NORTH SYDNEY BOYS HIGH SCHOOL

STAGE 5.3 MATHEMATICS

2021 YEAR 10 YEARLY EXAMINATION

Thursday, 28 October 2021.

General instructions

- Working time – **90 minutes**.
- Write using **blue or black pen**. Where diagrams are to be sketched, these may be done in pencil.
- **Board approved** calculators may be used.
- Attempt all questions.
- Commence each new question on a **new page.**
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NAME: ..... **# BOOKLETS USED:** .....

Class: (please ✓)

- ○ Ms Fu
- ○ Ms Lee
- ○ Ms Moss
- ○ Mr Umakanthan
- ○ Ms Vranesevic

Marker's use only

Question	1	2	3	4	5	6	7	Total	%
MARKS	/16	/13	/14	/16	/10	/13	/14	/96	/100

Question 1 (16 marks) START A NEW PAGE Marks

a) $2^{n+1} + 2^{n+1}$ equals (Select A, B, C, or D) 1

A) 2^{n+2}
B) 2^{2n+2}
C) 4^{2n+2}
D) 4^{2n+1}

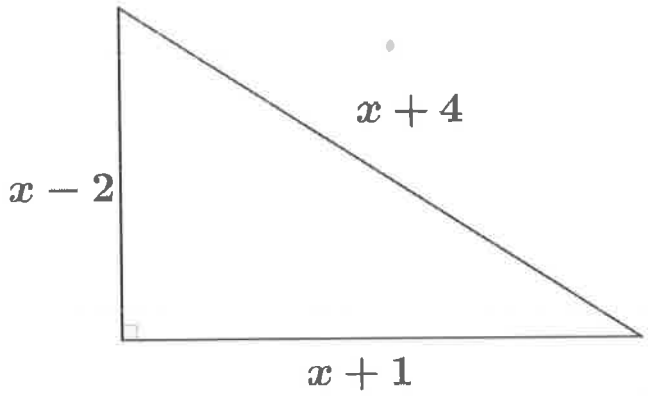
b) Simplify 2

$$\frac{4^m \times 8^{m+1}}{2^{3-m}}$$

c) Find all solutions of the equation 3

$(x-1)^2 + 2(x-1) - 3 = 0$

d) Find the value of x in the diagram below 2



e) Fully factorise

i) $2x^3 - 6x^2 + 4x$ 2

ii) $6x - 6y + x^2 - xy$ 2

iii) $x^4 - y^4$ 2

f) Solve for x

$$4^{2x+3} = 8^{x-5}$$ 2

Question 2 (13 marks) **START A NEW PAGE** **Marks**

a) Solve $(3x-1)^2 = 100$ 2

b) Solve $2x^2 - x - 15 = 0$ by factorisation 2

c) Solve $4x^2 - 8x + 1 = 0$ using the quadratic formula 2

d) Find the vertex of the parabola $y = 10x^2 - 9x - 9$ by completing the square 2

e) $6x^2 - 7x - 3 = 0$ has one solution of $x = \frac{3}{2}$. What is the other solution? 1

f) State the equation of the axis of symmetry of the parabola $y = 5x^2 - 4x + 2$ 1

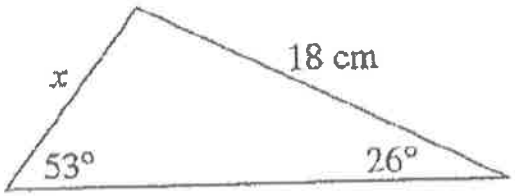
g) Solve the following simultaneous equations 3

$$y = x^2 - 3x + 7$$
$$2x - y + 3 = 0$$

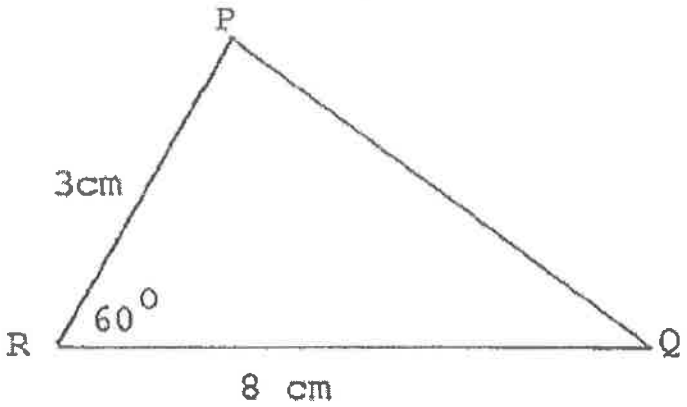
Question 3 (14 marks) START A NEW PAGE Marks

