Name:_____

Teacher:_____

The Scots College

Year 10 Mathematics 5.3 – Yearly Examination

Friday 10th November 2017

Weighting: 40%

Time allowed: 120 minutes + 5 minutes reading time.

General Instructions

- Show all necessary working.
- Marks may be awarded for working out
- Board approved calculators may be used.
- Begin a new labelled booklet for each question.
- Write your name on **EACH** new booklet.

Question	Possible Marks	Mark Achieved
PART A: MULTIPLE CHOICE	20	
PART B: EXTENDED RESPONSE		
1	23	
2	18	
3	26	
4	23	
TOTAL	110	
PERCENTAGE	**100%**	

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PART A: MULTIPLE CHOICE (20 MARKS)

Answer A. B. C. or D using the multiple-choice answer sheet provided.

There is only one correct answer per question

1) What is the value of x in $5^{x-2} = 625$

a) 5 b) 6 c) -4 d) -6

2) What is the result when you expand $(2\sqrt{5}+1)(2\sqrt{5}-1)$?

a) 9 b) 11 c) 19 d) 21

3) $A(x) = (x^2 - 4)(x^3 + 2x + 5)^4$. What is the degree of $A(x)$?

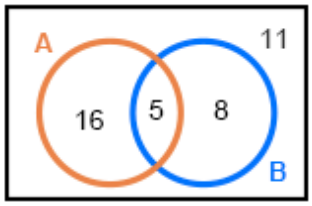
a) 12 b) 5 c) 14 d) 9

4) Rearrange the formula $A = \frac{h(a+b)}{2}$ to make b the subject.

a) $b = \frac{2A}{h} - a$ b) $b = \frac{A}{2h} - a$

c) $b = \frac{2A-a}{2}$ d) $b = \frac{h(A-a)}{2}$

5) There are two groups of students, labelled A and B. If a student is selected at random, what is the probability that the student is in both Group A and B?

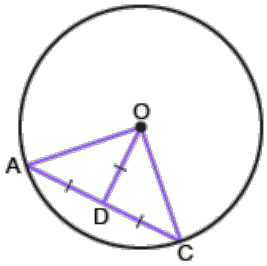


a) $\frac{11}{40}$ b) $\frac{3}{5}$ c) 0.725 d) 0.125

6) The expansion of $(4 - x^2)^2$ is:

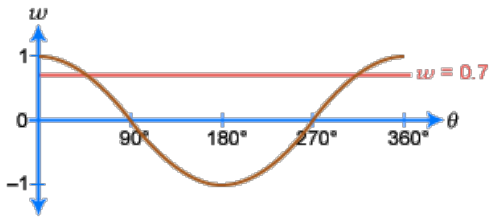
a) $(4^2 - x^2)^4$
b) $(4^4 - x^2)(x^2 + 4^4)$
c) $16 - 8x^2 + x^4$
d) $16 - 8x^2 - x^4$

7) Determine the size of angle AOC.



a) ∠AOC = 90°
b) ∠AOC = 45°
c) ∠AOC = 60°
d) ∠AOC= 135°

8) The graph shows the two equations $w = \cos\theta$, $w = 0.7$: For $0° \leq \theta \leq 360°$. Find the quadrants where the two solutions intercept.



a) 1st and 2nd
b) 2nd and 4th
c) 1st and 4th
d) 3rd and 4th

9) The equation $x^2 + y^2 = 125$ will graph to form a;

a) Parabola
b) Circle
c) Linear
d) Quadratic

10) Using the discriminant find how many solutions exist?

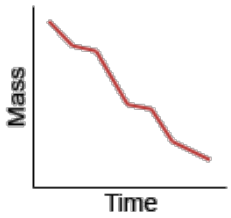
$$y = x^2 + 10x + 182$$

a) 0 b) 1 c) 2 d) 3

11) One of the solutions for the equation $(x - 5)(x + 4)(3x - 1) = 0$ is:

a) -5 b) 4 c) $\frac{1}{3}$ d) 1

12) This graph shows.



a) a downward trend
b) an upward trend
c) no trend
d) horizontal trend

13) Which expression is the factored form of $k^2 - 8k + 7$?

a) $(k-1)(k-7)$
b) $(k+1)(k-7)$
c) $(k-1)(k+7)$
d) $(k+1)(k+7)$

14) Which is the gradient intercept form of the equation $3y + 9x - 12 = 0$

a) $y = -9x + 12$
b) $y = 3x - 12$
c) $y = 9x - 12$
d) $y = -3x + 4$

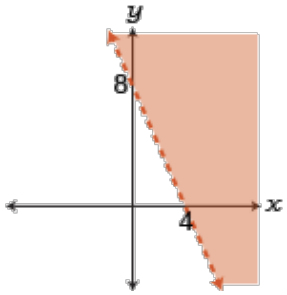
15) For the curve $y = \log_9 x$.
What are the coordinates of the x-intercept?

