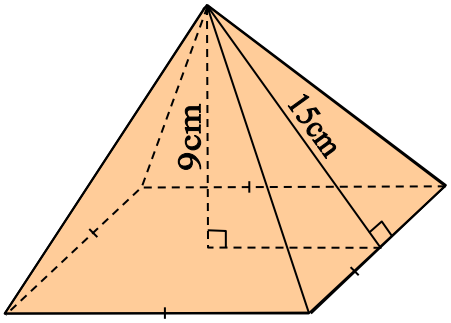


(a) Show that $\angle DAB = 120°$ **1**

(b) Find exact the length BD. **2**

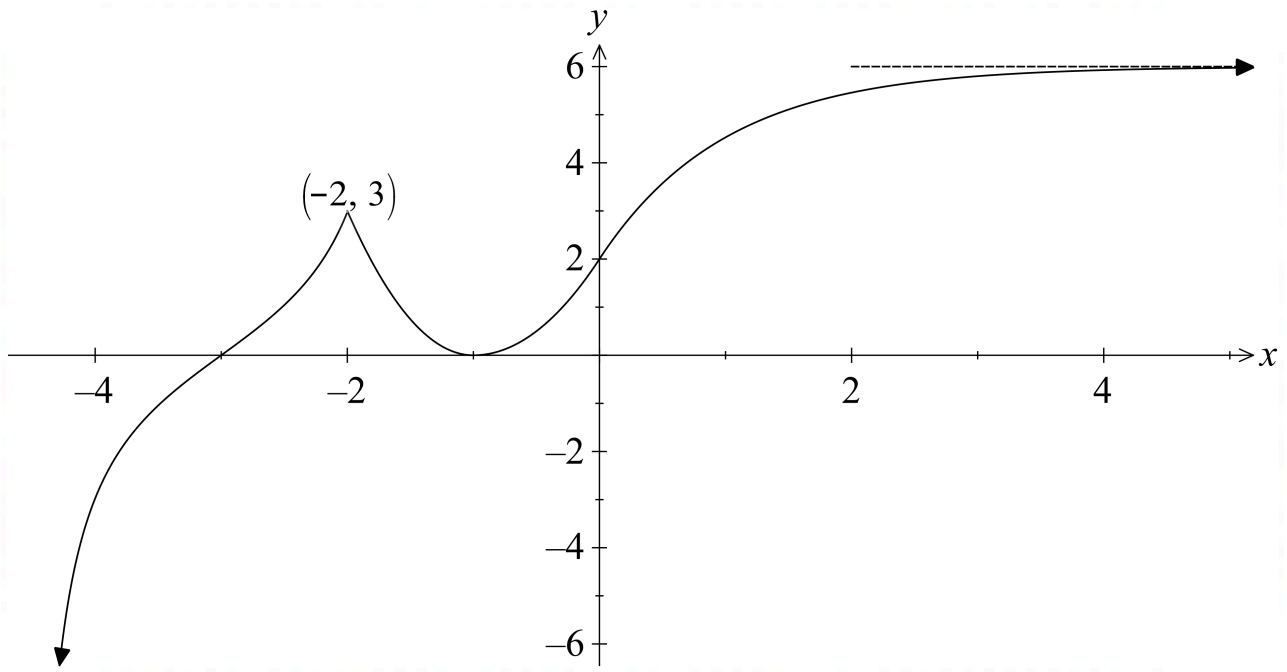
(c) Find the area of the garden $ABCD$ correct to one decimal place. **3**

3. The diagram shows a square pyramid.



Calculate its surface area. **2**

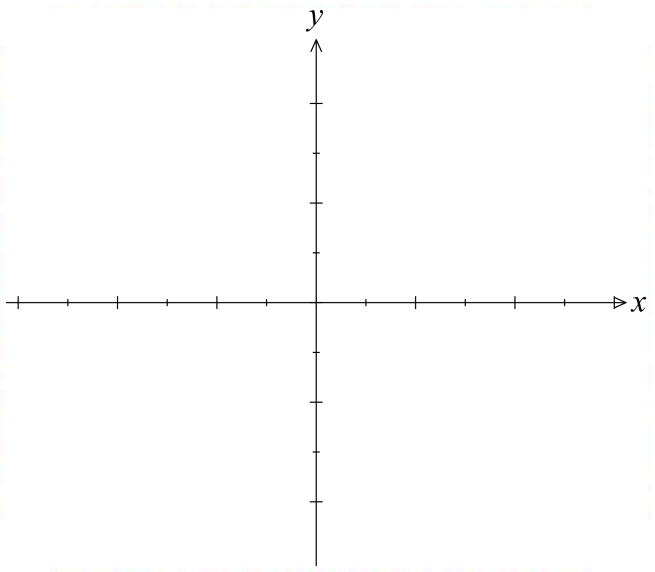
4. The diagram below shows the graph of an unknown function $w(x)$. **2**
The function $w(x)$ is transformed to give $-w(-x)$.
Sketch on the diagram the graph of $y = -w(-x)$, showing any intercepts with the axes and other important features.



Section E (15 Marks)

1. Sketch the graph of $y = \dfrac{1}{x-2} + 2$ on the number plane below. **2**

   Asymptotes and intercepts need to be shown.



2. Using the factor theorem, solve $x^3 - 5x^2 + 3x + 4 = 0$ **2**

   If necessary, leave answers in surd form.

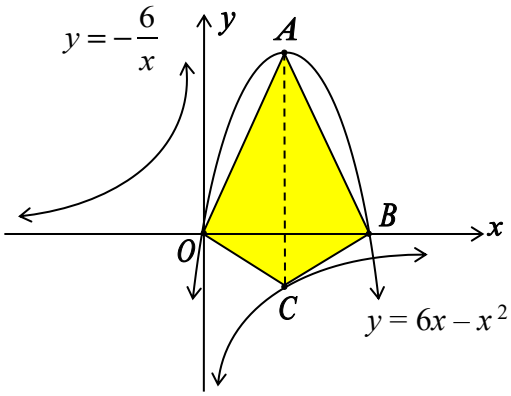
3. Consider the function $f(x) = 3^{x+2}$ and its inverse $f^{-1}(x)$. **2**

   What is the value of $f^{-1}(9)$?

4. The remainder when the polynomial $P(x) = x^4 - 8x^3 - 7x^2 + 3$ is divided by $x^2 + x$ is $kx + 3$. **2**

What is the value of k?

5.



In the number plane shown, the parabola $y = 6x - x^2$ has a vertex at A and crosses the x-axis at the origin O and at the point B.

The point C is vertically below A and lies on the curve $y = -\frac{6}{x}$.

(a) Find the coordinates of A. **1**

(b) Find the coordinates of B. **1**

(c) Find the coordinates of C. **1**

(d) Find the area of the kite $OABC$. **1**

6. A function is defined by $f(x) = -\sqrt{4-x^2}$ for $0 \le x \le 2$. **1**
Which of the following correctly represents the inverse function of $f(x)$.

A. $f^{-1}(x) = -\sqrt{4-x^2}$ for $0 \le x \le 2$

B. $f^{-1}(x) = \sqrt{4-x^2}$ for $0 \le x \le 2$

C. $f^{-1}(x) = -\sqrt{4-x^2}$ for $-2 \le x \le 0$

D. $f^{-1}(x) = \sqrt{4-x^2}$ for $-2 \le x \le 0$

7. A bunch of flowers is formed using single flowers from five different types: **2**

daffodils, tulips, roses, lilies and petunias

What is the minimum number of flowers in a bunch required to ensure that it contains at least four flowers of any one type? Justify.

Section F (15 marks) **Multiple Choice** **Circle the correct response**

1 Which one of the following is equivalent to 3.507×10^{-3}?

A. 3507×1000 B. 35.07×100

C. $3507 \div 1000$ D. $0.3507 \div 100$

2 Anne Vesta invests \$48 000 for 3 years at 9.6 % p.a compounded monthly.
Which of the following is a correct expression of her investment?

A. $\$48\,000 \times 1.096^{3}$ B. $\$48\,000 \times 1.008^{36}$

C. $\$48\,000 \times 1.8^{36}$ D. $\$48\,000 \times 1.0008^{36}$

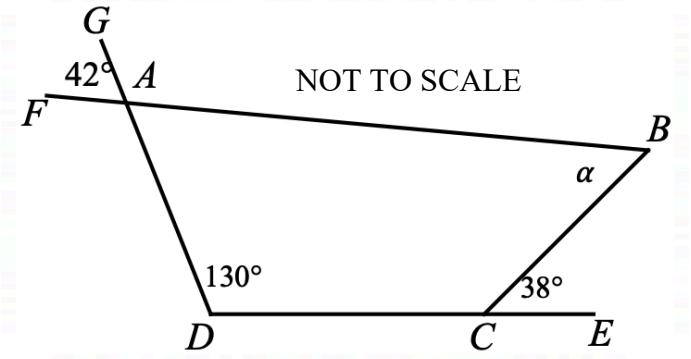
3 A bird cage contains 4 canaries, 2 of these are yellow and the other 2 are orange.
If two canaries escaped, find the probability that both are yellow.

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

4 Two years ago, Sam Tsung bought a phone. Its price depreciates at the rate of 60% p.a.
If the value of the phone now is \$384, how much did Sam pay for the phone initially?

A. \$983 B. \$1067 C. \$2000 D. \$2400

5 In the diagram $ABCD$ is a quadrilateral.
$\angle BCE = 38°$, $\angle ADC = 130°$ and $\angle FAG = 42°$.



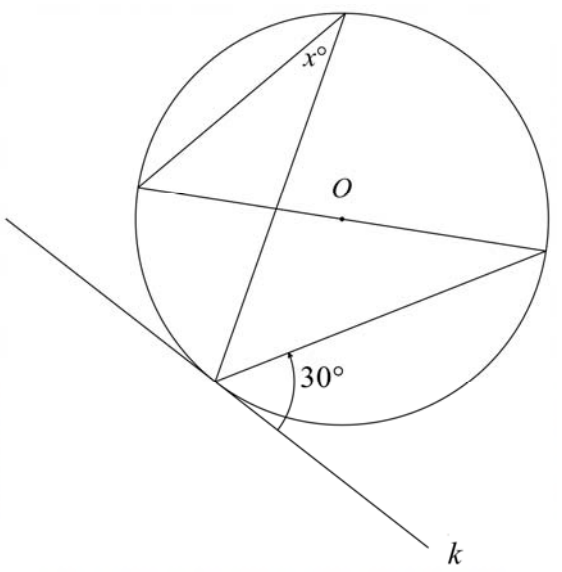
What is the size of $\angle ABC$?

