Name:_____

Teacher:_____

The Scots College
Year 10 5.3 Mathematics
16 November 2016
Yearly Examination
Weighting: 40%
2 hours plus 5 minutes reading time

Instructions:

- Show all necessary working.
- Approved non-programmable calculators can be used.
- Begin a new labelled booklet for each question.

Question	Maximum Mark	Mark
PART A: MULTIPLE CHOICE	20	
PART B: 1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	120	
PERCENTAGE		

PART A: MULTIPLE CHOICE (20 MARKS)

On the multiple choice answer sheet provided, shade in the most correct answer

1. The expanded form of $2\sqrt{5}(5-3\sqrt{2})$ is:

A. $2\sqrt{25}-6\sqrt{10}$ **B.** $7\sqrt{5}-5\sqrt{10}$ **C.** $10\sqrt{5}-6\sqrt{10}$ **D.** $2\sqrt{25}-5\sqrt{10}$

2. The simplified form of $\frac{(6xy^3)^2}{3x^3y^2\times 4x^4y^0}$ is:

A. $\frac{3y^4}{x^5}$ **B.** $\frac{3y^3}{x^{10}}$ **C.** $\frac{y^4}{2x^6}$ **D.** $\frac{y^6}{2x^6}$

3. $\frac{4}{x-1}+\frac{5}{2x-3}$ simplifies to

A. $\frac{-3x-4}{(x-1)(2x-3)}$ **B.** $\frac{13x-17}{(2x-3)(x-1)}$ **C.** $\frac{13x-4}{(x-1)(2x-3)}$ **D.** $\frac{13x-13}{(2x-3)(x-1)}$

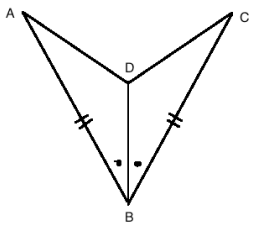
4. If (-1, 2) is a point on the line $ax-4y+11=0$, the value of a is:

A. -19 **B.** 3 **C.** $-\frac{15}{2}$ **D.** -1

5. The line that is perpendicular to the line with equation $y=-3x+7$ is:

A. $y=-3x+2$ **B.** $3y-x=4$ **C.** $3y=4-x$ **D.** $y=3x-3$

6. The test that proves $\Delta ABD \equiv \Delta CBD$ is:

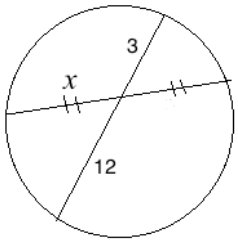


A. RHS **B.** SAS **C.** AAS **D.** SSS

7. A cyclic quadrilateral has one angle measuring $63^0$and another angle measuring 108^0. Another angle in the cyclic quadrilateral is:

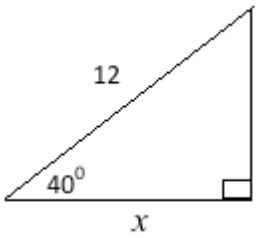
A. 63^0 **B.** 108^0 **C.** 117^0 **D.** 122^0

