NORTH SYDNEY BOYS HIGH SCHOOL

STAGE 5.3 MATHEMATICS

Year 10 Yearly Examination

Tuesday, 12 November 2019

General instructions

- Working time – 100 min.
- **Commence each new question on a new page**.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **all** questions.

Class (please ✔)

- ◯ 10MAT.A – Mr Umakanthan
- ◯ 10MAT.B – Ms Ziaziaris
- ◯ 10MAT.C – Mr Berry
- ◯ 10MAT.D – Mr Ireland
- ◯ 10MAT.E – Mr Hwang

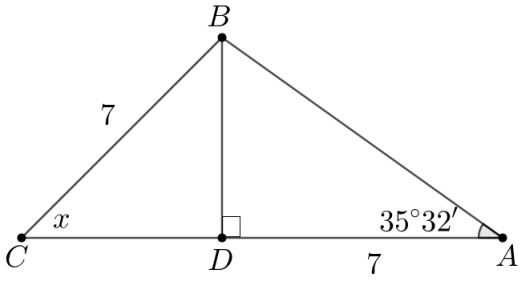
NAME ..... **# EXTRA BOOKLETS USED**: .....

Marker's use only.

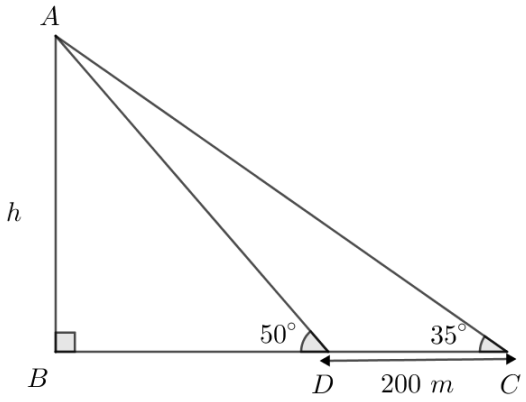
QUESTION	1	2	3	4	5	6	7	8	9	10	**Total**
MARKS	/12	/8	/5	/16	/13	/8	/10	/10	/13	/5	/100

Question 1 (12 Marks) Commence a NEW page. **Marks**

(a) Find x to the nearest minute. **2**



(b) In the diagram, $\angle ADB = 50°$, $\angle ACD = 35°$, $DC = 200m$. Let $AB = h$ metres.

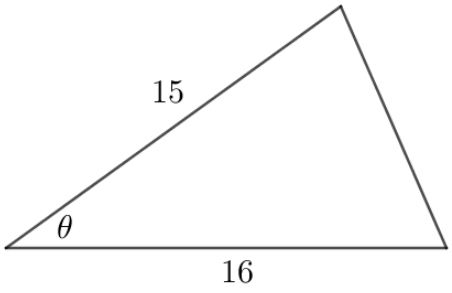


i. Use the sine rule in $\triangle ADC$ to show that : $AD = \dfrac{200 \sin 35°}{\sin 15°}$ **2**

ii. Hence, evaluate h correct to 2 decimal places. **2**

(c) A yacht sails due south from point P for a distance of 12 nautical miles and then 8 nautical miles on a bearing of 160 degrees. How far is the yacht from P? Answer correct to one-tenth of a nautical mile. **3**

(d) Find the angle θ, given that the triangle has area 75 cm^2. Give answer in radian measure. **3**



