Mrs Cooper
Mrs Gray
Ms Lau
Mrs Millar
Ms Smith
Mr Way
Mr Feng
Ms Hua
Mrs Lim
Ms Peng
Mrs Ward

Name:……

Teachers Name:……

Year 10 Mathematics

5.3 Pathway

YEARLY EXAMINATION

2023

General Instructions:

- Reading time – 5 minutes.
- Time allowed – 90 minutes.
- Put your name on the Multiple-Choice Answer sheet.
- Approved calculators may be used.
- Write using black or blue pen.
- Erasable pens are not to be used.
- Diagrams are NOT drawn to scale.
- Formulae sheets are provided.

Total Marks:

Section I	/10
Section II	/60
Total	**/70**

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Mathematics
Year 10/5.3
Yearly Examination
2023

Name:……

Teacher's Name: …….

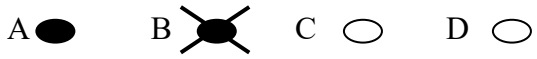
Section I – Multiple Choice Answer Sheet

Select the alternative (A), (B), (C) or (D) that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



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1 A ○ B ○ C ○ D ○

2 A ○ B ○ C ○ D ○

3 A ○ B ○ C ○ D ○

4 A ○ B ○ C ○ D ○

5 A ○ B ○ C ○ D ○

6 A ○ B ○ C ○ D ○

7 A ○ B ○ C ○ D ○

8 A ○ B ○ C ○ D ○

9 A ○ B ○ C ○ D ○

10 A ○ B ○ C ○ D ○

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SECTION I

Multiple Choice

10 marks

Answer using the Multiple-Choice Answer Sheet provided

---

1 Consider the two-way table below of 30 students.
What is the probability of choosing a boy given that the student is a junior?

	Boy	Girl
Junior	7	5
Senior	8	10

(A) $\frac{7}{12}$

(B) $\frac{7}{15}$

(C) $\frac{12}{30}$

(D) $\frac{15}{30}$

2 What are the values of x that satisfy the equation $(x+3)(x-2)=0$?

(A) $x=3$ or 2

(B) $x=3$ or -2

(C) $x=-3$ or 2

(D) $x=-3$ or -2

3 What is the solution set to the following pair of simultaneous equations?

$$3x-y=18$$
$$4x+y=10$$

(A) $(5,-3)$

(B) $(-3,5)$

(C) $(4,-6)$

(D) $(4,6)$

4 Which of the following statements is an example of negative correlation?

(A) The outdoor temperature and the number of people at the beach.

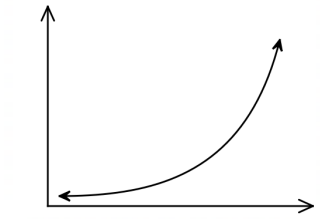
(B) The number of cars on the road during peak hour and the average speed of the vehicle.

(C) The height of a person and their arm-span.

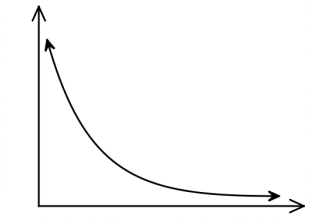
(D) The cost of a coffee and the height of a Jacaranda tree at Pymble Ladies' College.

5 The number of workers it takes to dig a ditch varies inversely with the time taken. Which graph best represents this?

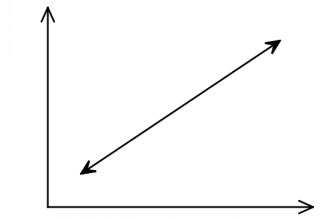
(A)



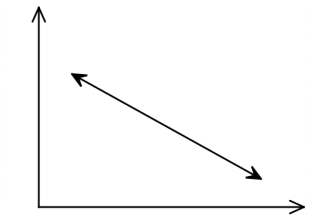
(B)



(C)



(D)



6 To the nearest degree, what is the smallest angle in a triangle with side lengths 8 cm, 13 cm and 19 cm?

(A) 19°

(B) 33°

(C) 52°

(D) 24°

7 The midpoint of $A(2, -6)$ and $B(x, y)$ is $(-3, -1)$. What are the coordinates of B?

(A) $(-0.5, -2.5)$

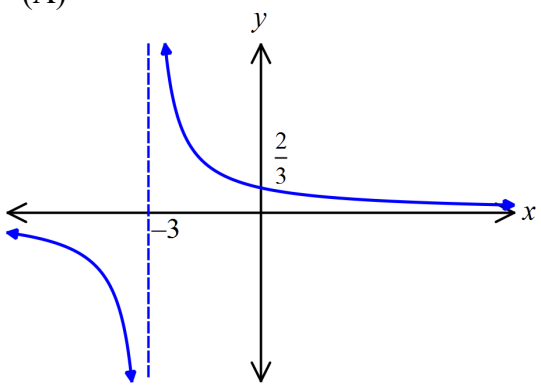
(B) $(2.5, -3.5)$

(C) $(-8, 4)$

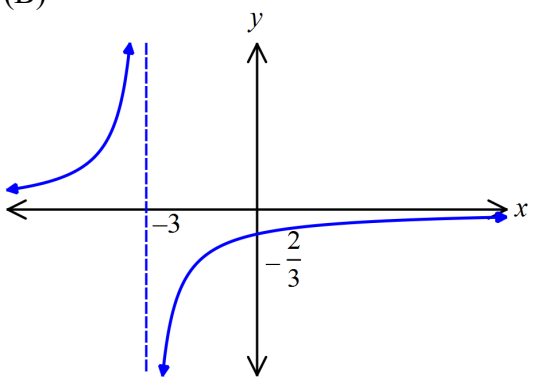
(D) $(7, -11)$

8 The graph with equation $y = \dfrac{2}{x-3}$ is best represented by which of the following graphs?

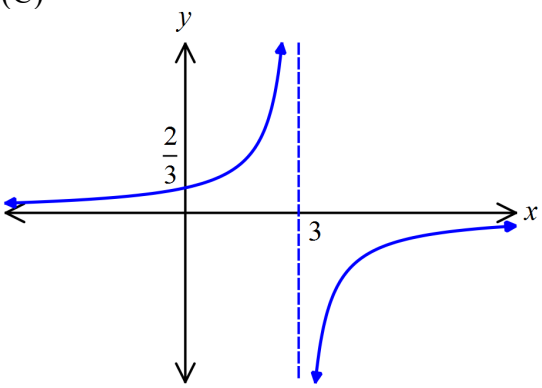
(A)



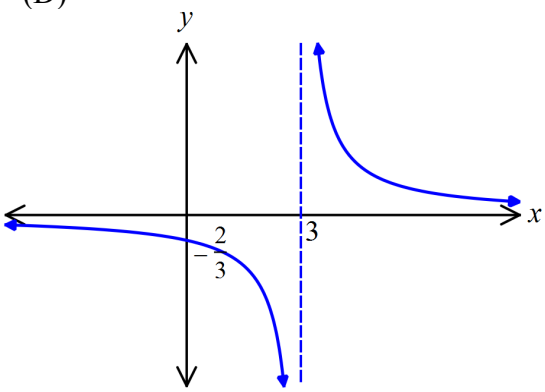
(B)



(C)



(D)



9 Indi purchases a car for \$9000. If it depreciates by 8% each year, what is it worth after 6 years, to the nearest dollar?

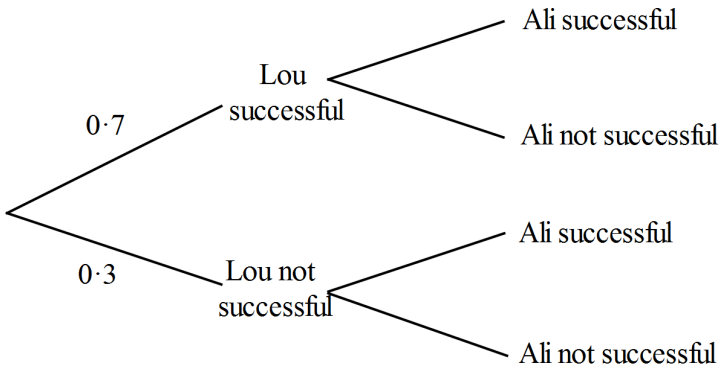
(A) \$4200

(B) \$4680

(C) \$5413

(D) \$5457

10 Lou and Ali are on a fitness program for one month. The probability that Lou will finish the program successfully is 0.7 while the probability that Ali will finish successfully is 0.6. The probability tree shows this information.



