NORTH SYDNEY BOYS HIGH SCHOOL

STAGE 5 MATHEMATICS

Year 10 Yearly Examination

Thursday 5 November 2020

General instructions

- Working time – 90 min.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **all** questions.

Class (please ✔)

- ◯ 10MAT.A –Ms Lee
- ◯ 10MAT.B –Ms Ma
- ◯ 10MAT.C –Mr Umakanthan
- ◯ 10MAT.D –Mr Lin
- ◯ 10MAT.E –Mr Hwang

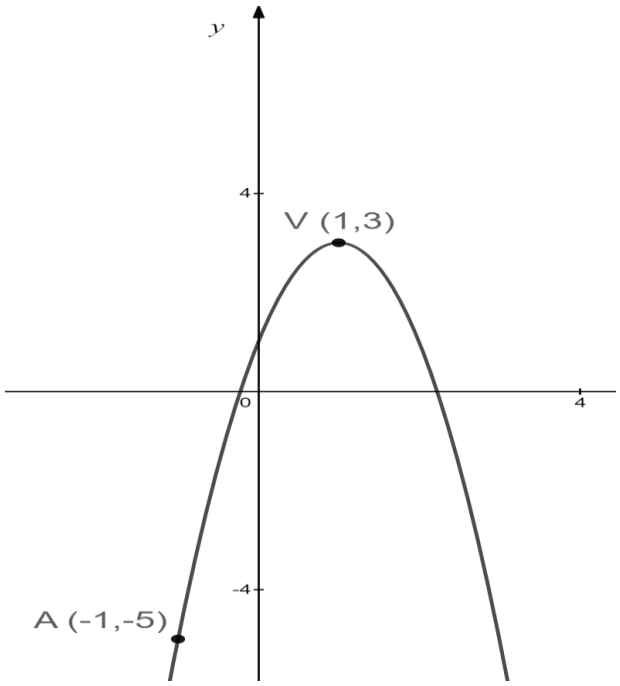
NAME .....

Marker's use only

QUESTION	1	2	3	4	5	6	7	8	9	10	Total
MARKS	/9	/12	/10	/9	/12	/10	/8	/8	/8	/18	/104

Question 1 Graphs and functions (9 Marks)

(a) **Marks**

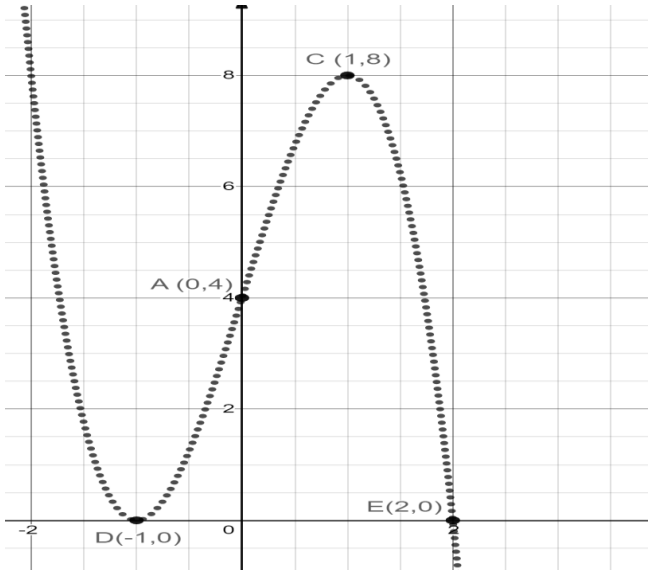


Write down the equation of the parabola. **3**

Question 1 continues on page 4

(c) Drawn in dotted line is the graph of the graph of $y = f(x)$ **Marks**

(i) Sketch the graph of $y = -f(x)$.

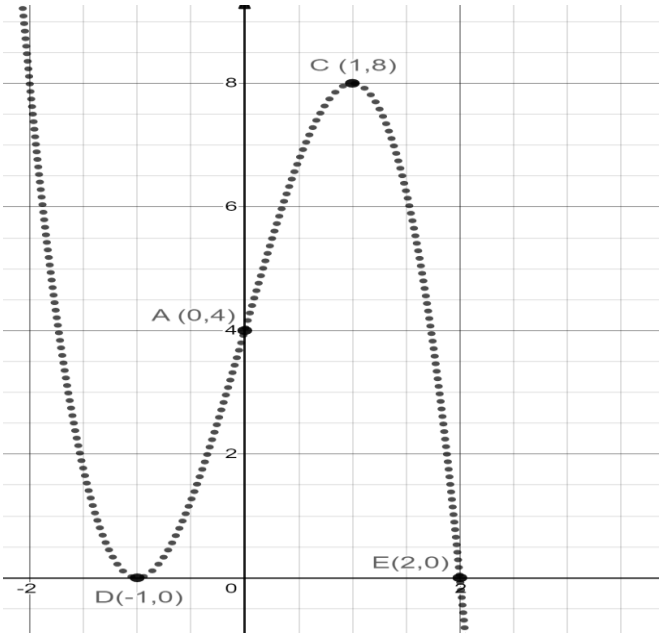


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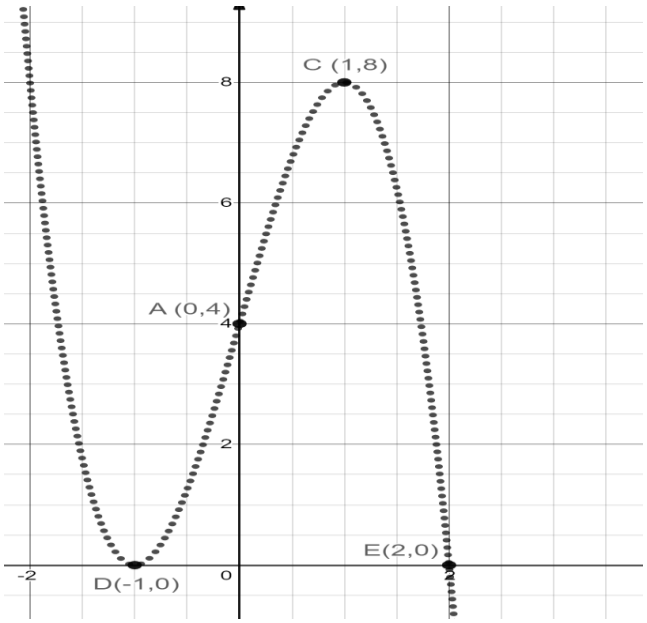
Question 1 continues on page 5

Marks

(i) Sketch the graph of $y = f(-x)$ **2**



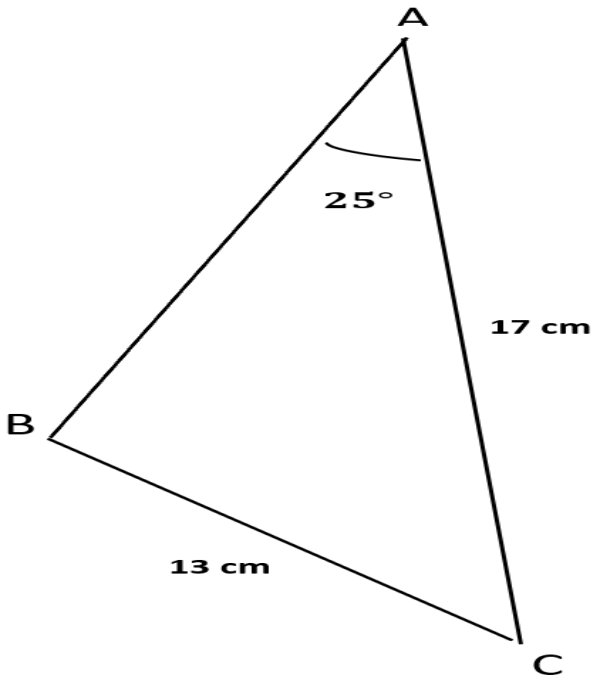
(ii) Sketch the graph of $y = f(x - 1)$. **2**



End of Question 1

Question 2 Trigonometry (12 Marks)

(a) **Marks 4**



In the Δ ABC above, A = 25°, AC = 17 cm and BC = 13 cm.
Find all possible values for the unknown sides and angles. (angles B and C; sides b and c).