8. The value of x in the diagram below is:



A. 7.5 **B.** 4 **C.** 3.8 **D.** 6

9. The value of x in the diagram below is equal to:



A. $\frac{12}{\cos 40^0}$ **B.** $12\cos 40^0$ **C.** $\frac{12}{\sin 40^0}$ **D.** $\frac{12}{\tan 40^0}$

10. A yacht is sailed from A to B on a bearing of 196^0 . To sail from B directly back to A, the bearing would be:

A. 016^0 **B.** 286^0 **C.** 074^0 **D.** 164^0

11. The obtuse angle θ such that cos θ = - cos 35^0 is:

A. $\theta = 125^0$ **B.** $\theta = 135^0$ **C.** $\theta = 145^0$ **D.** $\theta = 155^0$

12. The term that needs to be added to make $x^2 - 6x$ a perfect square is:

A. -9 **B.** 9 **C.** 3 **D.** -3

13. If $ax^2 + bx + c$ has exactly two solutions, then:

A. $b^2 - 4ac = 0$ **B.** $b^2 - 4ac \leq 0$ **C.** $b^2 - 4ac < 0$ **D.** $b^2 - 4ac > 0$

14. The coordinates and type of turning point of $y = -(x + 2)^2 + 1$ is:

A. a minimum at (-2, -1) **B.** a maximum at (2, 1) **C.** a minimum at (2, 1) **D.** a maximum at (-2, 1)

15. The circle with equation $(x + 1)^2 + (y - 3)^2 = 16$ has centre coordinates and radius:

A. $(-1, 3), r = 4$ **B.** $(1, -3), r = 16$ **C.** $(1, -3), r = 4$ **D.** $(-1, 3), r = 16$

16. Which of the following is NOT a measure of spread?

A. Range **B.** Mean **C.** Standard Deviation **D.** Interquartile range

17. The degree of the polynomial $5x^4 - x^3 + 7$ is:

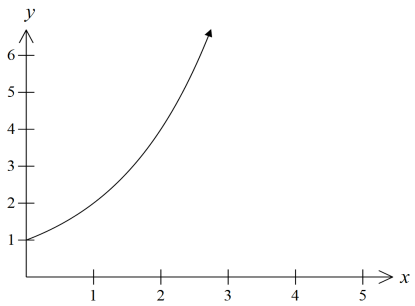
A. 5 **B.** 4 **C.** 7 **D.** 3

18. The remainder when $P(x) = x^3 + 2x^2 - 8x + 3$ is divided by $(x - 1)$ is:

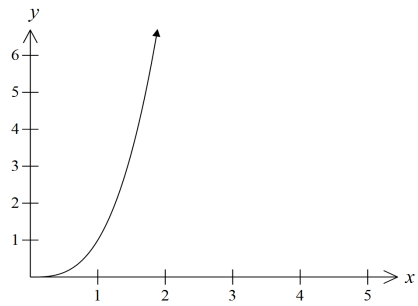
A. 3 **B.** - 2 **C.** 12 **D.** - 8

19 Which graph best represents $y = 2^x$?

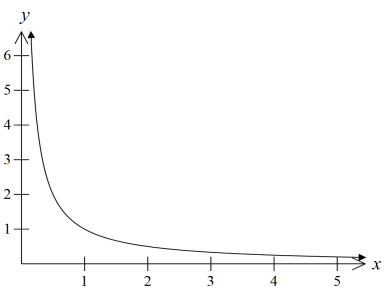
A.



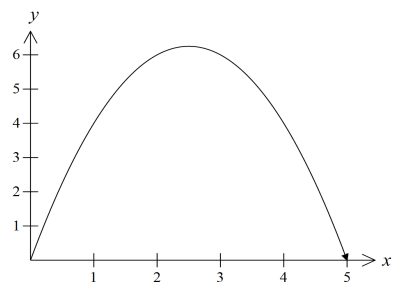
B.



C.



D.



20. Simplify $\frac{27^x}{3^{x-1}}$

A. 9 **B.** - 9 **C.** 3^{2x-1} **D.** 3^{2x+1}

END OF MULTIPLE CHOICE

PART B OVER THE PAGE

PART B **SHOW ALL WORKING**

Question 1 (20 marks) Begin in a new booklet

a. i. Write the equation $5x - 4y + 20 = 0$ in the form $y = mx + b$. **1**

ii. The midpoint of L (x, y) and M (0, -1) is (3, -4). Find the coordinates of L. **2**

iii. Showing all working, determine if the point (1, 4) lies on the line **2**

$$3x - 5y + 2 = 0.$$

iv. A (-1, 3), B (2, 4) and C(t, -1) are collinear. Find t. **3**

b. Consider the points A (-8, -1), B (8, 11) and C (2, -6).

i. Find the gradient of the line through A and B. **1**

ii. Show that the equation of the line through A and B is given by $3x - 4y + 20 = 0$ **2**

iii. Find the gradient of a line perpendicular to the line AB. **1**

c. Solve, leaving answers as exact values place where necessary:

i. $2x^2 - 5x = 0$ **2**

ii. $\frac{2x-1}{5} = \frac{1}{3x+2}$ **3**

iii. $(x^2 - 2x)^2 - 11(x^2 - 2x) + 24 = 0$ **3**

Question 2 over the page.