a) Find the exact value of $\cos 135°$ 1

b) Find the value of x, giving your answer correct to 1 decimal place 2



c) Find the exact area of the triangle 2



d) Find the largest angle in a triangle with sides $12cm, 16cm$ and $18cm$ 2

e) If $\tan\theta = \frac{7}{9}$ and $\cos\theta < 0$, find the exact value of $\sin\theta$ 2

f) When John is $70m$ away from a tree he notices a bird at an angle of elevation of $58°$ from his horizontal line of sight. The bird flies up the tree. What will be the angle of elevation after the bird has flown up $20m$ from its original position? 3

g) Solve the following trigonometric equation fo $0° \leq \theta \leq 360°$ 2

$$\cos\theta = -\frac{1}{\sqrt{2}}$$

Question 4 (16 marks) START A NEW PAGE Marks

a) Find the value of $\log_{10} 2^{1000}$ correct to 3 decimal places 2

b) If $x = \log_a 2, y = \log_a 3$ and $z = \log_a 5$, simplify the following (expressing in terms of x, y, z)

$$\log_a \frac{8}{15a^2}$$ 2

c) Simplify $6^{2\log_6 7}$ 2

d) If $\log_a 2 = 0.27$ and $\log_a 3 = 0.428$, find $\log_a 4.5$ 2

e) Evaluate $\log_5 25 - \log_5 \sqrt{5}$ 2

f) Solve for x: $\log_{10} x - \frac{3}{\log_{10} x} = 2$ 3

g) Given that $a^2 + b^2 = 23ab$, express $\left(\frac{a+b}{5}\right)^2$ in terms of a and b. 3

Hence show that $\log\left[\frac{1}{5}(a+b)\right] = \frac{1}{2}[\log a + \log b]$

Question 5 (10 marks) START A NEW PAGE Marks

A line k cuts the x-axis at $L(-4,0)$ and the y-axis at $M(0,3)$. N is a point on the line k and P is the point $(0,8)$.

a) Find the equation of the line k 1

b) Find the angle of inclination of k 1

c) Show that ΔLMP is isosceles 2

d) Find the gradient of PL 1

e) Find the coordinates of N if M is the midpoint of the interval LN 1

f) Show that $\angle NPL$ is $90°$ 2

g) Find the area of ΔLPN 2

Question 6 (13 marks) START A NEW PAGE Marks

a) The probability of rain on any given day in town A is given as 85%. What is the probability that it doesn't rain on any given day in town A? 1

b) How many two digit numbers can be formed from the numbers 1,2,3,4 if

i) Repetition is allowed 1

ii) Repetition is not allowed 1

c) From a group of 5 boys and 4 girls, one boy and one girl are chosen at random. What is the probability Anna and Bob are both chosen? 2

d) Two tetrahedron die with the numbers 1, 2, 3, 4 on each face is rolled and the numbers are added together. What is the probability the sum is

i) 1 1

ii) 3 1

iii) A prime number 1

e) A box contains 5 light gloves, 2 of which are faulty. Two globes are selected, one at a time, without replacement. Find the probability

i) Both globes are faulty 2

ii) Only one of the globes is faulty 2

f) Which of the following statements is false (Select A, B, C, or D) 1

I) $P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})$

II) $P(\overline{A \cup B}) = P(\bar{A} \cup \bar{B})$

A) I only
B) II only
C) both I and II
D) neither I nor II

Question 7 (14 marks) START A NEW PAGE Marks

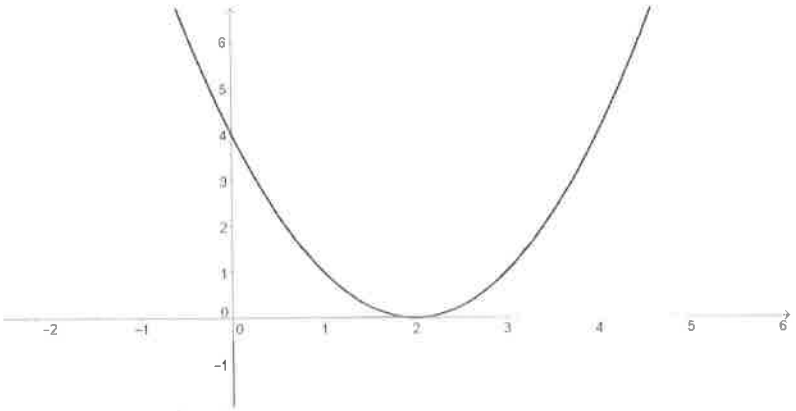
a) If $f(x) = 13 - 2x$, find the value of

