PENRITH SELECTIVE HIGH SCHOOL
MATHEMATICS DEPARTMENT

Assessment Task 4
Term 4 – 2024
Year 10 Mathematics

General Instructions:

Reading time – **5 minutes**
Working time – **75 minutes**

Total marks – 65

- Write using black or blue pen.
  Black pen is preferred.
- NESA approved calculators may be used.
- No correction tape or white out allowed
- Attempt ALL questions.
- All necessary working should be shown in each question.

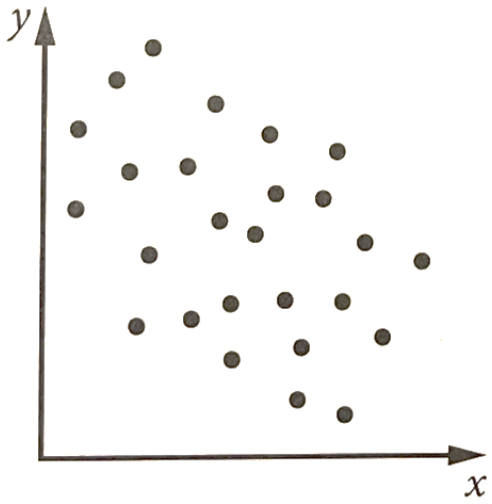
Topic	Section 1	Section 2	Section 3	Section 4	Section 5	TOTAL
Bivariate Data and Normal Distribution	**/3** 1, 3	**/3** 4		**/8** 1, 4	**/2** 1	**/16**
Direct and Indirect Variation	**/2** 4	**/1** 1	**/5** 1			**/8**
Logarithm		**/3** 2			**/6** 2	**/9**
Non-linear Relationships	**/2** 6	**/2** 3		**/2** 3		**/6**
Probability	**/2** 2	**/3** 5	**/5** 2	**/4** 2		**/14**
Quadratic Functions and Graphs	**/3** 5		**/4** 3		**/5** 3	**/12**
TOTAL	**/12**	**/12**	**/14**	**/14**	**/13**	**/65**
						%

Student Name: _____ Teacher: _____

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Section 1 (12 marks)

Q1) Describe the correlation of the scatterplot below: (Circle one letter) **1**



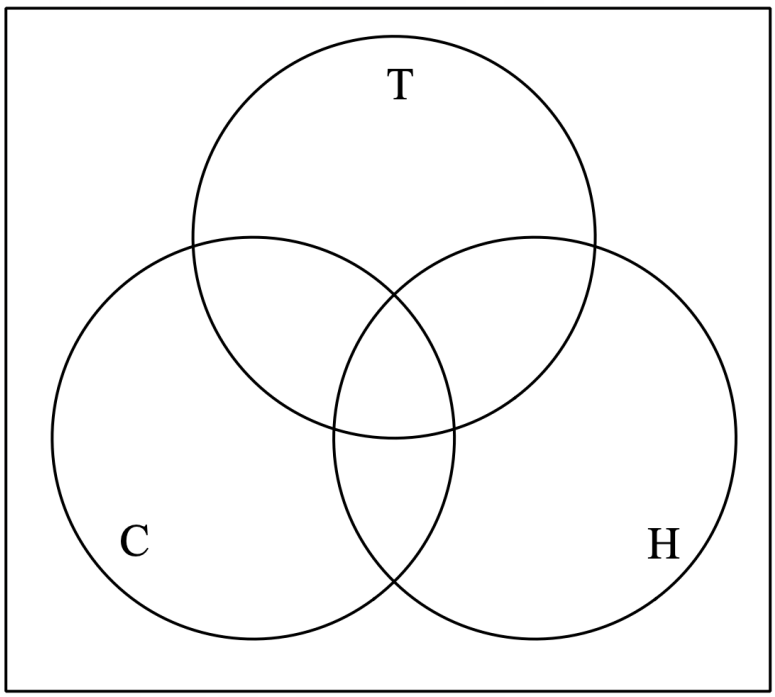
A Weak positive correlation

B Strong positive correlation

C Weak negative correlation

D Strong negative correlation

Q2) In the Venn Diagram below, shade in the region that is $C \cap (T \cup H)$. **2**



Q3) Carlisle scores 60 on his Music test and 58 on his Economics test. The class results of both tests are normally distributed, with the means and standard deviations as shown. **2**

Subject	Mean	Standard Deviation
Music	59	5
Economics	55	7.5

Carlisle claims that he performed better in Music. Is he correct? Justify your answer with appropriate calculation.

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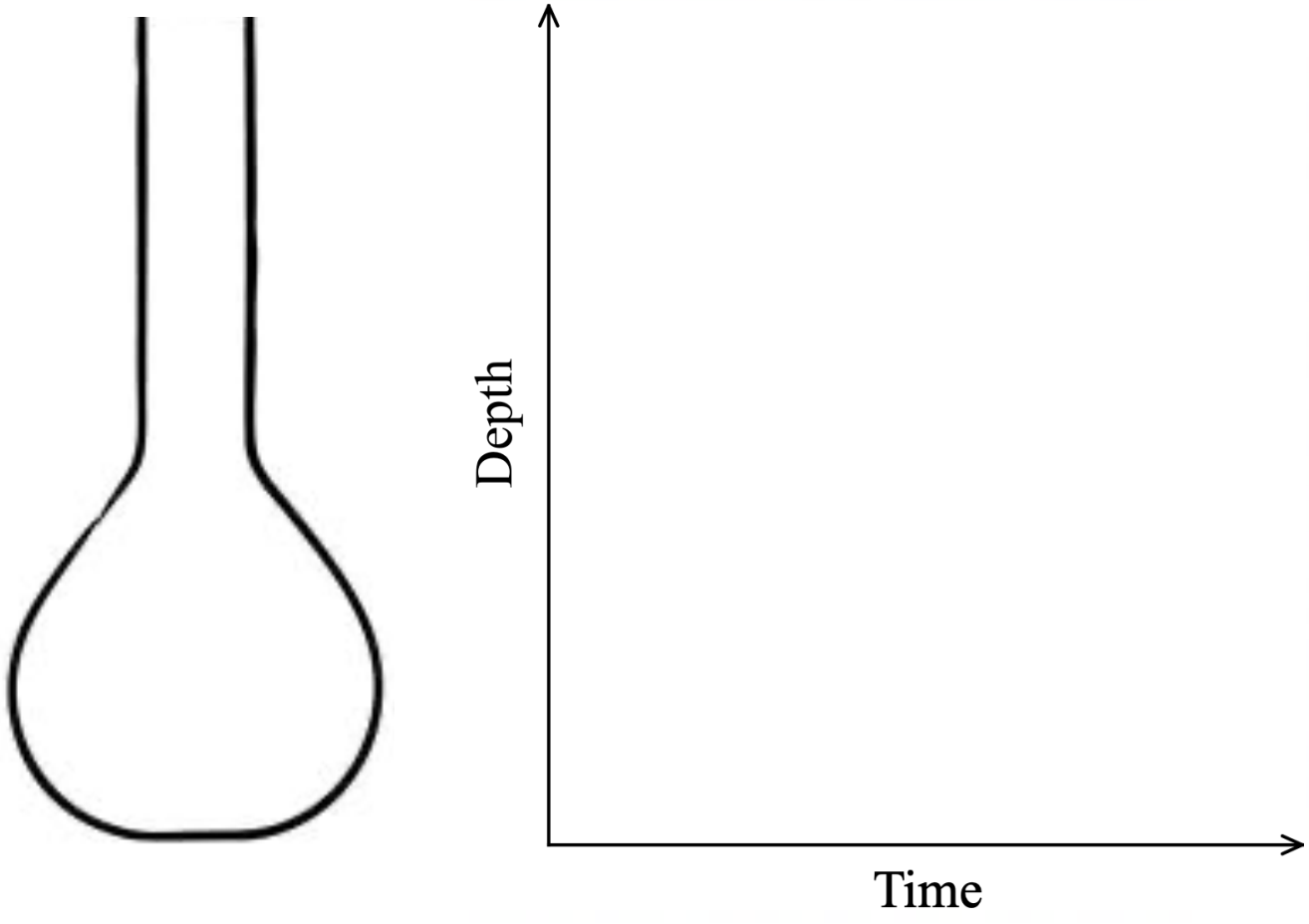
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Q4) A scientist is filling up a flask with a solution, pouring at **a constant rate**. Roughly sketch how the depth of the solution in the flask would change over time on the provided set of axes. (A scale is not required.) **2**