Question 2 continues on page

(b) Find the exact value of $\frac{\sin 150° + \cos 300°}{tan750° - tan120°}$

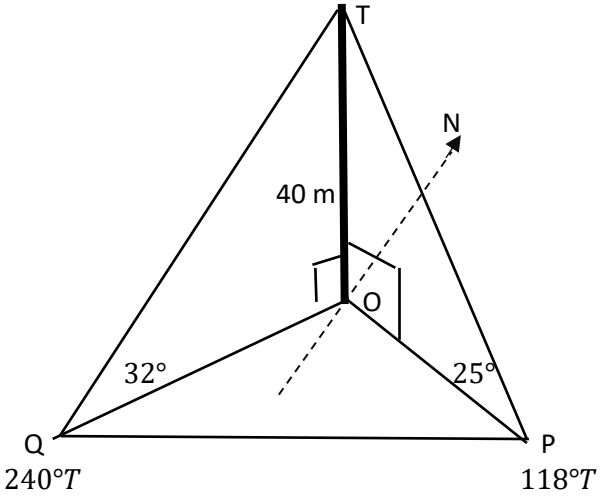
Marks
2

(c) Solve $cos\theta = -0.2351$ for $0° \leq \theta \leq 360°$. Answer to the nearest minute.

2

Question 2 continues on page 8

(d) **Marks**



OT is a vertical tower of height 40 m. The angles of elevation of T from two landmarks P and Q are $25°$ and $32°$ respectively. The bearings of P and Q from O are respectively $125°T$ and $240°T$.
Find the distance PQ. **4**

End of Question 2

Question 3 Polynomials (10 Marks)

Marks

(a) Use the **remainder theorem** to find the remainder when the polynomial $P(x) \equiv 4x^4 + 4x^3 + x^2 + 7x - 5$ is divided by $(x + 1)$. **2**

(b) (i) Use **long division** to find the remainder when the polynomial and write the result in Division Transformation form.

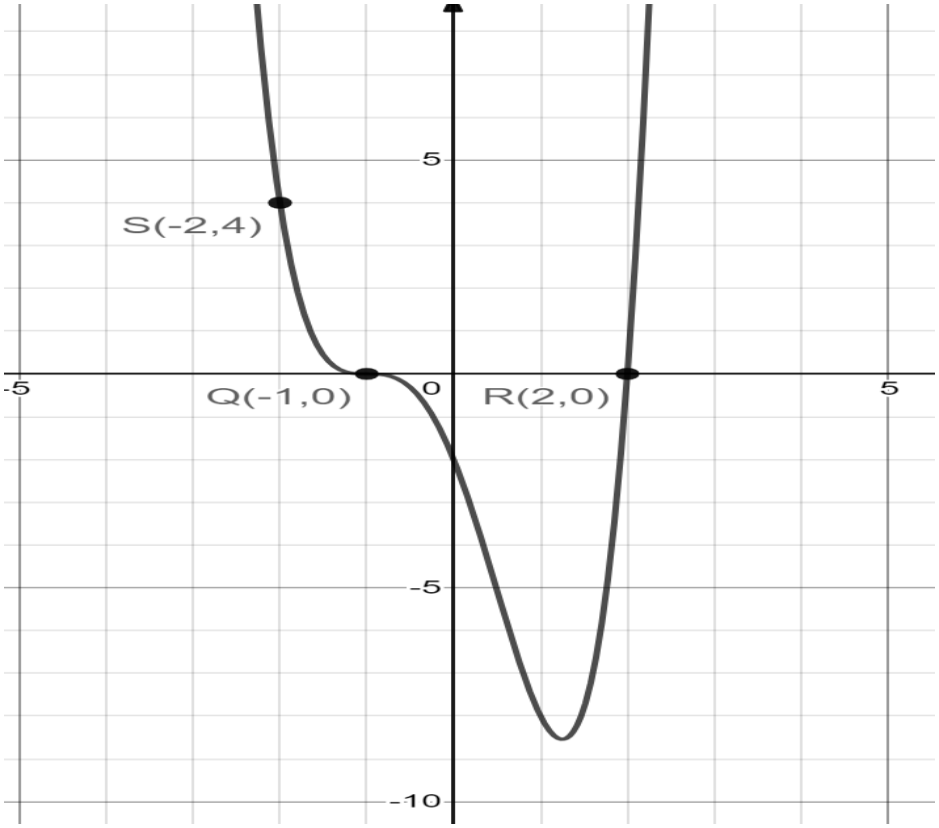
$P(x) \equiv 2x^3 - 7x^2 + 9x + 4$ is divided by $(x^2 + 2)$. **3**

(ii) If the polynomial $Q(x) \equiv 2x^3 - 7x^2 + ax + b$ is divisible by $(x^2 + 2)$, find the values of a and b. **2**

Question 3 continues on page 10

Marks

(c) Drawn below is the graph of the polynomial $y = P(x)$. **3**



Find the equation of the polynomial $P(x)$.

End of Question 3

Question 4 Probability (9 Marks)

(a) John bought 5 raffle tickets. 100 tickets are sold altogether. Two tickets are drawn in succession without replacement. Find the probability that John wins, All answers in simplest form. **Marks**

(i) The first prize **1**

(ii) Both prizes **1**

(iii) Only one prize **2**

Question 4 continues on page 12

(b) Serena and Venus are competing in a tennis tournament final. Records indicate that whenever they play a series of matches, Serena's probability of winning is 0.6 and that in subsequent matches in the series, Serena's probability of winning is 0.7 if she won the preceding match but only 0.5 if she lost the preceding match.
A match cannot be drawn and the first player to win two matches wins the championship. **Marks**

What is the probability that Serena wins the championship? **2**

Question 4 continues on page 13

(c) The two-way table below shows the results of the people taking a lie detector test. Some people are known to be telling the truth while others are known to be lying. **Marks**

Test Results

	Accurate	Not Accurate	Total
Truth teller	776	24	
Liar	153	47	
Total			

(i) How many people did the lie detector indicate were lying? **1**

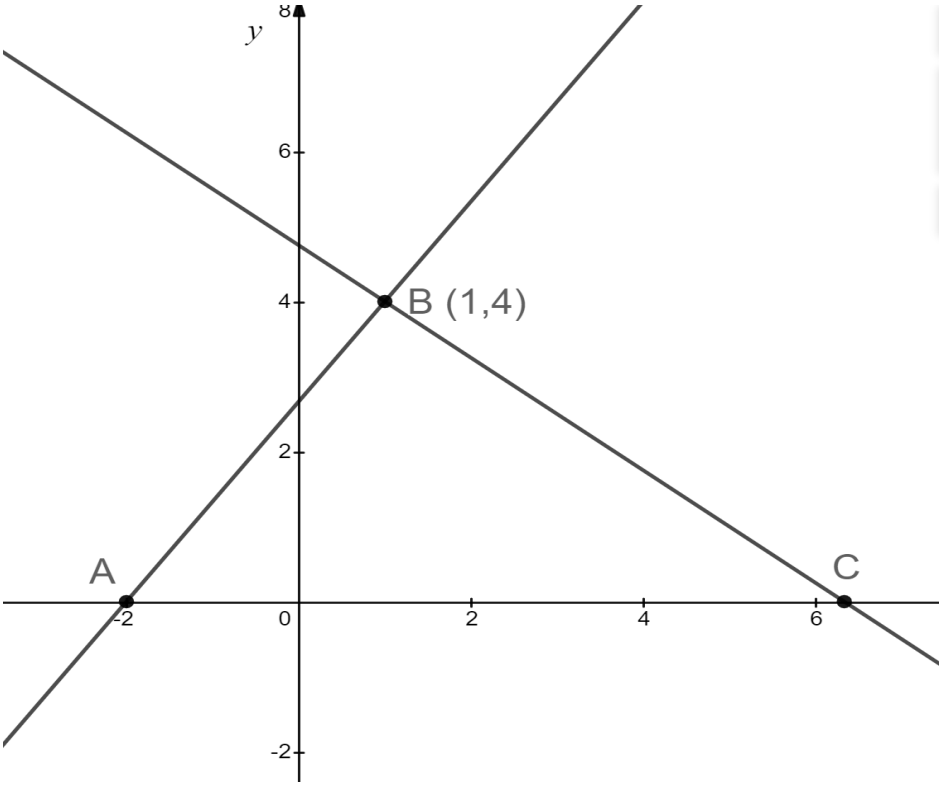
(ii) What percentage of tests were accurate? **1**