Question 2 (20 marks) Begin in a new booklet

a. i. Calculate θ, to the nearest degree, for which sin θ = 0.8631 and $90^0 \leq \theta \leq 180^0$. **1**

ii. What is the exact value of sin 300^0. **2**

iii. Find all values of θ, if cos $\theta = \frac{1}{\sqrt{2}}$ and $0^0 \leq \theta \leq 360^0$. **2**

b. From a point *H* in Sydney Harbour, the angle of elevation to the top of a tower, point *C*, is 40^0. After sailing 320 metres further away from the base of the tower, point *T*, to the point *S*, the angle of elevation of the point *C* is found to be $20^{0.}$

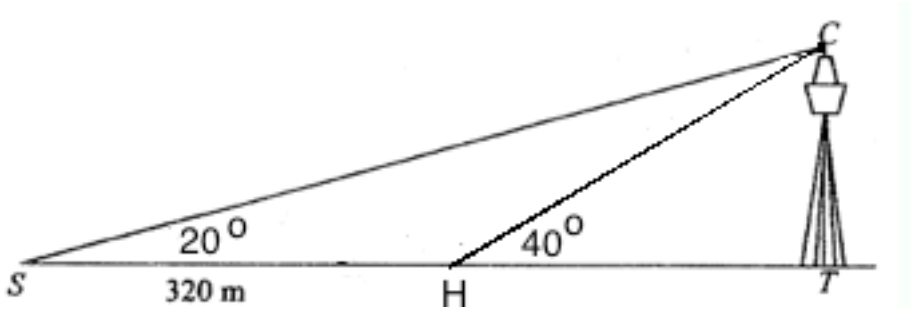
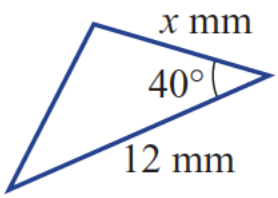


DIAGRAM NOT TO SCALE

i. Find the length of CH **1**

ii. Find the height of the tower CT, correct to the nearest metre. **2**

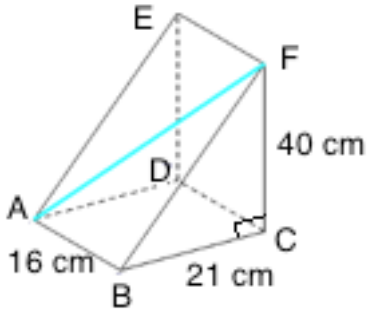
c. Find the value of x, correct to 1 decimal place, if the area of the triangle is 22 mm^2. **2**



d. This triangular prism shaped box labelled ABCDEF needs a diagonal support, AF, inserted as shown.

i. Find the length AF of this support, to 2 decimal places. **2**

ii. Find the angle this support, AF, makes with the base ABCD, to the nearest degree. **1**



e. A ship sailed 140 nautical miles from port P to port Q on a bearing of 050^0T. It then sailed 260 nautical miles from port Q to port R on a bearing of 130^0T.

i. Draw a diagram to represent the above information, labelling angles and lengths. **1**

ii. Explain why ∠PQR = 100^0. **1**

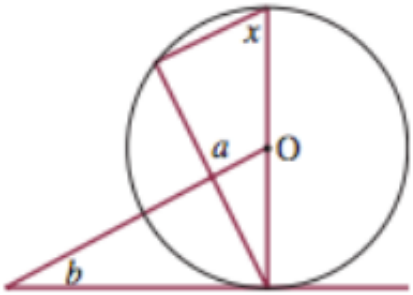
iii. Find the distance between ports R and P, correct to the nearest nautical mile. **2**

iv. Find the bearing of port R from port P, correct to the nearest degree. **3**

Question 3 over the page.

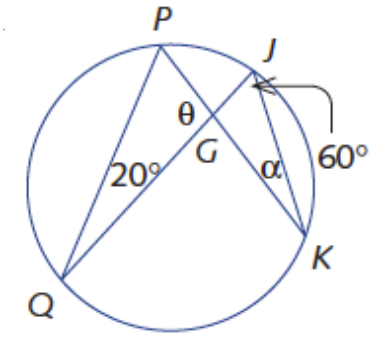
Question 3 (20 marks) Begin in a new booklet

a. In the diagram below, O is the centre of the circle. **3**
Using the diagram in your answer booklet to show all working, express x in terms of a and b, in simplest form.