A. 38° B. 42° C. 46° D. 50°

6 What is the value of $\log_2 3$ correct to 4 significant figures?

A. 1.584 B. 1.585 C. 0.6309 D. 0.6310

7 O is the centre of a circle. The line k is tangent to the circle.
What is the value of x?



A. 30° B. 45° C. 60° D. 90°

8 Solve $\cos x = -\frac{\sqrt{3}}{2}$ for $-180° \le x \le 180°$

A. $x = -60°$ or $60°$ B. $x = -120°$ or $120°$

C. $x = -30°$ or $30°$ D. $x = -150°$ or $150°$

9 How many solutions does the equation $6x\left(\frac{1}{x}+1\right)=0$ have?

A. 0 B. 1 C. 2 D. 3

10 What is the natural domain of the function $h(x) = \dfrac{1}{\sqrt{x^2 - 4}}$

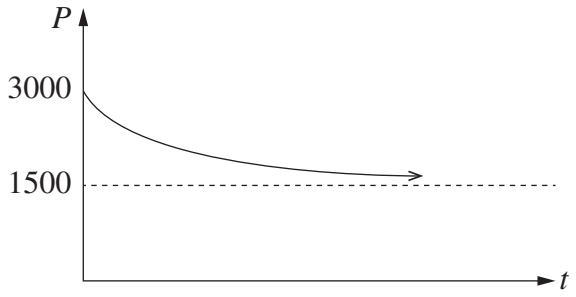
A. $-2 \le x \le 2$

B. $-2 < x < 2$

C. $x \le -2$ or $x \ge 2$

D. $x < -2$ or $x > 2$

11 The diagram below shows the number of penguins, $P(t)$, on an island at time t.



Which equation best represents this graph?

A. $P(t) = 1500 + 1500 \times 2.7^{-kt}$

B. $P(t) = 3000 - 1500 \times 2.7^{-kt}$

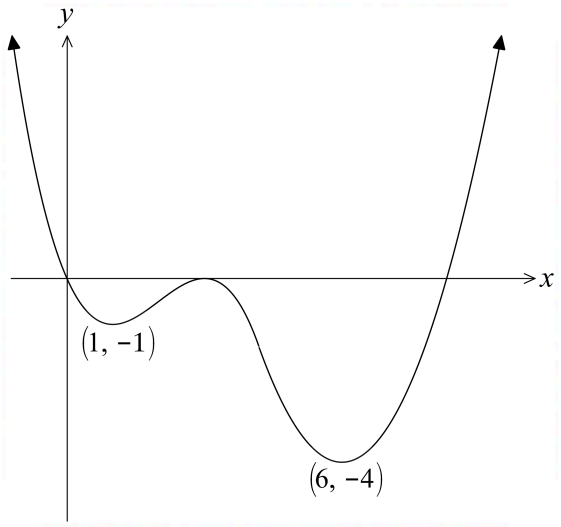
C. $P(t) = 3000 + 1500 \times 2.7^{-kt}$

D. $P(t) = 4500 - 1500 \times 2.7^{-kt}$

12 What is the value of $\log_{10} 5 + \log_{10} 50$, correct to two decimal places?

A. 1.74 B. 2.39 C. 2.40 D. 5.52

13 The graph below shows the function $y = f(x)$.



If $f(x) + k > 0$ for all x, then which of the following is correct?

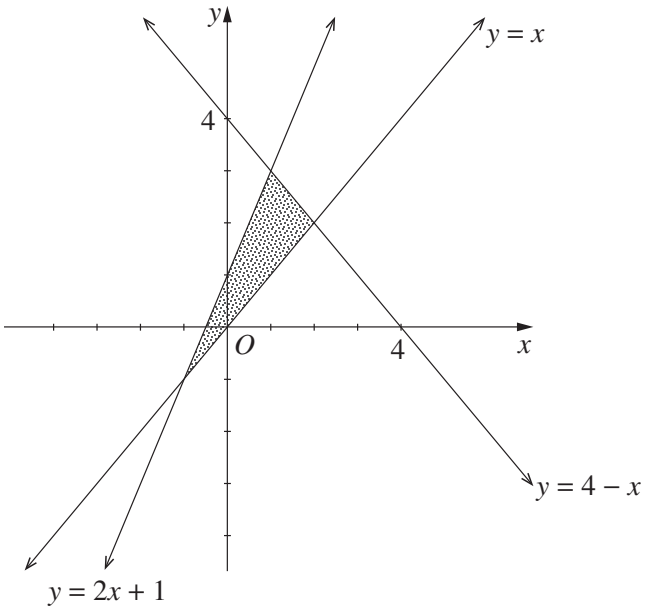
A. $1 < k < 6$

B. $-1 < k < -4$

C. $k > 4$

D. $k < 6$

14 The region enclosed by $y = 4 - x$, $y = x$ and $y = 2x + 1$ is shaded in the diagram below.



Which of the following defines the shaded region?

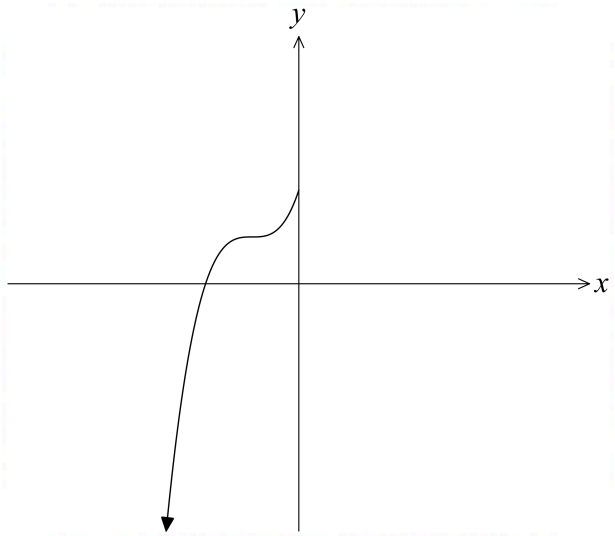
A. $y \leq 2x + 1, \quad y \leq 4 - x, \quad y \geq x$

B. $y \geq 2x + 1, \quad y \leq 4 - x, \quad y \geq x$

C. $y \leq 2x + 1, \quad y \geq 4 - x, \quad y \geq x$

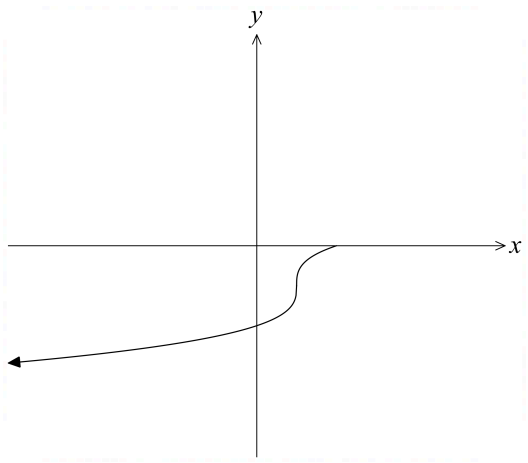
D. $y \geq 2x + 1, \quad y \geq 4 - x, \quad y \geq x$

15

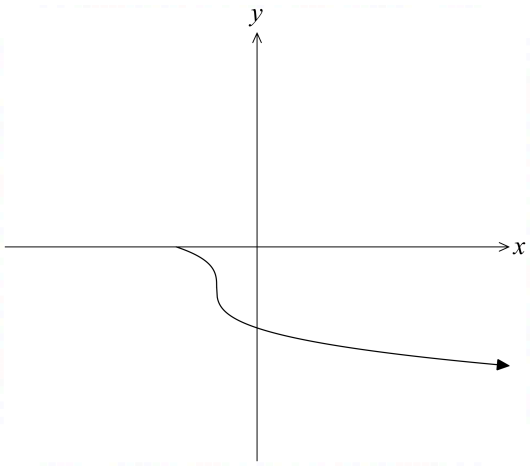


The graph above is that of $y = f(x)$. Which diagram below shows the graph of $y = f^{-1}(x)$?

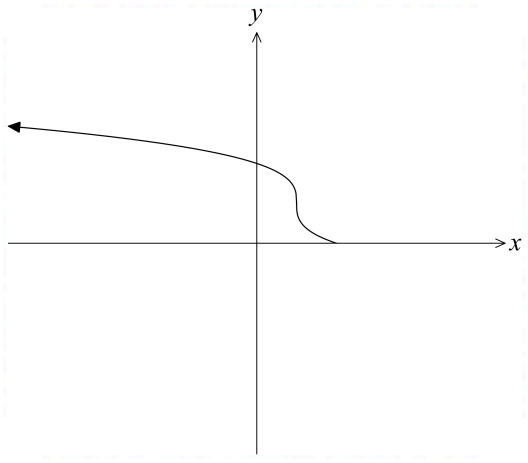
A.



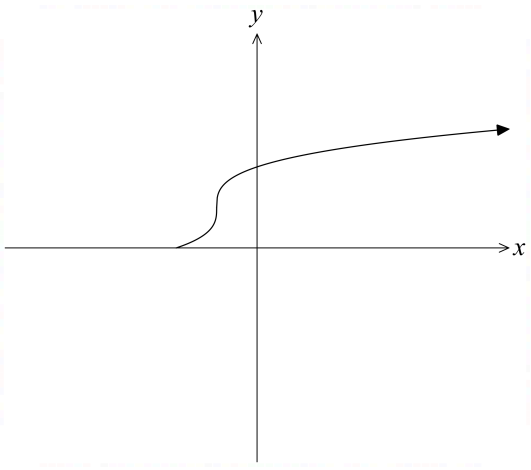
B.



C.



D.



Section G (15 Marks)

1. Atlas sat for three Geography tests during this term. The table below shows his results and the students' average mark and standard deviation for the tests.