(iii) What is the probability of a person who is lying successfully 'fooling' the lie detector? Express your answer as a percentage. **1**

End of Question 4

Question 5 Co-ordinate Geometry and Locus (12 Marks)

(a) **Marks**



The diagram shows the points A(-2,0), B(1,4) and the point C lies on the x-axis.

(i) Find the gradient of AB **1**

(ii) Find the equation of the line AB **2**

(iii) BC is perpendicular to AB. Find the gradient of BC. **1**

Question 5 continues on page 14

Marks

(iv) Find the equation of the line BC **2**

(v) Find the co-ordinates of C. (Exact Value) **1**

(vi) Calculate the area of $\Delta\ ABC$.(Exact value) **2**

(b) Show that the equation of the locus of a point P(x,y), which moves so that PA = 2PB, where A(-3,2) and B(3,-4) is a circle. **3**

End of Question 5

Question 6 Financial Mathematics (10 Marks)

(a) The table below shows monthly payments for each $1000 borrowed. **Marks**

INTEREST RATE (%p.a)	PERIOD OF LOAN				
	5 years	10 years	15 years	20 years	25 years
5	$18.87	$10.61	$7.91	$6.60	$5.85
6	$19.33	$11.10	$8.44	$7.10	$6.44
7	$19.80	$11.61	$9.00	$7.75	$7.07
8	$20.28	$12.13	$9.56	$8.36	$7,72
9	$20.76	$12.67	$10.14	$9.00	$8.39
10	$21.76	$13.22	$10.75	$9.65	$9.10
11	$21,74	$13.78	$11.37	$10.32	$9.80
12	$22.24	$14.35	$12.00	$11.01	$10.53
13	$22.75	$14.93	$12.65	$11.72	$11.28
14	$23.27	$15.53	$13.32	$12.44	$12.04
15	$23.79	$16.13	$14.00	$13.17	$12.81

David borrows $150 000 to buy a vacant land at 8%p.a over twenty-five years.

(i) Use this information in the table to calculate David's monthly payment on this loan. **2**

(ii) How much does David pay in total to repay the loan? **2**

Question 6 continues on page 17

(iii) How much extra per month would David pay if he were to repay the same loan over 20 years? **Marks 3**

(iv) Anna wants to buy a land for $150 000. She saved some money for deposit and will borrow the rest at 8% p.a will repay the loan over fifteen years, paying $1195 monthly.

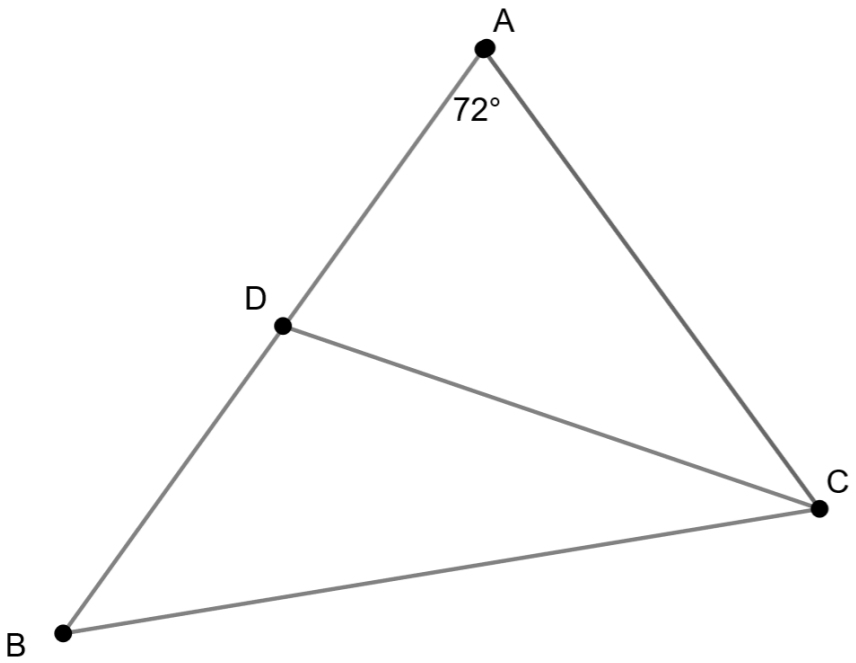
How much will she borrow? **2**

(v) How much has she saved for a deposit? **1**

End of Question 6

Question 7 Similarity (8 Marks)

Marks



ABC is an isosceles triangle with ∠BCA = ∠BAC = 72°. And sides AB = BC = 1 unit.
CD bisects ∠BCA and AC = x units.

(i) Show that ΔABC is similar to ΔDCA. **3**

Question 7 continues on page 18

(ii) By using part (i), find the value of x, giving your answer in exact form. **Marks 3**

(i) A cube is enlarged to a bigger size. If the surface area of the larger cube is $6\frac{1}{4}$ times the surface area of the smaller cube, find the ratio of volume of the smaller cube to that of the larger cube. Give your answer in simplest form. **2**

End of Question 7

Question 8 Single Data Analysis (8 Marks)

(a)

	Mean	Standard Deviation	Chris' Mark
Mathematics	61	12	83
English	72	5	83

Marks

(i) Using the table above, calculate Chris' z-scores for both English and Mathematics. **2**

(ii) It said in the report that Chris performed better in English than Mathematics. Chris could not understand this statement. Explain it to him. **1**

Question 8 continues on page 21

(iii) What should be the minimum marks in Mathematics that Chris should get, so that his performance in Mathematics is better than in English? **Marks 2**

(b) The petrol price, in cents per litre, is recorded eight days at two petrol stations in one suburb. **3**

Mobil: **93.9 97.9 92.4 93.9 98.5 92.3 97.9 99.9**

Ampol: 95.9 94.9 96.9 93.9 97.9 93.5 96.9 98.9

At which petrol station is the price of petrol more consistent? Explain.

End of Question 8

Quesdtion 9 Logarithms (8 Marks)

(a) If $log_2 3 = 1.6$ and $log_2 5 = 2.3$ evaluate without using a calculator: **Marks**

(i) $log_2 1.8$ **2**

(ii) $log_2 75$ **2**

(b) Solve for x:

(i) $log_5 x = \frac{log_5 64}{log_5 16}$ **2**

(c) (ii) Simplify $2log_{10} 2 + 4log_{10} 5 - 2log_{10} 50$ **2**

End of Question 9

Question 10 Algebra and Problem solving (18 Marks)

(a) Solve by factoring: $4x^2 - 20x + 21 = 0$ **Marks 2**

(b) Simplify: $\dfrac{\frac{1}{a} - 4a}{4 - 4a - \frac{1}{a}}$ **2**

(c) If $5^x = 1000$, evaluate 5^{x-3}. **2**

Question 10 continues on page 23

(d) **Marks 3**

If $\frac{8^{m-n}\times 14^{m+n}}{21^{m+n}\times 3^{m-2n}} = a^p$ find the values of a and p.