What is the probability that only one of them will finish the program successfully?

(A) 0.18

(B) 0.28

(C) 0.42

(D) 0.46

SECTION II

60 marks

Attempt all questions and answer in the spaces provided.
Extra writing space is available at the end of this section.

Marks as indicated. **Marks**

Question 11

Using the list given in the table, match each equation with its' type of curve. **2**

(a) $y = x^2 + 2$ .....

(b) $x^2 + y^2 = 9$ .....

(c) $y = \dfrac{2}{x+2}$ .....

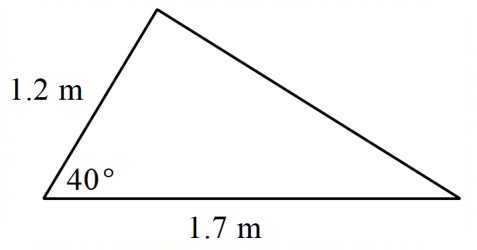
(d) $y = 6^{-x}$ .....

(e) $y = (x-2)^3 + 1$ .....

circle
cubic
exponential
parabola
hyperbola

Question 12

Find the area of the triangle below, correct to two decimal places. **2**



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Marks

Question 13

The point $(-3, -4)$ lies on a line parallel to $2x + 4y - 7 = 0$. **3**

What is the equation of the line?

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Question 14

Find the centre and radius of the circle $x^2 + y^2 - 6x + 2y - 26 = 0$. **3**

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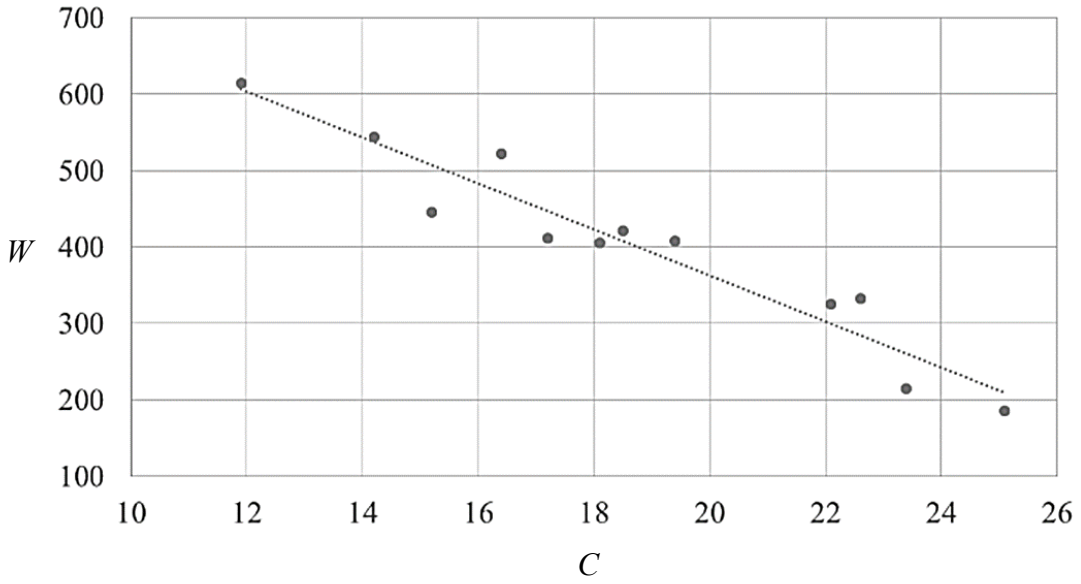
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Marks

Question 15

The relationship between the number of chickens, C, and the number of worms, W, in a chicken coop is shown on the scatterplot below.



(a) Describe the strength of the relationship between C and W. **1**

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(b) Use the graph to estimate the value of W when $C = 22$. **1**

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(c) What is the equation of the line of best fit? **2**

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(d) Why would it be unreasonable to use the line of best fit to estimate the value of W when C equals 50? **1**

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Marks

Question 16

A card is selected from a standard deck of 52 playing cards.

Event A – "the card is a heart"
Event B – "the card is a king".

(a) Draw a Venn Diagram for Events A and B. **2**

(b) Find:

(i) $n(A \cup B)$ **1**

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(ii) P(not A) **1**

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(iii) P(A and not B) **1**

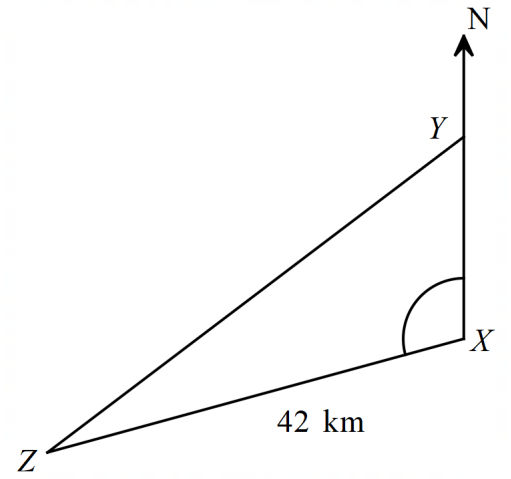
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Marks

Question 17

The bearing of Z from X is $220°$ and the distance between Z and X is 42 km.



(a) What is the size of $\angle YXZ$? **1**

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(b) If Y is 26 km due north of X, calculate the distance between Z and Y, correct to 2 decimal places. **2**

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Marks

Question 18

The distance, d km, it is possible to see on a clear day is proportional to the square root of the height, h m, above sea level.
Standing on a pier, 4 m above sea level, it is possible to see a distance of 10 km.

(a) Find a formula for d, in terms of h. **2**

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(b) Standing on top of the cliffs I can see a distance of 35 km. **1**
How high are the cliffs?

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Question 19

Solve $y = 3x^2 - 6x + 2$ using the quadratic formula. Leave your answer as a simplified surd. **2**

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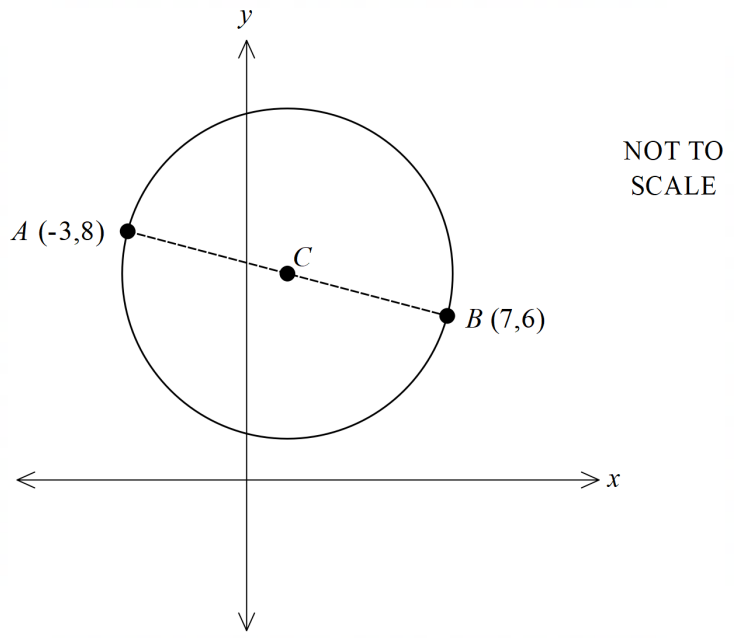
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Marks

Question 20



The circle, shown above, has

- centre, C
- diameter with end points A and B.