Q5) Given the function $f(x) = 7x^2 - 2x + 1$, find:

(a) $f(5)$ **1**

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(b) $f(3x - 4)$ **2**

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Q6) For the equation $x^2 - 14x + y^2 - 4y + 28 = 0$, state the domain and range of the graph of this equation. **2**

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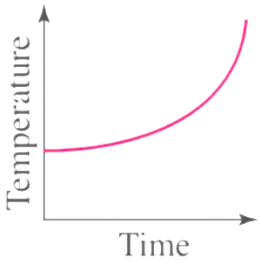
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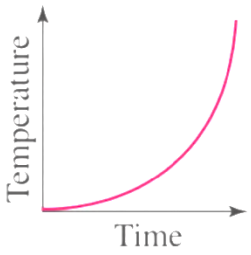
Section 2 (12 marks)

Q1) A pot of room temperature water is being brought to a boil to cook some pasta. It is known that when water boils, the temperature increases at a decreasing rate. Which graph best shows the temperature of the water against time? (Circle one letter) **1**

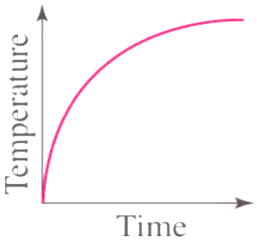
A



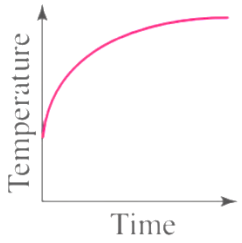
B



C



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Q2) Solve for x **3**

$$\log_{10} 2 + \log_{10} 5 + \log_{10} x - \log_{10} 3 = 2$$

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Q3) Find the equation of the function $f(x) = 3^x$ after it has been horizontally translated to the left by 4 units and vertically translated down by 1 unit. **2**

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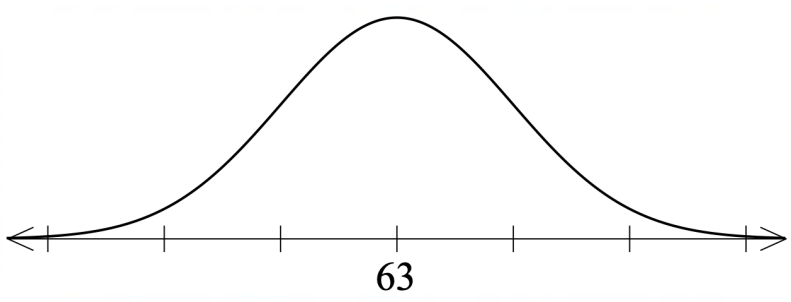
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Q4) The final examination results of 600 students are normally distributed, with a mean of 63 and standard deviation of 6.

(a) Complete the scale of the bell curve using the information above. **1**



(b) Using the empirical rule, find the number of students who scored more than 57? **2**

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Q5) For 2 independent events A and B, we are given that $P(B) = 40\%$ and $P(A \cup B) = 67\%$. Find $P(A \cap B)$. **3**

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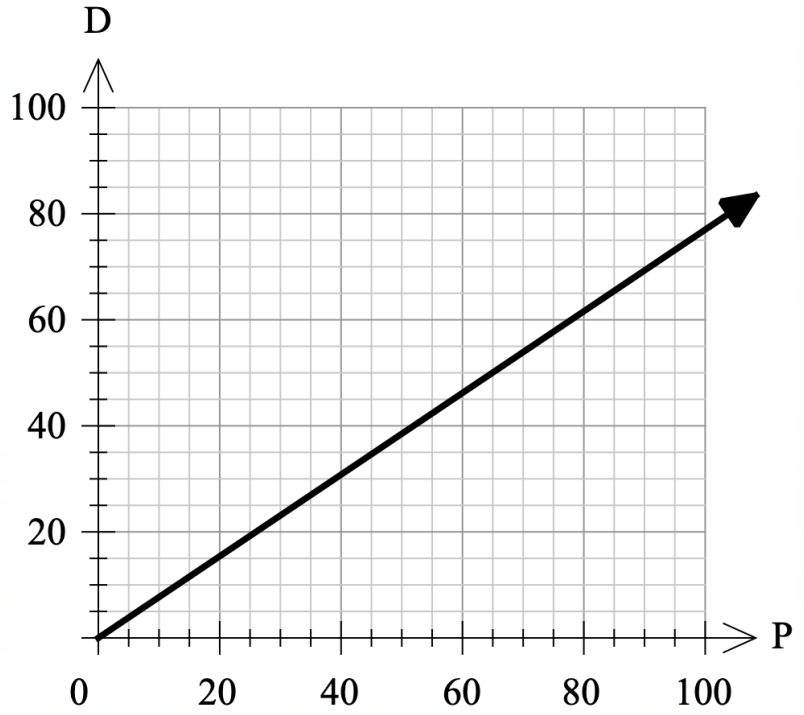
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Section 3 (14 marks)

Q1) The relationship between US Dollar (D) and British Pound Sterling (P) is given in the graph below.