a) 0 , 1) b) 1 , 0 c) -1 , -1 d) 9 , 0

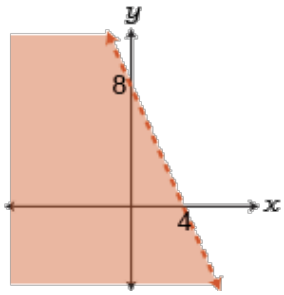
16) Which graph represents the inequality shown below.

$$2x + y - 8 > 0$$

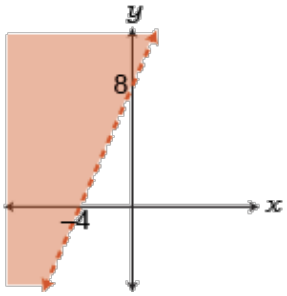
a)



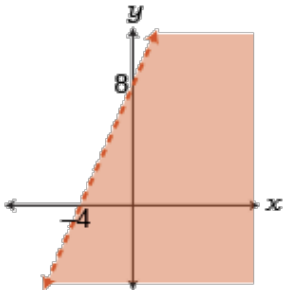
b)



c)



d)



17) A boat ferried passengers from point A to point B on a bearing of 250°. To ferry passengers back to point A directly the bearing would be.

a) 111° b) 072° c) 250° d) 070°

18) Which of the following is true for the data in this stem-and-leaf plot (correct to one decimal place)?

Stem	Leaf
1	1 3
2	2 2 4 8 9
3	2 3 3 4 6 9
4	2 3

a) $mean = 32 \quad \sigma = 9.2 \; Range = 29.4$

b) $mean = 29.4 \; \sigma = 9.2 \; Range = 32$

c) $mean = 29.4 \; \sigma = 3.8 \; Range = 32$

d) $mean = 32 \; \sigma = 29.4 \; Range = 29.4$

19) Simplify fully $\frac{10}{2x-3} + \frac{3}{x+4}$.

a) $\frac{12x+6}{(2x-3)(x+4)}$

b) $\frac{16x+31}{2x^2+5x-12}$

c) $\frac{23x-18}{2x^2+5x-12}$

d) $\frac{122+14x}{(2x-3)(x+4)}$

20) Simplify fully $\frac{10x^9z^{10}y^{-15}}{22x^{-1}y^{-15}z^5}$.

a) $\frac{10x^{10}z^5}{22}$

b) $\frac{5x^8z^5}{11}$

c) $\frac{10x^{10}z^5}{22y^{15}}$

d) $\frac{5x^{10}z^5}{11}$

PART B: EXTENDED RESPOSE (90 MARKS):

Question 1: (23 Marks) Start a new answer booklet. Remember to write your name at the top of each answer booklet.

a) What is the x-coordinate where the two lines intersect? (1)

$$y = 4x + 2$$
$$y = 3x - 1$$

b) The equation for a population, P, at a given time, t days, is given by: (2)

$$\mathrm{P} = 600 \times 2^{\mathrm{t}}$$

How long will it take for the population to reach $4{,}800$?

c) Evaluate $\log_{12} 72 + \log_{12} 2$ (2)

d) Simplify $\left(\frac{4x^2}{9y}\right)^{-\frac{1}{2}}$ without any negative or fractional indices. (2)

e) Evaluate this logarithm (2)

$$\log_x \sqrt[5]{x^2}$$

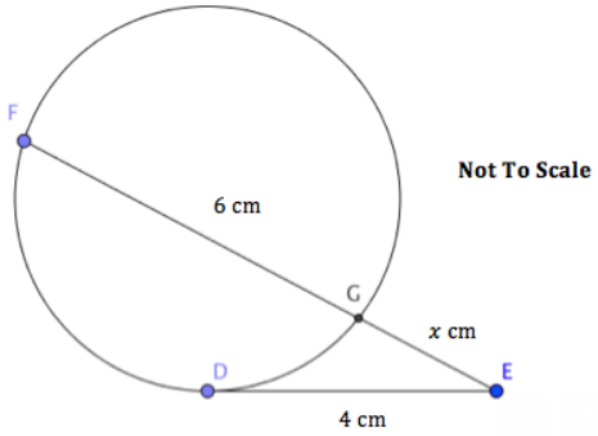
f) Express $\frac{5\sqrt{3}}{3+\sqrt{2}}$ with a rational denominator. (2)

g) Solve for x leaving your answer in simplified surd form. (3)

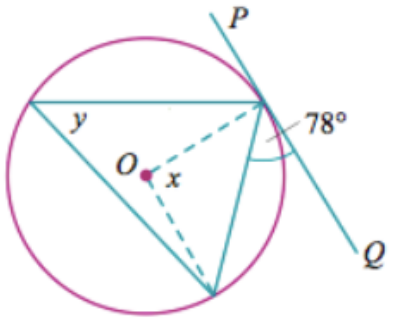
$$x^2 - 10 = 6x.$$

h) By completing the square, rewrite $y = x^2 - 4x + 6$ in the form $y = (x - h)^2 + k$ (2)

i) Find the value of x, justify your answer. (3)