(a) Find the coordinates of the centre C of the circle. **1**

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(b) Find the exact length of the radius of the circle. **2**

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(c) State the equation of the circle. **1**

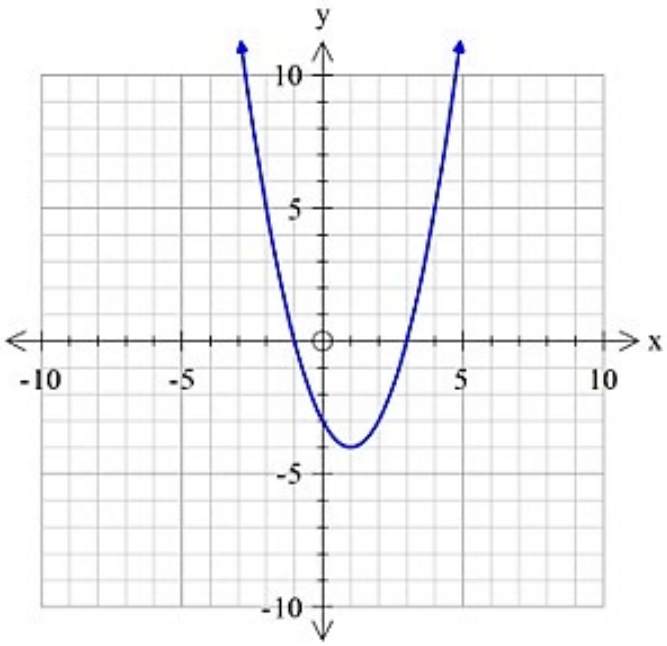
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Marks

Question 21

What is the equation of the following parabola in the form $y = a(x-h)^2 + k$? **2**



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Question 22

Solve the following simultaneous equations. **2**

$$5x - y = 1.5$$
$$2x - y = -2.1$$

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Marks

Question 23

If \$7420 is invested in a building society for 10 years with an interest rate of 8% p.a. compounding every six months, how much money is in the account after 10 years? **2**

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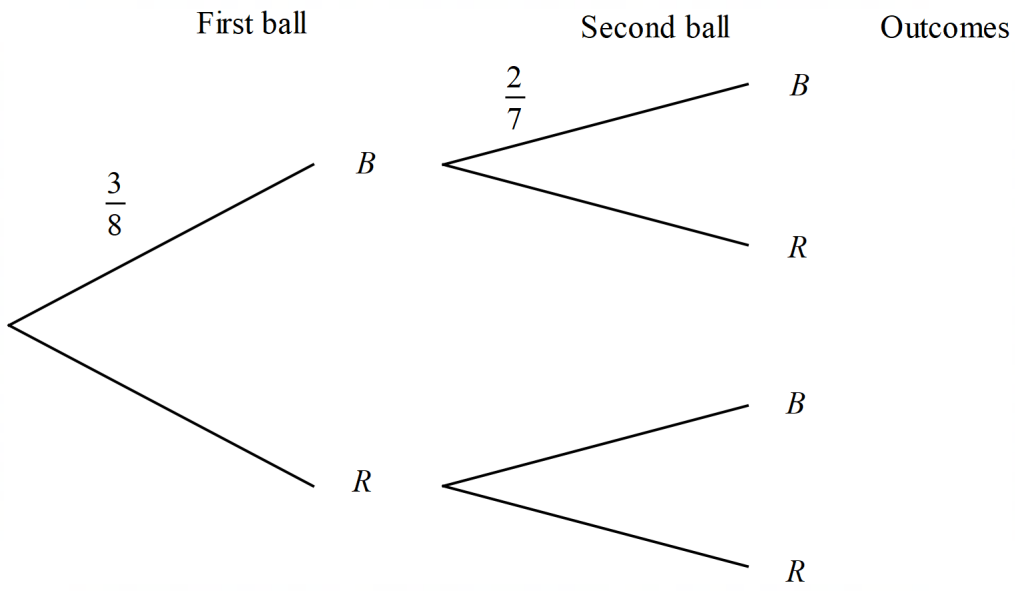
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Question 24

From a bag containing 5 red and 3 blue balls, a ball is taken at random and not replaced. A second ball is then withdrawn and placed beside it.

(a) Complete the following probability tree. **2**



(b) What is the probability that the second ball is blue given that the first ball is red? **1**

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Marks

Question 25

Harriett bought a motorbike for $7000. She paid a 15% deposit and agreed to pay the remaining balance with equal monthly payments over 5 years at 12% p.a simple interest.

(a) How much was owing on the motorbike after the deposit was paid? **1**

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(b) What was the monthly repayment? **2**

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(c) How much interest did Harriett have to pay? **1**

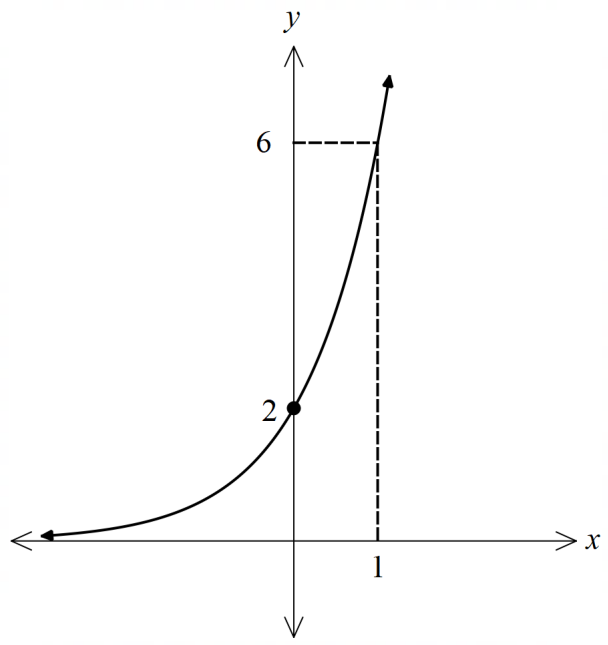
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Marks

Question 26



The diagram shows the graph of $y = m\left(a^x\right)$. **2**

Find the values of m and a.

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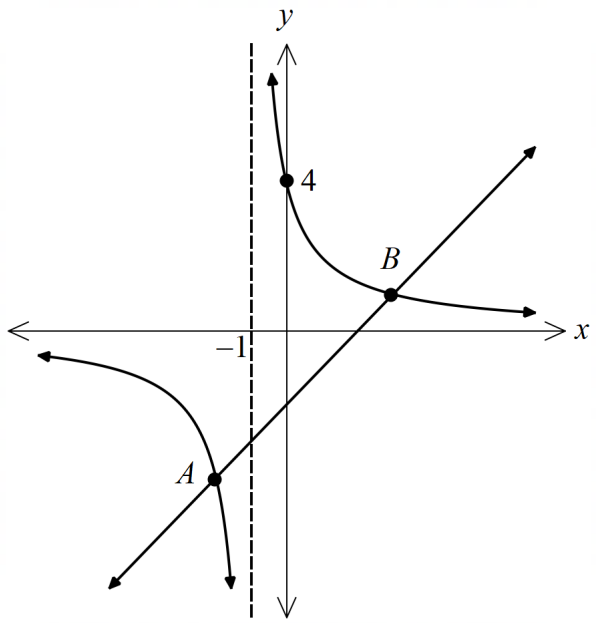
Marks

Question 27

On the diagram below the graphs of $y = \dfrac{4}{x+1}$ and $x - y - 2 = 0$ are drawn. **3**

Their points of intersection are labelled A and B.

Solve a pair of simultaneous equations to find the coordinates of A and B.



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Marks

Question 28

The distance between the points $(-2, -1)$ and $(a, 3)$ is $\sqrt{20}$. Find the values(s) of a. **2**

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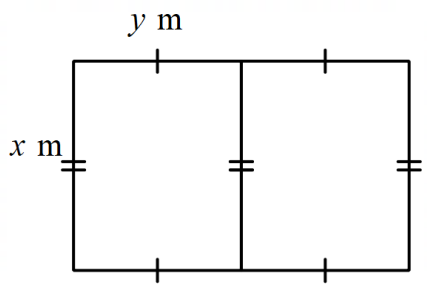
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Marks

Question 29

1600 metres of fencing is to be used to enclose a rectangular area and to divide it into two equal areas as shown.