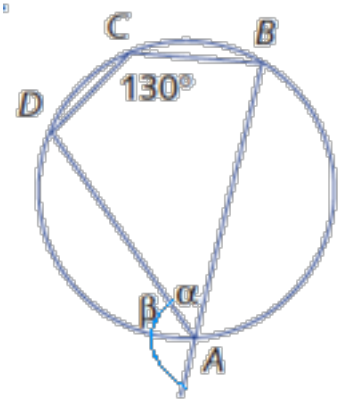
b. Find the values of the pronumerals in each of the following, with reasons:

i. Find α and θ **3**



3

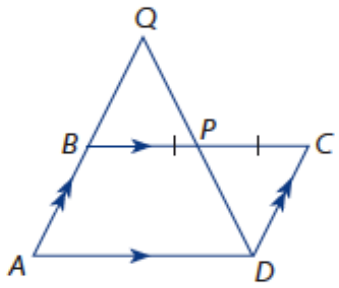
ii. Find α and β



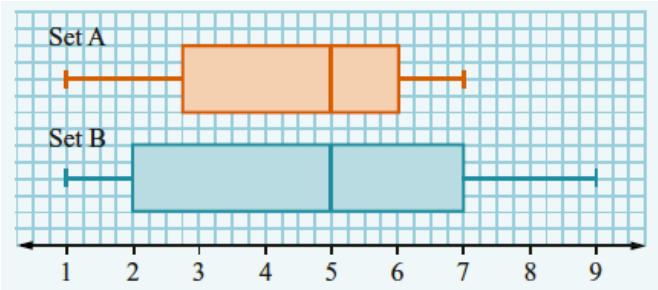
c. In parallelogram *ABCD* below, *P* is the midpoint of *BC.* Both *DP* and *AB* are produced to meet at *Q.*

i. Prove, giving reasons, that $\Delta PBQ \equiv \Delta PCD$ **3**

ii. Prove that *AQ* = 2*AB.* **2**



d. The diagram shows parallel box plots for the data in sets A and B.



i. List two similarities between these sets of data. **2**

ii. What is the interquartile range for each set? **2**

iii. If the box plots represent the marks of two classes on a test, which class do you think was more consistent and why? **2**

Question 4 over the page.

Question 4 (20 marks) Begin a new booklet

a. Express $x^2 + y^2 + 6x - 10y + 7 = 0$ in the form $(x-h)^2 + (y-k)^2 = r^2$ and state the centre and radius for the circle, in simplified exact form **3**

b. A parabola has x-intercepts of −2 and − 4 and a y-intercept of −8. Find its equation. **2**

c. The length of a rectangle is 5 cm greater than the breadth. If the area of the rectangle is 84 cm^2, find:

I. the dimensions of the rectangle **3**

II. the perimeter of the rectangle **2**

d. $P(x) = x^3 + kx + 2k$ and $A(x) = x - 1$.

I. Determine the quotient $Q(x)$ and the remainder $R(x)$ when $P(x)$ is divided by $A(x)$. **3**

II. Find the value of k for which $A(x)$ is a factor of $P(x)$. **2**

e. Factorise $y = x^3 + x^2 - 14x - 24$ using the factor theorem, then draw a neat sketch of the polynomial, showing the x and y intercepts and other important features, on the grid in the answer booklet provided. **5**

Question 5 over the page

Question 5 (20 marks) Begin a new booklet

a. Solve the following:

I. $2^{x+1} = 16$ **2**

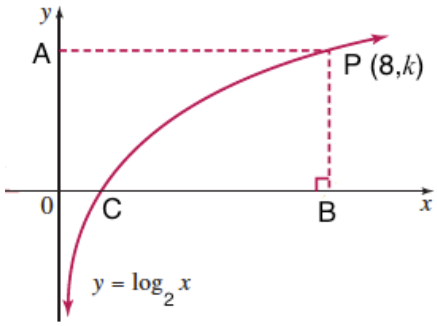
II. $\log_x\left(\frac{1}{16}\right) = -4$ **2**

b. Evaluate the following $\log_8 4$ **2**

c. Simplify fully: $\log 4p - 2\log 3q$ **2**

d. Solve for x: $\log_2(2x+6) - \log_2(x-3) = 3$ **3**

e. The graph of $y = \log_2 x$ is shown below.



Find the value of k. **1**

f. i. Make x the subject of the equation: $a - x = b + dx$ **2**

ii. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. **3**
By first constructing a pair of equations, solve to find the fraction.

g. On the axes provided in the answer booklet, graph $y = -\frac{1}{x-2}$ labelling intercepts, asymptotes and other important features. **3**

END OF EXAM

Name:_____

Teacher: Marking

The Scots College
Year 10 5.3 Mathematics
16 November 2016
Yearly Examination
Weighting: 40%
2 hours plus 5 minutes reading time

Instructions:

- Show all necessary working.
- Approved non-programmable calculators can be used.
- Begin a new labelled booklet for each question.