	Atlas' mark	Average mark	Standard deviation
Test 1	82	73	6
Test 2	87	75	10
Test 3	x	72	8

(a) In which test, Test 1 or Test 2 did Atlas perform better compared to the rest of the students? Justify your answer. **1**

(b) When compared to the rest of the students, Atlas' performance in Test 3 is equivalent to his performance in Test 1.
What is his mark for Test 3? **1**

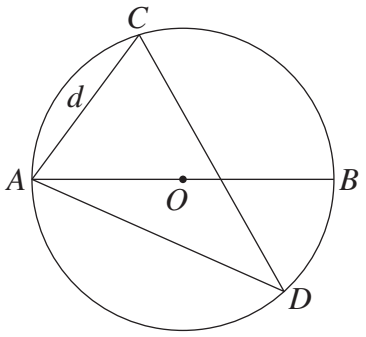
2. What are the exact solutions to $(x-2)^6 - 11\times(x-2)^3 + 10 = 0$. **2**

3. The pH scale is used to measure how acidic or basic a substance is using the formula $\text{pH} = -\log_{10}\left[\text{H}^+\right]$, where H^+ is the Hydrogen ion concentration in moles per litre.

(a) A brand of orange juice has a Hydrogen ion concentration of 10^{-3}. **1**
Find the pH of this orange juice.

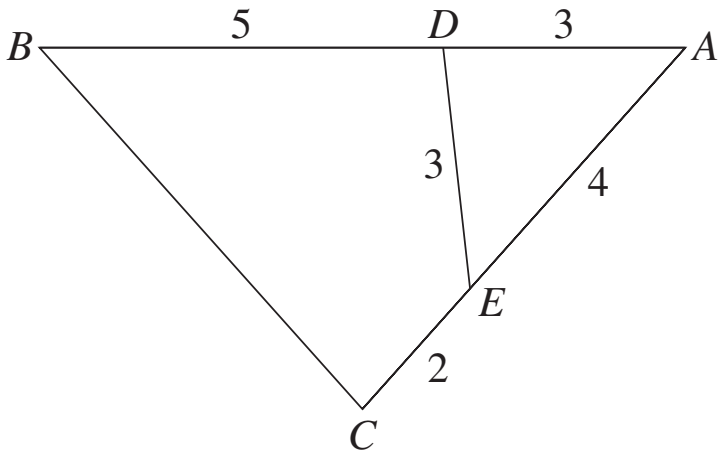
(b) A solution of soapy water is known to have a pH of 12. **1**
Find the Hydrogen ion concentration of this soapy water.

4. The circle centred at O with radius r has a diameter AB. Points C and D are chosen on the circle as shown in the diagram. The chord AC has length d. **3**



Show that $d = 2r \sin D$.

5. A triangle has vertices A, B and C. The point D lies on the interval AB such that $AD = 3$ and $DB = 5$.
The point E lies on the interval AC such that $AE = 4$, $DE = 3$ and $EC = 2$.



NOT TO SCALE

(a) State why $\Delta ABC \,|||\, \Delta AED$. **1**

(b) Hence, or otherwise, state why $BCED$ is a cyclic quadrilateral **1**

Question 5 continued on page 29

5. (c) Show that $CD = \sqrt{21}$. **2**

(d) Using the result of question 4, or otherwise, find the exact value of the radius of The circle passing through the points B, C, E and D. **2**

End of paper

Working out space

Working out space

SYDNEY
BOYS
HIGH
SCHOOL

2019

YEAR 10 YEARLY
STAGE 5.3

Mathematics

SUGGESTED SOLUTIONS

Section A (12 marks)

1. A function $f(x)$ is defined by $f(x) = x^3 - 2x^2 - 15x$.

(a) Evaluate $f(-3)$. 1

$f(-3) = (-3)^3 - 2(-3)^2 - 15(-3) = -27 - 18 + 45 = 0$

(b) What can be deduced about $x + 3$ from the answer in part (i)? 1

$x+3$ is a factor of $f(x)$

2. Solve the quadratic equation: $3x^2 - 7x - 10 = 0$ 2

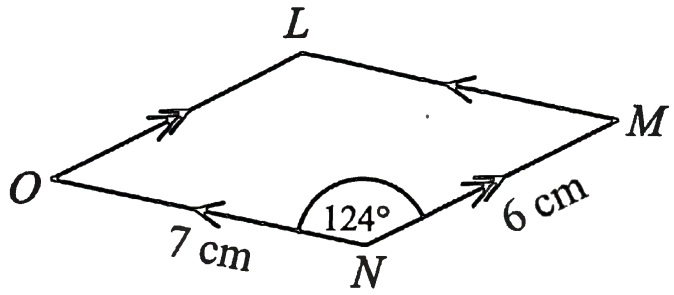
$(3x-10)(x+1) = 0$ (1) wrong signs (0.5)

$x = -1$ (0.5) surd form answer (1.5)

or

$x = \frac{10}{3}$ (0.5)

3. What is the area of quadrilateral $LMNO$ to the nearest square centimetre? 2



$2\left(\frac{1}{2} \times 6 \times 7 \sin 124°\right)$

incomplete equation (0.5)

$= (17.410) \times 2 = 35\ cm^2$ (nearest square centimeter)

answer (not expressed properly) (0.5)

4. Write down the equation of a circle with centre (0, 0) and radius 3 units. 1

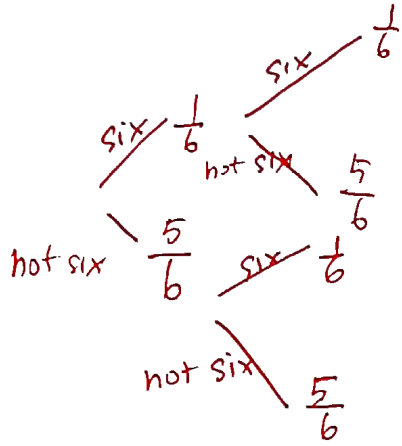
$x^2 + y^2 = 9$

5. The price of a fridge including 10% GST is $2 365. 1
What is the amount of GST charged of this fridge?

$\therefore \$2365 - \frac{\$2365}{1.1}$

$= \$215$

6. In a game, Iva Bett is to roll two dice, what is the probability that at least one of the dice shows a six? 2



$1 - P(\text{not } 6, \text{not } 6)$

$= 1 - \frac{5}{6} \times \frac{5}{6}$ (1.5)

$= \frac{11}{16}$ (0.5)

OR

$P(\text{not } 6, 6) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$ (0.5)

$P(6, \text{not } 6) = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$ (0.5)

$P(6, 6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ (0.5)

7. The polynomial $q(x) = x^4 - kx^3 - 2x + 33$ has $x - 3$ as a factor. What is the value of k? 2

$\therefore P(\text{at least one of the dice shows a six})$

$\frac{5}{36} \times 2 + \frac{1}{36}$

$= \frac{11}{36}$ (0.5)

$q(3) = 3^4 - k(3)^3 - 2(3) + 33$ (0.5)

$0 = 81 - 27k - 6 + 33$ (0.5)

$27k = 108$ (0.5)

$k = 4$ (0.5)

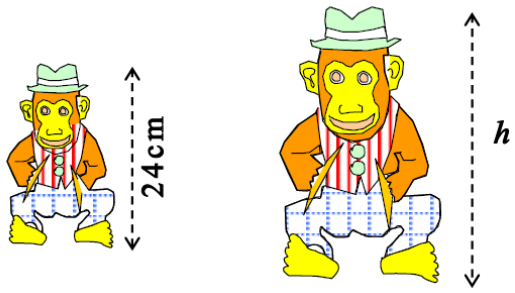
2019 SBHS YR10 Yearly Exam Section B Solutions and Comments

1. Find the standard deviation of the set of scores correct to 2 decimal places. 1

$$16, 10, 18, 12, 14, 20$$

Solution	Comments
By using the statistics mode on the calculator: $\sigma = 3.42\ (2\ d.p.)$	No half marks. Some students have poor handwriting, writing σ as *6*. Alternative solution: $\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$ $= 3.42\ (2\ d.p.)$

2. The diagram shows two similar monkey toys with volumes $3600\ cm^3$ and $12150\ cm^3$. 2



NOT TO SCALE

What is the height of the larger of the two monkey toys?

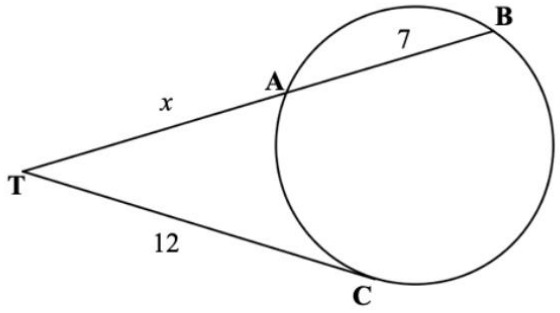
Solution	Comments
$\frac{V_{small}}{V_{big}} = \frac{3600}{12150}$ $= \frac{8}{27}$ $\frac{h_{small}}{h_{big}} = \sqrt[3]{\frac{V_{small}}{V_{big}}}$ $\frac{24}{h_{big}} = \sqrt[3]{\frac{8}{27}}$ $= \frac{2}{3}$ $\therefore h_{big} = 36\ cm$	Common error(s): • Not stating the units. • Not carrying the cube roots forward in calculations to get an exact ratio of sides. Students who incorrectly calculated the side ratio could only score a maximum of 1 mark.

2019 SBHS YR10 Yearly Exam Section B Solutions and Comments

3. What is the angle of inclination to the nearest minute, of the line $3x + 2y = 7$ with the positive direction of the x-axis? 2

Solution	Comments
$3x + 2y = 7$ $\therefore y = \dfrac{-3x + 7}{2}$ $m = -\dfrac{3}{2}$ $\therefore \tan\theta = -\dfrac{3}{2}$ $\theta = \arctan -\dfrac{3}{2}$ $= -56°19'$ Conventionally, the angle made with the positive direction of the x-axis is defined to be anticlockwise. $\therefore \theta = 180 - 56°19'$ $= 123°41'$	Common error(s): • Forgetting the angle a line makes with the positive x-axis is defined to be anticlockwise. • Forgetting $m = \tan\theta$. This question was very poorly done, due to the above errors.

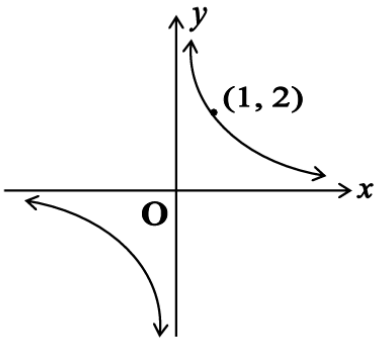
4. In the diagram below, the line TB is a secant intersecting the circle ABC at the points A and B. TC is a tangent at the point C. $AT = x$. What is the value of x? 2



Solution	Comments
By the theorem of intersecting tangents and secants: $CT^2 = AT \times BT$ $12^2 = x(x + 7)$ $x^2 + 7x - 144 = 0$ $(x + 16)(x - 9) = 0$ $x = -16$ or $x = 9$ x is a length, so the negative solution is rejected. $\therefore x = 9\,u$	Common error(s): • Not rejecting the negative solution. • Incorrectly remembering the theorem of intersecting tangents and secants. Students who wrote $12^2 = 7x^2$ or equivalent could not score any marks.