(a) Using the pronumerals given, show that the combined enclosed area A is given by $A = 800x - \frac{3}{2}x^2$. **2**

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(b) Hence, find the values of x and y for which the area enclosed is greatest. **3**

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Marks

Question 30

Two fair six-sided dice are rolled. The number on the upper face of the first die determines the coefficient b and the upper face of the second die determines the value of c in the quadratic equation $x^2 + bx + c = 0$. **3**

With the use of an array, or otherwise, determine the probability that the equation formed will have two distinct, real solutions.

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END OF EXAMINATION

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

5.3 Pathway MATHEMATICS FORMULA SHEET

Circumference of a circle

$C = 2\pi r$ or $C = \pi d$

Arc length of a circle

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

Circle

$A = \pi r^2$

Sector

$$A = \frac{\theta}{360} \times \pi r^2$$

Area of a Parallelogram

$A = bh$

Area of a Trapezium

$$A = \frac{h}{2}(a + b)$$

Area of a Rhombus

$$A = \frac{1}{2}xy$$

Surface Area

Closed cylinder

$A = 2\pi r^2 + 2\pi rh$

Cone

$SA = \pi r^2 + \pi rl$

Sphere

$A = 4\pi r^2$

Volume

Prism or cylinder

$V = Ah$

Pyramid or cone

$$V = \frac{1}{3}Ah$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

Volume and capacity

unit conversion: 1 m^3 = 1000 L

Pythagoras' Theorem

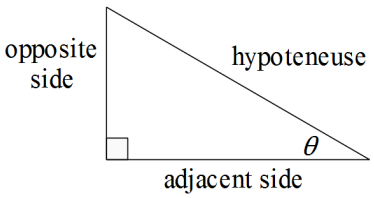
$c^2 = a^2 + b^2$

Distance, Speed, Time

$$D = ST, \quad S = \frac{D}{T}, \quad T = \frac{D}{S}$$

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Trigonometric Ratios



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine Rule

In ΔABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule

In ΔABC, $c^2 = a^2 + b^2 - 2ab\cos C$

or $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Area of a Triangle

In ΔABC, $A = \dfrac{1}{2}ab\sin C.$

Financial Mathematics

Simple Interest

$I = Prn$

Compound Interest

$A = P(1+r)^n$

Angle sum of a Polygon

$S = (n-2)\times 180^0$

Factorisation

$a^2 - b^2 = (a+b)(a-b)$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of two points

$$x = \frac{x_1 + x_2}{2},\ y = \frac{y_1 + y_2}{2}$$

Slope (gradient) of a line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Gradient-intercept form

$y = mx + c$

Point-gradient form

$y - y_1 = m(x - x_1)$

Equation of a circle

$(x-h)^2 + (y-k)^2 = r^2$

Data Analysis

Mean

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

Probability of an Event

The probability of an event where outcomes are equally likely is given by:

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Mrs Cooper
Mrs Gray
Ms Lau
Mrs Millar
Ms Smith
Mr Way
Mr Feng
Ms Hua
Mrs Lim
Ms Peng
Mrs Ward

Name: Solutions

Teachers Name:.....

Pymble Ladies' College

Year 10 Mathematics

5.3 Pathway

YEARLY EXAMINATION

2023

General Instructions:

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- Approved calculators may be used.
- Write using black or blue pen.
- Erasable pens are not to be used.
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Total Marks:

Section I	/10
Section II	/60
Total	**/70**

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Mathematics
Year 10/5.3
Yearly Examination
2023

Name:.....

Teacher's Name: .....

Section I – Multiple Choice Answer Sheet

Select the alternative (A), (B), (C) or (D) that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.



1	A ●	B ○	C ○	D ○
2	A ○	B ○	C ●	D ○
3	A ○	B ○	C ●	D ○
4	A ○	B ●	C ○	D ○
5	A ○	B ●	C ○	D ○
6	A ●	B ○	C ○	D ○
7	A ○	B ○	C ●	D ○
8	A ○	B ○	C ○	D ●
9	A ○	B ○	C ○	D ●
10	A ○	B ○	C ○	D ●

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SECTION I

Multiple Choice

10 marks

Answer using the Multiple-Choice Answer Sheet provided

---

1 Consider the two-way table below of 30 students.
What is the probability of choosing a boy given that the student is a junior?

	Boy	Girl
Junior	7	5
Senior	8	10

Total 12

(A) $\frac{7}{12}$

(B) $\frac{7}{15}$

(C) $\frac{12}{30}$

(D) $\frac{15}{30}$

2 What are the values of x that satisfy the equation $(x+3)(x-2)=0$?

(A) $x=3$ or 2

(B) $x=3$ or -2

(C) $x=-3$ or 2

(D) $x=-3$ or -2

3 What is the solution set to the following pair of simultaneous equations?

$$3x-y=18$$
$$4x+y=10$$

(A) $(5,-3)$ ×

(B) $(-3,5)$ ×

(C) $(4,-6)$ ✓

(D) $(4,6)$ ×

4 Which of the following statements is an example of negative correlation?

(A) The outdoor temperature and the number of people at the beach.

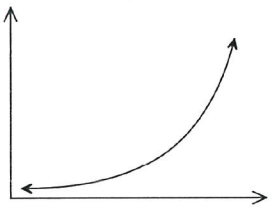
(B) The number of cars on the road during peak hour and the average speed of the vehicle.

(C) The height of a person and their arm-span.

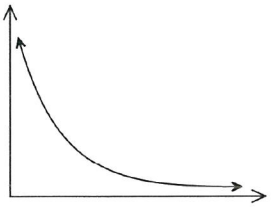
(D) The cost of a coffee and the height of a Jacaranda tree at Pymble Ladies' College.

5 The number of workers it takes to dig a ditch varies inversely with the time taken. Which graph best represents this?

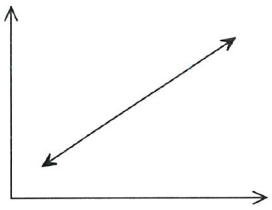
(A)



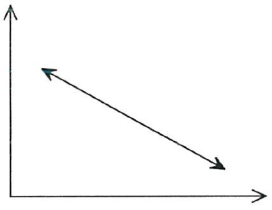
(B)



(C)



(D)



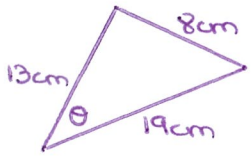
6 To the nearest degree, what is the smallest angle in a triangle with side lengths 8 cm, 13 cm and 19 cm?

(A) 19°

(B) 33°

(C) 52°

(D) 24°



$\cos\theta = \frac{13^2 + 19^2 - 8^2}{2(13)(19)}$

$\cos\theta = \frac{233}{247}$

$\theta = \cos^{-1}\left(\frac{233}{247}\right)$

$\theta = 19°$

7 The midpoint of $A(2, -6)$ and $B(x, y)$ is $(-3, -1)$. What are the coordinates of B?

(A) $(-0.5, -2.5)$

(B) $(2.5, -3.5)$

(C) $(-8, 4)$

(D) $(7, -11)$

$M(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$(-3,-1) = \left(\frac{2+x}{2}, \frac{-6+y}{2}\right)$