(a) Write the relationship between D and P in the form of $D = kP$, express k in exact value. **2**

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(b) The current exchange rate for USD to Australian Dollar (A) is $D = 1.49A$. An Australian tourist is looking to receive £1000 (British Pound Sterling). How much AUD will need to be exchanged? (Correct to 2 decimal places). **3**

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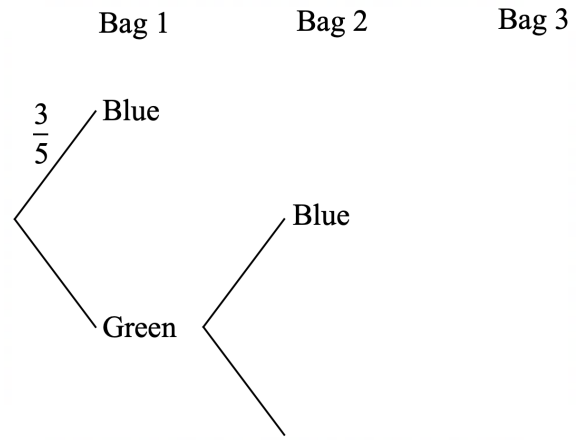
Q2) At the Penrith High School Annual Summer Carnival, a popular game involves picking out coloured balls from 3 bags. The rule of the game is as follows:

If a green ball is chosen from the bag, the round continues, and the player moves on to the next bag. If a blue ball is chosen, the game ends. To win the game, the player must choose a green ball from all three bags.

The content of the bag is as following:

- Bag 1: 2 green balls, 3 blue balls
- Bag 2: 2 green balls, 4 blue balls
- Bag 3: 2 green ball, 5 blue balls

(a) Complete the probability tree diagram below to represent the events. **2**



(b) Find the probability of winning the game in simplest fraction form. **1**

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(c) To win the jackpot, the player will need to win a second round, where the balls from the first round are not replaced. Find the probability of picking 3 green balls in the second round, hence find the probability of winning the jackpot. **2**

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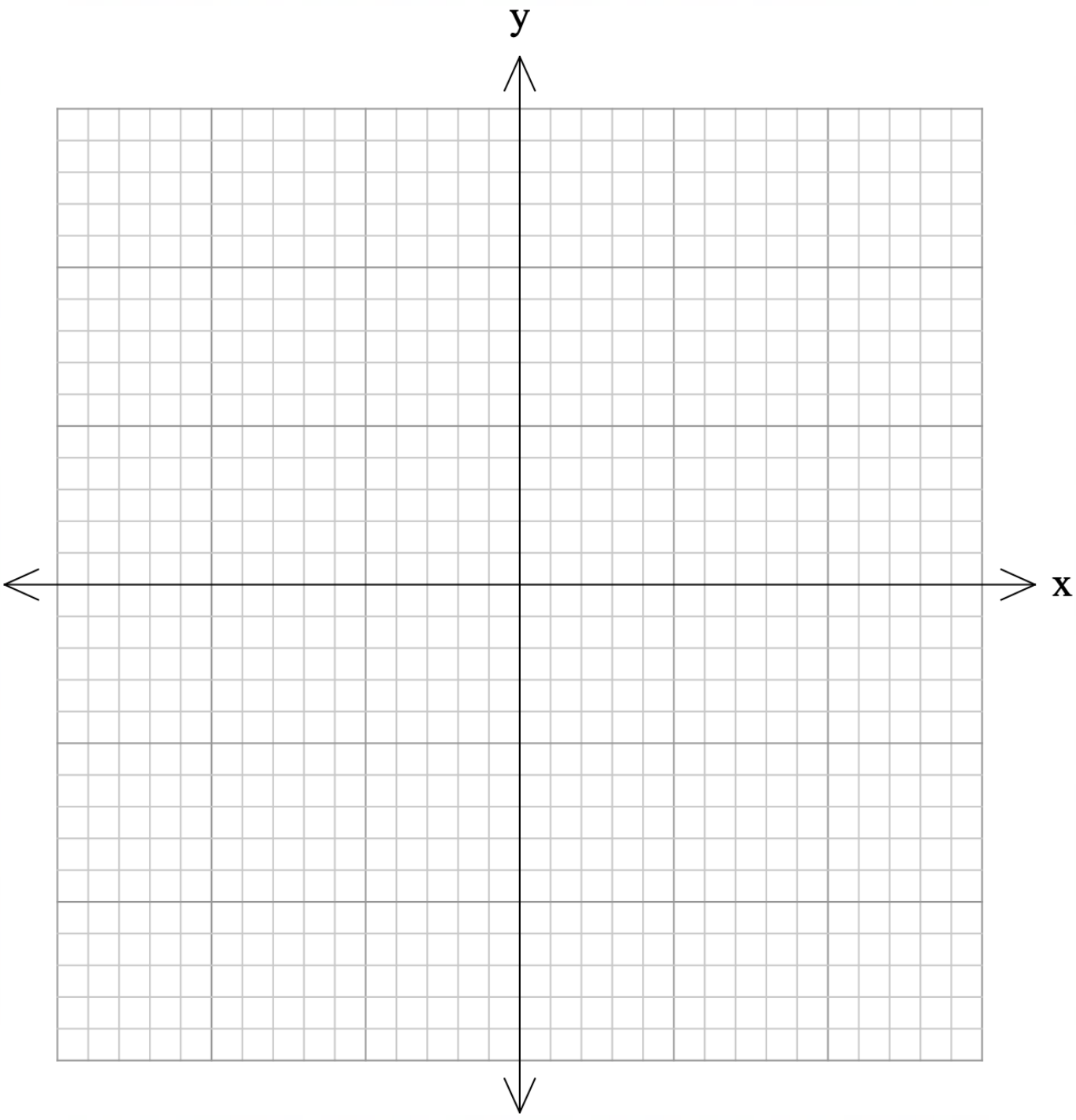
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Q3) Sketch the graph of $y = -x^2 + 5x + 6$, labelling all important features (vertex, axis of symmetry, all intercepts). **4**



Section 4 (14 marks)

Q1) Henry works at a database company that collects information on advertising spending and total revenue of Big Tech companies in the Greater Sydney area. The data for the financial year 2022-2023 is recorded in the table below.

Company	Advertising Spend ($ '000)	Total Revenue ($ '000)
1	14.5	59
2	19	64
3	22.4	89
4	28.9	86
5	30	94
6	32	104
7	29	89
8	28	82

(a) Calculate Pearson's Correlation Coefficient and use it to describe the linear correlation between the amount spent on advertising and total revenue. **2**

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(b) Isaac is the CEO at Ferrarin Tech and he wants to know what his company's revenue would be if he spend $25 000 in advertising. By calculating the equation of the least squares regression line, predict the revenue correct to the nearest thousand dollars. **2**

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(c) Is the prediction likely to be accurate? Justify your answer. **2**

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Q2) In a survey, 55% of the respondents liked to watch movies (M), 32% liked to play sport (S), and 30% likes neither.