Question 2 (8 Marks) Commence a NEW page. **Marks**

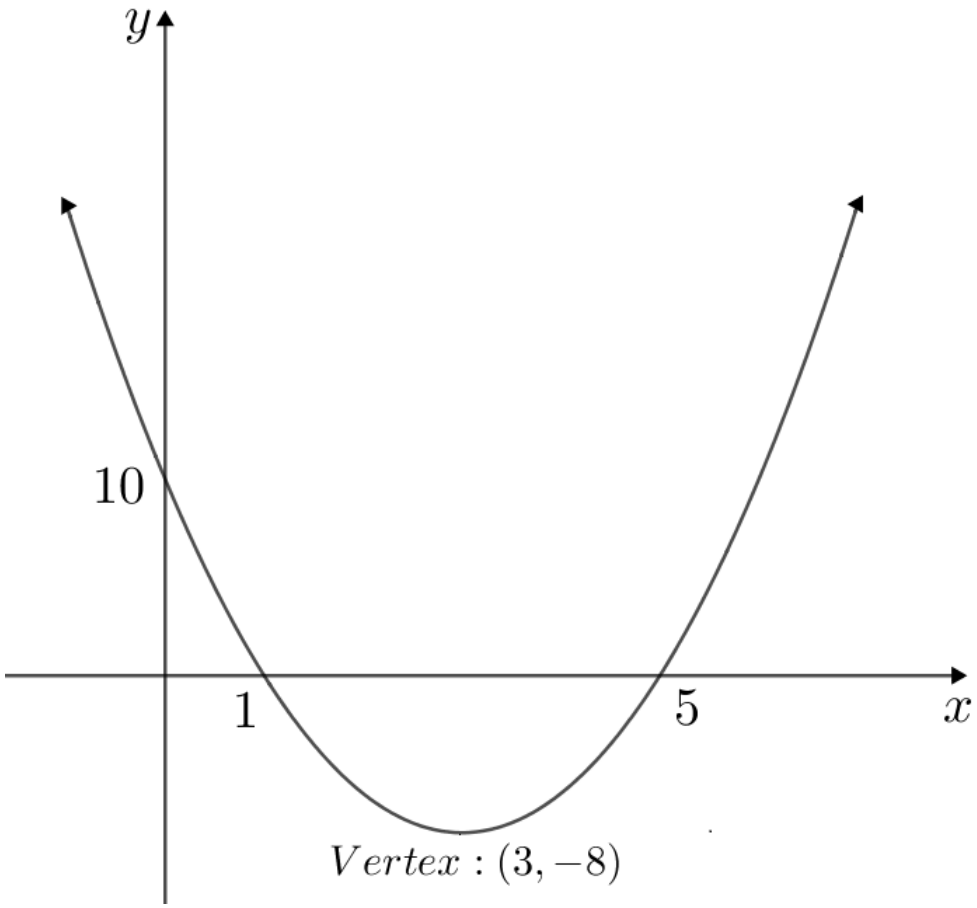
(a) Express the following equation in the form $y = a(x-h)^2 + k$, then sketch the curve showing all important features. **3**

$$y = 2x^2 - 5x + 2$$

(b) Solve the following pair of equations simultaneously. Hence find the coordinates of the point(s) of intersection of the two graphs if there are any. **3**

$$y = x^2 + 7x - 6 \text{ and } y = 5x + 2$$

(c) State the equation of the quadratic function sketched below. **2**



Question 3 (5 Marks) Commence a NEW page. **Marks**

(a) Chris and James each invested \$9 000. James' investment depreciated at a rate of 4% pa for 3 years, while during that time Chris' investment grew at a rate of 8% (compounded yearly). Find the difference in the value of their investments at the end of the 3 years. **2**

(b) The following table is the set of future values of \$1 invested into an annuity for different interest rates and period of time. **Use the table to answer parts i and ii**.

Future value of an annuity of \$1

Interest rate per period / Period	4	4.5	5	5.5	6
2	2.0400	2.0450	2.0500	2.0550	2.0600
3	3.1216	3.1370	3.1525	3.1680	3.1836
4	4.2465	4.2782	4.3101	4.3423	4.3746
5	5.4163	5.4707	5.5256	5.5811	5.6371
6	6.6330	6.7169	6.8019	6.8881	6.9753
7	7.8983	8.0192	8.1420	8.2669	8.3938

i. What would be the future value of an annuity of \$4 500 invested per year for 5 years at 5.5% pa with interest compounded annually? **1**

ii. After the second year of the above investment, an additional \$1 500 is invested into the fund. Find the increase in the future value in comparison to part i. **2**