$-3 = \frac{2+x}{2}$

$-6 = 2 + x$

$x = -8$

$-1 = \frac{-6+y}{2}$

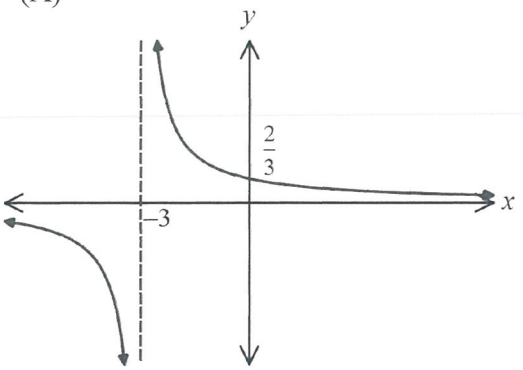
$-2 = -6 + y$

$y = 4$

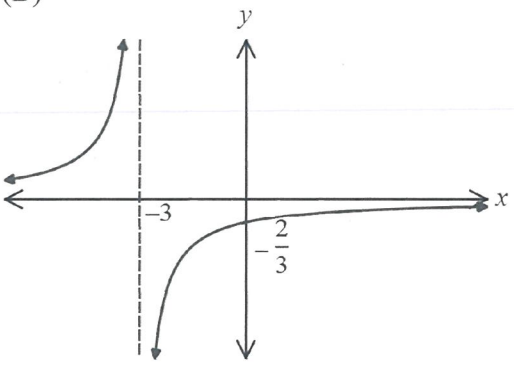
$\therefore (-8, 4)$

8 The graph with equation $y = \dfrac{2}{x-3}$ is best represented by which of the following graphs?

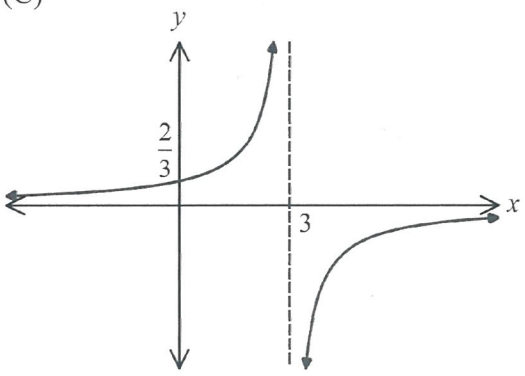
(A)



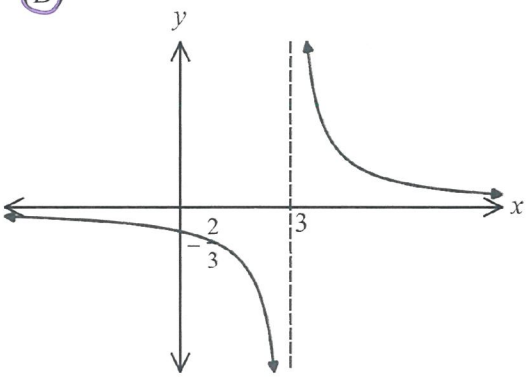
(B)



(C)



(D)



9 Indi purchases a car for \$9000. If it depreciates by 8% each year, what is it worth after 6 years, to the nearest dollar?

(A) \$4200

(B) \$4680

(C) \$5413

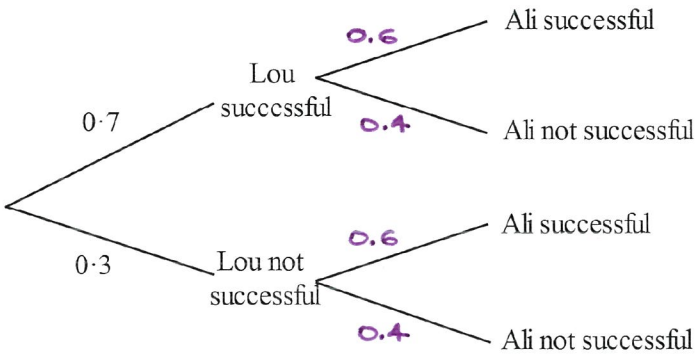
(D) \$5457

$A = P(1-r)^n$

$= \$9000(1-8\%)^6$

$\approx \$5457.20$

10 Lou and Ali are on a fitness program for one month. The probability that Lou will finish the program successfully is 0.7 while the probability that Ali will finish successfully is 0.6. The probability tree shows this information.



What is the probability that only one of them will finish the program successfully?

(A) 0.18

(B) 0.28

(C) 0.42

(D) 0.46

P(only one successful) = (0.7 × 0.4) + (0.3 × 0.6)
= 0.46

SECTION II

60 marks

Attempt all questions and answer in the spaces provided.
Extra writing space is available at the end of this section.

Marks as indicated. **Marks**

Question 11

Using the list given in the table, match each equation with its' type of curve. **2**

(a) $y = x^2 + 2$ parabola

(b) $x^2 + y^2 = 9$ circle

(c) $y = \dfrac{2}{x+2}$ hyperbola

(d) $y = 6^{-x}$ exponential

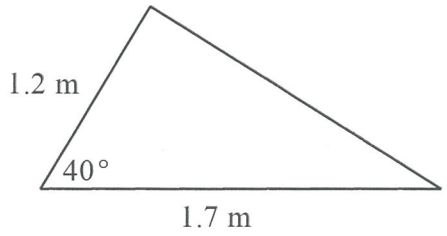
(e) $y = (x-2)^3 + 1$ cubic

circle
cubic
exponential
parabola
hyperbola

2	All correct
1	2 errors
0	3 or more errors

Question 12

Find the area of the triangle below, correct to two decimal places. **2**



$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 1.2 \times 1.7 \times \sin 40°$ ✓

$= 0.655643361 9...$

$\approx 0.66 m^2$ ✓

2	Correct answer
1	Correct substitution

Marks	
3	Correct $y=-\frac{1}{2}x-\frac{11}{2}$
2	Correct $m_2=-\frac{1}{2}$
1	Correct $y=-\frac{1}{2}x+\frac{7}{4}$

Question 13

The point $(-3,-4)$ lies on a line parallel to $2x+4y-7=0$. **3**

What is the equation of the line?

$2x+4y-7=0$

$4y=-2x+7$

$y=-\frac{1}{2}x+\frac{7}{4}$ ✓

$m_1=m_2=-\frac{1}{2}$ ✓

$y-y_1=m(x-x_1)$

$y-(-4)=-\frac{1}{2}[x-(-3)]$

$y+4=-\frac{1}{2}(x+3)$

$y+4=-\frac{1}{2}x-\frac{3}{2}$

$y=-\frac{1}{2}x-\frac{11}{2}$ or $x+2y+11=0$ ✓

Question 14

Find the centre and radius of the circle $x^2+y^2-6x+2y-26=0$. **3**

$x^2+y^2-6x+2y-26=0$

$x^2-6x+(3)^2+y^2+2y+(1)^2-26-(3)^2-(1)^2=0$ ✓

$(x-3)^2+(y+1)^2-36=0$

$(x-3)^2+(y+1)^2=36$

$\therefore$ centre is $(3,-1)$ ✓

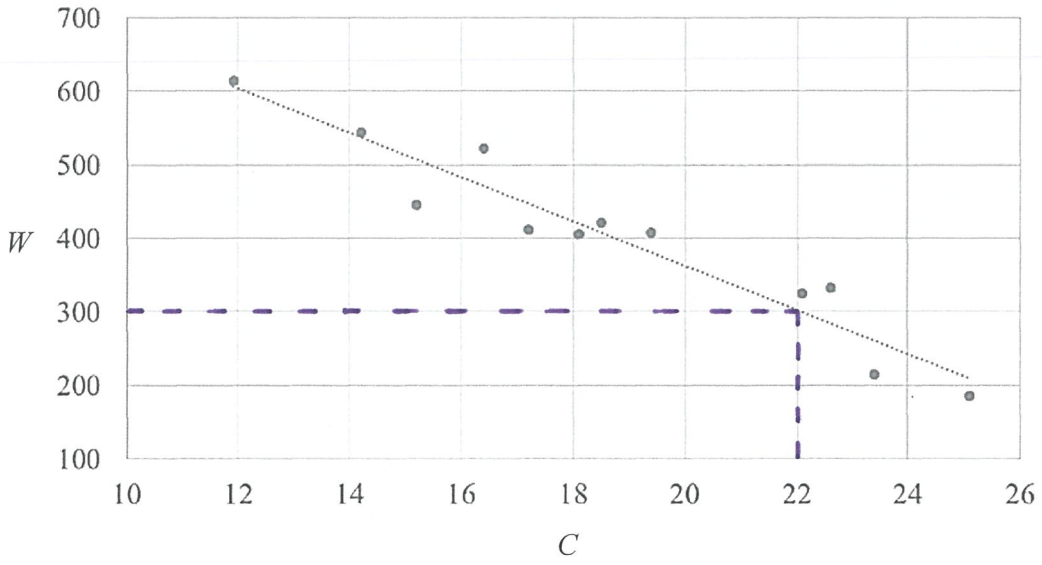
radius is 6 units ✓

3	Correct centre and radius
2	Correct centre or radius
1	Correct completing the square

Marks

Question 15

The relationship between the number of chickens, C, and the number of worms, W, in a chicken coop is shown on the scatterplot below.