(e) **3**

Simplify $\frac{(x-y)^2-z^2}{xy-y^2-yz} \times \frac{z}{x^2+xy-xz} \div \frac{xz-yz+z^2}{x^2-(y-z)^2}$

Question 10 continues on page 24

(f) A family has a large back yard and decides to construct a swimming pool. Here is a plan of the yard which measures 40 metres by 30 metres; and the proposed swimming pool, which is to measure 12 metres by 10 metres, with a depth of 2 metres. **Marks**

(i) How much earth will need to be removed from the hole? **1**

(ii) If the earth is spread evenly over the rest of the yard, by how much would the height of the yard increase? **2**

(iii) How deep should the hole be, If the homeowner insists that the depth of the pool should be 2 metres and that the earth should be spread over the rest of the yard. **3**

End of Paper

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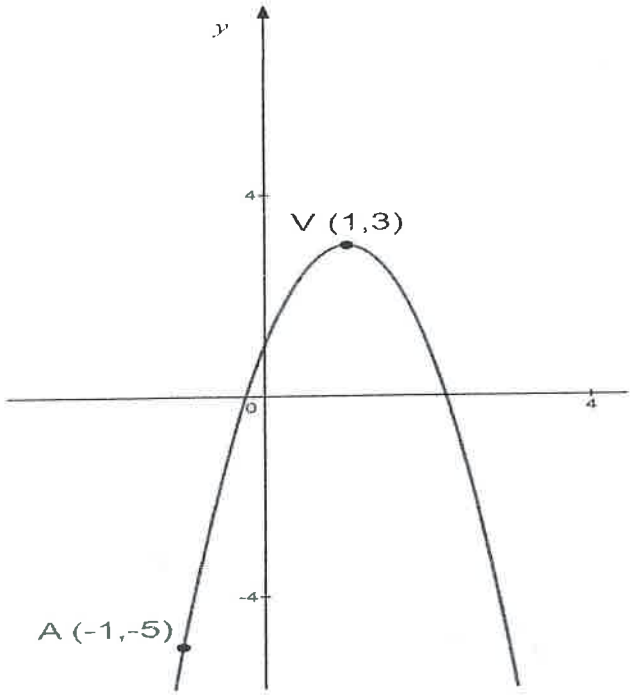
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QUESTION	1	2	3	4	5	6	7	8	9	10	Total
MARKS	/9	/12	/10	/9	/12	/10	/8	/8	/8	/18	/104

Question 1 Graphs and functions (9 Marks)

a)

Marks



Write down the equation of the parabola. 3

Let the equation of the parabola be $(y-3)=k(x-1)^2$

It passes through $A(-1,-5)$

$\therefore (-5-3)=k(-1-1)^2 \Rightarrow k=-2$

$\therefore$ The required equation is

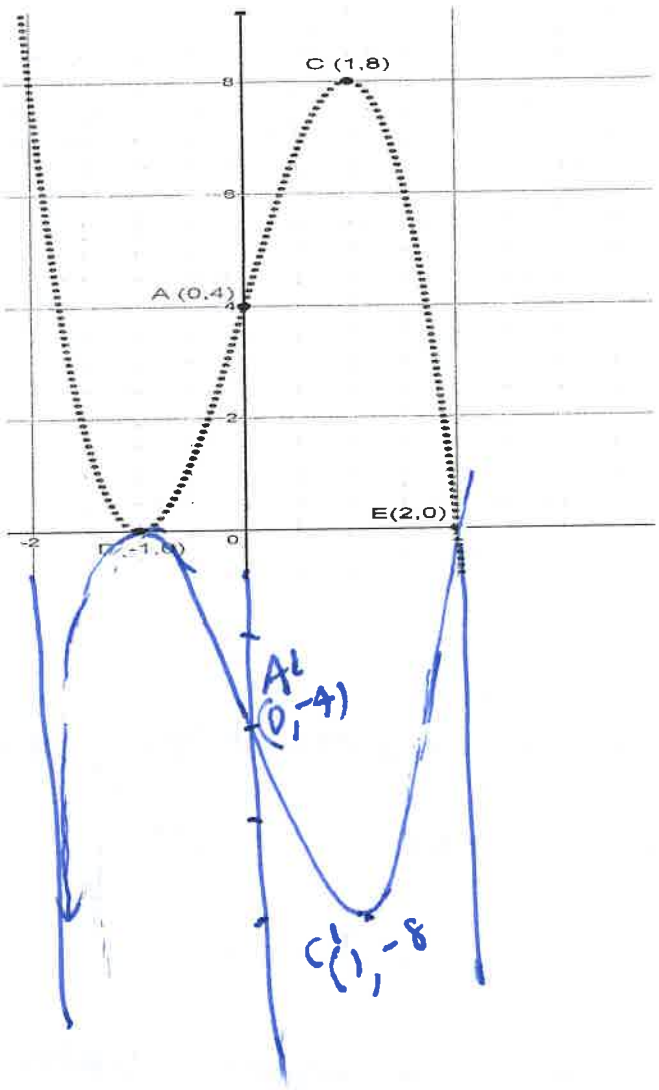
$y-3=-2(x-1)^2$ or in any other form

Question 1 continues on page 4

(c) Drawn in dotted line is the graph of the graph of $y = f(x)$

Marks

(i) Sketch the graph of $y = -f(x)$.

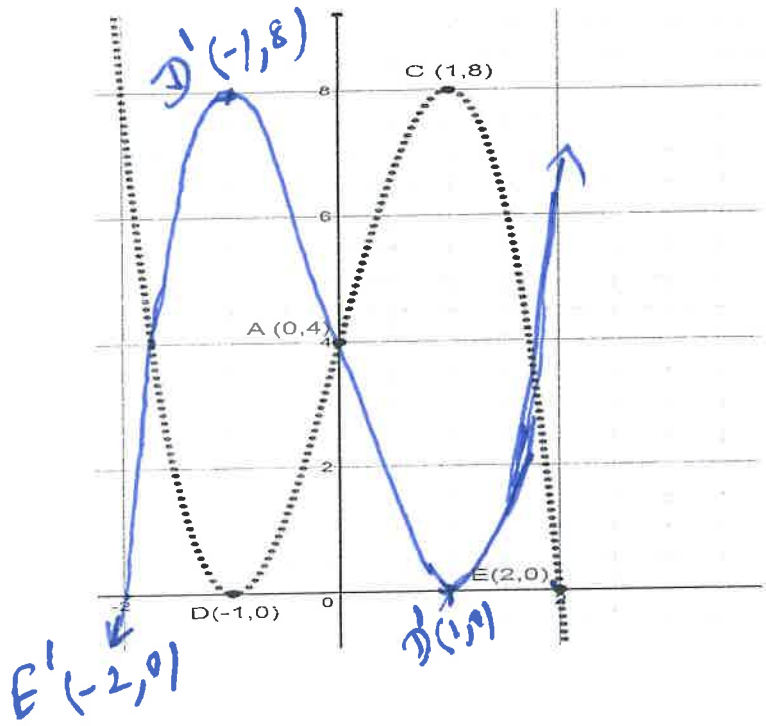


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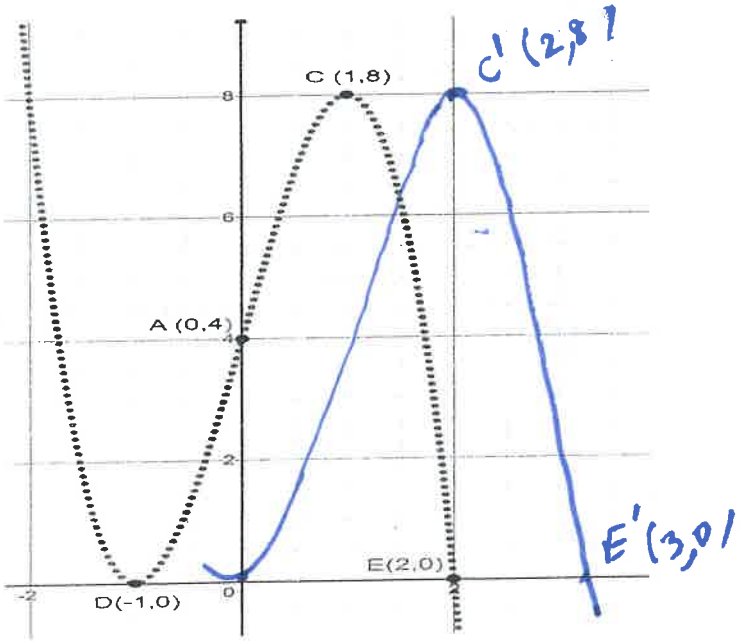
Question 1 continues on page 5