Question 4 (16 Marks) Commence a NEW page. **Marks**

(a) Simplify the expression fully, writing each pronumeral in the answer with a positive index. **2**

$$\frac{a^5b^{-3}+b^5a^{-3}}{b^8+a^8}$$

(b) Factorise fully:

i. $12x^2-7xy-12y^2$ **2**

ii. $(a-2)^2-(a+2)^2$ **2**

iii. $x^2-25y^2+2x+10y$ **2**

(c) Simplify fully:

i. $\dfrac{2}{\sqrt{5}-2}+\dfrac{2}{\sqrt{5}+2}$ **1**

ii. $\dfrac{6^{n+1}-6^n}{12^n+12^{n+2}}$ **2**

iii. $\dfrac{x^3+2x^2+4x+8}{2x^3-8x}\times\dfrac{x^3-4x^2+4x}{x^4-16}$ **2**

iv. $\dfrac{4b}{a^2+2ab}-\dfrac{3a}{ab+2b^2}+\dfrac{3a-2b}{ab}$ **3**

Question 5 (13 Marks) Commence a NEW page. **Marks**

(a) Sketch the following, showing all important features (each on a separate set of axes using at least $\frac{1}{3}$ of the page)

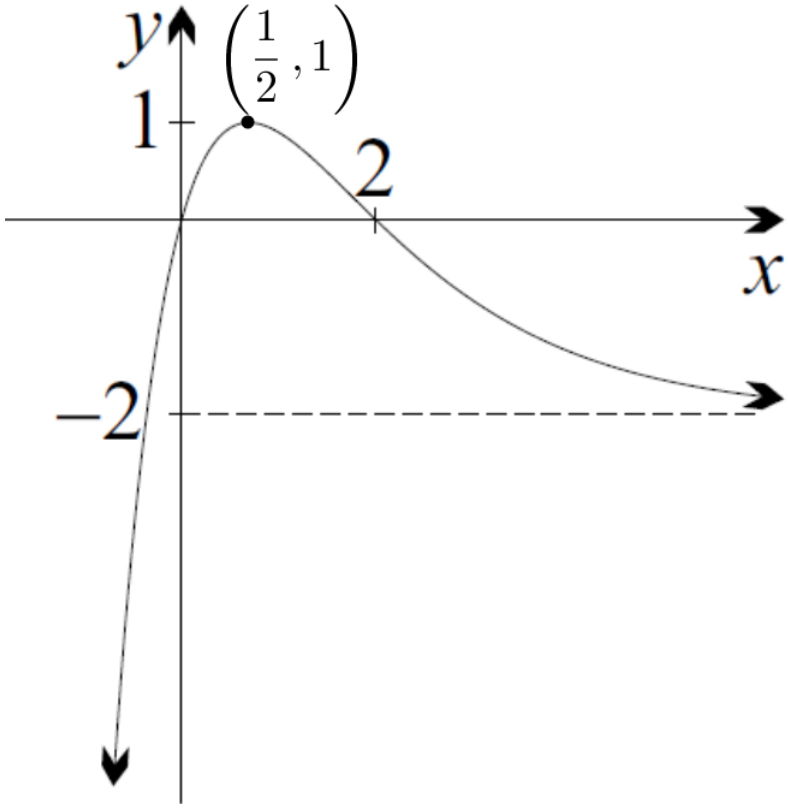
i. $y = 2x^3$ **2**

ii. $y = \dfrac{-3}{x} + 1$ **2**

iii. 3^{3-x} **2**

(b) Find the inverse function of $y = \dfrac{ax+b}{x+c}$ **2**

(c) The graph of $y = f(x)$ is shown below



Draw a separate half-page diagram for each of the following functions, showing all important points.

i. $y = f(x) - 1$ **1**

ii. $y = 1 - f(x)$ **2**

iii. $y = 3f(-x)$ **2**

Question 6 (8 Marks) Commence a NEW page. **Marks**

(a) The Mathematics and English marks for a class of 10 students are given below.