(a) Describe the strength of the relationship between C and W. **1**

Strong negative linear correlation ✓ R/W

(b) Use the graph to estimate the value of W when $C = 22$. **1**

W = 300 ✓ R/W

(c) What is the equation of the line of best fit? **2**

(12, 600) and (22, 300)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{300 - 600}{22 - 12} = -30$ ✓

$y - y_1 = m(x - x_1)$

$W - 600 = -30(C - 12)$

$W - 600 = -30C + 360$

$W = -30C + 960$ ✓

2 Correct $W = -30C + 960$

1 Correct $m = -30$

(d) Why would it be unreasonable to use the line of best fit to estimate the value of W when C equals 50? **1**

When C = 50, W = -540, since there cannot be a negative number of worms it would be unreasonable to use the line of best fit. ✓

(1) Reasonable answer

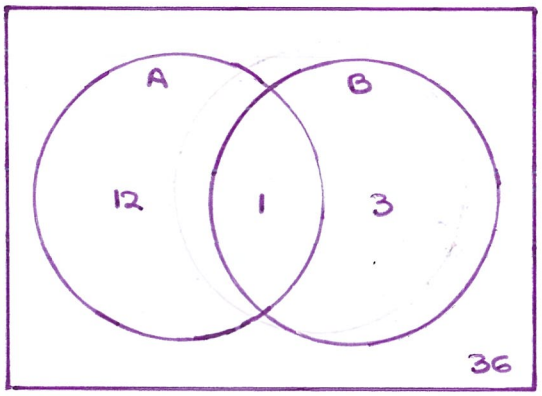
Marks

Question 16

A card is selected from a standard deck of 52 playing cards.

Event A – "the card is a heart"
Event B – "the card is a king".

(a) Draw a Venn Diagram for Events A and B. **2**



(b) Find:

(i) $n(A \cup B)$ **1**

$n(A \cup B) = 12 + 1 + 3$

$= 16$ ✓ R/W

(ii) P(not A) **1**

$P(\text{not } A) = \frac{3+36}{52}$

$= \frac{39}{52}$

$= \frac{3}{4}$ ✓ R/W

(iii) P(A and not B) **1**

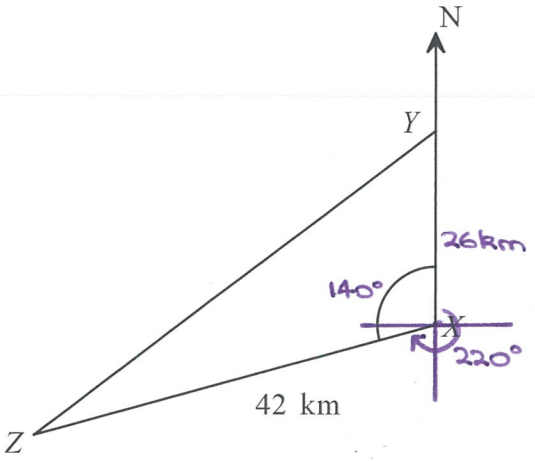
$P(A \text{ and not } B) = \frac{12}{52}$

$= \frac{3}{13}$ ✓ R/W

Marks

Question 17

The bearing of Z from X is $220°$ and the distance between Z and X is 42 km.



(a) What is the size of $\angle YXZ$? **1**

$\angle YXZ = 360° - 220°$

$= 140°$ ✓ R/W

(b) If Y is 26 km due north of X, calculate the distance between Z and Y, correct to 2 decimal places. **2**

$c^2 = a^2 + b^2 - 2ab\cos C$

$YZ^2 = 26^2 + 42^2 - 2(26)(42)\cos 140°$ ✓

$YZ^2 = 4113.041064...$

$YZ = \sqrt{4113.041064...}$

$YZ = 64.13299513...$

$YZ \approx 64.13\text{ km}$ ✓

2	Correct $YZ = 64.13$km
1	Correct substitution

Marks

Question 18

The distance, d km, it is possible to see on a clear day is proportional to the square root of the height, h m, above sea level.
Standing on a pier, 4 m above sea level, it is possible to see a distance of 10 km.

(a) Find a formula for d, in terms of h. **2**

$d \propto \sqrt{h}$ ..... When $d=10$ and $h=4$

$d = k\sqrt{h}$ ..... $10 = k \times \sqrt{4}$

..... $10 = 2k$

..... $k = 5$

.....

..... $\therefore d = 5\sqrt{h}$

2	Correct $d=5\sqrt{h}$
1	Working towards a proportion statement

(b) Standing on top of the cliffs I can see a distance of 35 km. **1**
How high are the cliffs?

Let $d=35$

$35 = 5\sqrt{h}$

$7 = \sqrt{h}$

$h = 7^2 = 49$ m ✓ R/W

Question 19

Solve $y = 3x^2 - 6x + 2$ using the quadratic formula. Leave your answer as a simplified surd. **2**

$y = 3x^2 - 6x + 2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$ ✓

$= \dfrac{6 \pm \sqrt{12}}{6}$

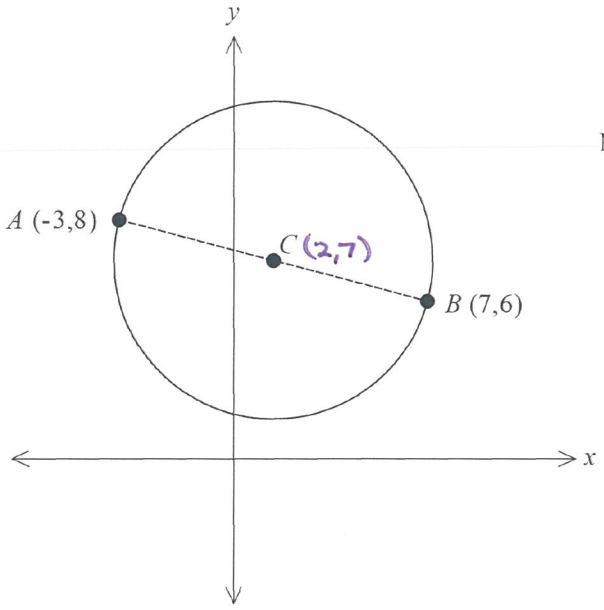
$= \dfrac{6 \pm 2\sqrt{3}}{6}$

$= \dfrac{3 \pm \sqrt{3}}{3}$ ✓

2	Correct answer
1	Correct substitution

Marks

Question 20



NOT TO SCALE

The circle, shown above, has

- centre, C
- diameter with end points A and B.