i) $f(-5)$ 1

ii) x if $f(x) = 100$ 1

b) Write the equation of the circle with centre $(-3,2)$ and radius 4 2

c) What is the equation of the parabola below? (Select A, B, C, or D) 1



A) $y = x^2 + 2$

B) $y = (x-4)^2 + 2$

C) $y = x^2 - 2$

D) $y = (x-2)^2$

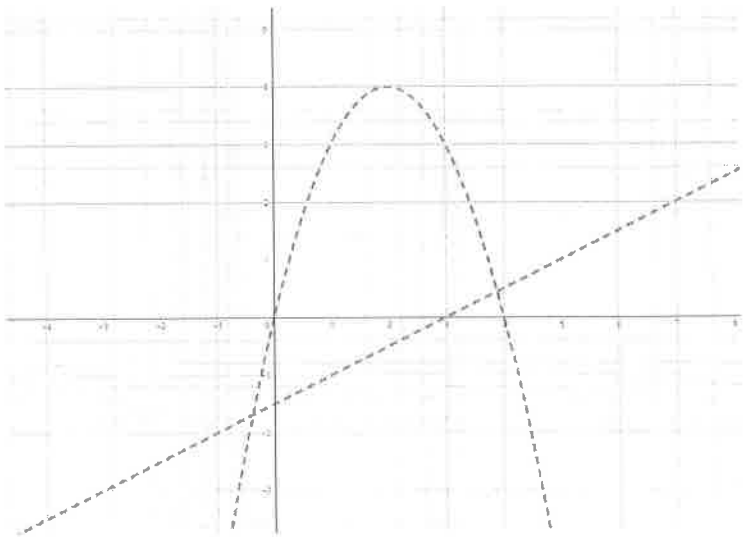
d) Graph the following curves showing all important features

i) $y = (x+1)^3$ 2

ii) $y = 1 - \dfrac{2}{x-2}$ 3

e) Graph $y = 2\sin x$ for $0° \leq x \leq 270°$ showing all important features 2

f) Shade the region on the number plane given below defined by $2x - 3y - 6 \geq 0$ AND $y < 4 - x^2$ 2



2021 410 Task 4 (Yearly)

(Suggested Solutions)

Q1

(a) $2^{n+1} + 2^{n+1} = 2(2^{n+1})$

$= 2^{n+2}$ $\therefore$ (A) ✓

(b) $\dfrac{4^m \times 8^{m+1}}{2^{3-m}} = \dfrac{(2^2)^m \times (2^3)^{m+1}}{2^{3-m}}$ ✓