Mathematics : 78 85 89 91 87 77 95 87 93 91
English : 88 98 79 87 67 77 92 78 83 86

i. Calculate, correct to two decimal places, the mean and standard deviation for each set of marks. **2**

ii. If a student scored 87 for the Mathematics test and 83 for the English test, which is the better mark relative to the class results? Give a clear explanation. **2**

(b) In a group of 150 students, 60 study Physics, 54 study Chemistry and 23 study both Physics and Chemistry. One student is selected at random from the group. What is the probability that the student studies

i. Physics but not Chemistry **1**

ii. Chemistry, given that the student also studies Physics. **1**

(c) From a group of 12 students made up of 9 junior students and 3 senior students, a random sample of 3 students are selected. What is the probability that:

i. all selected students are junior students **1**

ii. at least one is a senior student **1**

Question 7 (10 Marks) Commence a NEW page. **Marks**

(a) Find the quotient and remainder in the following

i. $(4x^2 - 3x + 5) \div (x + 2)$ **2**

ii. $(-2x^4 + 7x^2 - 3x + 5) \div (x^2 - 3)$ **2**

(b) Find the remainder of the following division without actual division. **1**

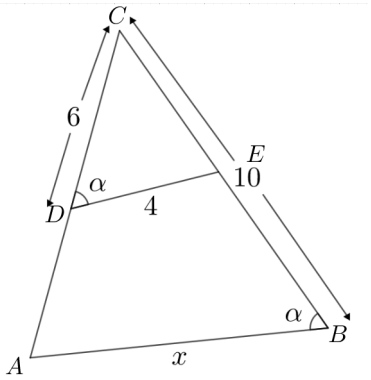
$$(5x^4 - 2x^3 - 3x) \div (x + 4)$$

(c) When the polynomial $P(x) = x^3 - ax^2 + bx - 7$ is divided by $(x + 3)$, the remainder is 5. When $P(x)$ is divided by $(x - 2)$, the remainder is 5. Find the value of a and b. **2**

(d) Use the factor theorem to fully factorise $P(x) = x^3 - 7x^2 + 14x - 8$ **3**

Question 8 (10 Marks) Commence a NEW page. **Marks**

(a) In the triangle $\triangle ABC$ a line DE is drawn. Given that $CD = 6, BC = 10, DE = 4$ and $\angle ABC = \angle CDE$.



i. Prove $\triangle ABC \;|||\triangle EDC$ **2**

ii. Hence, find the value of x **1**

(b) An architect designed a building and had a model made to a scale of 1:300.

i. The external surface area of the building is 90 000 m^2. Calculate the external surface area of the model to the nearest m^2. **1**

ii. The volume of the building is 605 000m^3. Calculate the volume of the model in m^3. **1**

(c) Sketch the region defined by: **3**

$$x \leq 3 - y$$
$$4x + 3y > -3$$

(d) Find the locus of a point P if it moves equidistantly from points $(4, 3)$ and $(-2, -1)$. **2**

Question 9 (13 Marks) Commence a NEW page. **Marks**

(a) Simplify **1**

$$\log_2 3 + \log_2 5 - \log_2 7$$

(b) Given that $\log_x 2 = a, \log_x 3 = b$ and $\log_x 5 = c$, express the following in terms of a, b and c

i. $\log_x 30$ **1**

ii. $\log_x 1875$ **2**

(c) Sketch the graphs of $y = \log_3 x$ and $y = \log_5 x$ on the same set of axes clearly labelling each graph and showing all important features. **3**

(d) Solve for x, correct to two decimal places.