Marks

(i) Sketch the graph of $y = f(-x)$ **2**



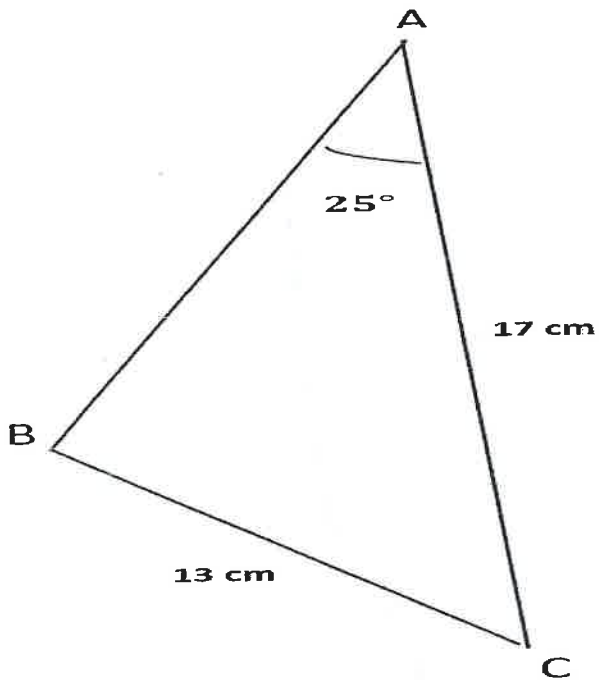
(ii) Sketch the graph of $y = f(x-1)$. **2**



End of Question 1

Question 2 Trigonometry (12 Marks)

(a) Marks 4



In the Δ ABC above, A = 25°, AC = 17 cm and BC = 13 cm.
Find all possible values for the unknown sides and angles. (angles B and C; sides b and c).

$\sin B = \frac{17\sin 25^\circ}{13}$

$= 0.5527$

$B = 35.55^\circ$ or $B = 180^\circ - 35.55^\circ = 146.45^\circ$

$C = 119.45^\circ$ $C = 8.55^\circ$

$c = 17.88$ cm $c = 4.57$ cm

Question 2 continues on page

) Find the exact value of $\frac{\sin 150°+\cos 300°}{tan750°-tan120°}$

Marks
2

$$\frac{\frac{1}{2}+\frac{1}{2}}{\frac{1}{\sqrt{3}}-(-\sqrt{3})}$$

$$= \frac{\sqrt{3}}{4} //$$

c) Solve $cos\theta = -0.2351$ for $0° \leq \theta \leq 360°$. Answer to the nearest minute.

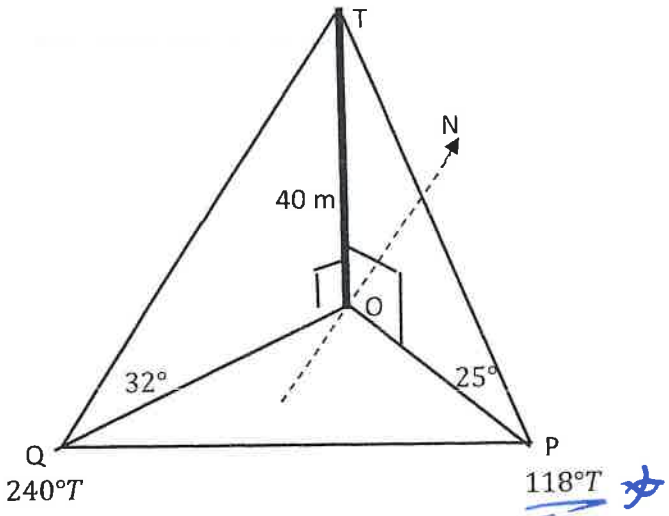
2

$\theta = 103°36'$ or $\theta = 283°36'$

Question 2 continues on page 8

(d)

Marks



OT is a vertical tower of height 40 m. The angles of elevation of T from two landmarks P and Q are 25° and 32° respectively. The bearings of P and Q from O are respectively $125°T$ and $240°T$.
Find the distance PQ. 4

$OQ = \frac{40}{\tan 32}$

$OP = \frac{40}{\tan 25}$

$QP = \sqrt{\left(\frac{40}{\tan 32}\right)^2 + \left(\frac{40}{\tan 25}\right)^2 - 2 \times \frac{40}{\tan 32} \times \frac{40}{\tan 25} \times \cos 122}$

$= 131.4 \text{ m}$ (1 dp)

or

$QP = \sqrt{\left(\frac{40}{\tan 32}\right)^2 + \left(\frac{40}{\tan 25}\right)^2 - 2 \times \frac{40}{\tan 32} \times \frac{40}{\tan 25} \times \cos 115°}$

=

End of Question 2

Question 3 Polynomials (10 Marks)

Marks

a) Use the **remainder theorem** to find the remainder when the polynomial $P(x) \equiv 4x^4 + 4x^3 + x^2 + 7x - 5$ is divided by $(x + 1)$. **2**

By the remainder theorem, the remainder $= P(-1)$

$= 4(-1)^4 + 4(-1)^3 + (-1)^2 + 7(-1) - 5$

$= -11$

b) (i) Use **long division** to find the remainder when the polynomial and write the result in Division Transformation form. **3**

$P(x) \equiv 2x^3 - 7x^2 + 9x + 4$ is divided by $(x^2 + 2)$.

$$\begin{array}{r|l} & 2x - 7 \\ \hline x^2 + 2 & 2x^3 - 7x^2 + 9x + 4 \\ & 2x^3 \quad\quad + 4x \\ \hline & -7x^2 + 5x + 4 \\ & -7x^2 \quad\quad - 14 \\ \hline & 5x + 18 \end{array}$$

$\therefore P(x) \equiv 2x^3 - 7x^2 + 9x + 4 \equiv (x^2 + 2)(2x - 7) + (5x + 18)$

(ii) If the polynomial $Q(x) \equiv 2x^3 - 7x^2 + ax + b$ is divisible by $(x^2 + 2)$, find the values of a and b. **2**