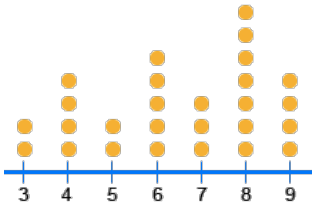


j) Find the values of x and y giving reasons. (4)



Question 2: (18 Marks) Start a new booklet for your answer. Remember to write your name at the top of each answer booklet.

a) A dot plot is shown below.

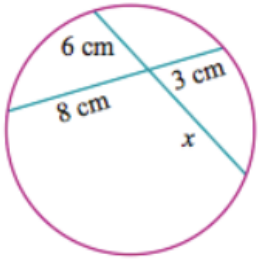


Find:

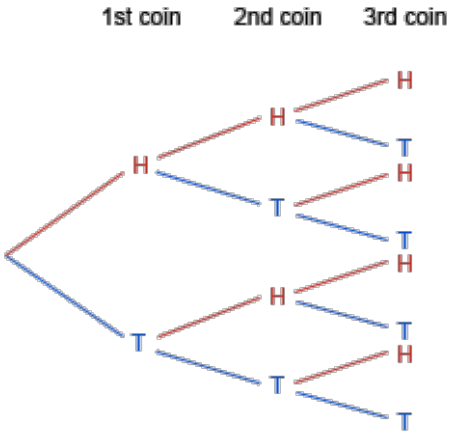
i. Median (1)

ii. Interquartile range (1)

b) Find the value of x in the diagram below (2)



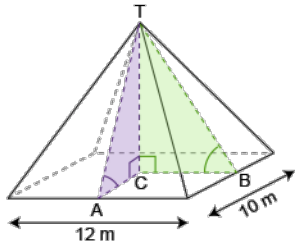
c) The tree diagram shows the outcomes when three coins are tossed.



i. Calculate the probability of getting two heads **THEN** a tail. (1)

ii. Calculate the probability of getting two heads **AND** a tail. (1)

d) The perpendicular height of this rectangular pyramid is 24 m. Find the size of the angle $\angle$TBC to the nearest degree. (2)



e) Consider the curve $y = x^2 - 7x + 10$

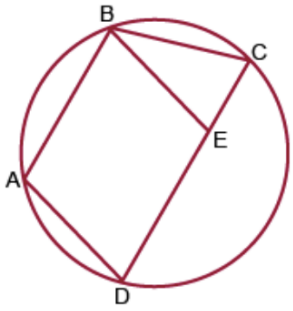
i. Find the x-intercept. (2)

ii. Find the y-intercept (1)

iii. Find the vertex. (2)

iv. Hence sketch the curve labelling all of the above features. (1)

f) $ABED$ is a parallelogram. Prove ΔBCE is isosceles, let angle $BAD = x$ (4)



Question 3: (26 Marks) Start a new answer booklet. Remember to write your name at the top of each answer booklet.

a) For this question refer to the function $f(x)$

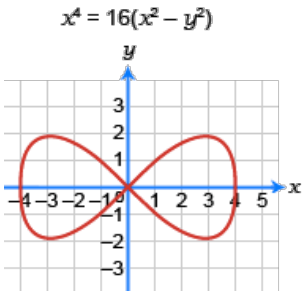
$$f(x) = \frac{1}{x-1} + 3$$

i. Write down the equation of the vertical and horizontal asymptotes. (2)

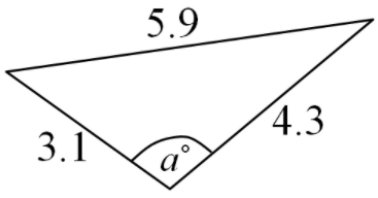
ii. Sketch $f(x)$ showing all important features. (3)

iii. Find an expression for the inverse function $f^{-1}(x)$. (2)

b) Find the domain and range of the graph below. (2)

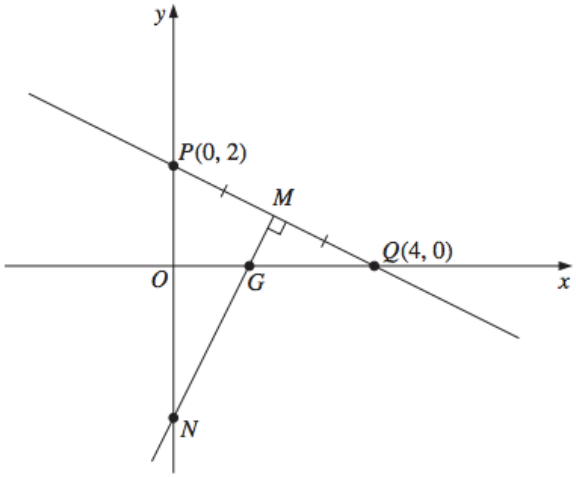


c) Determine the size of angle $a°$ correct to the nearest degree. (2)



d) Find all possible solutions to $sin\,\theta = -0.82$ given that $0° \leq \theta \leq 360°$ to the nearest degree. (3)

e) The diagram shows the points $P(0,2)$ and $Q(4,0)$. The point M is the midpoint of PQ. The line MN is perpendicular to PQ and meets the x axis at G and the y axis at N.