i. $5^x = 7$ **2**

ii. $2^{2-x} = 13^{3x+1}$ **2**

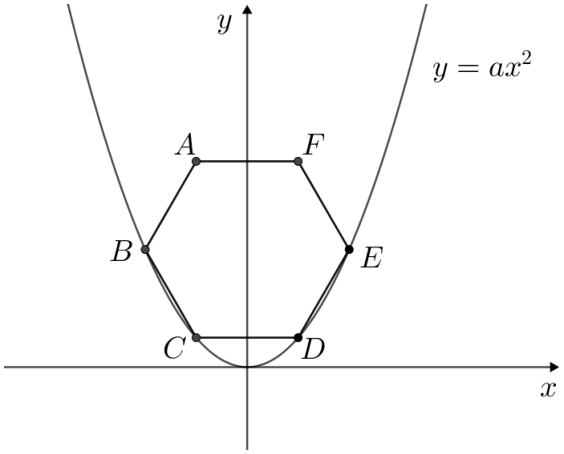
iii. $(\log_2 x)^2 - 8 = \log_2 x^2$ **2**

Question 10 (5 Marks) Commence a NEW page. **Marks**

(a) If $f(n) = \sqrt{n+1} - \sqrt{n}$, find the value of **2**

$$\frac{1}{f(1)} - \frac{1}{f(2)} + \frac{1}{f(3)} - \frac{1}{f(4)} + \cdots - \frac{1}{f(98)} + \frac{1}{f(99)}$$

(b) Inside the parabola $y = ax^2$, four points of regular hexagon, with side length of 2 units, B, C, D and E are touching the parabola.



i. Find the value of a. **1**

ii. Let P be the point where the diagonals of the hexagon intersect. When a line $y = mx + n$ passes through the point P and bisects the quadrilateral $ABCF$, find the exact values of m and n. **2**

End of Examination

Suggested Solutions

Question 1

(a) (2 marks)

$$BD = 7\tan 35°32'$$
$$x = \sin^{-1}\left(\frac{7\tan 35°32'}{7}\right)$$
$$x = 45°35'$$

(b) i. (2 marks)

$$\angle DAC = 50° - 35° = 15°$$
$$\frac{AD}{\sin 35°} = \frac{200}{\sin 15°}$$
$$AD = \frac{200\sin 35°}{\sin 15°}$$
as required

ii. (2 marks)

$$h = AD\sin 50°$$
$$h = \frac{200\sin 35° \times \sin 50°}{\sin 15°}$$
$$\approx 339.53\ m\ (2\ d.p)$$

(c) (3 marks)

$$d = \sqrt{12^2 + 8^2 - 2 \times 9 \times 12\cos 160°}$$
$$= 19.7\ nautical\ miles$$

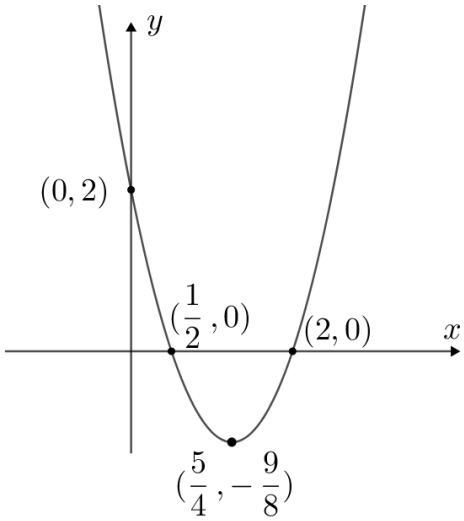
(d) (3 marks)

$$75 = \frac{1}{2} \times 16 \times 15 \times \sin\theta$$
$$\sin\theta = \frac{5}{8}$$
$$\theta = 0.675\ or\ 2.466$$

Question 2

(a) (3 marks)

$$y = 2x^2 - 5x + 2$$
$$= 2\left(x^2 - \frac{5x}{2} + \frac{25}{16}\right) - \frac{25}{8} + 2$$
$$= 2\left(x - \frac{5}{4}\right)^2 - \frac{9}{8}$$



(b) (3 marks)

$$x^2 + 7x - 6 = 5x + 2$$
$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$\therefore (-4, -18), (2, 12)$$