From (i) $P(x) - (5x + 18)$ is divisible by $(x^2 + 2)$

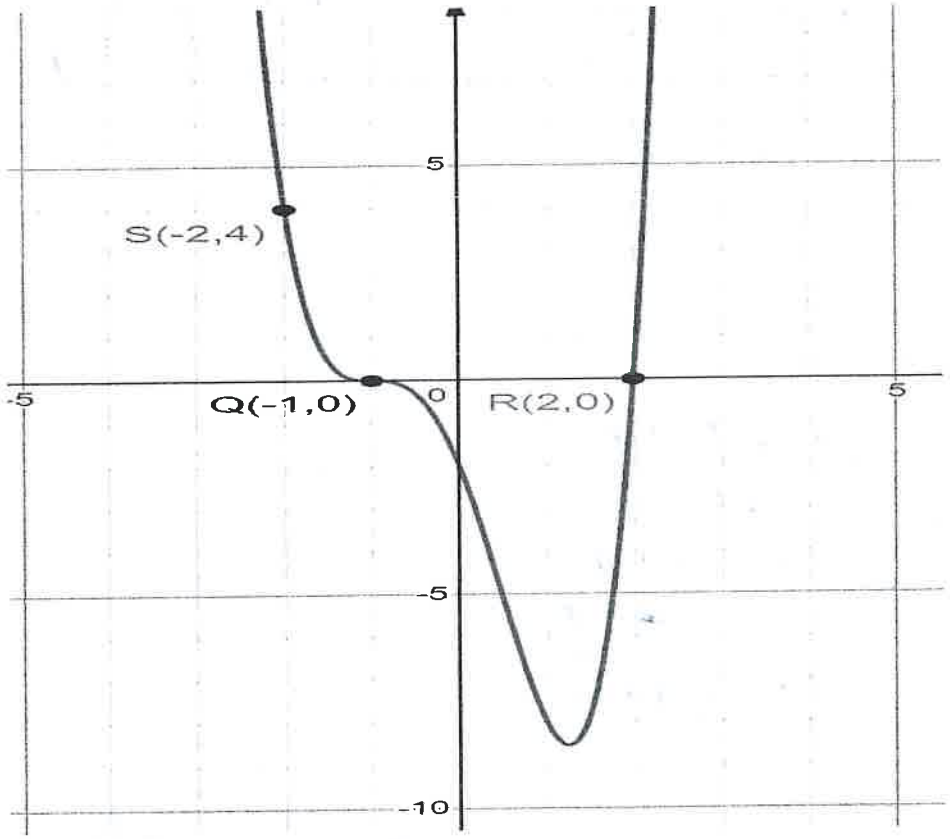
$\therefore Q(x) \equiv P(x) - (5x + 18) = 2x^3 - 7x^2 + 4x - 14$

Matching the coefficients, $a = 4, b = -14$

Question 3 continues on page 10

Marks
3

(c) Drawn below is the graph of the polynomial $y = P(x)$.



Find the equation of the polynomial $P(x)$.

Let the eqn be $P(x) = k(x+1)^3(x-2)$

Sub in $x = -2, y = 4$,

$4 = k(-2+1)^3(-2-2)$

$\therefore k = 1$

$\therefore P(x) = (x+1)^3(x-2)$

Note: Bold answer (-1 mark)

End of Question 3

Question 4 Probability (9 Marks)

a) John bought 5 raffle tickets. 100 tickets are sold altogether. Two tickets are drawn in succession without replacement. Find the probability that John wins, All answers in simplest form. **Marks**

(i) The first prize **1**

$P(\text{John wins the 1st prize}) = \frac{5}{100} = \frac{1}{20}$

(ii) Both prizes **1**

$P(\text{John wins 1st and 2nd}) = \frac{1}{20} \times \frac{4}{99} = \frac{1}{495}$

(iii) Only one prize **2**

$P(\text{John wins the 1st but not 2nd}) + P(\text{John wins 2nd but not 1st})$

$= \left(\frac{5}{100} \times \frac{95}{99}\right) + \left(\frac{95}{100} \times \frac{5}{99}\right)$

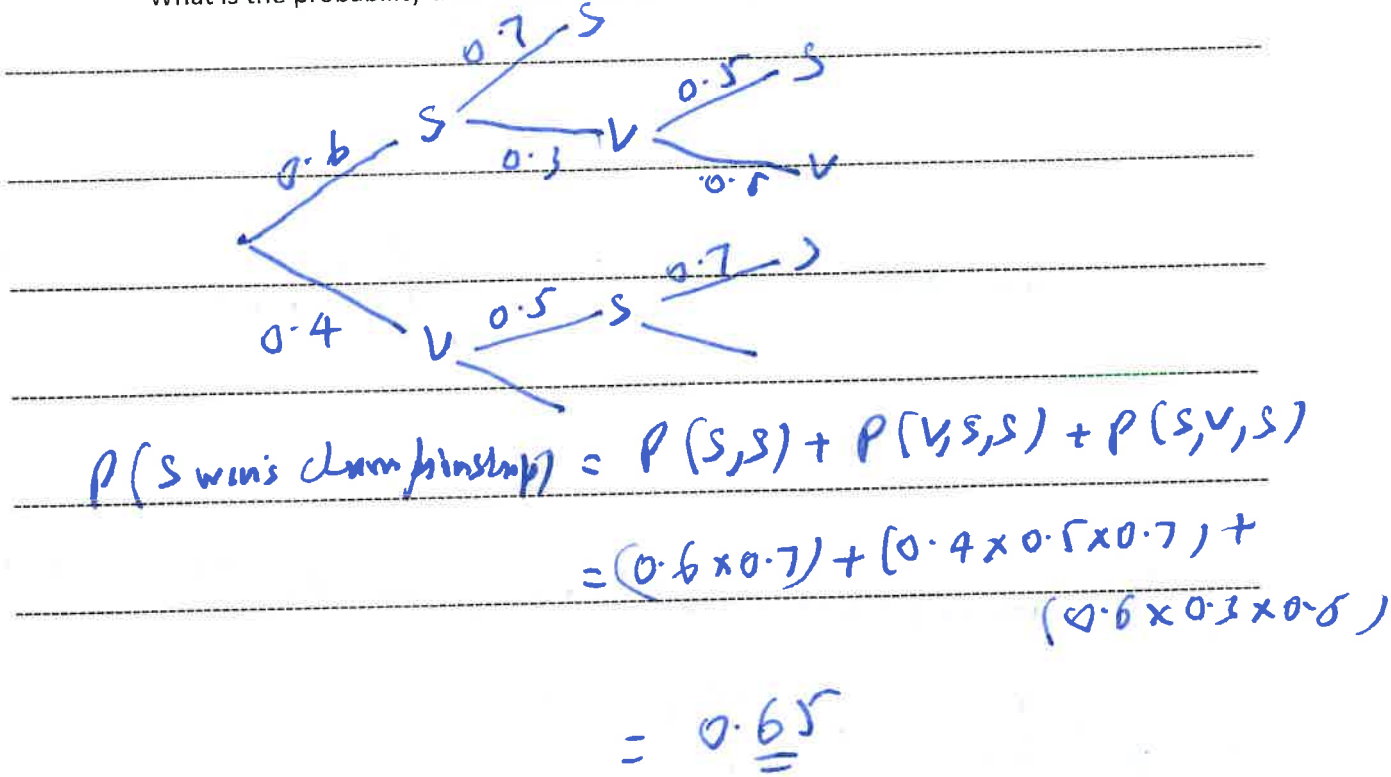
$= \frac{19}{198}$

Question 4 continues on page 12

(b) Serena and Venus are competing in a tennis tournament final. Records indicate that whenever they play a series of matches, Serena's probability of winning is 0.6 and that in subsequent matches in the series, Serena's probability of winning is 0.7 if she won the preceding match but only 0.5 if she lost the preceding match.
A match cannot be drawn and the first player to win two matches wins the championship.

Marks

What is the probability that Serena wins the championship? **2**



$P(S \text{ wins championship}) = P(S,S) + P(V,S,S) + P(S,V,S)$

$= (0.6 \times 0.7) + (0.4 \times 0.5 \times 0.7) + (0.6 \times 0.3 \times 0.5)$

$= 0.65$

Question 4 continues on page 13

c) The two-way table below shows the results of the people taking a lie detector test. Some people are known to be telling the truth while others are known to be lying.