(a) Find the probability of choosing a respondent who liked to play sport, given that they also liked to watch movies. **3**

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(b) Are the events M and S independent of each other? Justify your answer. **1**

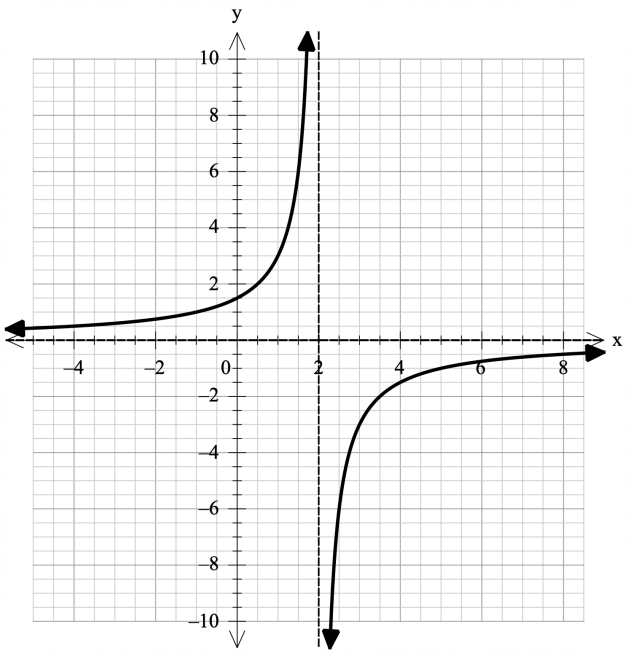
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Q3) Find the equation of the graph below. **2**



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Q4) A packet of "Healthy Choice" cereal is labelled as having a mass of 500 g. However, the machine that fills the packets actually follows a normal distribution with a mean of 510 g and a standard deviation of 5 g. What percentage of packets will have a mass greater than 500 g? **2**

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Section 5 (13 marks)

Q1) A set of scores is normally distributed. Out of 2000 scores, only 3 of the scores are greater than 28. The standard deviation of the data set is 2. What is the mean of the scores? **2**

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Q2) Jacinta is a seismologist. She is working on calculating the energy released by the recent earthquake in Indonesia that had a Richter magnitude of 5.3 . She uses the formula $\log_{10} E = 11.8 + 1.5M$ where E is the energy in kJ released and M is the Richter magnitude.

(a) Calculate the energy released by the 5.3 magnitude earthquake in Indonesia. Give your answer in scientific notation correct to 3 significant figures. **2**

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(b) Another earthquake 4 times as intense as this earthquake has been forecasted by the supercomputer. Calculate the Richter magnitude of this earthquake, hence find the energy release that it will have. Note: **4**

$$M = \log_{10}\left(\frac{I}{I_0}\right)$$

where I is the intensity of an earthquake and I_0 is the reference intensity.

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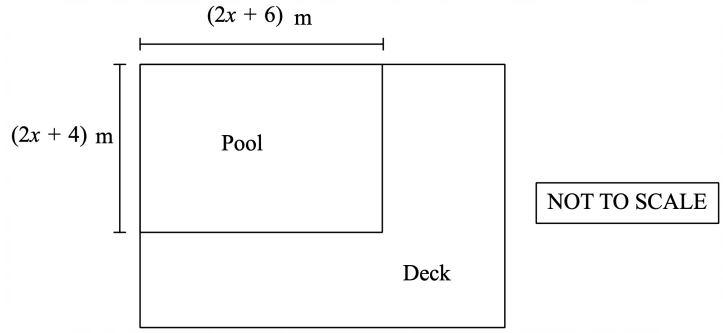
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Q3) Elena is designing a new rectangular pool. The pool will be surrounded by a deck on two sides. The length of the pool is 2 m more than the breadth (as shown). The length and breadth of the combined pool and deck is 50% more than the length and breadth of the pool.



(a) Show that the area of the desk in terms of x is given as $A = 5x^2 + 25x + 30$. **2**

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(b) If the area of the deck is 100 m^2, find the value of x. **3**

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End of Paper

Extra Writing Paper

PENRITH SELECTIVE HIGH SCHOOL
MATHEMATICS DEPARTMENT

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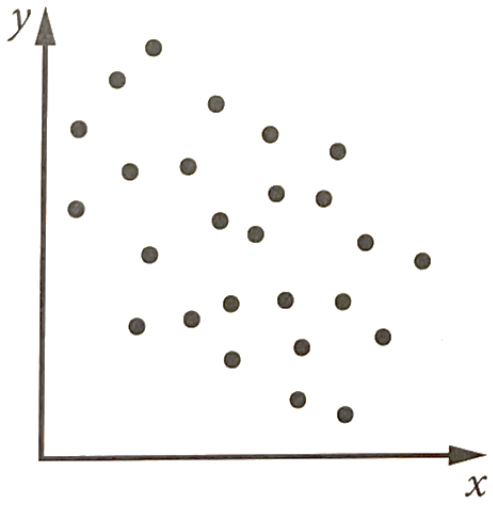
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Direct and Indirect Variation	**/2** 4	**/1** 1	**/5** 1			**/8**
Logarithm		**/3** 2			**/6** 2	**/9**
Non-linear Relationships	**/2** 6	**/2** 3		**/2** 3		**/6**
Probability	**/2** 2	**/3** 5	**/5** 2	**/4** 2		**/14**
Quadratic Functions and Graphs	**/3** 5		**/4** 3		**/5** 3	**/12**
TOTAL	**/12**	**/12**	**/14**	**/14**	**/13**	**/65**
						%

Student Name: _____ Teacher: _____

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Section 1 (12 marks)

Q1) Describe the correlation of the scatterplot below: (Circle one letter) **1**



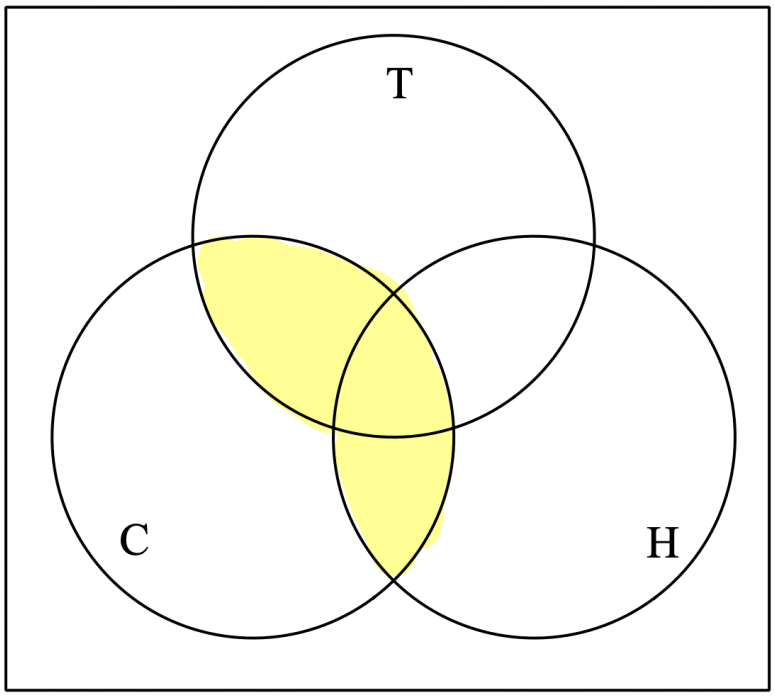
A Weak positive correlation

B Strong positive correlation

C Weak negative correlation

D Strong negative correlation

Q2) In the Venn Diagram below, shade in the region that is $C \cap (T \cup H)$. **2**