2019 SBHS YR10 Yearly Exam Section B Solutions and Comments

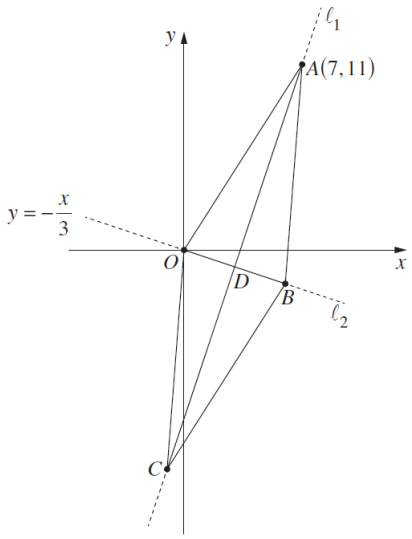
5. What is a possible equation for the graph below? 1



Solution	Comments
The graph as a hyperbola, which has the general equation: $y = \frac{k}{x}$ Given the point $(1, 2)$: $2 = \frac{k}{1}$ $k = 2$ $\therefore y = \frac{2}{x}$	No half marks.

2019 SBHS YR10 Yearly Exam Section B Solutions and Comments

6. The diagram shows the rhombus $OABC$.
The diagonal from the point $A(7,11)$ to the point C lies on the line l_1.
The other diagonal, from the origin O to the point B, lies on the line l_2 which has equation $y = -\frac{x}{3}$.



NOT TO SCALE

(a) Show that the equation of the line l_1 is $y = 3x - 10$. 2

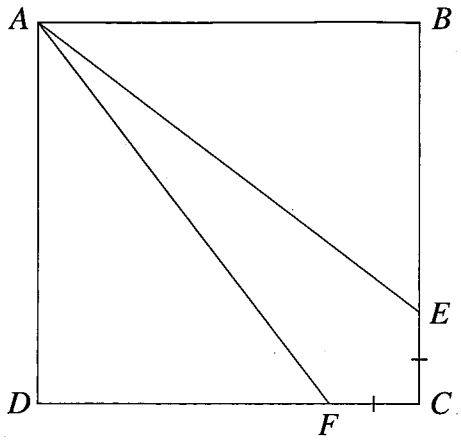
Solution	Comments
The diagonals of a rhombus are perpendicular bisectors of each other. $\therefore m_1 \times m_2 = -1$ $m_1 \times -\frac{1}{3} = -1$ $m_1 = 3$ By the point-gradient formula: $3 = \frac{y-11}{x-7}$ $y - 11 = 3x - 21$ $y = 3x - 10$	Students who attempted to prove by substituting A into the equation for l_1 could only score a maximum of 1 mark.

(b) The lines l_1 and l_2 intersect at the point D. Find the coordinates of D. 2

Solution	Comments
$-\frac{x}{3} = 3x - 10$ $3x + \frac{x}{3} = 10$ $\frac{10x}{3} = 10$ $x = 3$ Substitute $x = 3$ to solve for y: $y = 3 \times 3 - 10$ $= -1$ $\therefore D = (3, -1)$	Common error(s): • Forgetting to find the y-coordinate.

Section C (12 marks)

1. The diagram shows the square $ABCD$. The point E is chosen on BC and the point F is chosen on CD so that $EC = FC$.



(a) Gio Meta wrote below his proof of $\Delta ADF \equiv \Delta ABE$. Unfortunately, parts of his proof were obscured. Fill in the blanks. **2**

AIM: $\Delta ADF \equiv \Delta ABE$ WELL DONE

In ΔADF and ΔABE

$AD = AB$ (equal sides of a square $\frac{1}{2}$)

$\angle ADF = \angle ABE = \underline{90}$ (equal $\angle$'s of a square are 90° $\frac{1}{2}$)

BOTH MUST BE REASONED

$DF = DC - FC$

$= \underline{BC - EC}$ (FC=EC given, DC=BC equal sides of a square $\frac{1}{2}$)

$= BE$

$\therefore \Delta ADF \equiv \Delta ABE$ (SAS $\frac{1}{2}$)

(b) The side length of the square is 14 cm and EC has length 4 cm. Find the area of $AECF$. **2**

36

$ACEF = ABCD - 2 \times ADF$

$= 14^2 - 2[\frac{1}{2}(14)(10)]$ — (1)

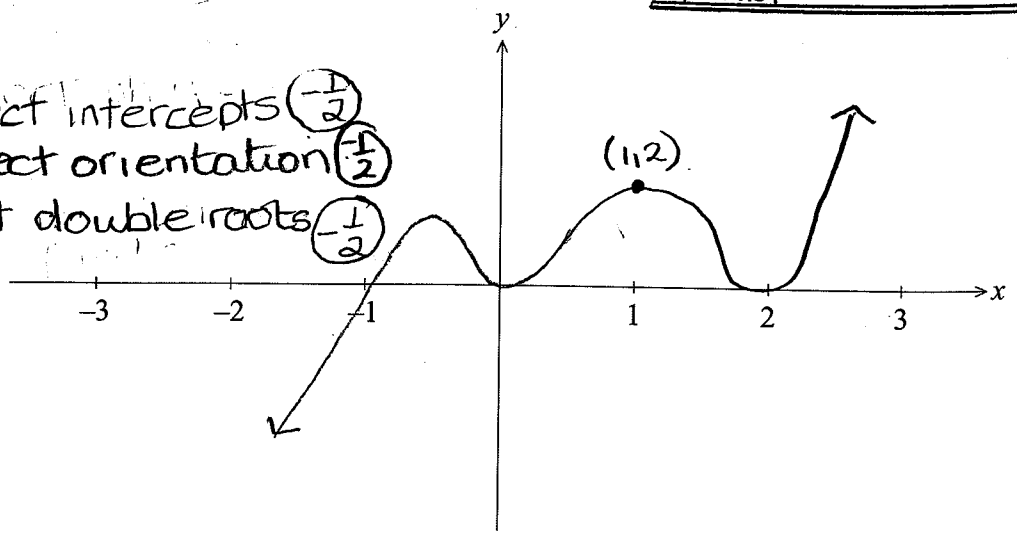
$= 56\ cm^2$ — (1)

WELL DONE
Some tried other methods and then struggled with rounding errors OR finding perpendicular of the triangle.

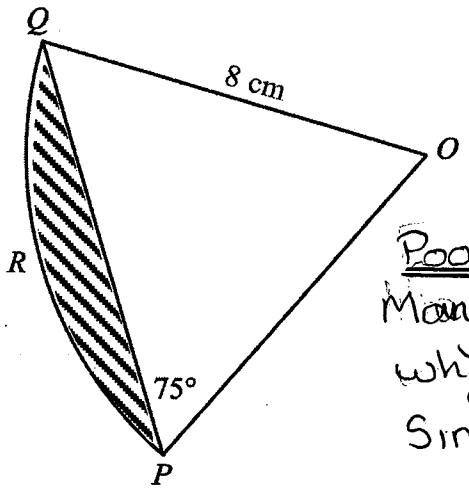
2. On the number plane below sketch $y = x^2(x+1)(2-x)^2$. **2**

FAIRLY WELL DONE

Incorrect intercepts $(-\frac{1}{2})$
Incorrect orientation $(-\frac{1}{2})$
Incorrect double roots $(-\frac{1}{2})$



3. A sector OPQ of a circle is shown in the diagram. The circle has its centre at O and a radius of 8 cm. $\angle OPQ = 75°$



POORLY COMPLETED
Many did not show why 30°
Sine rule well done

(a) Show that $PQ = \dfrac{4}{\sin 75°}$ cm. **2**

$\angle PQO = 75°$ (base $\angle$s of isosceles $\triangle$)
$\angle POQ = 180 - 2\times 75$ ($\angle$sum $\triangle$) } (1)
$= 30°$

$\dfrac{PQ}{\sin 30} = \dfrac{8}{\sin 75}$, $PQ = \dfrac{8\sin 30}{\sin 75}$, $PQ = \dfrac{8(\frac{1}{2})}{\sin 75}$, $PQ = \dfrac{4}{\sin 75}$ (1)

(b) Calculate the perimeter of the segment PQR, correct to 1 decimal place. **1**

$P = \dfrac{30}{360} \times 2\pi \times 8 + \dfrac{4}{\sin 75}$

FAIRLY WELL DONE
Some did not add PQ

$\doteq 8.3299$
$\doteq 8.3$cm (1dp)

2 4. At Inner City High School, the senior students were surveyed about wearing school uniform.

	Year 10	Year 11	Year 12	Total
In favour	50	55	25	130
Not in favour	70	35	25	130
Uncertain	20	10	10	40
Total	140	100	60	300

What is the probability that a student chosen at random from this school

(a) is in Year 10 and in favour of the uniform? **1**

$P(10 \cap \text{in favour}) = \frac{50}{300} = \frac{1}{6}$ (1)

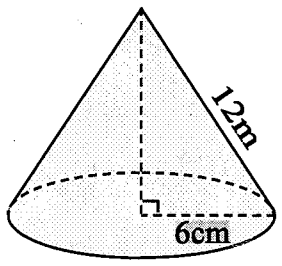
VERY WELL DONE
A few got total of 300 incorrect
whole marks only.