i. Show that the gradient of PQ is $-\frac{1}{2}$ (1)

ii. Find that coordinates of the midpoint M. (2)

iii. Show that the equation of the line MN is $y = 2x - 3$. (2)

iv. Find the distance of NQ, given that $N = (0, -3)$. (1)

v. Write down the equation of the circle with centre N and radius NQ (2)

f) Brad measured the angle of elevation to the top of a tower as 45°. He walked 35 m towards the tower and measured the angle of elevation as 70°.
Calculate the height of the tower to the nearest metre. (4)

Question 4: (23 Marks) Start a new answer booklet. Remember to write your name at the top of each answer booklet.

a) $P(\mathrm{x}) = \frac{x^5 - x^2}{x}$
Evaluate $P(2)$, giving your answer as a fraction in its simplest form. (2)

b) If $P(x) = 5x^3 + 6x$ and $Q(x) = 1 + x - x^3$ simplify $P(x) - Q(x)$ (2)

c) $P(x) = x^3 - x^2 - 22x + 40$

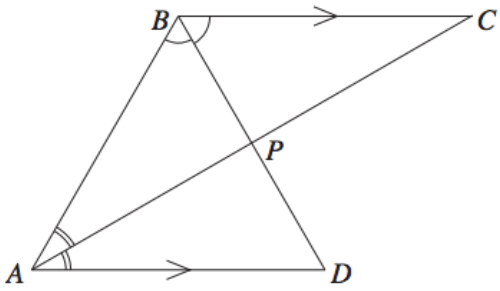
i. If $(x - 2)$ is a factor complete the factorisation using algebraic long division. Give your answer in the form $P(x) = (x - 2)Q(x)$, where $Q(x)$ is the quotient. (3)

ii. Hence solve $P(x) = 0$ (2)

d) Sketch $P(x) = -(x + 2)^2(x - 2)$ showing the x-intercepts and the y-intercept and other important features. (3)

e) $P(x) = x^3 + 10x^2 + 15x + \mathrm{c}$
When $P(x)$ is divided by $(x + 8)$ the remainder is 10.
Using remainder theorem find the value of *c*. (2)

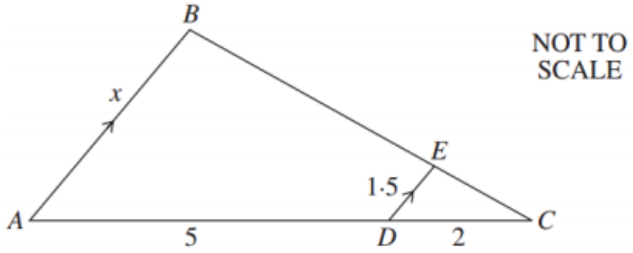
f) In the diagram, AD is parallel to BC, AC bisects $\angle BAD$ and BD bisects $\angle ABC$. The lines AC and BD intersect at P.



i. Prove that $\angle BAC = \angle BCA$ (2)

ii. Prove that $\Delta ABP \equiv \Delta CBP$ (3)

g) In the diagram, AB is parallel to DE, AD is 5cm , DC is 2cm and DE is 1.5cm.



i. Prove that triangle $\Delta ABC ||| \Delta DEC$ are similar. (2)

ii. Find the length AB. (2)

END OF EXAMINATION

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Q1

a) $x_{\text{LORD}} = -3$ (1)

b) $4800 = 600 \times 2^{t}$ (1)

$8 = 2^{t}$

$t = 3$ (1)

c) $\log_{12} 144$ (1)

2 (1)

d) $\left(\frac{9y}{4x^2}\right)^{\frac{1}{2}}$ (1)

$\frac{3\sqrt{y}}{2x}$ (1)

Q1

e) $\log x^{x^{\frac{2}{5}}}$ (1)

$x \frac{2}{5}$ (1)

f) $\frac{5\sqrt{3}}{(3+\sqrt{2})} \times \frac{(3-\sqrt{2})}{(3-\sqrt{2})}$ (1)

$\frac{15\sqrt{3} - 5\sqrt{6}}{7}$. (1)

g) $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times (-10)}}{2}$ (1)

$x = 3 \pm \frac{\sqrt{76}}{2}$ (1)