(c) (2 marks)

$$y = a(x-1)(x-5)$$
$$10 = a(0-1)(0-5)$$
$$10 = 5a$$
$$\therefore a = 2$$
$$y = 2(x-1)(x-5)$$
$$= 2x^2 - 12x + 10$$
$$= 2(x-3)^2 - 8$$

Question 3

(a) (2 marks)

$$\begin{aligned} James &= 9000(1-0.04)^3 \\ &= \$7962.62 \\ Chris &= 9000(1+0.08)^3 \\ &= \$11337.41 \\ \therefore Difference &= 11337.41 - 7962.62 \\ &= \$3374.79 \end{aligned}$$

(b) i. (1 mark)

$$4500 \times 5.5811 = \$25114.95$$

ii. (2 marks)

$$\begin{aligned} 4500 \times 5.5811 + 1500 \times 3.168 &= 29866.95 \\ Increase &= 29866.95 - 25114.95 \\ &= \$4752 \end{aligned}$$

Question 4

(a) (2 marks)

$$\begin{aligned} \frac{a^5b^{-3}+b^5a^{-3}}{b^8+a^8} &= \frac{\frac{a^5}{b^3}+\frac{b^5}{a^3}}{b^8+a^8} \\ &= \frac{\frac{a^8+b^8}{a^3b^3}}{b^8+a^8} \\ &= \frac{1}{a^3b^3} \end{aligned}$$

(b) i. (2 marks)

$$\begin{aligned} &12x^2 - 7xy - 12y^2 \\ &= (4x+3y)(3x-4y) \end{aligned}$$

ii. (2 marks)

$$\begin{aligned} &(a-2)^2 - (a+2)^2 \\ &= [(a-2)+(a+2)] \\ &\times [(a-2)-(a+2)] \\ &= (2a)(-4) \\ &= -8a \end{aligned}$$

iii. (2 marks)

$$\begin{aligned} &x^2 - 25y^2 + 2x + 10y \\ &= (x+5y)(x-5y) + 2(x+5y) \\ &= (x+5y)(x-5y+2) \end{aligned}$$

(c) i. (1 mark)

$$\begin{aligned} &\frac{2}{\sqrt{5}-2} + \frac{2}{\sqrt{5}+2} \\ &= \frac{2(\sqrt{5}+2)}{5-4} + \frac{2(\sqrt{5}-2)}{5-4} \\ &= 2\sqrt{5}+4+2\sqrt{5}-4 \\ &= 4\sqrt{5} \end{aligned}$$

ii. (2 marks)

$$\begin{aligned} &\frac{6^{n+1}-6^n}{12^n+12^{n+2}} \\ &= \frac{6\times 6^n - 6^n}{12^n + 144\times 12^n} \\ &= \frac{6^n(6-1)}{12^n(1+144)} \\ &= \frac{\cancel{6^n}^{1}(5)}{\cancel{6^n}\times 2^n(\cancel{145}^{29})} \\ &= \frac{1}{29\times 2^n} \end{aligned}$$

iii. (2 marks)

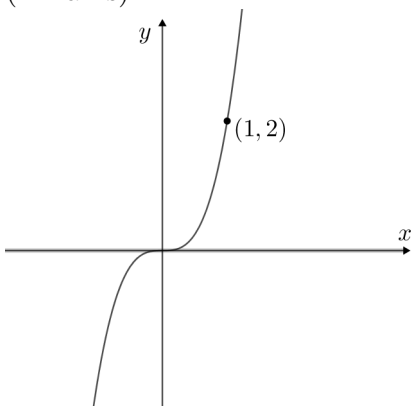
$$\begin{aligned} &\frac{x^3+2x^2+4x+8}{2x^3-8x} \times \frac{x^3-4x^2+4x}{x^4-16} \\ &= \frac{x^2(x+2)+4(x+2)}{2x(x^2-4)} \times \frac{x(x^2-4x+4)}{(x^2+4)(x^2-4)} \\ &= \frac{\cancel{(x^2+4)}\cancel{(x+2)}}{\cancel{2x}\cancel{(x+2)}\cancel{(x-2)}} \times \frac{\cancel{x}\cancel{(x-2)^2}}{\cancel{(x^2+4)}\cancel{(x+2)}\cancel{(x-2)}} \\ &= \frac{1}{2(x+2)} \end{aligned}$$

iv. (3 marks)