(b) is in Year 10, given that they are in favour of the uniform? **1**

$P(10 \text{ given in favour}) = \frac{50}{130} = \frac{5}{13}$ (1)

WELL DONE
Some had 140 not 130
whole marks only

5. Find the curved surface area of this cone shown in the diagram, correct to one decimal place. **1**



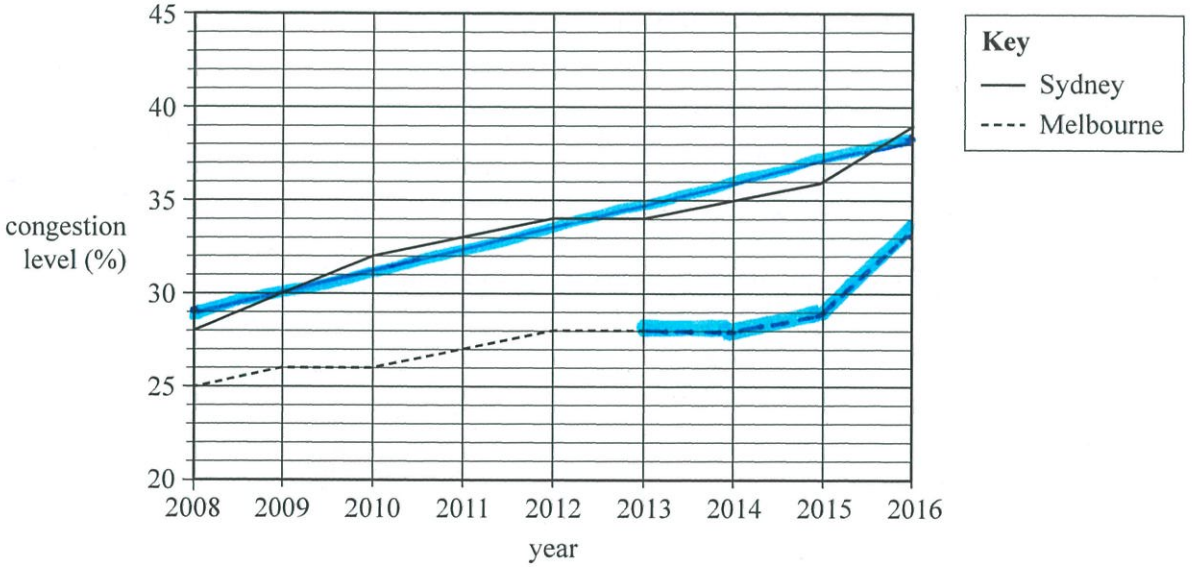
POORLY DONE
Some found total S.A
Some did not know formula.
A lot did not recognise cm and m.

$SA = \pi rs$
$= \pi \times 6 \times 1200$
$= 7200\pi$
$\doteq 22619.46711$
$\doteq 22619.5 \text{ cm}^2 \text{ (1dp)}$
$\doteq 2.3 \text{ m}^2 \text{ (1dp)}$

12 not 1200 $(-\frac{1}{2})$

Section D

1.



Data: TomTom International BV, <www.tomtom.com/en_gb/trafficindex>

Very well done.

1. (b) (ii)

$$= \frac{38.4 - 29.2}{2016 - 2008}$$

$$= 1.15$$

[1]

Very poorly done.

(iii) $43 = -2280 + 1.15 \times y$

$1.15y = 2323$

$y = 2020$

[1]

Very well done.

2. (a) $\angle ABD = 180° - 150°$ (angles of a straight line)

$= 30°$

$(= \angle ADB)$ (corresponding base angles in an isosceles Δ)

(b) $$\frac{BD}{\sin 120°} = \frac{3}{\sin 30°}$$

$$BD = \frac{3 \sin 120°}{\sin 30}$$

$$= \frac{3 \times \left(\sqrt{3}/2\right)}{1/2} = 3\sqrt{3}$$

(c) $$\text{Area} = \frac{1}{2} \times 3 \times 3 \times \sin 120 + \frac{150}{360} \times \pi \times (3\sqrt{3})^2$$

$$= 39.2 \ m^2$$

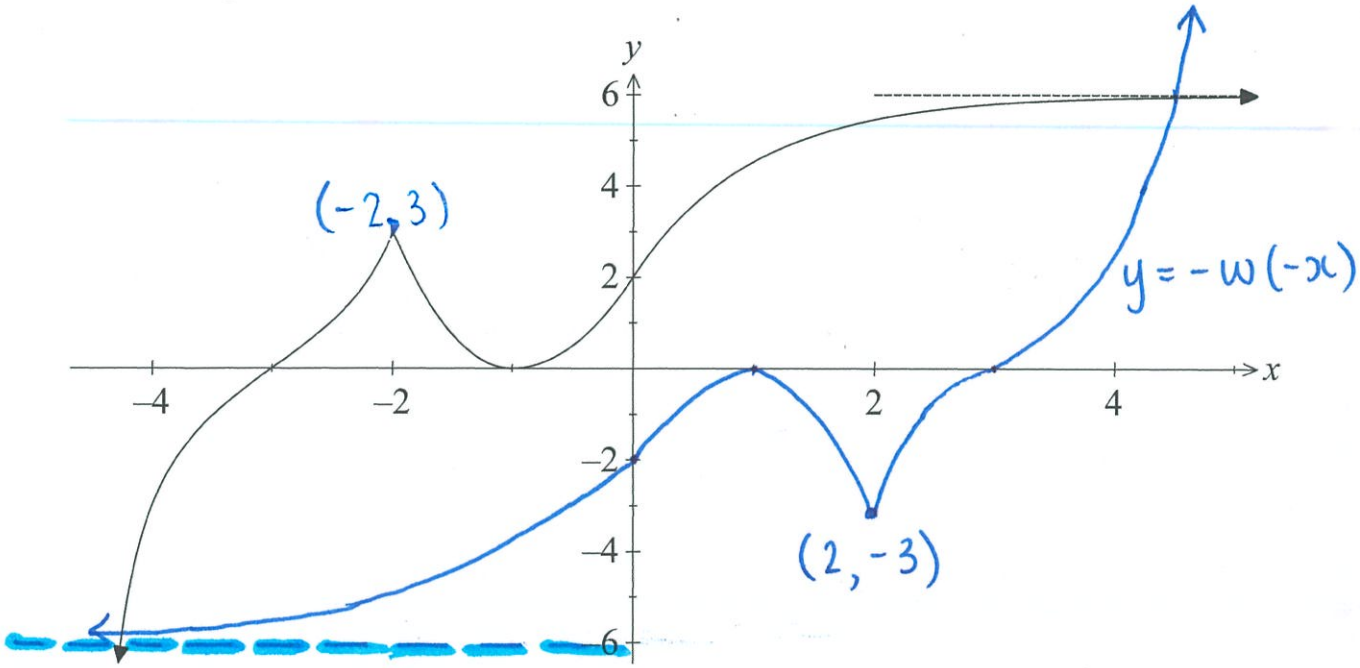
Very well done.
Although some candidates forgot to simplify there answer for part (b).

3. $SA = 24 \times 24 + \frac{1}{2} \times 4 \times 15 \times 24$

$= 1296\ cm^2$

Very well done.
However, some common mistakes were that candidate forgot to double the base value for the base, some calculated the volume instead, and some incorrectly calculated the areas of the triangular faces.

4.



Very poorly done.

Section E

1. Vertical asymptote $x = 2$

   Horizontal asymptote $y = 2$

   x-intercept, $y = 0$

   $0 = \frac{1}{x-2} + 2$

   $-2 = \frac{1}{x-2}$

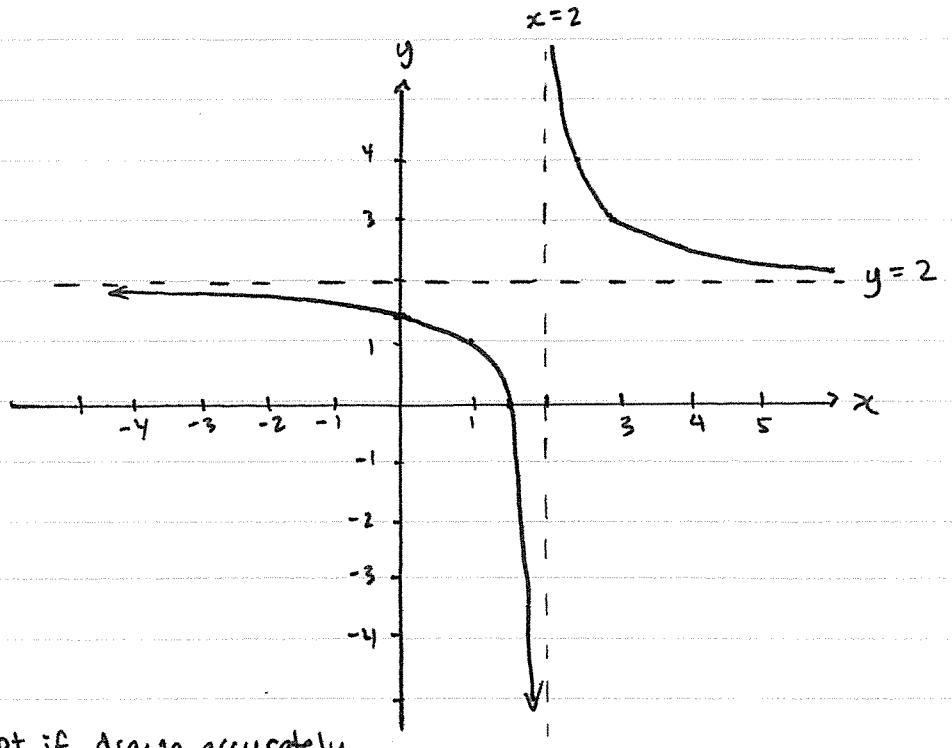
   $-\frac{1}{2} = x - 2$

   $x = \frac{3}{2}$

   y-intercept, $x = 0$

   $y = \frac{1}{-2} + 2$

   $= \frac{3}{2}$



$\frac{1}{2}$ for each asymptote and intercept if drawn accurately + labelled. Branches of hyperbola must be correct

2. Test $x = \pm 1, \pm 2, \pm 4$

   When $x = 4$,

   $4^3 - 5 \times 4^2 + 3 \times 4 + 4 = 0$

   $\therefore x - 4$ is a factor

① If $x = 4$ found

$\frac{1}{2}$ If $x^2 - x - 1$ factor found

② complete solution

$$
\begin{array}{r}
x^2 - x - 1 \\
x-4 \overline{)\ x^3 - 5x^2 + 3x + 4} \\
x^3 - 4x^2 \\
\hline
-x^2 + 3x \\
-x^2 + 4x \\
\hline
-x + 4 \\
-x + 4 \\
\hline
0
\end{array}
$$

Solving $x^2 - x - 1 = 0$

$$x = \frac{1 \pm \sqrt{1^2 + 4 \times 1}}{2}$$

$$\therefore \quad x = 4, \ \frac{1+\sqrt{5}}{2}, \ \frac{1-\sqrt{5}}{2}$$

3. Solving $f(x) = 3^{x+2} = 9 = 3^2$, $x = 0$.

Since $f(0) = 9$, $f^{-1}(9) = 0$

① For substantial correct working.