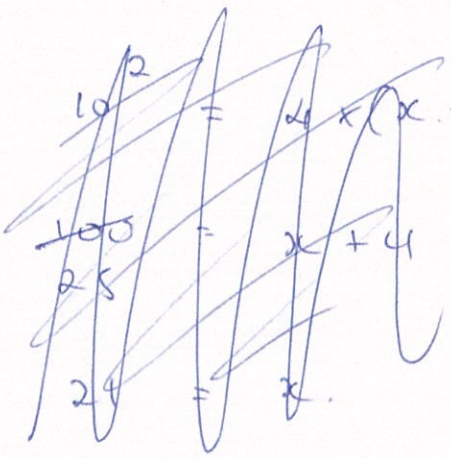
$x = 3 \pm \sqrt{19}$ (1)

Q1.

h). $y = x^2 - 4x + 4 - 4 + 6$ (1)

$y = (x-2)^2 + 2$ (1)

i)



$4^2 = x(x+6)$ (1)

$16 = x^2 + 6x$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = 2$ ~~not 8~~ or -8 (1)

but $x > 0$ (length)

$\therefore x = 2$cm only. (1)

j). $y = 78^\circ$

$\therefore x = 156^\circ$

Q2

a) i) 7

ii) 3.

b) $6x = 8 \times 3$

$x = 4$

c). i) $\frac{1}{8}$

ii) $\frac{3}{8}$

d) $\tan\theta = \frac{24}{6}$.

$\theta = \tan^{-1}\left(\frac{24}{6}\right)$

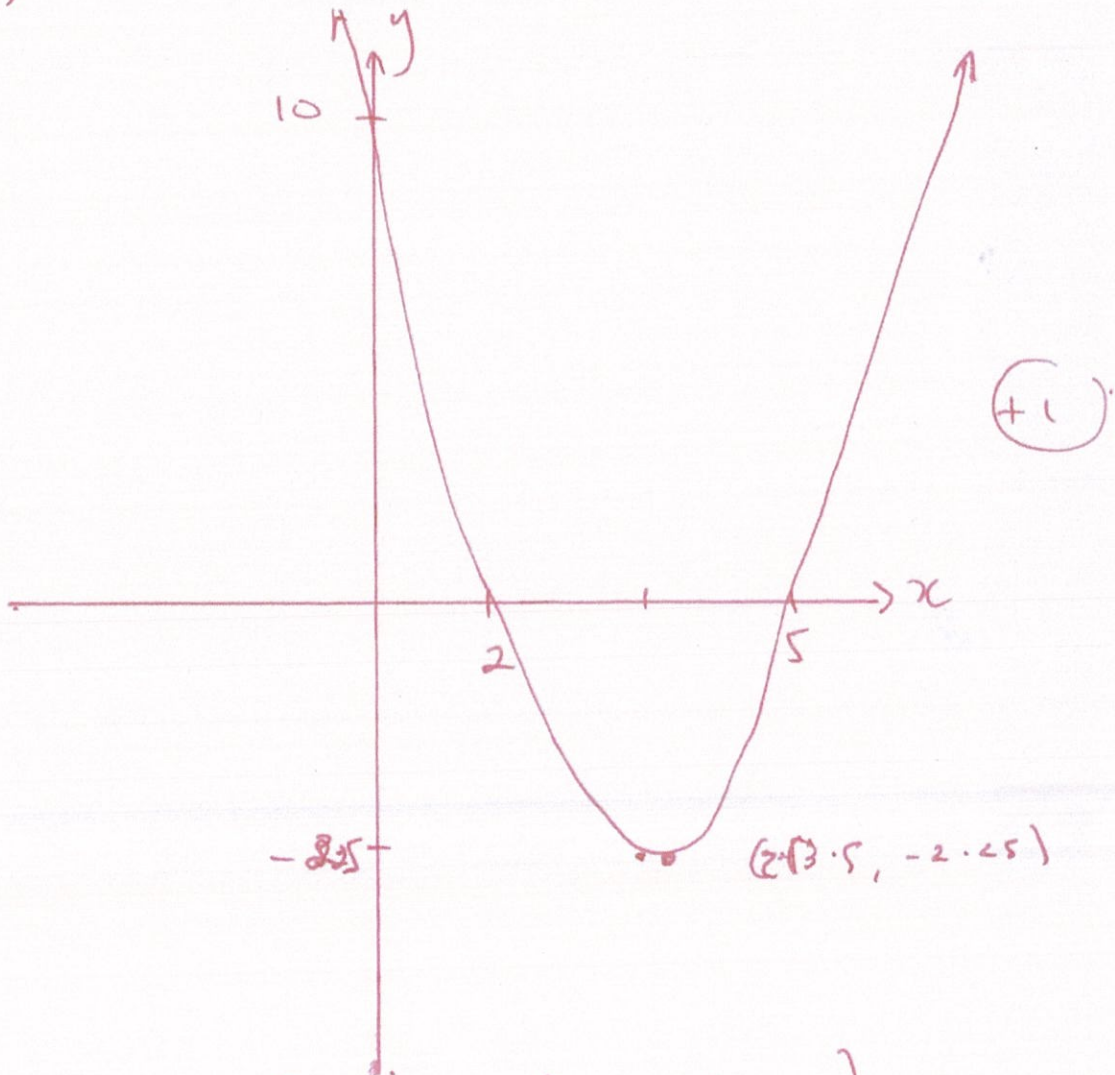
$\theta = 76^\circ$ (nearest degree).