Marks

Test Results

	Accurate	Not Accurate	Total
Truth teller	776	24	800
Liar	153	47	200
Total	929	71	1000

(i) How many people did the lie detector indicate were lying? **1**

24 + 153 = 177

(ii) What percentage of tests were accurate? **1**

$\frac{929}{1000} \times 100 = 92.9\%$

(iii) What is the probability of a person who is lying successfully 'fooling' the lie detector? Express your answer as a percentage. **1**

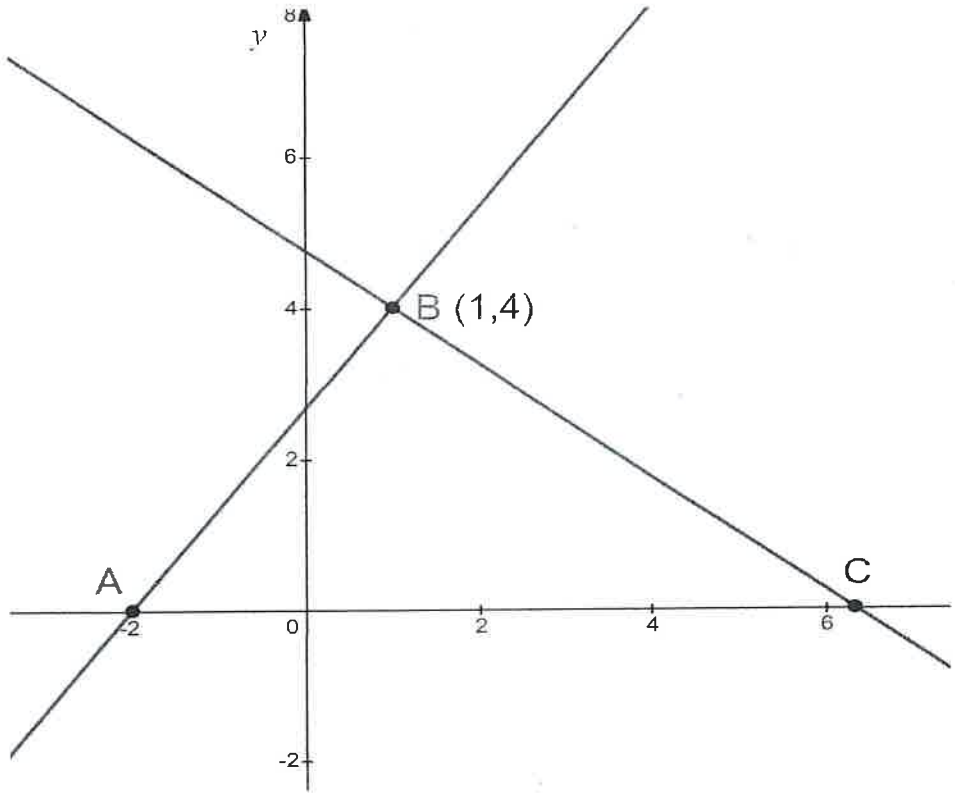
$\frac{47}{200} \times 100$

$= 23.5\%$

End of Question 4

Question 5 Co-ordinate Geometry and Locus (12 Marks)

(a)



Marks

The diagram shows the points A(-2,0), B(1,4) and the point C lies on the x-axis.

(i) Find the gradient of AB

$\frac{4-0}{1-(-2)}$

$= \frac{4}{3}$

1

(ii) Find the equation of the line AB

$y - 0 = \frac{4}{3}(x+2)$ $\quad 4x - 3y + 8 = 0$

2

(iii) BC is perpendicular to AB. Find the gradient of BC.

$m \times \frac{4}{3} = -1$

$m = -\frac{3}{4}$

1

Question 5 continues on page 14

Marks

(iv) Find the equation of the line BC **2**

$y - 4 = -\frac{3}{4}(x-1)$

$3x + 4y - 19 = 0$

1

(v) Find the co-ordinates of C. (Exact Value)

In the eqn of BC, when $y = 0$

$x = \frac{19}{3} = 6\frac{1}{3} \quad \therefore C = (6\frac{1}{3}, 0)$

(vi) Calculate the area of ΔABC.(Exact value) **2**

$= \frac{1}{2} AC \times h$

$= \frac{1}{2} \times 8\frac{1}{3} \times 4 = 16\frac{2}{3}$ units^2

(b) Show that the equation of the locus of a point P(x,y), which moves so that PA = 2PB, where A(-3,2) and B(3,-4) is a circle. **3**

$PA = 2PB \Rightarrow PA^2 = 4PB^2$

$(x+3)^2 + (y-2)^2 = 4[(x-3)^2 + (y+4)^2]$

$\Rightarrow (x-3)^2 + (y+9)^2 = (\sqrt{61})^2$

Circle with centre at $(3, -9)$ and radius $\sqrt{61}$ units

End of Question 5

Question 6 Financial Mathematics (10 Marks)

(a) The table below shows monthly payments for each $1000 borrowed. **Marks**

INTEREST RATE (%p.a)	PERIOD OF LOAN				
	5 years	10 years	15 years	20 years	25 years
5	$18.87	$10.61	$7.91	$6.60	$5.85
6	$19.33	$11.10	$8.44	$7.10	$6.44
7	$19.80	$11.61	$9.00	$7.75	$7.07
8	$20.28	$12.13	$9.56	$8.36	$7,72 $7.72
9	$20.76	$12.67	$10.14	$9.00	$8.39
10	$21.76	$13.22	$10.75	$9.65	$9.10
11	$21,74	$13.78	$11.37	$10.32	$9.80
12	$22.24	$14.35	$12.00	$11.01	$10.53
13	$22.75	$14.93	$12.65	$11.72	$11.28
14	$23.27	$15.53	$13.32	$12.44	$12.04
15	$23.79	$16.13	$14.00	$13.17	$12.81

David borrows $150 000 to buy a vacant land at 8%p.a over twenty-five years.

(i) Use this information in the table to calculate David's monthly payment on this loan. **2**

Repayment = $7.72 × ($150000 ÷ 1000)

= $1158

(ii) How much does David pay in total to repay the loan? **2**

$1158 × 12 × 25

= $347 400

Question 6 continues on page 17

(iii) How much extra per month would David pay if he were to repay the same loan over 20 years? **Marks 3**

$8.36 × (150000 ÷ 1000)

= $1254

Extra payment per months = $1254 − $1158

= $96

(iv) Anna wants to buy a land for $150 000. She saved some money for deposit and will borrow the rest at 8% p.a will repay the loan over fifteen years, paying $1195 monthly. **2**

How much will she borrow?

Let it be $x

$9.56 × (x ÷ 1000) = $1195

x = $125000

(v) How much has she saved for a deposit? **1**

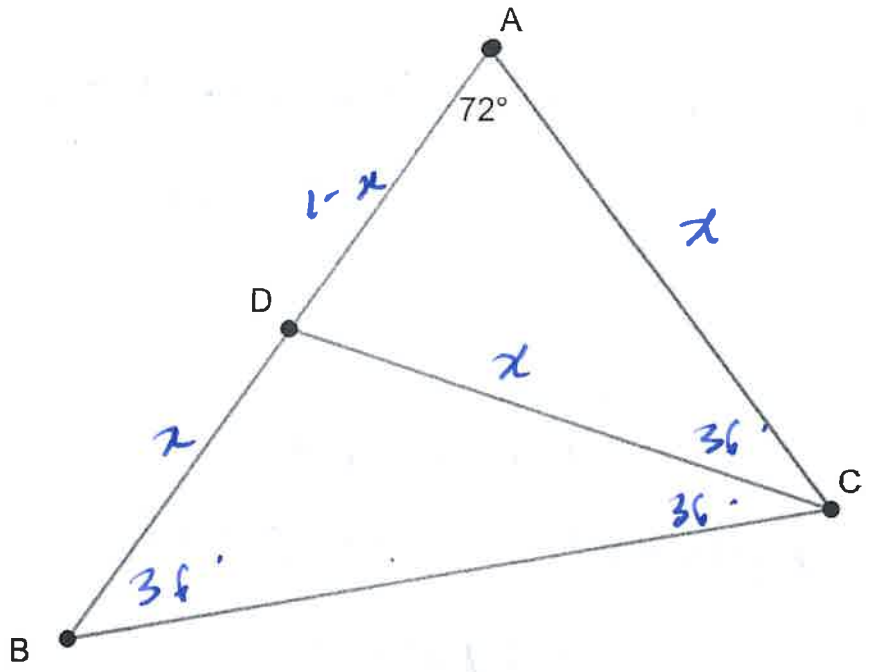
Deposit = $150000 − $125000

= $25000

End of Question 6

Question 7 Similarity (8 Marks)

Marks



ABC is an isosceles triangle with $\angle$ BCA = $\angle$ BAC = 72°. And sides AB = BC = 1 unit.
CD bisects $\angle$ BCA and AC = x units.