No marks just for $x = 3^{y+2}$

Alternate Solution

For the inverse: $x = 3^{y+2}$

when $x = 9$, $9 = 3^{y+2}$

$3^2 = 3^{y+2}$

$y = f^{-1}(9) = 0$

4. Since the roots of x^2+x are $x = -1, 0$, and $P(x)$ can be written

$P(x) = (x^2+x)\,Q(x) + kx + 3$

$P(-1) = -k+3$

Also $P(-1) = (-1)^4 - 8\times(-1)^3 - 7\times(-1)^2 + 3$

$= 5$

$-k+3 = 5$

$k = -2$

Alternate Solution

$\frac{1}{2}$ If division set up but error made in first step

① If error made later

$1\frac{1}{2}$ If error make in last step

$$
\begin{array}{r|l}
 & x^2 - 9x + 2 \\
\hline
x^2+x & x^4 - 8x^3 - 7x^2 + 0.x + 3 \\
 & \underline{x^4 + x^3} \\
 & -9x^3 - 7x^2 \\
 & \underline{-9x^3 - 9x^2} \\
 & 2x^2 + 0.x \\
 & \underline{2x^2 + 2x} \\
 & -2x + 3
\end{array}
$$

since $-2x+3 = kx+3$,

$k = -2$

5. a) $y = 6x - x^2$

$= -x^2 + 6x$

at axis of symmetry, $x = \frac{-b}{2a}$

$= \frac{-6}{-2} = 3$

$y = 6 \times 3 - 3^2 = 9$

$\therefore$ A = (3, 9)

b) $y = -x(x-6)$

$x = 6$ at second x-intercept

B = (6, 0)

c) When $x = 3$, $y = -\frac{6}{3}$

$= -2$

C = (3, -2)

d) OB = 6 units

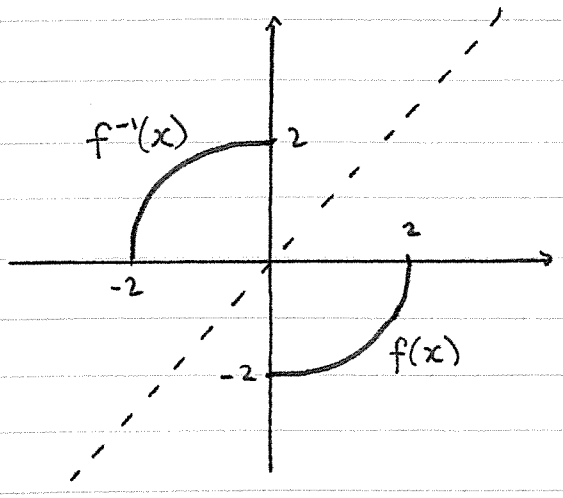
AC = 11 units

Area $= \frac{1}{2} xy$

$= \frac{1}{2} \times 6 \times 11$

$= 33$ units^2 $\left(\frac{1}{2}\right)$ for $6 \times 11 = 66$

6.



(D)

7. The maximum number of flowers you can have without 4 of a kind is:
3 daffodils + 3 tulips + 3 roses + 3 lilies + 3 petunias = 15 flowers.
Therefore a bunch with 16 flowers must contain at least 4 of any one type.
① For 16. ① For clear, correct explanation of why it is 16.

General Comments for Section E

1. Well done. Some issues with accuracy, and many students not clearly labelling the x-intercept and y-intercept as $\frac{3}{2}$ or similar.

2. Students found this challenging. Some struggled with the division, or correctly factorised but didn't solve for the equation.

3. Many struggled with logarithmic expressions, or making index law mistakes such as $3^{x+2} \rightarrow 3^x + 9$.

4. Two methods given. Most students attempted division. Many made arithmetic errors, or stopped at $2x^2+3$ rather than dividing further.

5. Done very well. Students had difficulty applying the area of a kite formula.

6. Done very poorly. A graph would have helped many students.

7. Done very poorly. The majority of the grade misunderstood the question or what was asked of them.

Section F **Multiple Choice** **SOLUTIONS**

QUICK ANSWERS

1 D

2 B

3 A

4 D

5 C

6 B

7 C

8 D

9 B

10 D

11 A

12 C

13 C

14 A

15 A

SOLUTIONS

1 Which one of the following is equivalent to 3.507×10^{-3}?

A. 3507×1000 B. 35.07×100

C. $3507 \div 1000$ D. $0.3507 \div 100$

$3.507 \times 10^{-3} = 0.003507 = 0.3507 \div 100$

2 Anne Vesta invests \$48 000 for 3 years at 9.6 % p.a compounded monthly.
Which of the following is a correct expression of her investment?

A. $\$48\,000 \times 1.096^{3}$ B. $\$48\,000 \times 1.008^{36}$

C. $\$48\,000 \times 1.8^{36}$ D. $\$48\,000 \times 1.0008^{36}$

3 years = 3 × 12 = 36 months; 9.6% p.a. = 0.096 ÷ 12 = 0.008 per month

3 A bird cage contains 4 canaries, 2 of these are yellow and the other 2 are orange.
If two canaries escaped, find the probability that both are yellow.

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

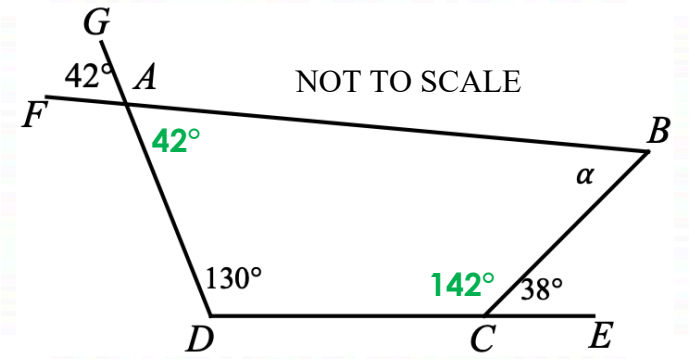
$P(YY) = \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

4 Two years ago, Sam Tsung bought a phone. Its price depreciates at the rate of 60% p.a.
If the value of the phone now is \$384, how much did Sam pay for the phone initially?

A. \$983 B. \$1067 C. \$2000 D. \$2400

$384 = S(1-0.6)^2 \Rightarrow S = \frac{384}{0.4^2}$

5 In the diagram $ABCD$ is a quadrilateral.
$\angle BCE = 38°$, $\angle ADC = 130°$ and $\angle FAG = 42°$.



What is the size of $\angle ABC$?

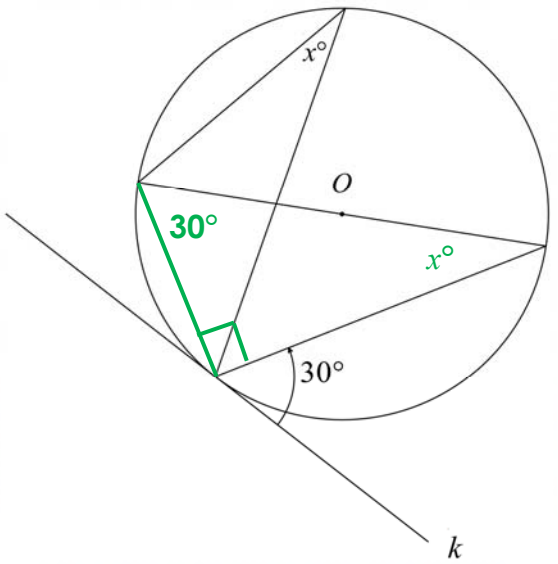
A. 38° B. 42° C. 46° D. 50°

$\alpha = 360 - (42 + 130 + 142)$

6 What is the value of $\log_2 3$ correct to 4 significant figures?

A. 1.584 B. 1.585 C. 0.6309 D. 0.6310

7 O is the centre of a circle. The line k is tangent to the circle.
What is the value of x?



A. 30° B. 45° C. 60° D. 90°

8 Solve $\cos x = -\frac{\sqrt{3}}{2}$ for $-180° \le x \le 180°$

A. $x = -60°$ or $60°$ B. $x = -120°$ or $120°$

C. $x = -30°$ or $30°$ D. $x = -150°$ or $150°$

$\cos x < 0 \Rightarrow$ only second and third quadrant answers also $\cos 30° = \frac{\sqrt{3}}{2}$

9 How many solutions does the equation $6x\left(\frac{1}{x}+1\right)=0$ have?

A. 0 B. 1 C. 2 D. 3

$x = 0$ is not a solution. Only $x = -1$

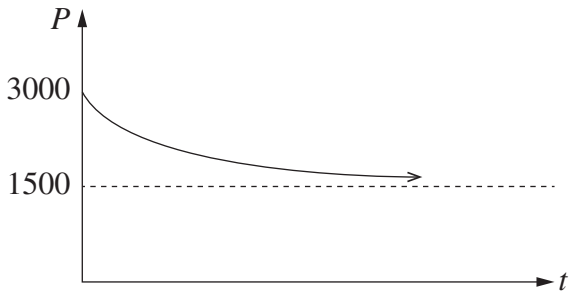
10 What is the natural domain of the function $h(x) = \frac{1}{\sqrt{x^2-4}}$

A. $-2 \le x \le 2$ B. $-2 < x < 2$

C. $x \le -2$ or $x \ge 2$ D. $x < -2$ or $x > 2$

Domain: $x^2 - 4 > 0$

11 The diagram below shows the number of penguins, $P(t)$, on an island at time t.



Which equation best represents this graph?

A. $P(t) = 1500 + 1500 \times 2.7^{-kt}$ B. $P(t) = 3000 - 1500 \times 2.7^{-kt}$

C. $P(t) = 3000 + 1500 \times 2.7^{-kt}$ D. $P(t) = 4500 - 1500 \times 2.7^{-kt}$

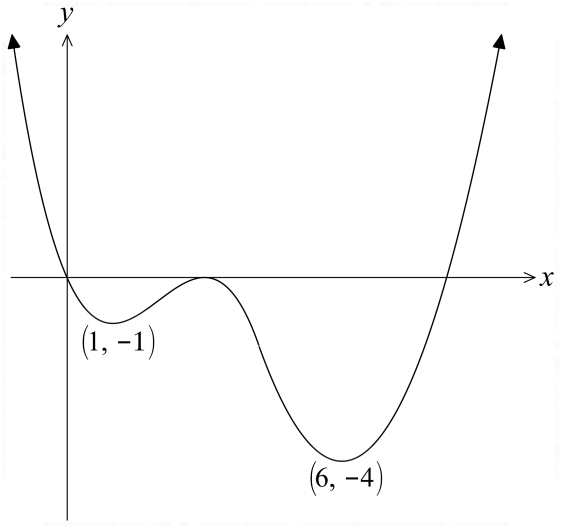
The graph is that of $y = B + Ae^{-kx}$ and the 'y-intercept' is 3000.