Question	Maximum Mark	Mark
PART A: MULTIPLE CHOICE	20	MOBR.
PART B: 1	20	FER
2	20	G2L
3	20	SHN
4	20	RIL
5	20	ROD.
TOTAL	120	
PERCENTAGE		

Name: _____

Teacher: Solutions

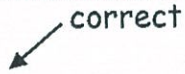
Year 10 5.3 Mathematics Exam
Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



A ● B ⊗ C ○ D ○

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1. A ○ B ○ C ● D ○
2. A ● B ○ C ○ D ○
3. A ○ B ○ C ● D ○
4. A ○ B ● C ○ D ○
5. A ○ B ● C ○ D ○
6. A ○ B ● C ○ D ○
7. A ○ B ○ C ● D ○
8. A ○ B ○ C ○ D ●
9. A ○ B ● C ○ D ○
10. A ● B ○ C ○ D ○
11. A ○ B ○ C ● D ○
12. A ○ B ● C ○ D ○
13. A ○ B ○ C ○ D ●
14. A ○ B ○ C ○ D ●
15. A ● B ○ C ○ D ○
16. A ○ B ● C ○ D ○
17. A ○ B ● C ○ D ○
18. A ○ B ● C ○ D ○
19. A ● B ○ C ○ D ○
20. A ○ B ○ C ○ D ●

Part B

Question 1

a) i) $5x - 4y + 20 = 0$

$4y = 5x + 20$

$y = \frac{5}{4}x + 5$ (1)

ii) $(3, -4) = \left(\frac{x+0}{2}, \frac{y-4}{2}\right)$ (1)

$\frac{x}{2} = 3 \quad -4 = \frac{y-4}{2}$

$x = 6 \quad -8 = y - 4$

($\frac{1}{2}$) $\quad y = -4$ ($\frac{1}{2}$)

$L(6, -4)$

iii) LHS $= 3x - 5y + 2 \quad x = 1, y = 4$

$= 3 - 20 + 2$ (1)

$\neq 0$

$\therefore (1, 4)$ does NOT lie on line (1)

iv) $m_{AB} = \frac{4-3}{2+1}$

$= \frac{1}{3}$ (1)

$y = \frac{1}{3}x + b \quad x = 2, y = 4$

$4 = \frac{2}{3} + b$

$b = \frac{10}{3}$ (1)

$\therefore$ equation $y = \frac{1}{3}x + \frac{10}{3} \quad x = t, \; y = -1$

$-1 = \frac{t}{3} + \frac{10}{3}$

$-3 = t + 10$

$t = -13.$ (1)

b) i) $m_{AB} = \frac{11+1}{8+8}$

$= \frac{12}{16}$

$= \frac{3}{4}$ (1)

ii) $y = \frac{3}{4}x + b \quad x = 8, y = 11$

$11 = \frac{3}{4} \times 8 + b$

$5 = b$

$y = \frac{3}{4}x + 5$ (1)

$4y = 3x + 20$

$3x - 4y + 20 = 0$ (1)

iii) $m_{\perp AB} = -\frac{4}{3}$ (1)

c)

i) $2x^2 - 5x = 0$

$x(2x - 5) = 0$ (1)

$x = 0 \quad$ or $\quad 2x - 5 = 0$

($\frac{1}{2}$) $\quad 2x = 5$ ($\frac{1}{2}$)

$x = \frac{5}{2}$

ii) $\frac{2x-1}{5} = \frac{1}{3x+2}$

$(2x - 1)(3x + 2) = 5$

$6x^2 + 4x - 3x - 2 = 5$

$6x^2 + x - 7 = 0$ (1)