(a) Find the coordinates of the centre C of the circle. **1**

$M(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$C(x,y) = \left(\frac{-3+7}{2}, \frac{8+6}{2}\right)$

$= (2,7)$

$\therefore C(2,7)$ ✓ R/W

(b) Find the exact length of the radius of the circle. **2**

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$AC = \sqrt{(2--3)^2 + (7-8)^2}$ ✓

$AC = \sqrt{26}$

$\therefore$ radius is $\sqrt{26}$ units ✓

2	Correct $\sqrt{26}$ units
1	Correct substitution

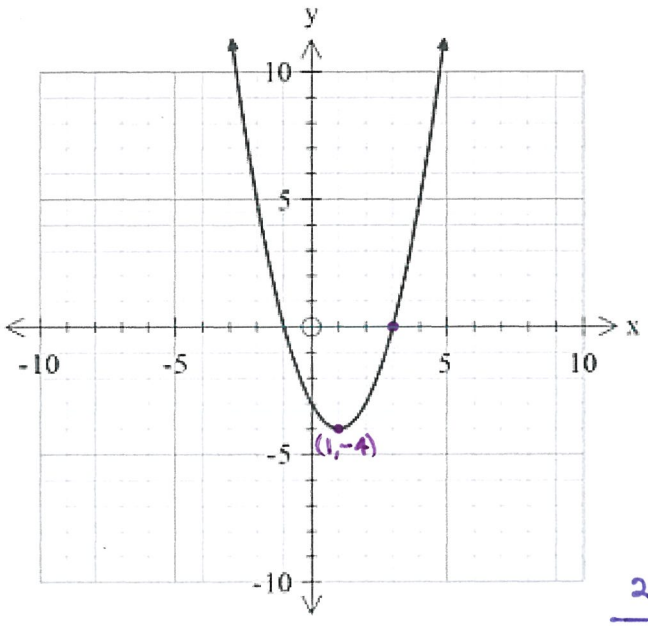
(c) State the equation of the circle. **1**

$(x-2)^2 + (y-7)^2 = 26$ ✓ R/W

Marks

Question 21

What is the equation of the following parabola in the form $y = a(x-h)^2 + k$? **2**



2	Correct $y=(x-1)^2-4$
1	Correct $a=1$

Vertex is $(1,-4)$ ... Let $x=3$, $y=0$

$y=a(x-h)^2+k$ ... $0=a(3-1)^2-4$

$y=a(x-1)^2-4$ ... $4=4a$

... $a=1$ ✓

... $\therefore y=(x-1)^2-4$ ✓

Question 22

Solve the following simultaneous equations. **2**

$$5x - y = 1.5 \quad [1]$$
$$2x - y = -2.1 \quad [2]$$

[1] − [2] : $3x = 3.6$

$x = 1.2$ ✓

Substitute $x=1.2$ into [1]

$5(1.2) - y = 1.5$

$y = 4.5$ ✓

$\therefore x = 1.2$ and $y = 4.5$

2	Correct $y=4.5$
1	Correct $x=1.2$

Marks

Question 23

If \$7420 is invested in a building society for 10 years with an interest rate of 8% p.a. compounding every six months, how much money is in the account after 10 years? **2**

$A = P(1+r)^n$

$= \$7420\left(1+\frac{8\%}{2}\right)^{10\times 2}$ ✓

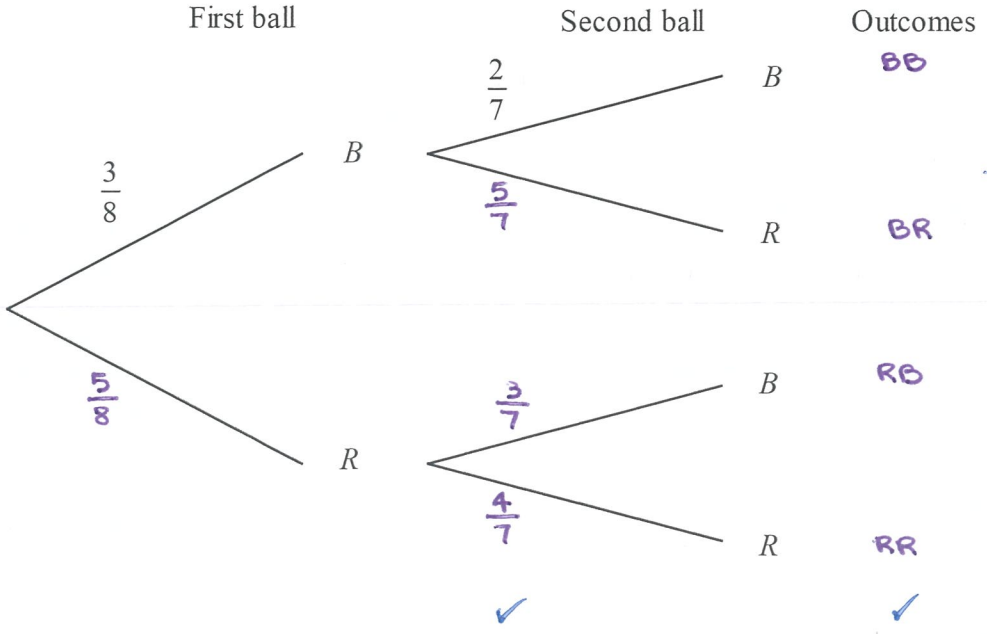
$\approx \$16258.13$ ✓

2	Correct answer
1	Correct rate and number of time period conversion

Question 24

From a bag containing 5 red and 3 blue balls, a ball is taken at random and not replaced. A second ball is then withdrawn and placed beside it.

(a) Complete the following probability tree. **2**



✓ ✓

2	Correct outcomes
1	Correct probability

(b) What is the probability that the second ball is blue given that the first ball is red? **1**

$P(B|R) = \frac{3}{7}$ ✓ R/W

Marks

Question 25

Harriett bought a motorbike for $7000. She paid a 15% deposit and agreed to pay the remaining balance with equal monthly payments over 5 years at 12% p.a simple interest.

(a) How much was owing on the motorbike after the deposit was paid? **1**

Amount owing = $7000 - (15% × $7000)
= $7000 - $1050
= $5950 ✓ R/W

(b) What was the monthly repayment? **2**

Total owing = $5950 + ($5950 × 12% × 5)
= $5950 + $3570
= $9520 ✓

Monthly replacement = $\frac{\$9520}{5 \times 12}$
= $\frac{\$9520}{60}$
≈ $158.67 ✓

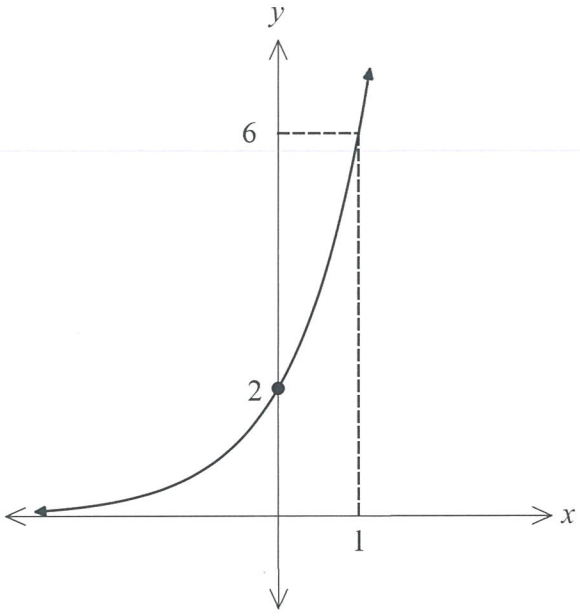
2	Correct monthly repayment
1	Correct total owing

(c) How much interest did Harriett have to pay? **1**

Interest = $5950 × 12% × 5
= $3570 ✓ R/W

Marks

Question 26



The diagram shows the graph of $y = m(a^x)$. **2**

Find the values of m and a.

Let $x=0$, $y=2$ ..... Let $x=1$, $y=6$

$2 = m(a^0)$ ..... $6 = 2(a^1)$

$m = 2$ ✓ ..... $a = 3$ ✓

$\therefore y = 2(a^x)$ ..... $\therefore y = 2(3^x)$

2	Correct $a=3$
1	Correct $m=2$

Marks

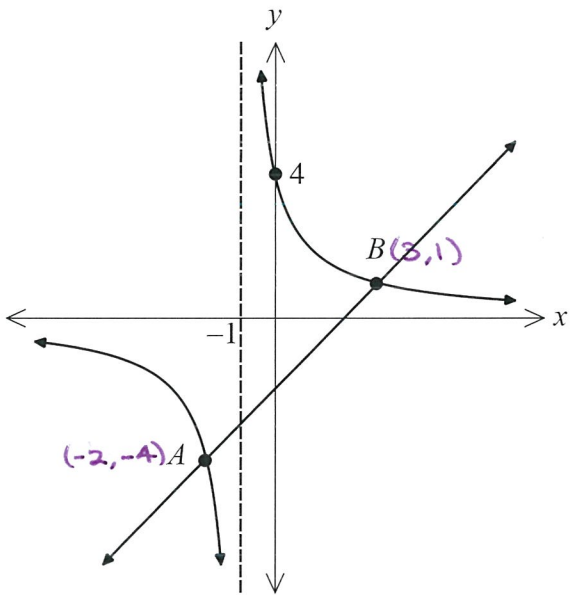
Question 27

On the diagram below the graphs of $y = \dfrac{4}{x+1}$ and $x - y - 2 = 0$ are drawn. **3**

Their points of intersection are labelled A and B.