Q3) Carlisle scores 60 on his Music test and 58 on his Economics test. The class results of both tests are normally distributed, with the means and standard deviations as shown. **2**

Subject	Mean	Standard Deviation
Music	59	5
Economics	55	7.5

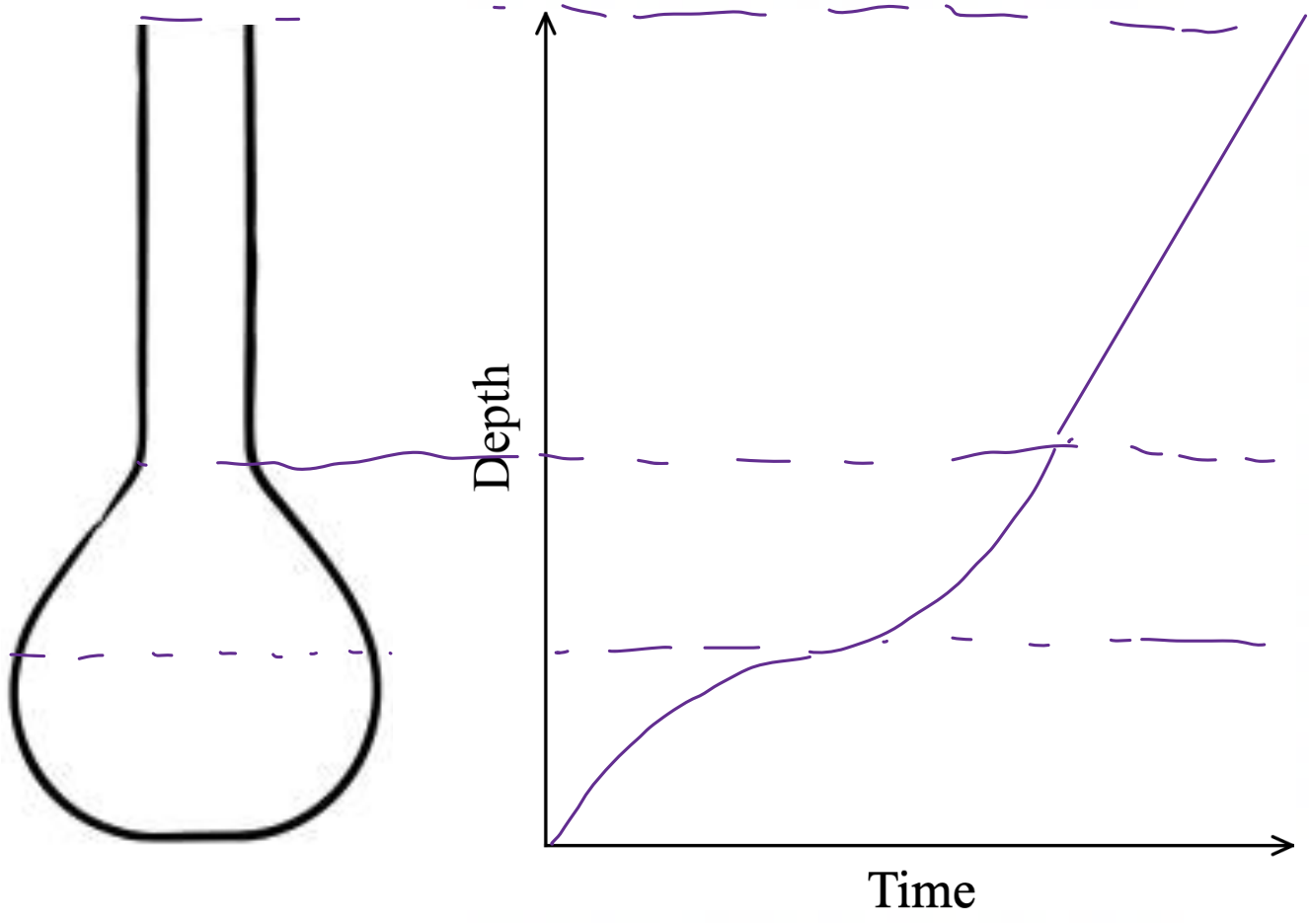
Carlisle claims that he performed better in Music. Is he correct? Justify your answer with appropriate calculation.

$z_m = \frac{x-\mu}{\sigma}$ $z_e = \frac{58-55}{7.5}$

$= \frac{60-59}{5}$ $= \frac{2}{5}$

$= \frac{1}{5}$ Carlisle is wrong as $z_m < z_e$. He performed better in Economics.

Q4) A scientist is filling up a flask with a solution, pouring at **a constant rate**. Roughly sketch how the depth of the solution in the flask would change over time on the provided set of axes. (A scale is not required.) **2**



Q5) Given the function $f(x) = 7x^2 - 2x + 1$, find:

(a) $f(5)$ **1**

$f(5) = 7(5)^2 - 2(5) + 1$

$= 175 - 10 + 1$

$= 166$

(b) $f(3x - 4)$ **2**

$f(3x-4) = 7(3x-4)^2 - 2(3x-4) + 1$

$= 7(9x^2 - 24x + 16) - 6x + 8 + 1$

$= 63x^2 - 168x + 112 - 6x + 9$

$= 63x^2 - 174x + 121$

Q6) For the equation $x^2 - 14x + y^2 - 4y + 28 = 0$, state the domain and range of the graph of this equation. **2**

$x^2 - 14x + (7)^2 + y^2 - 4y + (2)^2 = -28 + 7^2 + 2^2$

$(x-7)^2 + (y-2)^2 = 25$

Centre $(7, 2)$

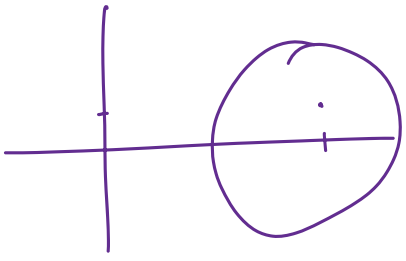
$r = 5$

Domain: $7 - 5 \leq x \leq 7 + 5$

$2 \leq x \leq 12$

Range: $2 - 5 \leq y \leq 2 + 5$

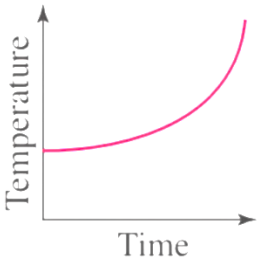
$-3 \leq y \leq 7$



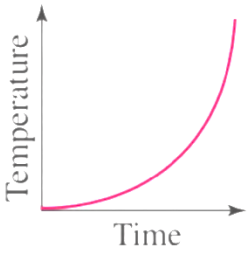
Section 2 (12 marks)

Q1) A pot of room temperature water is being brought to a boil to cook some pasta. It is known that when water boils, the temperature increases at a decreasing rate. Which graph best shows the temperature of the water against time? (Circle one letter) **1**