12 What is the value of $\log_{10} 5 + \log_{10} 50$, correct to two decimal places?

A. 1.74 B. 2.39 C. 2.40 D. 5.52

$\log_{10} 5 + \log_{10} 50 = \log_{10}(5 \times 50) \doteqdot 2.397940009...$

13 The graph below shows the function $y = f(x)$.



If $f(x) + k > 0$ for all x, then which of the following is correct?

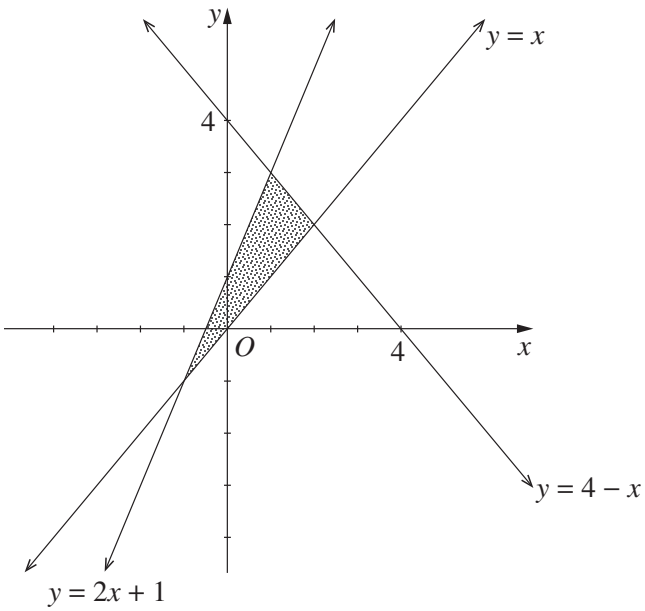
A. $1 < k < 6$ B. $-1 < k < -4$

C. $k > 4$ D. $k < 6$

The minimum value of $f(x)$ is -4.

$f(x) \geq -4 \Rightarrow f(x) + 4 \geq 0$

14 The region enclosed by $y = 4 - x$, $y = x$ and $y = 2x + 1$ is shaded in the diagram below.



Which of the following defines the shaded region?

A. $y \leq 2x + 1, \quad y \leq 4 - x, \quad y \geq x$

B. $y \geq 2x + 1, \quad y \leq 4 - x, \quad y \geq x$

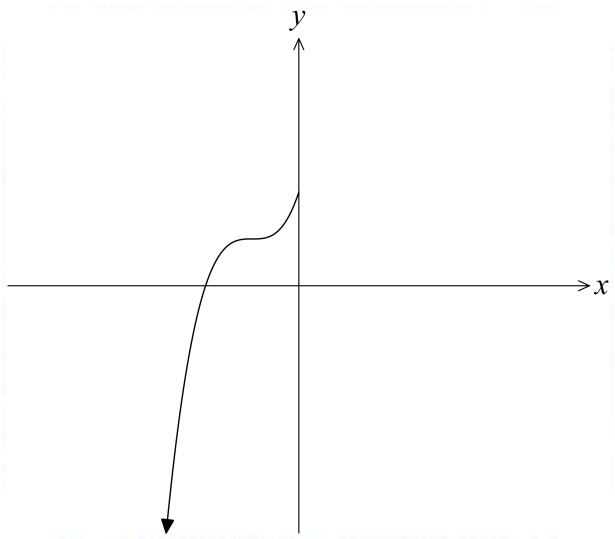
C. $y \leq 2x + 1, \quad y \geq 4 - x, \quad y \geq x$

D. $y \geq 2x + 1, \quad y \geq 4 - x, \quad y \geq x$

Above $y = x$, below $y = 2x + 1$ and $y = 4 - x$
$\geq$ $\leq$ $\leq$

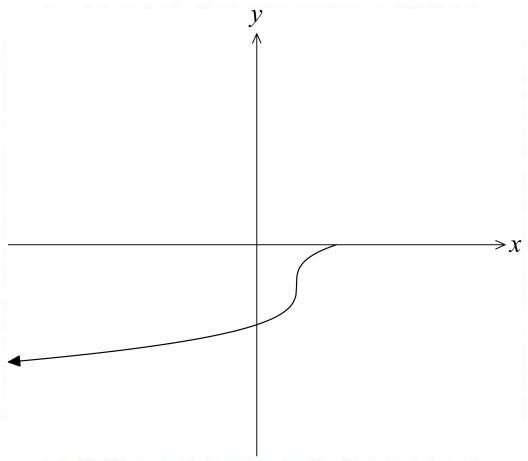
Check with (1, 2) if worried.

15

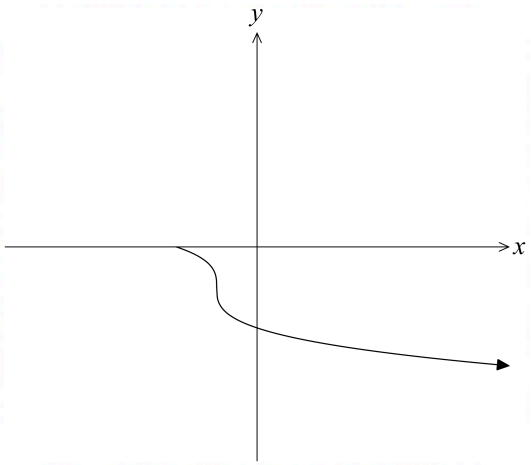


The graph above is that of $y = f(x)$. Which diagram below shows the graph of $y = f^{-1}(x)$?

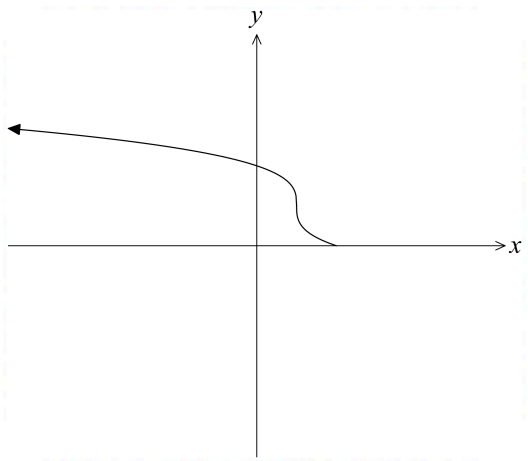
A.



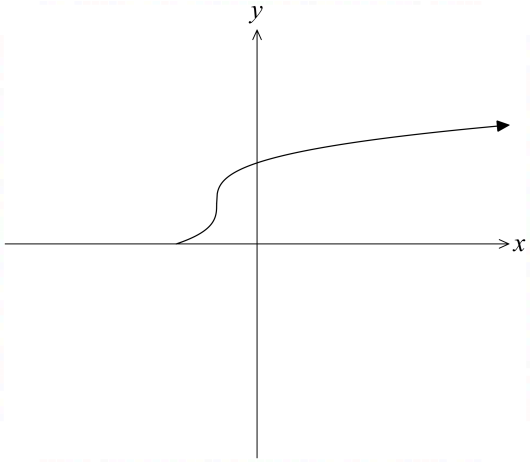
B.



C.



D.



Reflect in the line $y = x$. Note $(a, 0) \rightarrow (0, a)$ and $(0, b) \rightarrow (b, 0)$

1. Atlas sat for three Geography tests during this term. The table below shows his results and the students' average mark and standard deviation for the tests.

	Atlas' mark	Average mark	Standard deviation
Test 1	82	73	6
Test 2	87	75	10
Test 3	x	72	8

(a) In which test, Test 1 or Test 2 did Atlas perform better compared to the rest of the students? Justify your answer. **1**

To compare, one needs to find the z-scores for each result.

Test 1 z-score $= \dfrac{82-73}{6} \rightarrow \dfrac{3}{2}$

Test 2 z-score $= \dfrac{87-75}{10} \rightarrow \dfrac{6}{5}$

Note: half a mark awarded for getting the correct z-score for both tests

Answer and justification: The results are better for test 1 since its z-score is higher than the result for test 2.

Note: half a mark awarded for the correct answer **and** justification

Feedback: many students simply wrote (as justification) either: 'Atlas did better in Test 1' or 'the standard deviation was higher' – neither of these were good enough to get the half mark for the justification.

(b) When compared to the rest of the students, Atlas' performance in Test 3 is equivalent to his performance in Test 1. What is his mark for Test 3? **1**

Equate the z-score for test 3 to the same for test 1:

$$\frac{3}{2} = \frac{x-\mu}{\sigma}$$
$$= \frac{x-72}{8}$$

Solving for x: $x = 8 \times \dfrac{3}{2} + 72 \rightarrow 84$ (1 mark awarded for the correct answer)

Feedback: This was generally done very well

2. What are the exact solutions to $(x-2)^6 - 11\times(x-2)^3 + 10 = 0$. **2**

Many students unsuccessfully tried to expand this out to find the solution. This would be extremely difficult to solve at high school level. The polynomial only has 2 real solutions and only one of these is rational. The simple solution is as follows:

Let $u = (x-2)^3$, then $u^2 = (x-2)^6$ and the equation becomes:

$u^2 - 11u + 10 = 0$ and the factorisation of this is: $(u-10)(u-1) = 0$

which means $u = 10$ or 1

But $u = (x-2)^3$ so $(x-2)^3 = 10$ or 1

We must solve **both**: $(x-2)^3 = 10 \rightarrow x - 2 = 10^{\frac{1}{3}}$, thus $x = 2 + 10^{\frac{1}{3}}$

Now solve $(x-2)^3 = 1 \rightarrow x - 2 = 1$, thus $x = 3$

All the solutions are $x = 3$ and $2 + 10^{\frac{1}{3}}$

Marks awarded as per bullet points below:

- Half a mark for the writing a new equation correctly with the correct substitution.
- Half a mark for factorising correctly
- Half a mark each for solving the solutions for x

Feedback: sadly the only way to find both solutions is to do the method above. The solution x =3 can be found by inspection and anyone who only found that solution with no working was awarded half a mark only (as per the marking guide). There was a reasonably even spread of non-attempts, students trying to expand then solve and students solving it the correct way. A handful gave the obvious solution x = 3 only. No marks could be given for the expanding method, as it was only making the problem harder, not leading to a solution.