Solve a pair of simultaneous equations to find the coordinates of A and B.

3	Correct $A(-2,-4)$ $B(3,1)$
2	Correct $x=-2$, $x=3$
1	Correct line [1]



$y = \dfrac{4}{x+1}$ [1]

$x - y - 2 = 0$ [2]

Substitute [1] into [2]

$x - \dfrac{4}{x+1} - 2 = 0$ ✓ [1]

$x(x+1) - 4 - 2(x+1) = 0$

$x^2 + x - 4 - 2x - 2 = 0$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$\therefore x = -2, \; x = 3$ ✓

Substitute $x = -2$ into [1]

$y = \dfrac{4}{-2+1}$

$= -4$

Substitute $x = 3$ into [1]

$y = \dfrac{4}{3+1}$

$= 1$

$\therefore$ the coordinates of A and B are $A(-2,-4)$ and $B(3,1)$ ✓

Marks

Question 28

The distance between the points $(-2, -1)$ and $(a, 3)$ is $\sqrt{20}$. Find the values(s) of a. **2**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{20} = \sqrt{(a - -2)^2 + (3 - -1)^2}$ ✓

$20 = (a+2)^2 + 16$

$4 = (a+2)^2$

$\pm 2 = a + 2$

$a = -2 \pm 2$

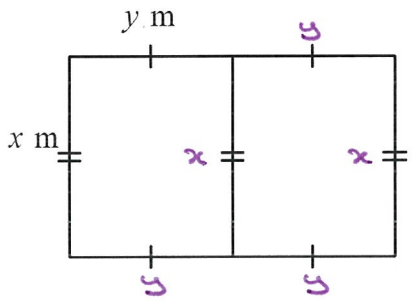
$\therefore a = 0$ or $a = -4$ ✓

2	Correct $a = 0$ $a = -4$
1	Correct substitution

Marks

Question 29

1600 metres of fencing is to be used to enclose a rectangular area and to divide it into two equal areas as shown.



(a) Using the pronumerals given, show that the combined enclosed area A is given by $A = 800x - \frac{3}{2}x^2$. **2**

Fencing: $3x + 4y = 1600$

$4y = 1600 - 3x$

$y = 400 - \frac{3}{4}x$ ✓

$A = 2xy$

$= 2 \times x\left(400 - \frac{3}{4}x\right)$

$= 2\left(400x - \frac{3}{4}x\right)$ ✓

$= 800x - \frac{3}{2}x^2$

1	$3x + 4y = 1600$
2	sub y into $A = 2x\left(400 - \frac{3}{4}x\right)$

(b) Hence, find the values of x and y for which the area enclosed is greatest. **3**

$A = 800x - \frac{3}{2}x^2$

Axis of symmetry: $x = -\frac{b}{2a}$

$x = -\frac{800}{2\left(-\frac{3}{2}\right)}$

$x = \frac{800}{3}$ ✓

For $A = 800x - \frac{3}{2}x^2$, $a = -\frac{3}{2} < 0$, which is concave down, therefore the parabola has a maximum turning point. ✓

$y = 400 - \frac{3}{4}\left(\frac{800}{3}\right)$

$= 200$ ✓

$\therefore x = \frac{800}{3}$, $y = 200$

1	• $x = \frac{800}{3}$ uses $x = \frac{-b}{2a}$
2	$x = \frac{800}{3}$ and $y = 200$
3	$x = \frac{800}{3}$ and $y = 200$ and $a = -\frac{3}{2}$ ∴ conc down for max or sketch showing max value

Marks

Question 30

Two fair six-sided dice are rolled. The number on the upper face of the first die determines the coefficient b and the upper face of the second die determines the value of c in the quadratic equation $x^2 + bx + c = 0$. **3**

With the use of an array, or otherwise, determine the probability that the equation formed will have two distinct, real solutions.

Number of event space

2nd die (c) \ 1st die (b)	1	2	3	4	5	6
1	×	×	✓	✓	✓	✓
2	×	×	✓	✓	✓	✓
3	×	×	×	✓	✓	✓
4	×	×	×	×	✓	✓
5	×	×	×	×	✓	✓
6	×	×	×	×	✓	✓

[1] – , – , – • uses $\Delta = b^2 - 4ac$, $\Delta > 0$ (or) • 36 outcomes (or) • starts an array

[2] 2 of above and progress towards soln

[3] correctly calcs $P(E) = \frac{17}{36}$

$x^2 + bx + c = 0$

$\Delta = b^2 - 4ac$

$= b^2 - 4(1)c$

$= b^2 - 4c$

Required event: $\Delta > 0$ ✓

Number of sample space $= 6 \times 6$

$= 36$

OR

Number of event space

$b = 1$, $1 - 4c > 0$, no c

$b = 2$, $4 - 4c > 0$, no c

$b = 3$, $9 - 4c > 0$, $c = 1, 2$

$b = 4$, $16 - 4c > 0$, $c = 1, 2, 3$

$b = 5$, $25 - 4c > 0$, $c = 1, 2, 3, 4, 5, 6$

$b = 6$, $36 - 4c > 0$, $c = 1, 2, 3, 4, 5, 6$ ✓

$\therefore P(\text{2 distinct real solutions}) = \frac{17}{36}$ ✓

END OF EXAMINATION

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

5.3 Pathway MATHEMATICS FORMULA SHEET

Circumference of a circle

$C = 2\pi r$ or $C = \pi d$

Arc length of a circle

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

Circle

$A = \pi r^2$

Sector

$$A = \frac{\theta}{360} \times \pi r^2$$

Area of a Parallelogram

$A = bh$

Area of a Trapezium

$$A = \frac{h}{2}(a+b)$$

Area of a Rhombus

$$A = \frac{1}{2}xy$$

Surface Area

Closed cylinder

$A = 2\pi r^2 + 2\pi rh$

Cone

$SA = \pi r^2 + \pi rl$

Sphere

$A = 4\pi r^2$

Volume

Prism or cylinder

$V = Ah$

Pyramid or cone

$$V = \frac{1}{3}Ah$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

Volume and capacity

unit conversion: 1 m^3 = 1000 L

Pythagoras' Theorem

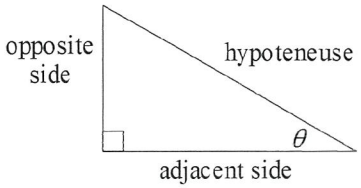
$c^2 = a^2 + b^2$

Distance, Speed, Time

$$D = ST, \quad S = \frac{D}{T}, \quad T = \frac{D}{S}$$

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Trigonometric Ratios



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine Rule

In ΔABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule

In ΔABC, $c^2 = a^2 + b^2 - 2ab\cos C$

or $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Area of a Triangle

In ΔABC, $A = \dfrac{1}{2}ab\sin C.$

Financial Mathematics

Simple Interest

$I = Prn$

Compound Interest

$A = P(1+r)^n$

Angle sum of a Polygon

$S = (n-2)\times 180^0$

Factorisation

$a^2 - b^2 = (a+b)(a-b)$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of two points

$$x = \frac{x_1 + x_2}{2},\quad y = \frac{y_1 + y_2}{2}$$

Slope (gradient) of a line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Gradient-intercept form

$y = mx + c$

Point-gradient form

$y - y_1 = m(x - x_1)$

Equation of a circle

$(x-h)^2 + (y-k)^2 = r^2$

Data Analysis

Mean

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

Probability of an Event

The probability of an event where outcomes are equally likely is given by:

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$