$P = -42$
$S = +1$
$-6, +7$

$6x^2 - 6x + 7x - 7 = 0$

$6x(x - 1) + 7(x - 1) = 0$

$(x - 1)(6x + 7) = 0$

$x - 1 = 0 \quad$ or $\quad 6x + 7 = 0$

$x = 1$ (1) $\quad x = \frac{-7}{6}$ (1)

iii) $(x^2-2x)^2-11(x^2-2x)+24=0$

let $u=(x^2-2x)$ ½

$u^2-11u+24=0$

$P=+24$ $1\ 24\ 2\ 12\ 3\ 8\ 4\ 6$

$S=-11$ ½

$u^2-3u-8u+24=0$

$u(u-3)-8(u-3)=0$

$(u-3)(u-8)=0$

$u-3=0$ or $u-8=0$

$u=3$ ½ $u=8$ ½

$\therefore x^2-2x=3$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x-3=0$ or $x+1=0$

$x=3$ $x=-1$

½

$x^2-2x=8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x-4=0$ or $x+2=0$

$x=4$ $x=-2$

½

Question 2

a) i) acute $\theta = 59.66\ldots$
$= 60^\circ$

$180 - 60 = 120^\circ$ ①

ii) $\sin 300 = -\sin 60^\circ$ ①

$= -\frac{\sqrt{3}}{2}$ ①

iii) $\cos\theta = \frac{1}{\sqrt{2}}$ ++ V V

$\theta = 45^\circ, 315^\circ$
① ①

b) i) 

isosceles ①

$\therefore CH = 320m$

ii) 

$\sin 40^\circ = \frac{x}{320}$ ①

$x = 320 \times \sin 40^\circ$

$= 205.69$ ①

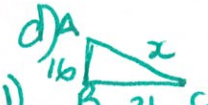
$= 206m$

c) $A = \frac{1}{2}ab\sin C$

$22 = \frac{1}{2} \times 12 \times x \times \sin 40^\circ$ ①

$x = \frac{22 \times 2}{12 \sin 40^\circ}$

$= 5.7\ mm$ ①

d) i) 

By Pythag
$x^2 = 16^2 + 21^2$ ①

$x^2 = 697$

$x = 26.40m$



By Pythag ①

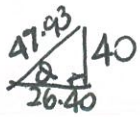
$y^2 = (26.40)^2 + 40^2$

$= 697 + 1600$

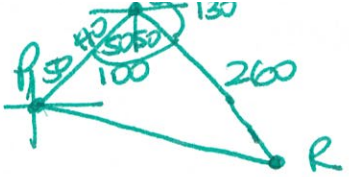
$= 2297$ ①

$y = 47.93\ m$

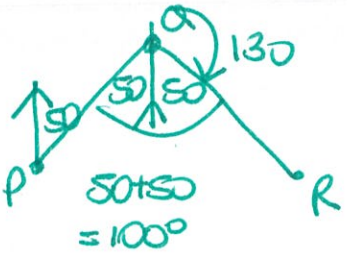
ii)



$40 \sin\theta = \frac{40}{47.97}$ $\theta = 56^\circ$ ①

e) i) 

①

ii) 

$50 + 50 = 100^\circ$ ①

iii) $x^2 = 140^2 + 260^2 - 2 \times 140 \times 260 \times \cos 100^\circ$ ①

$= 99841.587\ldots$ ①

$x = 315.977\ldots = 316$ nautical miles

iv) 

$\frac{316}{\sin 100} = \frac{260}{\sin\theta}$ ①

$316 \sin\theta = 260 \sin 100$

$\sin\theta = \frac{260 \sin 100}{316}$

$\theta = \sin^{-1}\left(\frac{260 \sin 100}{316}\right)$

$= 54^\circ$ ①

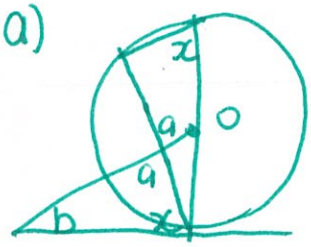
$\therefore$ bearing of R from P

$= 50 + 54$

$= 104^\circ$ ①

Question 3

a)



'a'

① vertically opp

① 'x' exterior angle

① $x = 180 - (a+b)$

b) i) $\alpha = 20^\circ$ ($\frac{1}{2}$) (angles on circumference standing on same arc) ①

$\theta = 180 - (60+20) = 100^\circ$ ($\frac{1}{2}$) (angle sum $\Delta = 180^\circ$) ①

ii) $\alpha = 180 - 130$ ①

$= 50^\circ$ ($\frac{1}{2}$) (opp angles cyclic quad add 180°)

$B = 130^\circ$ ($\frac{1}{2}$) (straight angle $= 180^\circ$) ①

c) i) In ΔPBQ & ΔPCD

② { $BP = CP$ (given)
$\angle BQP = \angle DPC$ (alt. $\angle$'s parallel lines equal)
$\angle QPB = \angle CPD$ (vertically opp angles equal).