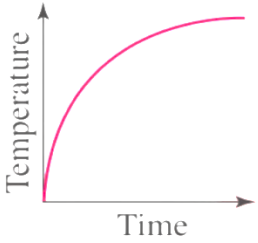
A



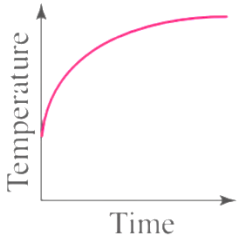
B



C



D



Q2) Solve for x **3**

$$\log_{10} 2 + \log_{10} 5 + \log_{10} x - \log_{10} 3 = 2$$

$\log_{10}\left(\frac{2 \times 5 \times x}{3}\right) = 2$

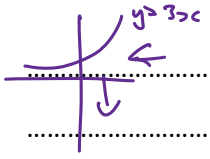
$\log_{10}\left(\frac{10x}{3}\right) = 2$

$10^2 = \frac{10x}{3}$

$3 \times 10^2 = 10x$

$x = 30$

Q3) Find the equation of the function $f(x) = 3^x$ after it has been horizontally translated to the left by 4 units and vertically translated down by 1 unit. **2**

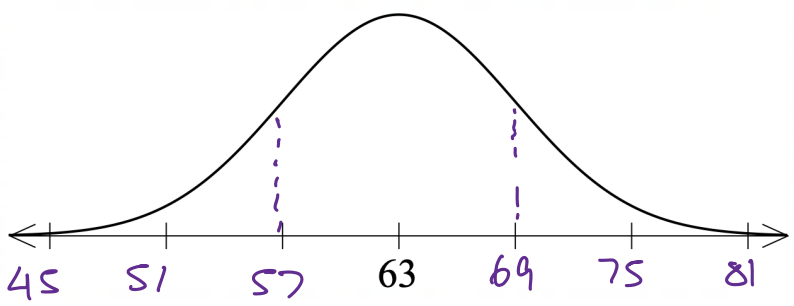


$f(x) = 3^{x+4}$

$f(x) = 3^{x+4} - 1$

Q4) The final examination results of 600 students are normally distributed, with a mean of 63 and standard deviation of 6.

(a) Complete the scale of the bell curve using the information above. **1**



(b) Using the empirical rule, find the number of students who scored more than 57? **2**

From 57 → 63 = 68 ÷ 2 = 34%

63 onward = 50%

34 + 50 = 84%.

Q5) For 2 independent events A and B, we are given that $P(B) = 40\%$ and $P(A \cup B) = 67\%$. Find $P(A \cap B)$. **3**

$P(A \cap B) = P(A) \times P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(B) = 0.4$ $P(A \cup B) = 0.67$

$P(A \cap B) = 0.4\,P(A)$

$0.67 = P(A) + 0.4 - 0.4P(A)$

$0.27 = 0.6\,P(A)$

$P(A) = 0.45$

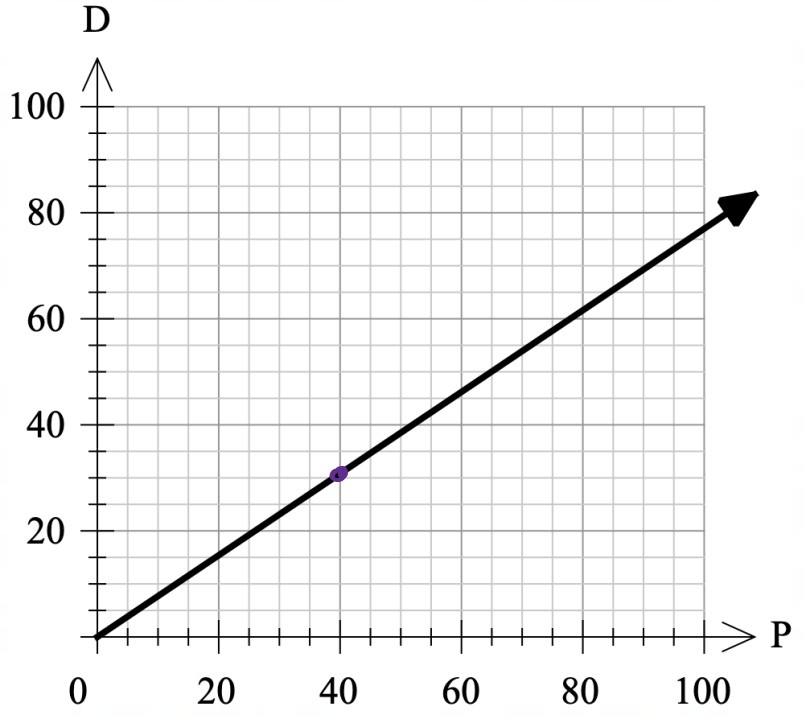
$P(A \cap B) = 0.4 \times 0.45$

$= 0.18$

$\therefore P(A \cap B) = 18\%$.

Section 3 (14 marks)

Q1) The relationship between US Dollar (D) and British Pound Sterling (P) is given in the graph below.



(a) Write the relationship between D and P in the form of $D = kP$, express k in exact value. **2**

$D = kP$

$k = \frac{D}{P}$

$= \frac{30}{40}$

$= \frac{3}{4}$

$\therefore D = \frac{3}{4}P$

(b) The current exchange rate for USD to Australian Dollar (A) is $D = 1.49A$. An Australian tourist is looking to receive £1000 (British Pound Sterling). How much AUD will need to be exchanged? (Correct to 2 decimal places). **3**

$D = \frac{3}{4}P$ - ①

$D = 1.49A$ - ②

$\frac{3}{4}P = 1.49A$ ① = ②

$P = \frac{149}{75}A$

$1000 = \frac{149}{75}A$

$A = 503.36$

$\therefore A = \$503.36$

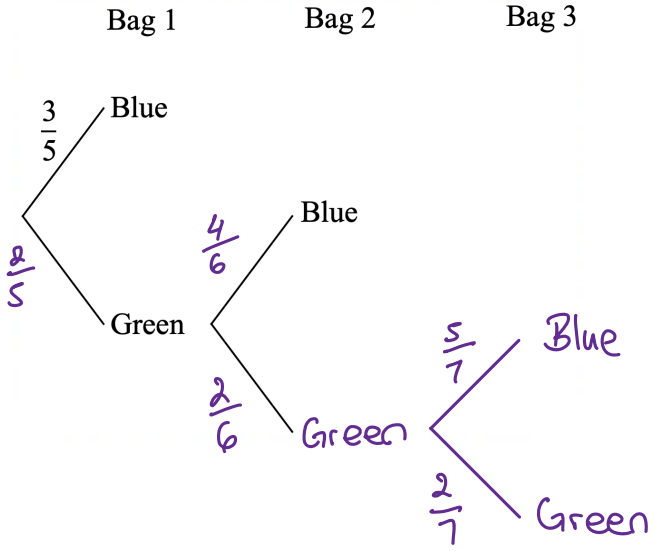
Q2) At the Penrith High School Annual Summer Carnival, a popular game involves picking out coloured balls from 3 bags. The rule of the game is as follows:

> *If a green ball is chosen from the bag, the round continues, and the player moves on to the next bag. If a blue ball is chosen, the game ends. To win the game, the player must choose a green ball from all three bags.*

The content of the bag is as following:

- Bag 1: 2 green balls, 3 blue balls
- Bag 2: 2 green balls, 4 blue balls
- Bag 3: 2 green ball, 5 blue balls

(a) Complete the probability tree diagram below to represent the events. **2**



(b) Find the probability of winning the game in simplest fraction form. **1**

$P(GGG) = \frac{2}{5} \times \frac{2}{6} \times \frac{2}{7}$

$= \frac{4}{105}$

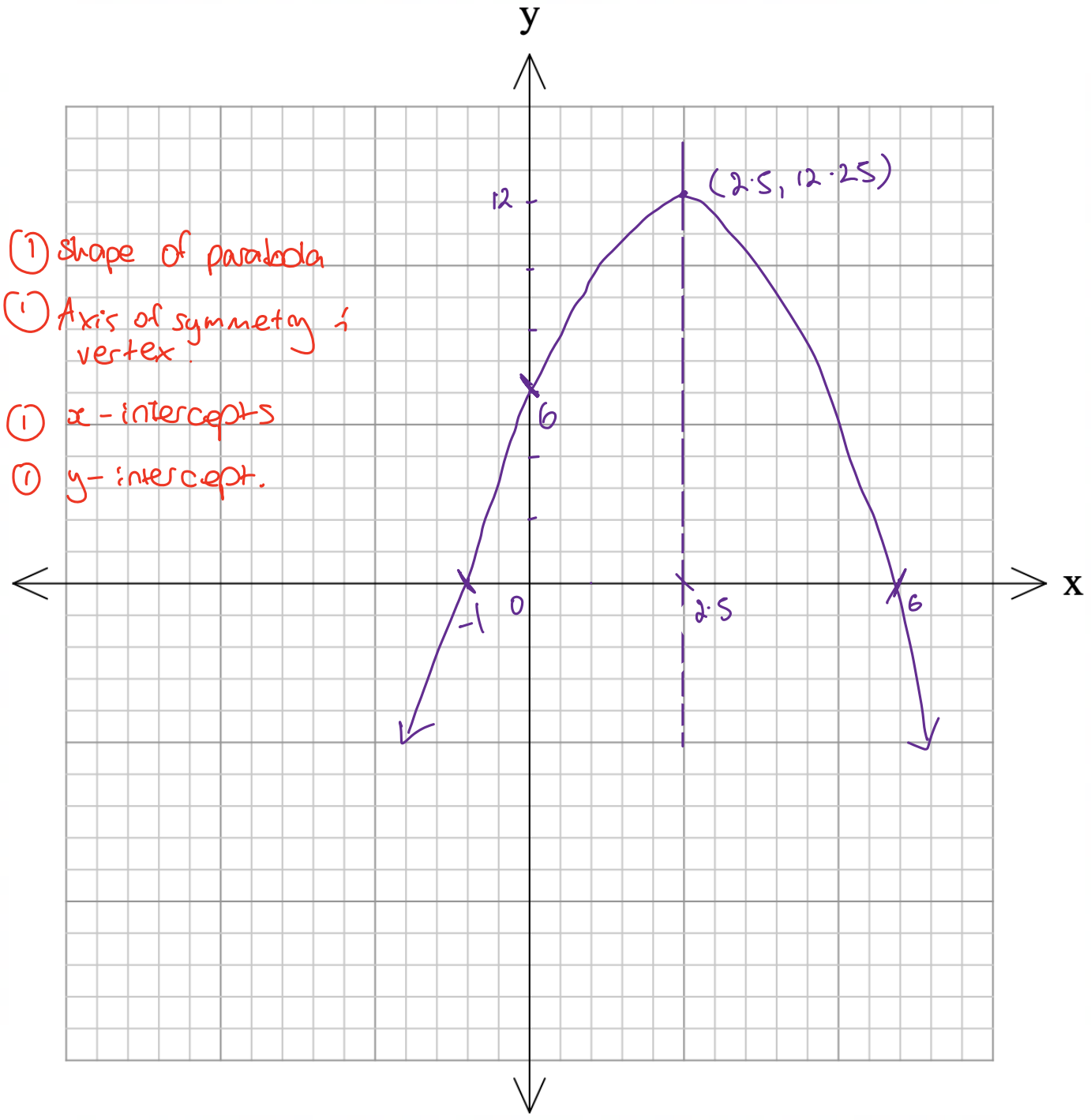
(c) To win the jackpot, the player will need to win a second round, where the balls from the first round are not replaced. Find the probability of picking 3 green balls in the second round, hence find the probability of winning the jackpot. **2**

$P(GGG \mid GGG) = \frac{4}{105} \times \left(\frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}\right)$

$= \frac{4}{105} \times \frac{1}{120}$

$= \frac{1}{3150}$

Q3) Sketch the graph of $y = -x^2 + 5x + 6$, labelling all important features (vertex, axis of symmetry, all intercepts). **4**



① Shape of parabola

① Axis of symmetry & vertex

① x-intercepts

① y-intercept.

y-int : $y = 6$

x-int : $y = 0$

$0 = -x^2 + 5x + 6$

$x^2 - 5x - 6 = 0$

$(x - 6)(x + 1) = 0$

$\therefore x = 6$ or $x = -1$

$x = \frac{-b}{2a}$

$= \frac{-5}{2(1)}$

$= \frac{5}{2} = 2{\cdot}5$

$y = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) + 6$

$= 12{\cdot}25$

$(2{\cdot}5, 12{\cdot}25)$

Section 4 (14 marks)

Q1) Henry works at a database company that collects information on advertising spending and total revenue of Big Tech companies in the Greater Sydney area. The data for the financial year 2022-2023 is recorded in the table below.

Company	Advertising Spend ($ '000)	Total Revenue ($ '000)
1	14.5	59
2	19	64
3	22.4	89
4	28.9	86
5	30	94
6	32	104
7	29	89
8	28	82

(a) Calculate Pearson's Correlation Coefficient and use it to describe the linear correlation between the amount spent on advertising and total revenue. **2**

$r = 0.901$ (3dp)

Strong positive linear correlation.

(b) Isaac is the CEO at Ferrarin Tech and he wants to know what his company's revenue would be if he spend $25 000 in advertising. By calculating the equation of the least squares regression line, predict the revenue correct to the nearest thousand dollars. **2**

$y = A + Bx$

$y = 27.353 + 2.199x$

$y = 27.353 + 2.199(25)$

$= 82.328$

$\therefore$ \$82 000 (nearest thousand)

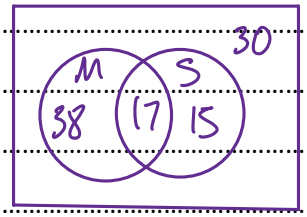
(c) Is the prediction likely to be accurate? Justify your answer. **2**

Likely to be accurate

- interpolation
- r-value is high.

Q2) In a survey, 55% of the respondents liked to watch movies (M), 32% liked to play sport (S), and 30% likes neither.