$= \dfrac{2^{2m} \times 2^{3m+3}}{2^{3-m}}$

$= 2^{5m+3-(3-m)}$

$= 2^{6m}$ ✓ ✓

(c) $(x-1)^2 + 2(x-1) - 3 = 0$

Let $u = x-1$

$\therefore u^2 + 2u - 3 = 0$ ✓

$(u+3)(u-1) = 0$

$\therefore u = -3$ or $+1$

$\therefore x-1 = -3$ or $x-1 = 1$

Thus $x = -2$ or 2. ✓✓

(d) $(x+4)^2 = (x-2)^2 + (x+1)^2$

$x^2 + 8x + 16 = x^2 - 4x + 4 + x^2 + 2x + 1$

$0 = x^2 - 10x - 11$ ✓

$\therefore (x-11)(x+1) = 0$ $\therefore x = 11$ or -1

But lengths > 0 $\therefore x = 11$ ✓

(Q1) – ctd

(e) (i) $2x^3 - 6x^2 + 4x = 2x[x^2 - 3x + 2]$

$= \boxed{2x\ (x-2)(x-1)}$ ✓✓

(ii) $6x - 6y + x^2 - xy = 6(x-y) + x(x-y)$

$= \boxed{(x-y)(6+x)}$ ✓✓

(iii) $x^4 - y^4 = (x^2-y^2)(x^2+y^2)$

$= \boxed{(x-y)(x+y)(x^2+y^2)}$ ✓✓

(f) $4^{2x+3} = 8^{x-5}$

$\therefore (2^2)^{2x+3} = (2^3)^{x-5}$ ✓

$2^{4x+6} = 2^{3x-15}$

$\therefore 4x + 6 = 3x - 15$

$\boxed{x = -21}$ ✓

16

Q2

(a) $(3x-1)^2 = 100$

$3x-1 = 10 \quad \text{or} \quad -10$

$3x = 11 \quad \text{or} \quad -9$

$\therefore x = \frac{11}{3} \text{ or } -3$ ✓✓

(b) $2x^2 - x - 15 = 0$

$(2x+5)(x-3) = 0$

$x = -\frac{5}{2} \text{ or } 3$ ✓✓

(c) $4x^2 - 8x + 1 = 0$

$\therefore x = \frac{8 \pm \sqrt{64 - 4(4)(1)}}{8}$

$= \frac{8 \pm \sqrt{48}}{8}$

$= \frac{8 \pm 4\sqrt{3}}{8}$

$\therefore x = \frac{2 \pm \sqrt{3}}{2}$ ✓✓

(d) $y = 10x^2 - 9x - 9 = 10\left[x^2 - \frac{9}{10}x\right] - 9$

$= 10\left[x^2 - \frac{9}{10}x + \frac{81}{400} - \frac{81}{400}\right] - 9$

$= 10\left(x - \frac{9}{20}\right)^2 - \frac{81}{40} - 9$

$\therefore y = 10\left(x - \frac{9}{20}\right)^2 - \frac{441}{40}$

Thus $V = \left(\frac{9}{20}, -\frac{441}{40}\right)$ ✓✓

(Q2) – ctd

(e) If $x = \frac{3}{2}$ is a solution, $\therefore (2x-3)$ is factor

$\therefore (2x-3)(3x+1) = 0$

$\therefore$ other solution is $\boxed{-\frac{1}{3}}$. ✓

(f) $y = 5x^2 - 4x + 2$

$\therefore$ axis $x = \frac{-b}{2a} = \frac{-(-4)}{10}$

$\therefore \boxed{x = \frac{2}{5}}$ ✓

(g) $y = x^2 - 3x + 7$ — (1)

$2x - y + 3 = 0$ — (2)

Sub. (1) into (2) →

$2x - (x^2 - 3x + 7) + 3 = 0$

$2x - x^2 + 3x - 7 + 3 = 0$

$\therefore x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$\therefore x = 4$ or $x = 1$ ✓

$y = 2x + 3$ $\quad$ $y = 2x + 3$

$= 11$ $\quad$ $= 5$

$\therefore$ solutions are $\boxed{x = 4, y = 11 \text{ or } x = 1, y = 5.}$ ✓✓

(ie (4, 11) or (1, 5))

13

Q3

(a) $\cos 135^\circ = -\cos 45^\circ = \boxed{-\frac{1}{\sqrt{2}}}$ ✓

(b) $\frac{x}{\sin 26} = \frac{18}{\sin 53}$ ✓

$\therefore x = \frac{18 \sin 26}{\sin 53}$

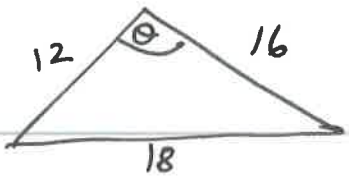
$\doteq 9.88020\ldots$

$\therefore \boxed{x = 9.9 \text{ (1 d.p.)}}$ ✓

(c) Area $= \frac{1}{2} \cdot 3 \cdot 8 \cdot \sin 60^\circ$ ✓

$= \frac{1}{2} \cdot 3 \cdot 8 \cdot \frac{\sqrt{3}}{2}$

$\boxed{\text{Area} = 6\sqrt{3} \text{ cm}^2}$ ✓

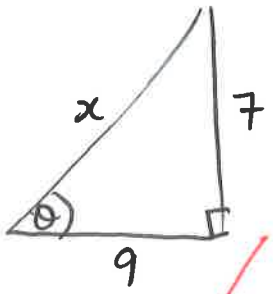
(d) 

$18^2 = 12^2 + 16^2 - 2(12)(16)\cos\theta$ ✓

$\therefore \cos\theta = \frac{12^2 + 16^2 - 18^2}{2(12)(16)}$

$\doteq 0.1979\,1666\ldots$ ✓

$\boxed{\theta = 78^\circ 35' \text{ (nearest min.)}}$.