e) i) 2 & 5 (+2)

ii) 10 (+1)

iii) (3·5, −2·25) (+2)

iv)



(+1)

(+c) f). ∠BED = x (opposite ∠'s parallelogram)

(+1) ∠BEC = 180° − x (adjacent supplementary ∠'s)

(+c) ∠BCE = 180 − x (opposite ∠'s in a cyclic quadrilateral)

∴ ∠BCE = ∠BEC

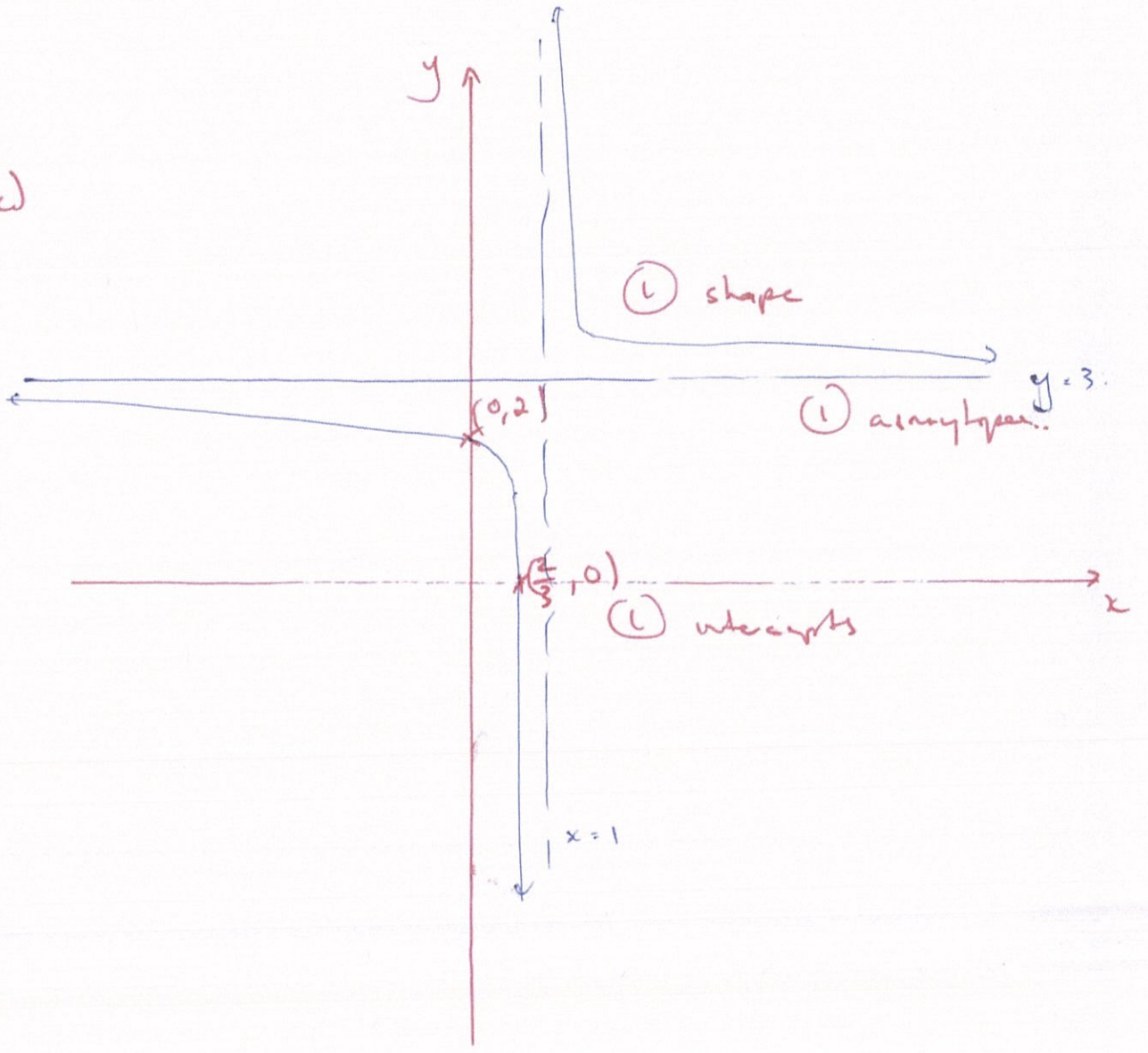
(+1) ∴ ΔBCE is isosceles (angle base of a triangle are equal).