$\therefore$ by test AAS ①

$\Delta PBQ \equiv \Delta PCD$.

ii) $QB = CD$ as they are corresponding sides in congruent triangles ①

$CD = AB$ as they are opposite sides of parallelogram

$\therefore QB = AB$ ①

$AQ = AB + QB$

$= AB + AB = 2AB$.

d) i) same median (5) ①

same min. value (1) ①

ii) IQR set A $= 6 - 2.8$

$= 3.2$ ①

set B $= 7 - 2$

$= 5$ ①

iii) Set A. ①

– lower IQR while having same median. ①

Question 4

a) $x^2+y^2+6x-10y+7=0$

$x^2+6x+\left(\frac{6}{2}\right)^2+y^2-10y+\left(\frac{10}{2}\right)^2=-7+\left(\frac{6}{2}\right)^2+\left(\frac{10}{2}\right)^2$ ①

$(x+3)^2+(y-5)^2=-7+9+25$

$(x-(-3))^2+(y-5)^2=27$

∴ Centre $(-3,5)$ ① $r=\sqrt{27}$ ①

$=3\sqrt{3}$ units

b) $y=k(x+2)(x+4)$ when $x=0$, $y=-8$ ①

$-8=8k$

$k=-1$

∴ $y=-1(x+2)(x+4)$ ①

c) $A=x(x+5)$



i) $x(x+5)=84$ ①

$x^2+5x-84=0$

$(x+12)(x-7)=0$

$x=-12, \quad x=7$ ①

as length cannot be negative

$x=7$

∴ dimension length = 12cm

width = 7cm ①

ii) $P=(2\times 7)+(2\times 12)$ ①

$=14+24$

$=38$cm ①

d) $x-1 \overline{)\, x^3+kx+2k}$ ①

x^3-x^2 ↓

x^2+kx

x^2-x

$(k+1)x+2k$

$(k+1)x-(k+1)$

$2k+(k+1)$

$=3k+1$

$Q(x)=x^2+x+(k+1)$ ①

$R(x)=3k+1$ ①

ii) if $A(x)$ is a factor then ①

$R(x)=0$

$3k+1=0$

$k=-\frac{1}{3}$ ①

e) $y=x^3+x^2-14x-24$

$P(1)=1+1-14-24$

$\neq 0$

$P(2)=8+4-28-24$

$\neq 0$

$P(-2)=-8+4+28-24$ ①

$=0$ ∴ $x+2$ is a factor

$x+2 \overline{)\, x^3+x^2-14x-24}$

x^2-x-12

x^3+2x^2 ↓

$-x^2-14x$

$-x^2-2x$

$-12x-24$

$-12x-24$

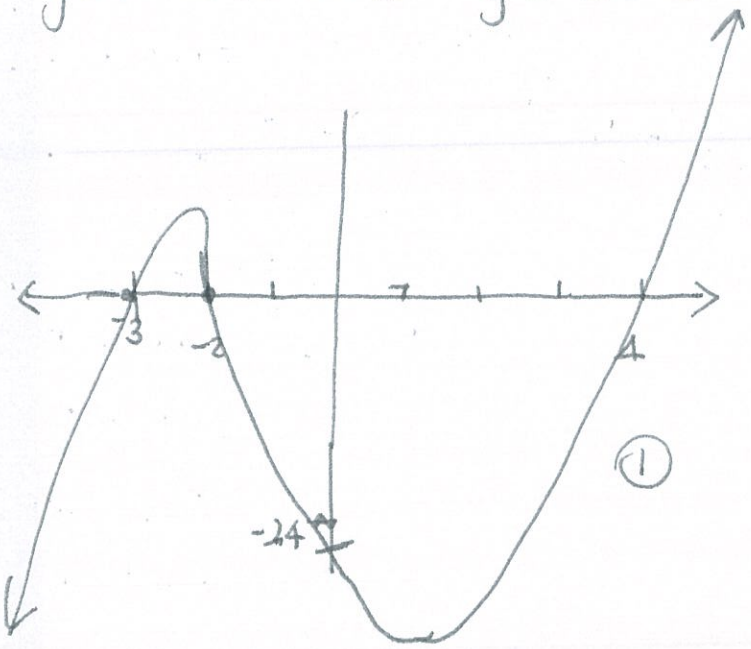
$=(x+2)(x^2-x-12)$ ①