(e) 

$x^2 = 7^2 + 9^2 = 130$

$x = \sqrt{130}$

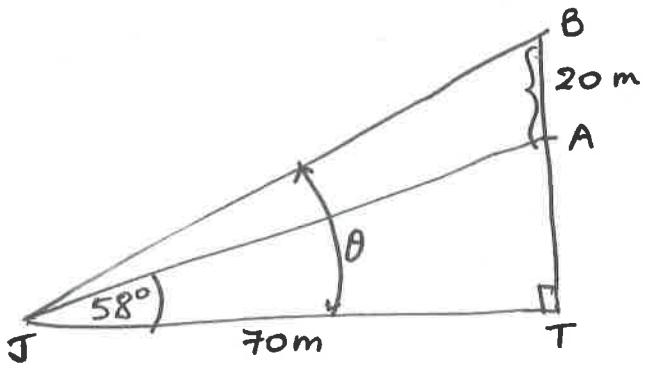
tan > 0, cos < 0

$\therefore$ Q3

$\therefore \boxed{\sin\theta = -\frac{7}{\sqrt{130}}}$ ✓

(Q3) ctd

(f)



$\frac{AT}{70} = \tan 58 \quad \therefore \quad AT = 70 \tan 58$ ✓

$\therefore \tan\theta = \frac{BT}{70} = \frac{70\tan 58 + 20}{70} \doteq \frac{132.0234}{70}$ ✓

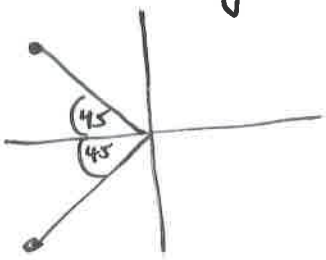
$\doteq 1.88605$

$\therefore$ $\theta = 62^\circ$ (nearest degree). ✓

(g) $\cos\theta = -\frac{1}{\sqrt{2}}, \quad 0^\circ \leq \theta \leq 360^\circ$.

$\cos < 0 \quad \therefore$ in Q3, or Q2.

related angle is 45° (as $\cos 45^\circ = \frac{1}{\sqrt{2}}$)



$\therefore$ $\theta = 135^\circ$ or 225°. ✓✓

14

Q4

(a) $\log_{10} 2^{1000} = 1000 \log_{10} 2$ ✓

$\doteq 301.029995\ldots$

$= \boxed{301.030 \quad (3\text{ d.p.})}$. ✓

(b) $\log_a \frac{8}{15a^2} = \log_a 8 - \log_a (15a^2)$

$= 3\log_a 2 - \left[\log_a 3 + \log_a 5 + 2\log_a a\right]$ ✓

$= 3x - (y + z + 2)$

$= \boxed{3x - y - z - 2}$. ✓

(c) $6^{2\log_6 7} = \left(6^{\log_6 7}\right)^2$ ✓

$= 7^2$

$= \boxed{49}$ ✓

(d) $\log_a 4.5 = \log_a \frac{9}{2}$

$= \log_a 9 - \log_a 2 \quad = \log_a 3^2 - \log_a 2$

$= 2\log_a 3 - \log_a 2$ ✓

$= 2(0.428) - 0.27$

$= \boxed{0.586}$. ✓

(Q4) – ctd

(e) $\log_5 25 - \log_5 \sqrt{5} = \log_5(5^2) - \log_5(5^{\frac{1}{2}})$ ✓

$= 2 - \frac{1}{2}$

$= \frac{3}{2}$ ✓

(f) $\log_{10} x - \dfrac{3}{\log_{10} x} = 2$

Let $u = \log_{10} x$ $\therefore$ $u - \frac{3}{u} = 2$

$\therefore$ ✓

$u^2 - 2u - 3 = 0$

$(u-3)(u+1) = 0$

$u = 3$ or -1

$\therefore \log_{10} x = 3$ or $\log_{10} x = -1$ $\therefore$ $x = 10^3$ or 10^{-1}

$\therefore$ $x = 1000$ or $\frac{1}{10}$. ✓✓

(g) $a^2 + b^2 = 23ab$ $\therefore$ $\left(\frac{a+b}{5}\right)^2 = \dfrac{a^2+b^2+2ab}{25}$