Q3

a). i) $x = 1$ (1)

$y = 3$. (1)

ii)



iii) $x = \frac{1}{y-1} + 3$ (1)

$y - 1 = \frac{1}{x-3}$

$y = \frac{1}{x-3} + 1$

$f'(x) = \frac{1}{x-3} + 1$. (1)

Q3. b) $-4 \leq x \leq 4$ (1)

$-2 \leq y \leq 2$ (1)

c)

$$\cos(a^\circ) = \frac{(3.1)^2 + (4.3)^2 - (5.9)^2}{2 \times (3.1) \times (4.3)}$$ (1)

$a = \cancel{104^\circ}$ 105° (1)

d) acute $= 55^\circ$ (1)

$\theta = \cancel{124^\circ}$ 235° (1)

$\theta = 305^\circ$ (1)

K)

i) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{0-2}{4-0}$ ← (1)

$= -\frac{1}{2}$

ii) $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$= \left(\frac{0+4}{2}, \frac{2+0}{2} \right)$ (1)

$= (2, 1)$

iii) $y = -\frac{1}{2}x + b$

$1 = -\frac{1}{2} \times 2 + b$

$b = 2$

$2 \times y = 2 \times -\frac{1}{2}x \times 2 \times 2$

$2y = x + 4$

$y = \frac{2}{1}x + b$ (1)

$1 = 2 \times 2 + b$

$b = -3$. (1)

$y = 2x - 3$.

Q3

iv) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-4)^2 + (-3-0)^2}$

$= \sqrt{25}$

$= 5$ units (1)

v) $(x-0)^2 + (y+3)^2 = 5^2$ (1)

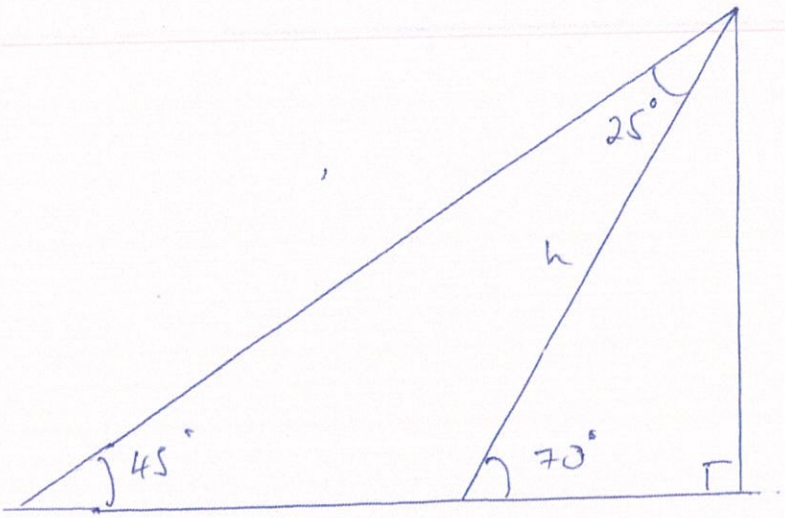
$x^2 + (y+3)^2 = 25$ (1)

(1) centre

(1) radius

Q3.

a).



~~Sin~~ 45

$$\frac{h}{\sin 45} = \frac{35}{\sin 25} \quad (1)$$

$$h = \frac{35 \times \sin 45}{\sin 25} \quad (1).$$

~~Sin θ~~ $\sin 70^\circ = \frac{x}{h}$ (1)

$$x = \frac{35 \times \sin 45}{\sin 25} \times \sin 70.$$

$x = 55m.$ (1)

Q.4

a) $p(2) = \frac{2^5 - 2^2}{2}$ (1)

$p(2) = 14$ (1).

b) $5x^3 + 6x + (-1 + x + x^3)$ (1)

$6x^3 + 7x - 1$. (1).

c)