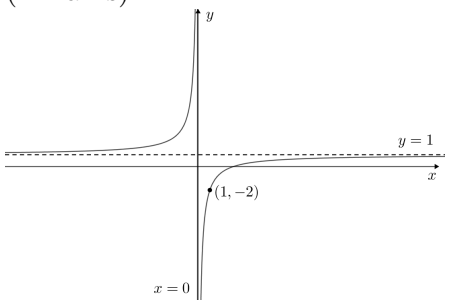
$$\begin{aligned} &\frac{4b}{a^2+2ab} - \frac{3a}{ab+2b^2} + \frac{3a-2b}{ab} \\ &= \frac{4b^2-3a^2+(3a-2b)(a+2b)}{ab(a+2b)} \\ &= \frac{4b^2-3a^2+3a^2+6ab-2ab-4b^2}{ab(a+2b)} \\ &= \frac{4}{a+2b} \end{aligned}$$

Question 5

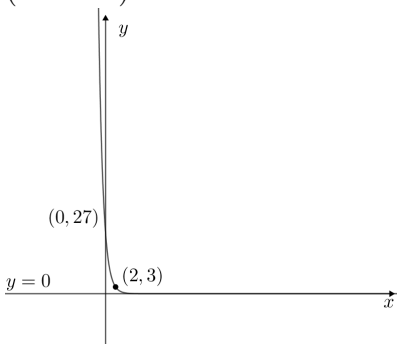
(a) i. (2 marks)



ii. (2 marks)



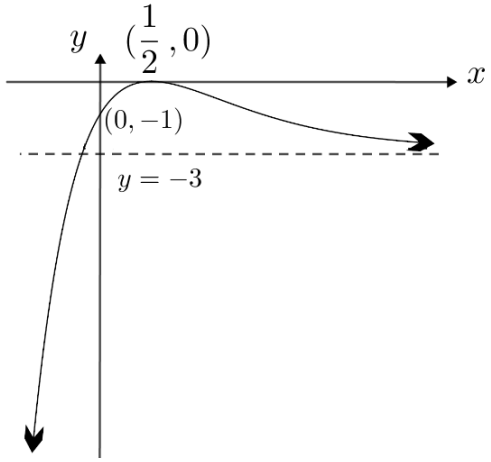
iii. (2 marks)



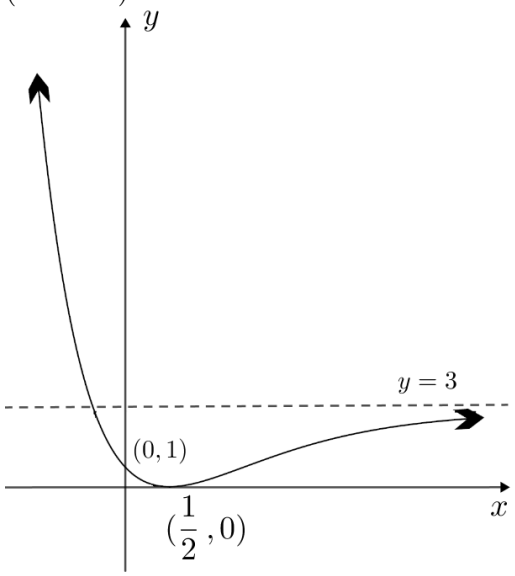
(b) (2 marks)

$$
\begin{aligned}
x &= \frac{ay+b}{y+c} \\
x(y+c) &= ay+b \\
xy+xc &= ay+b \\
xy-ay &= b-xc \\
y(x-a) &= b-xc \\
y &= \frac{b-xc}{x-a} \\
or \quad &= \frac{xc-b}{a-x}
\end{aligned}
$$