3. The pH scale is used to measure how acidic or basic a substance is using the formula $\text{pH} = -\log_{10}\left[\text{H}^+\right]$, where H^+ is the Hydrogen ion concentration in moles per litre.

(a) A brand of orange juice has a Hydrogen ion concentration of 10^{-3}. **1**
Find the pH of this orange juice.

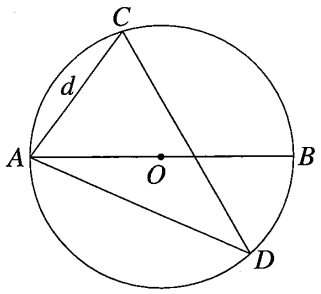
$$pH = -\log_{10}\left[10^{-3}\right]$$
$$= --3\log_{10}[10] \text{ (1 mark for the correct answer)}$$
$$= 3 \times 1 \rightarrow 3$$

(b) A solution of soapy water is known to have a pH of 12. **1**
Find the Hydrogen ion concentration of this soapy water.

$$12 = -\log_{10}(x)$$
$$-12 = \log_{10}(x)$$
$$10^{-12} = 10^{\log_{10}(x)} \quad \text{(1 mark for the correct answer)}$$
$$x = 10^{-12}$$

Feedback for 3: This was generally done well; however, some students left off the negative sing for part b).

4. The circle centred at O with radius r has a diameter AB. Points C and D are chosen on the circle as shown in the diagram. The chord AC has length d. **3**



Show that $d = 2r\sin D$.

Draw a line from C to B to form a new triangle: ΔABC

$\angle ACB = 90°$ (The angle in a semi-circle is a right angle)

$$\therefore \sin B = \frac{d}{2r}$$

Making d the subject: $d = 2r\sin B$

$\angle ABC = \angle ADC$ (Angles on the circumference that subtend equal chords in the same segment are equal)

Then $\sin B = \sin D$ and we can rewrite the expression for d:

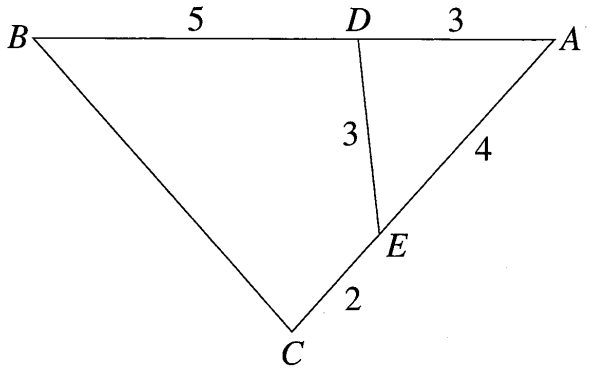
$d = 2r\sin D$.

Feedback for 4: Many students tried using the angle from the centre and this rarely got resolved unless they used mathematics beyond this course. In almost every case it was not leading to a solution and therefore the work done did not receive any marks.

Of those that used the method above, a common problem was that they assumed $\angle ACB$ was a right-angle without stating the reasons why.

A show question must not assume anything and you must give reasons as to why you did this. Equating angles and opposite lines in 2 different triangles cannot be done without a very good reason – but most students seemed to do this with impunity. Actually, there is a consequence, marks were deducted.

5. A triangle has vertices A, B and C. The point D lies on the interval AB such that $AD = 3$ and $DB = 5$.
The point E lies on the interval AC such that $AE = 4$, $DE = 3$ and $EC = 2$.



NOT TO SCALE

(a) State why $\Delta ABC \,|||\, \Delta AED$. **1**

$\angle DAE = \angle CAB$ (common) – half a mark for this statement

$\frac{AE}{AB} = \frac{1}{2} \& \frac{AD}{AC} = \frac{1}{2}$ – half a mark for this statement

$\therefore \Delta ABC \,|||\, \Delta AED$ (2 pairs of corresponding sides have the same scale factor and their inclusive angles are equal)

Feedback: regardless of the fact that this is a one mark question, students still needed to show this is similar (one mark because it is easy).

A handful of students seemed to think that simply writing a similarity statement is enough. Many students wrote equi-proportional or equi-angular which can only be known if you first prove similarity.

(b) Hence, or otherwise, state why $BCED$ is a cyclic quadrilateral **1**

Method 1: $\angle ADE = \angle ACB$ (corresponding angle in similar triangles) – half a mark for this statement.

$\therefore$ $BCED$ is a cyclic quadrilateral (The exterior angle at a vertex of a cyclic quadrilateral equals the opposite interior angle) – half a mark for this statement.

Feedback: generally well done, although as in part a) some just wrote a general definition of cyclic quad which was not enough – you had to show *BCED* was cyclic.

Method 2: $\angle BDE$ is supplementary to $\angle ADE$ (angles on a line add up to $180°$)

$\angle BCE = \angle ADE$ (corresponding angle in similar triangles)

$\therefore \angle BCE$ is supplementary to $\angle BDE$

Thus $BCED$ is a cyclic quadrilateral (If the opposite angles in a quadrilateral are supplementary then the quadrilateral is cyclic)

(c) Show that $CD = \sqrt{21}$. **2**

Method 1: Draw a perpendicular line from D to AE and call the point it meets AE X. Since ΔADE is Isosceles, AX has length 2 (i.e. half of length AE).

Using Pythagoras:

$$\begin{aligned} ED^2 &= DX^2 + EX^2 \\ 3^2 &= DX^2 + 2^2 \\ DX^2 &= 9 - 4 \Rightarrow 5 \\ DX &= \sqrt{5} \end{aligned}$$ (1 mark for this result)

The triangle formed by the vertices C, D and X is also a right-angle triangle, and we can find CD using Pythagoras on this triangle.

$$\begin{aligned} CD^2 &= CX^2 + DX^2 \\ CD^2 &= (2+2)^2 + \sqrt{5}^2 \\ CD^2 &= 16 + 5 \Rightarrow 21 \\ CD &= \sqrt{21} \end{aligned}$$ (1 mark for this result)

Feedback: being one of the last questions, there were a lot of non-attempts.

There were too many students who tried to use the length *CD* to show the result. This is not a proof and no marks were awarded.

All these students achieved was the magnitude of $\angle CED$ given *CD* is the square root of 21. This did not show *CD* must be $\sqrt{21}$.

(c) (Continued)

Method 2:

We can draw a line from C to D. We'll obtain 2 triangles with a common side. Use the fact that $BCED$ is a cyclic quadrilateral. Let the opposite angle to the common side of $\Delta BCD = \theta$. Then the angle opposite the common side $\Delta ECD = 180° - \theta$.

Recall: $\cos(180° - \theta) = -\cos\theta$ *

From part a) $BC = 6$ (Isosceles, where AB is the base of the triangle: BC must equal AC)

Let $x = CD$

Then for ΔBCD:

$$x^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos\theta$$
$$\cos\theta = \frac{5^2 + 6^2 - x^2}{60}$$

And for ΔECD:

$$x^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos(180° - \theta)$$
$$-\cos(180° - \theta) = \frac{x^2 - (2^2 + 3^2)}{12}$$

where $-\cos(180° - \theta) = \cos\theta$ *

Equate these two equations:

$$\frac{5^2 + 6^2 - x^2}{60} = \frac{x^2 - (2^2 + 3^2)}{12}$$
$$5^2 + 6^2 - x^2 = 5\left(x^2 - (2^2 + 3^2)\right)$$
$$61 - x^2 = 5x^2 - 65$$
$$6x^2 = 126$$
$$x^2 = 21$$

Since x is a length, $x = \sqrt{21}$

(d) Using the result of question 4, or otherwise, find the exact value of the radius of The circle passing through the points B, C, E and D. **2**

Method 1: Recall that $d = 2r\sin D$

Let $d = \sqrt{21}$, and D be E such that it is $\angle DEC$ and we get: $\sqrt{21} = 2r\sin E$.

Sine is positive in the first and second quadrants and the obtuse angle $\angle BCE$ is equivalent to the acute angle $\angle AED$.

From part c) and a bit of Pythagoras' Theorem we get that $\sin(\angle AED) = \dfrac{\sqrt{5}}{3}$

Note: 1 mark for this result

Then:

$$\sqrt{21} = 2r\frac{\sqrt{5}}{3}$$

$$r = \frac{3\sqrt{105}}{10}$$

Note: 1 mark for successfully finding an exact value for *r,* however only half a mark awarded here if the expression for r is correct but still uses some sort of trigonometric function.

Half marks may be awarded instead of full marks if mistakes in arithmetic were made or similar

An alternative way to find the Sine ratio is to find an expression for $\cos\theta$ using the Cosine rule.

$$\cos\theta = \frac{5^2 + 6^2 - \sqrt{21}^2}{2\times 5\times 6}$$

$$= \frac{25 + 36 - 21}{60}$$

$$= \frac{2}{3}$$

Using Pythagoras (or otherwise), $\sin\theta = \dfrac{\sqrt{5}}{3}$, i.e. $\sin^2\theta + \cos^2\theta = 1$

Feedback: of those that attempted this question, many could not resolve the trigonometric ratios into an exact value. They gave the answer with an inverse trigonometric ratio. This is not in exact form. As shown above, these are able to be worked out, using methods known to students (like Pythagoras' Theorem). If students everything correctly but failed to put the solution in exact form – they only lost an half mark in total.

(d) **(continued)** Method 2: Use the Cosine Rule on a triangle from the centre of the circle with sides of length r (radius) and base CD. The angle from the centre is twice that of $\angle ABC$.

$\angle ABC = \angle AED$ (corresponding angles in similar triangles)

$$\cos(\angle AED) = \frac{2}{3}$$

$$\angle AED = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\Rightarrow \cos(COD) = \cos(2 \times \angle AED)$$

$$= \cos\left(2\cos^{-1}\left(\frac{2}{3}\right)\right)$$

Putting this in your calculator gives the result as $-0.\dot{1}$ which is $-\frac{1}{9}$ as a fraction.

Thus the Cosine of the angle from the centre O, of the circle that subtends the line CD is $-\frac{1}{9}$.

Then $\sqrt{21}^2 = r^2 + r^2 - 2 \times r \times r \times \cos(\angle COD)$

$$21 = 2r^2 - 2r^2\left(-\frac{1}{9}\right)$$

$$21 = 2r^2\left(1 + \frac{1}{9}\right)$$

$$r^2 = \frac{21 \times 9}{2 \times 10} \Rightarrow \frac{189}{20}$$

$$r = \frac{3\sqrt{105}}{10}$$