$$x-2 \;\overline{)\; x^3 - x^2 - 22x + 40}$$

$$x^2 + x - 20$$

$$-x^3 + 2x^2$$

$$0 + x^2$$

$$-x^2 + 2x$$

$$20x$$

$$-20x - 40.$$

0

(1) for correct approach

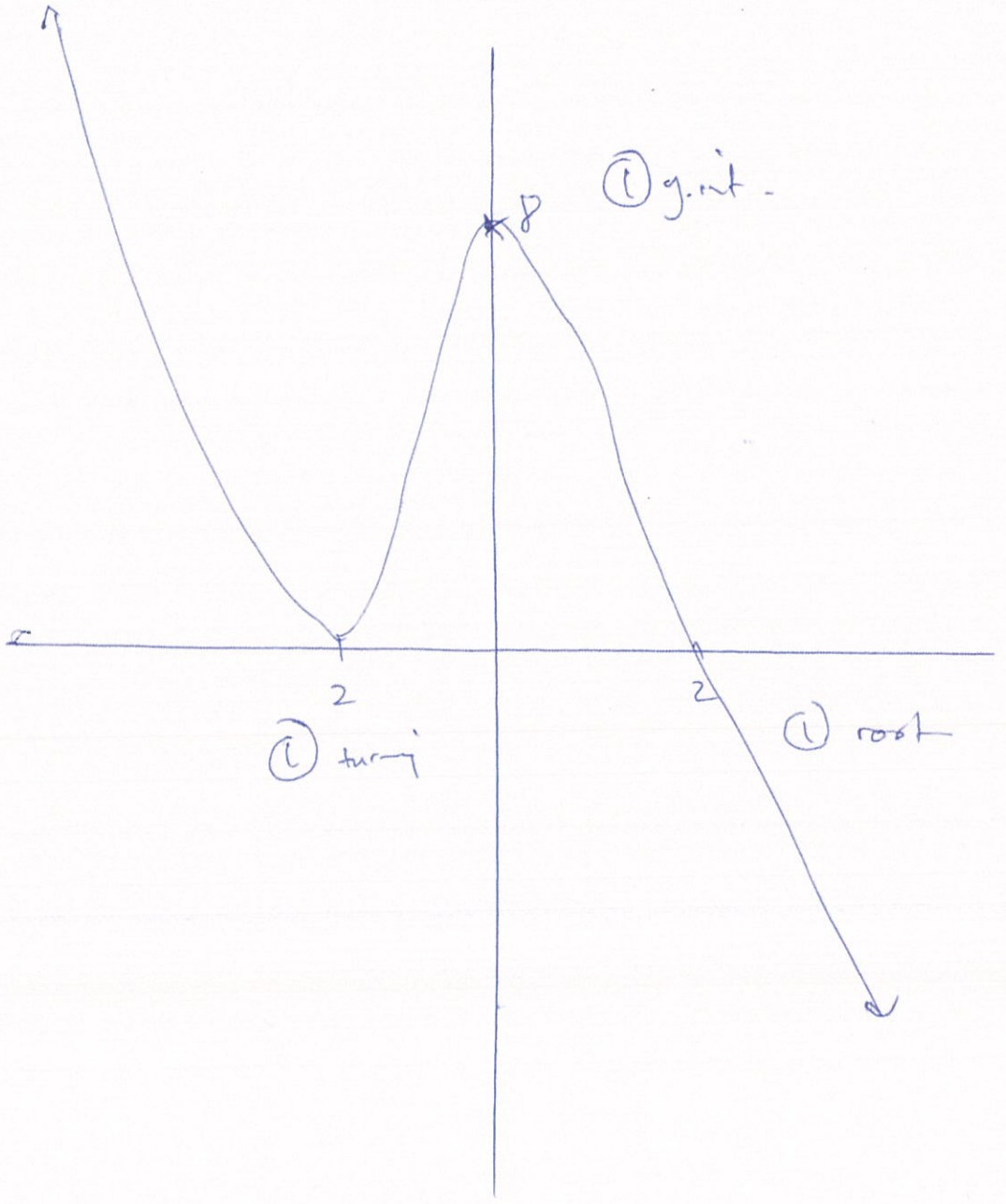
(1) fully correct

$p(x) = (x-2)(x^2 + x - 20)$. (1)

c ii) $(x-2)(x+5)(x-4) = 0$ ① factorised

$x = 2, -5, 4$ ① solving

d)



① y-int.

① turning

① root

Q4

e)

$$10 = (-8)^3 + 10(-8)^2 + 15(-8) + c \quad (1)$$

$$2 = c. \quad (1)$$

f).

i) ~~$\angle BAC = \angle BCA$~~

~~$\angle BA$~~ $\angle BCA = \angle DAC$ (alternate $\angle$'s & $\parallel$ lines) (2)

$\angle BAP = \angle DAP$ (given) (1)

$\therefore \angle BAC = \angle BCA.$

Δ

ii) $\Delta ABP \equiv \Delta CBP$

(1) $\angle BAC = \angle BCA$ (i)

$\angle ABP = \angle CBP$ (given) (1).

BP is common (1)

$\therefore \Delta ABP \equiv \Delta CBP$ (AAS) (1)

4.

g) i) $\angle ECD = \angle BCA$ (common $\angle$'.).

$\angle BAC = \angle EDC$ corresponding $\angle$'s on parallel lines (1).

or

ii) $\therefore \triangle ABC \,|||\, \triangle DEC$ equiangular. (1)

$$\frac{x}{1.5} = \frac{7}{2} \quad (1)$$

$$x = 5.25\text{cm}. \quad (1)$$