$= \dfrac{23ab + 2ab}{25}$

$= ab$ ✓

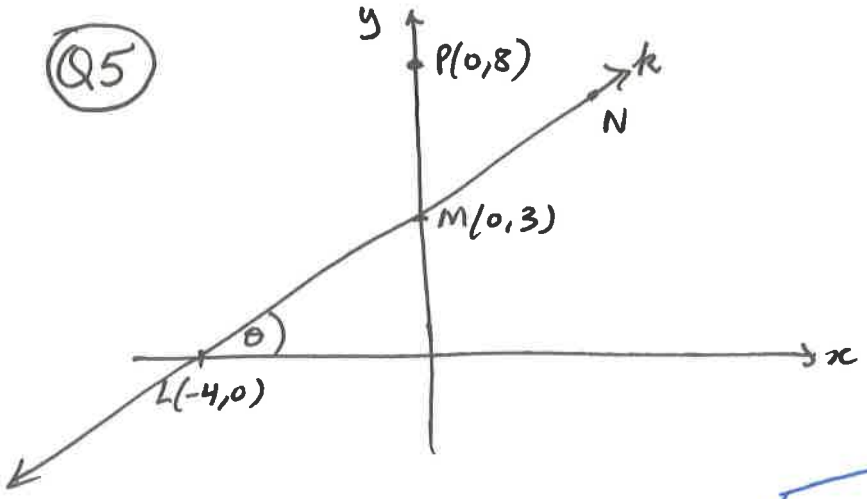
Thus $\log\left(\frac{a+b}{5}\right) = \frac{1}{2} \times 2\log\left(\frac{a+b}{5}\right) = \frac{1}{2}\log\left(\frac{a+b}{5}\right)^2$ ✓

$= \frac{1}{2}\log ab$

$= \frac{1}{2}(\log a + \log b)$ ✓

16

Q5



(a) $m = \frac{3}{4}$, $b = 3$ $\therefore$ $y = \frac{3}{4}x + 3$ ✓

(b) $\theta = \tan^{-1}\frac{3}{4} = 37°$ (nearest degree). ✓

(c) $PM = 8 - 3 = 5$,
& $LM = \sqrt{4^2+3^2} = 5 = PM$ ✓✓
$\therefore \Delta LMP$ is isosceles (2 sides equal length)

(d) $m_{PL} = \frac{8}{4} = 2$ ✓

(e) For N, $x = 0+4$, $y = 3+3$
$\therefore N = (4,6)$. ✓

(f) $m_{NP} \times m_{LP} = \frac{8-6}{0-4} \times \frac{8-0}{0-(-4)}$ ✓

$= \frac{2}{-4} \times \frac{8}{4} = -1$ ✓
$\therefore NP \perp LP$ $\therefore$ $\angle NPL = 90°$.

(g) Area $= \frac{1}{2} \cdot PN \cdot PL$
$= \frac{1}{2} \cdot \sqrt{4^2+2^2} \cdot \sqrt{4^2+8^2} = \frac{1}{2}\sqrt{20} \cdot \sqrt{80}$ ✓
$\therefore$ area $= \frac{1}{2}\sqrt{1600} = 20\ u^2$. ✓

10

Q6

(a) $P(R) = 0.85 \quad \therefore P(\bar{R}) = 1 - 0.85$

$= 0.15$ ✓

(ie 15%).

(b) (i) $4 \times 4 = 16$ ✓

(ii) $4 \times 3 = 12$ ✓

(c) $p = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ ✓✓, as events P(A) & P(B) are independent.

(alt: there are $5 \times 4 = 20$ possible pairings $\therefore p = \frac{1}{20}$).

(d)

A \ B	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

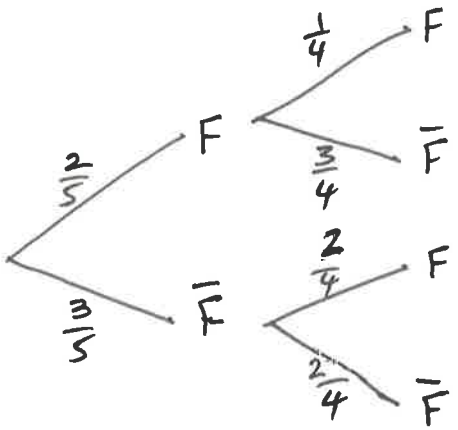
(i) $P(\text{sum} = 1) = 0$ ✓

(ii) $P(\text{sum} = 3) = \frac{2}{16} = \frac{1}{8}$ ✓

(iii) $P(\text{prime sum}) = \frac{9}{16}$ ✓

Q6 – ctd

(e)



(i) $P(FF) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$ ✓

(ii) $P(F\bar{F}) + P(\bar{F}F) = \frac{2}{5} \cdot \frac{3}{4} + \frac{3}{5} \cdot \frac{2}{4}$ ✓

$= \frac{12}{20}$

$= \frac{3}{5}$ ✓

(f)

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\therefore P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})$ $\therefore$ (A) ✓