$=(x+2)(x-4)(x+3)$

∴ Roots $x=-2 \quad x=4 \quad x=-3$ ①

cut when $x = 0$ $y = -24$ ①

$y = (x+2)(x+3)(x-4)$



$x = 1$ $y = 3 \times 4 \times -3$

$= -36$

$x = 2$ $y = 4 \times 5 \times -2$

$= -40$

$x = 3$ $y = 5 \times 6 \times -1$

$= -30$

$x = 5$ $y = 7 \times 8 \times 1$

$= 56$

Question 5

a) i) $2^{x+1} = 16$

$2^{x+1} = 2^4$ ①

$x+1 = 4$

$x = 3$. ①

ii) $\log_x\left(\frac{1}{16}\right) = -4$

$x^{-4} = \frac{1}{16}$ ①

$x^{-4} = 2^{-4}$

$x = 2$ ①

b) $\log_8 4 = x$

$8^x = 4$ ①

$2^{3x} = 2^2$

$3x = 2$

$x = \frac{2}{3}$ ①

c) $\log 4p - 2\log 3q$

$= \log 4p - \log (3q)^2$ ①

$= \log\left(\frac{4p}{9q^2}\right)$ ①

or $\log\left(\frac{4p}{(3q)^2}\right)$

d) LHS $= \log_2(2x+6) - \log_2(x-3)$

$= \log_2 2(x+3) - \log_2(x-3)$

$= \log_2 \frac{2(x+3)}{(x-3)}$ ①

$\therefore \log_2\left(\frac{2(x+3)}{(x-3)}\right) = 3$

$2^3 = \frac{2(x+3)}{(x-3)}$ ①

$8(x-3) = 2(x+3)$

$4x - 12 = x + 3$

$3x = 15$ ①

$x = 5$

e) $y = \log_2 8$

$= \log_2 2^3$

$= 3\log_2 2$,

$= 3 \times 1$

$y = 3$ $\therefore k = 3$. ①

f) i) $a - x = b + dx$

$a - b = dx + x$ ①

$a - b = x(d+1)$

$x = \frac{(a-b)}{(d+1)}$ ①

ii) $\frac{x}{y}$ ① $x + y = 12$ (½)

② $\frac{x}{y+3} = \frac{1}{2}$ (½)

① $x + y = 12$ ② $2x = y + 3$

$2x - 3 = y$

$\therefore x + 2x - 3 = 12$

$3x = 15$

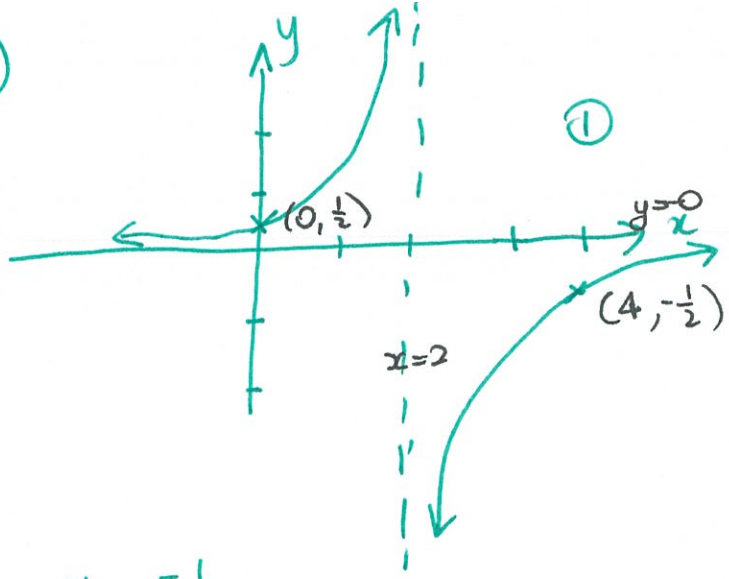
$x = 5$ ①

$5 + y = 12$

$y = 7$

$\therefore$ fraction $\frac{5}{7}$ ①

g)



①

$y = \frac{-1}{x-2}$

asymptote $\Rightarrow$ $x=2$ ①
$y=0$

y int when $x=0$ $y=\frac{1}{2}$ ①