(a) Find the probability of choosing a respondent who liked to play sport, given that they also liked to watch movies. **3**



$M = 55$

$S = 32$

$M + S - M \cap S = 70$

$55 + 32 - (M \cap S) = 70$

$(M \cap S) = 17$

$P(S|M) = \frac{P(S \cap M)}{P(M)}$

$= \frac{0.17}{0.55}$

$= \frac{17}{55}$

(b) Are the events M and ~~X~~ S independent of each other? Justify your answer. **1**

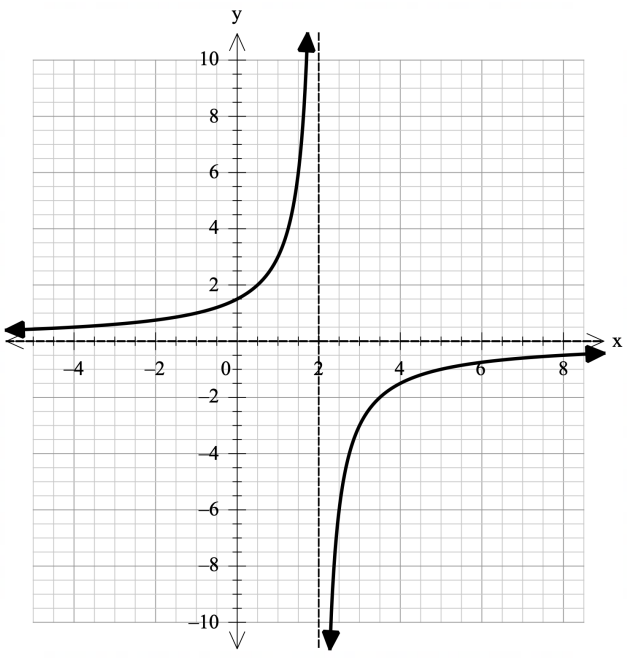
$P(S) = \frac{32}{100} = \frac{8}{25}$

$P(S|M) = \frac{17}{55}$

since $P(S) \neq P(S|M)$

∴ The events are not independent.

Q3) Find the equation of the graph below. **2**



$y = \frac{a}{x-b} + c$

asymptote at $x = 2$

$y = 0$

$y = \frac{a}{x-2}$

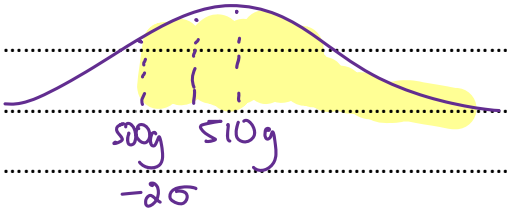
at $x = 0$, $y = \frac{3}{2}$

$\frac{3}{2} = \frac{a}{0-2}$

$a = -3$

$\therefore y = -\frac{3}{x-2}$

Q4) A packet of "Healthy Choice" cereal is labelled as having a mass of 500 g. However, the machine that fills the packets actually follows a normal distribution with a mean of 510 g and a standard deviation of 5 g. What percentage of packets will have a mass greater than 500 g? **2**



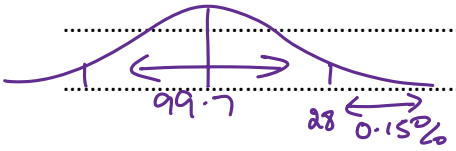
$50 + \frac{95}{2} = 97.5\%$

Section 5 (13 marks)

Q1) A set of scores is normally distributed. Out of 2000 scores, only 3 of the scores are greater than 28. The standard deviation of the data set is 2. What is the mean of the scores? **2**

$\frac{3}{2000} = 0.0015$

$= 0.15\%$.



28 is 3σ away from μ

$\mu + 3\sigma = 28$

$\mu + 3(2) = 28$

$\mu = 22$

Q2) Jacinta is a seismologist. She is working on calculating the energy released by the recent earthquake in Indonesia that had a Richter magnitude of 5.3 . She uses the formula $\log_{10} E = 11.8 + 1.5M$ where E is the energy in kJ released and M is the Richter magnitude.

(a) Calculate the energy released by the 5.3 magnitude earthquake in Indonesia. Give your answer in scientific notation correct to 3 significant figures. **2**

$M = 5.3$

$\log_{10} E = 11.8 + 1.5(5.3)$

$\log_{10} E = 19.75$

$E = 10^{19.75}$

$= 5.62 \times 10^{19}$ (3 sig fig)

(b) Another earthquake 4 times as intense as this earthquake has been forecasted by the supercomputer. Calculate the Richter magnitude of this earthquake, hence find the energy release that it will have. Note: **4**

$$M = \log_{10}\left(\frac{I}{I_0}\right)$$

where I is the intensity of an earthquake and I_0 is the reference intensity.

For $M = 5.3$

$5.3 = \log_{10}\left(\frac{I}{I_0}\right)$

$10^{5.3} = \frac{I}{I_0}$

$I = 10^{5.3} I_0$

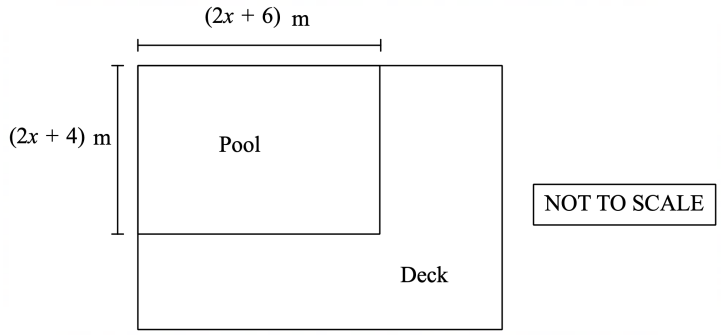
$4I = 4 \times 10^{5.3} I_0$

$M = \log_{10}\left(\frac{4 \times 10^{5.3} I_0}{I_0}\right)$

$M = \log_{10} 4 \times 10^{5.3}$

$\therefore M = 5.9$

Q3) Elena is designing a new rectangular pool. The pool will be surrounded by a deck on two sides. The length of the pool is 2 m more than the breadth (as shown). The length and breadth of the combined pool and deck is 50% more than the length and breadth of the pool.



(a) Show that the area of the desk in terms of x is given as $A = 5x^2 + 25x + 30$. **2**

Length = $(2x+6) \times \frac{3}{2}$

$= 3x + 9$

Breadth = $(2x+4) \times \frac{3}{2}$

$= 3x + 6$

Area = $(3x+9)(3x+6) - (2x+6)(2x+4)$

$= 9x^2 + 45x + 54 - (4x^2 + 20x + 24)$

$= 9x^2 - 4x^2 + 45x - 20x + 54 - 24$

$\therefore$ Area = $5x^2 + 25x + 30$

(b) If the area of the deck is 100 m^2, find the value of x, ~~correct to 3 decimal places~~. **3**

$5x^2 + 25x + 30 = 100$

$5x^2 + 25x - 70 = 0$

$x^2 + 5x - 14 = 0$

$(x+7)(x-2) = 0$

$\therefore x = 2$ or $x = -7$

since $x > -2$

$\therefore x = 2$

End of Paper

Extra Writing Paper