/13

Q7

(a) $f(x) = 13 - 2x$

(i) $f(-5) = 13 - 2(-5)$

$= \boxed{23}$ ✓

(ii) $f(x) = 100 \Rightarrow 13 - 2x = 100$

$2x = -87$

$\therefore \boxed{x = \frac{-87}{2}}$ ✓

(b) $C = (-3, 2)$, $r = 4$

$\therefore \boxed{(x+3)^2 + (y-2)^2 = 16}$ ✓✓

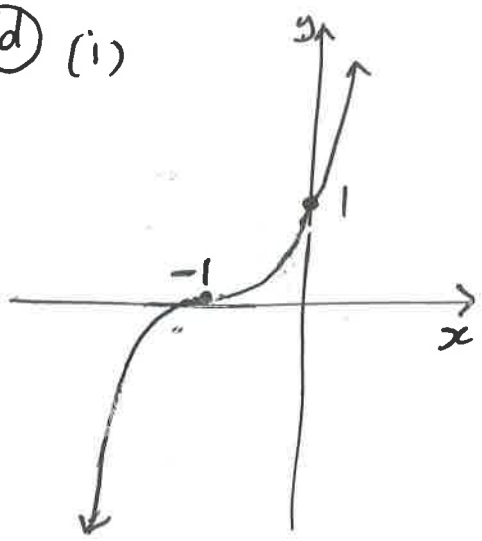
(c) $V = (2, 0)$ $\therefore$ $y = a(x-2)^2$

goes thru $(0, 4)$ $\therefore$ $4 = a(4-2)^2$

$\therefore a = 1$

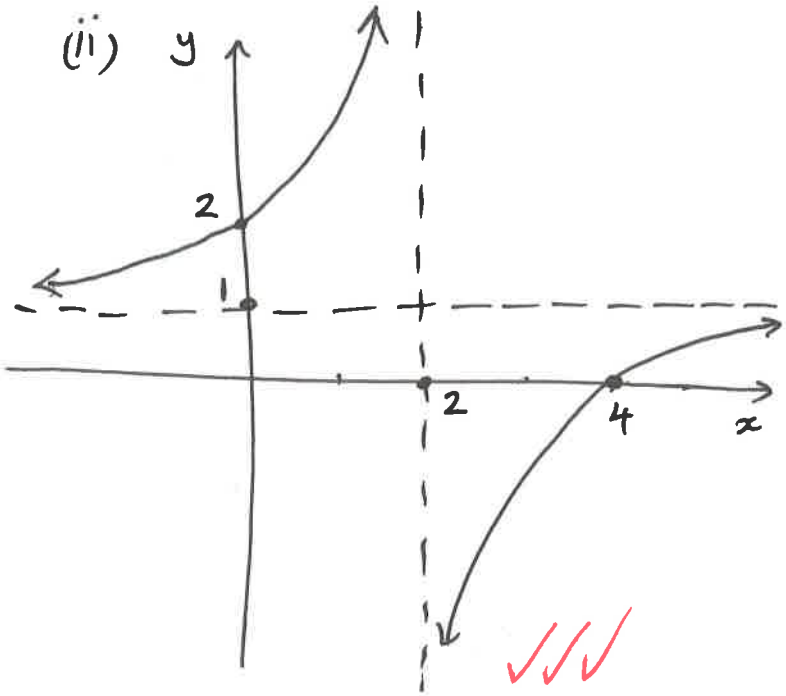
$\therefore$ (D) ✓

(d) (i)



$y = (x+1)^3$ ✓✓

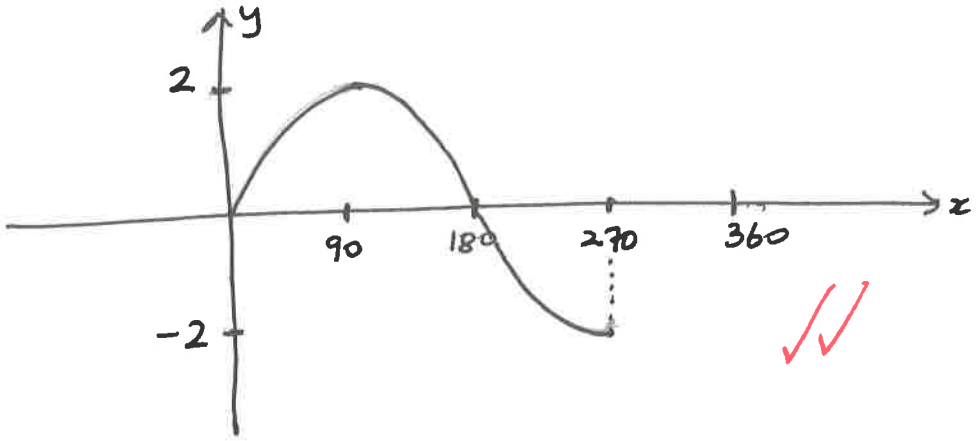
(ii)