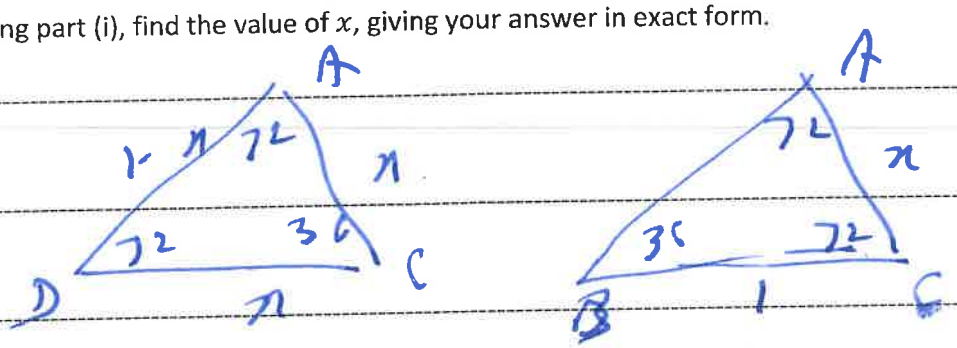
(i) Show that ΔABC is similar to ΔDCA. **3**

$\triangle ABC$	$\triangle DCA$
$\angle ABC = 36°$	$\angle DCB = 36°$
$\angle ACB = 72°$	$\angle CAD = 72°$

$\triangle ABC \parallel\!\parallel \triangle ADC$ (equiangular)

Question 7 continues on page 18

(ii) By using part (i), find the value of x, giving your answer in exact form. **Marks 3**



$$\frac{AC}{BC} = \frac{AD}{AC}$$

$$\frac{x}{1} = \frac{1-x}{x} \Rightarrow x^2 + x - 1 = 1$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

but $x > 0$ $\therefore x = \left(\frac{-1+\sqrt{5}}{2}\right)$ units

(i) A cube is enlarged to a bigger size. If the surface area of the larger cube is $6\frac{1}{4}$ times the surface area of the smaller cube, find the ratio of volume of the smaller cube to that of the larger cube. Give your answer in simplest form. **2**

$$A_1 : A_2 = 4 : 25$$

$$l_1 : l_2 = \sqrt{4} : \sqrt{25} = 2 : 5$$

$$V_1 : V_2 = 2^3 : 5^3$$

$$= 8 : 125$$

End of Question 7

Question 8 Single Data Analysis (8 Marks)

(a)

	Mean	Standard Deviation	Chris' Mark
Mathematics	61	12	83
English	72	5	83

Marks

(i) Using the table above, calculate Chris' z-scores for both English and Mathematics. **2**

Z Score for English $= \frac{83 - 72}{5} = 2.20$

.. .. Maths $= \frac{83 - 61}{12} = 1.83$

(ii) It said in the report that Chris performed better in English than Mathematics. Chris could not understand this statement. Explain it to him. **1**

As the z-score for English is higher than that for Maths

English marks is within the top 5% of the students

Question 8 continues on page 21

(iii) What should be the minimum marks in Mathematics that Chris should get, so that his performance in Mathematics is better than in English? **Marks 2**

$\frac{x-61}{12} = 2.2$

$x = 87.4$

$\therefore$ He should score 88 mark

(b) The petrol price, in cents per litre, is recorded eight days at two petrol stations in one suburb. **3**

Mobil:	**93.9**	**97.9**	**92.4**	**93.9**	**98.5**	**92.3**	**97.9**	**99.9**
Ampol:	**95.9**	**94.9**	**96.9**	**93.9**	**97.9**	**93.5**	**96.9**	**98.9**

At which petrol station is the price of petrol more consistent? Explain.

Mobil: SD = 2.83

Ampol: SD = 1.71

Lower SD $\Rightarrow$ more consistent

$\therefore$ Ampol

End of Question 8

Quesdtion 9 — Logarithms (8 Marks)

(a) If $log_2 3 = 1.6$ and $log_2 5 = 2.3$ evaluate without using a calculator: **Marks**

(i) $log_2 1.8$ **2**

$= \log_2\left(\frac{9}{5}\right) = 2\log_2 3 - \log_2 5$

$= 2(1.6) - 2.3$

$= 0.9$

(ii) $log_2 75$ **2**

$= \log_2(3 \times 25) = \log_2 3 + 2\log_2 5$

$= 1.6 + 2 \times 2.3$

$= 6.2$

(b) Solve for x:

(i) $log_5 x = \frac{log_5 64}{log_5 16}$ **2**

$= \frac{3\log_5 4}{2\log_5 4} = \frac{3}{2}$

$\therefore x = 5^{3/2} = 5\sqrt{5}$

(c) (ii) Simplify $2log_{10}2 + 4log_{10}5 - 2log_{10}50$ **2**

$\log_{10}\left[\frac{2^2 \times 5^4}{50^2}\right] = \log_{10}(1) = 0$

End of Question 9

Question 10 Algebra and Problem solving (18 Marks)

(a) Solve by factoring: $4x^2 - 20x + 21 = 0$ **Marks 2**

$(2x-7)(2x-3)=0$

$x = \frac{7}{2}$ or $x = \frac{3}{2}$

(b) Simplify: $\dfrac{\frac{1}{a}-4a}{4-4a-\frac{1}{a}}$ **2**

$\frac{1-4a^2}{-1+4a-4a^2}$

$= \frac{(2a-1)(2a+1)}{(2a-1)^2}$

$= \frac{2a+1}{2a-1}$

(c) If $5^x = 1000$, evaluate 5^{x-3}. **2**

$5^{x-3} = \frac{5^x}{5^3} = \frac{1000}{125} = 8$

Question 10 continues on page 23

(d) Marks 3

If $\frac{8^{m-n}\times 14^{m+n}}{21^{m+n}\times 3^{m-2n}} = a^p$ find the values of a and p.

$$\frac{2^{3m-3n}\times 2^{m+n}\times 7^{m+n}}{3^{m+n}\times 7^{m+n}\times 3^{m-2n}} \div \frac{2^{4m-2n}}{3^{2m-n}}$$

$$= \left(\frac{4}{3}\right)^{2m-n}$$

$a = \frac{4}{3}$, $p = 2m-n$

(e) Simplify $\frac{(x-y)^2-z^2}{xy-y^2-yz} \times \frac{z}{x^2+xy-xz} \div \frac{xz-yz+z^2}{x^2-(y-z)^2}$ 3

$$\frac{(x-y-z)(x-y+z)\times z\times (z-y+z)(x+y-z)}{y(x-y-z)\times x(x+y-z)\times z(x-y+z)}$$

$$= \frac{x-y+z}{xy}$$

Question 10 continues on page 24

(f) A family has a large back yard and decides to construct a swimming pool. Here is a plan of the yard which measures 40 metres by 30 metres; and the proposed swimming pool, which is to measure 12 metres by 10 metres, with a depth of 2 metres.

Marks

1

(i) How much earth will need to be removed from the hole?

$12 \times 10 \times 2$

$= 240 m^3$

(ii) If the earth is spread evenly over the rest of the yard, by how much would the height of the yard increase?

2

$240 \div (30 \times 40 - 12 \times 10)$

$= \frac{2}{7} m$

(iii) How deep should the hole be, If the homeowner insists that the depth of the pool should be 2 metres and that the earth should be spread over the rest of the yard.

3

$\frac{120x}{(30 \times 40 - 12 \times 10)} = 2 - x$

$x = \frac{1080 \times 2}{(120 + 1080)} = 1.8 m$

End of Paper