$y = 1 - \frac{2}{x-2}$ ✓✓✓

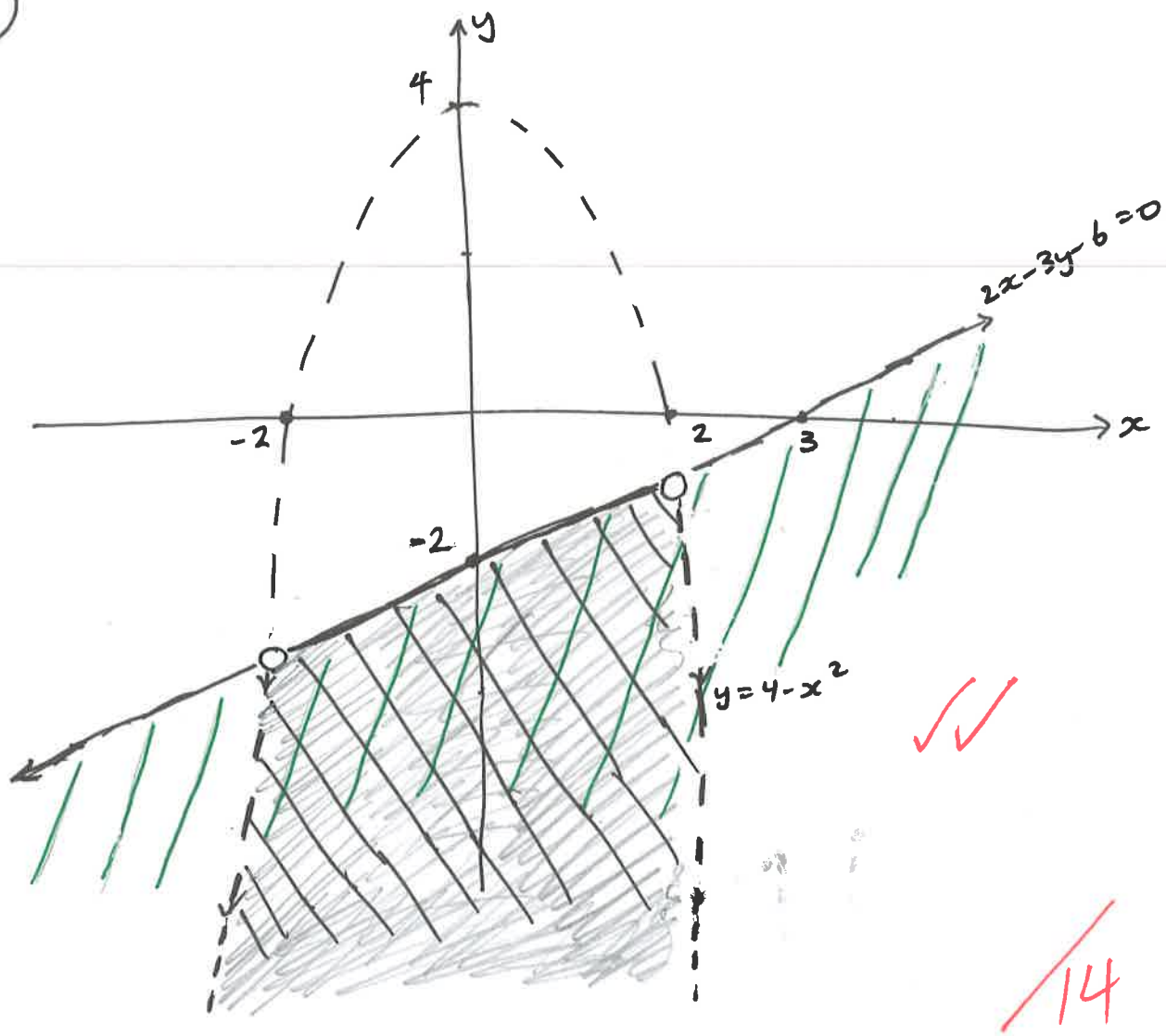
Q7 – ctd

(e) $y = 2\sin x$, $0° \leq x \leq 270°$



√√

(f)



√√

/14