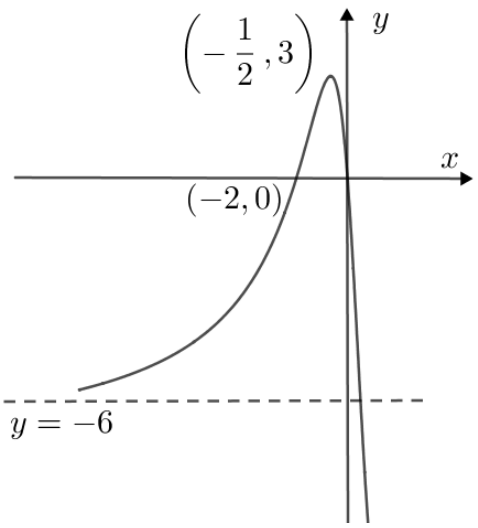
(c) i. (1 mark)



ii. (2 marks)



iii. (2 marks)



Question 6

(a) i. (2 marks)

	English	Maths
Mean	83.5	87.3
Standard Deviation	8.29 (2 d.p)	5.66 (2 d.p)

ii. (2 marks)

$$Mathematics\ z-score = \frac{87-87.3}{5.66} = -0.053$$
$$English\ z-score = \frac{83-83.5}{8.29} = -0.060$$

The student scored better in Mathematics as z-score is higher for Mathematics than in English.

(b) i. (1 mark)

$$60-23=37$$
$$\therefore \frac{37}{150}$$

ii. (1 mark)

$$\frac{23}{60}$$

(c) i. (1 mark)

$$\frac{9}{12}\times\frac{8}{11}\times\frac{7}{10}=\frac{21}{55}$$

ii. (1 mark)

$$1-\frac{21}{55}=\frac{34}{55}$$

Question 7

(a) i. (2 marks)

$$\begin{array}{r} 4x-11 \\ x+2\overline{)\ 4x^2-3x+5} \\ \underline{-4x^2-8x\ \ \ } \\ -11x+5 \\ \underline{11x+22} \\ 27 \end{array}$$

ii. (2 marks)

$$\begin{array}{r} -2x^2+1 \\ x^2-3\overline{)\ -2x^4+7x^2-3x+5} \\ \underline{2x^4-6x^2\ \ \ } \\ x^2-3x+5 \\ \underline{-x^2\ \ \ +3} \\ -3x+8 \end{array}$$

(b) (1 mark)

$$5(-4)^4-2(-4)^3-3(-4)=1420$$

(c) (2 marks)

$$P(-3)=-27-9a-3b-7=5$$
$$3a+b=-13$$
$$P(2)=8-4a+2b-7=5$$
$$2a-b=-2$$
$$\therefore 5a=-15$$
$$\therefore a=-3, b=-4$$

(d) (2 marks)

$$\begin{array}{r} x^2-6x+8 \\ x-1\overline{)\ x^3-7x^2+14x-8} \\ \underline{-x^3+x^2\ \ \ } \\ -6x^2+14x \\ \underline{6x^2-6x\ \ \ } \\ 8x-8 \\ \underline{-8x+8} \\ 0 \end{array}$$

$\therefore (x-1)(x-4)(x-2)$

Question 8

(a) i. (2 marks)
In $\triangle ABC$ & $\triangle EDC$
$\angle C$ is common
$\angle ABC=\alpha=\angle CDE$
$\angle BAC=\angle DEC$(angle sum of $\triangle$)
$\triangle ABC|||\triangle EDC$(Equiangular Similarity)

ii. (1 mark)

$$\frac{AB}{DE}=\frac{BC}{DE}(ratio\ of\ sides\ in\ similar\ \triangle s)$$
$$\frac{x}{4}=\frac{10}{6}$$
$$x=\frac{20}{3}$$

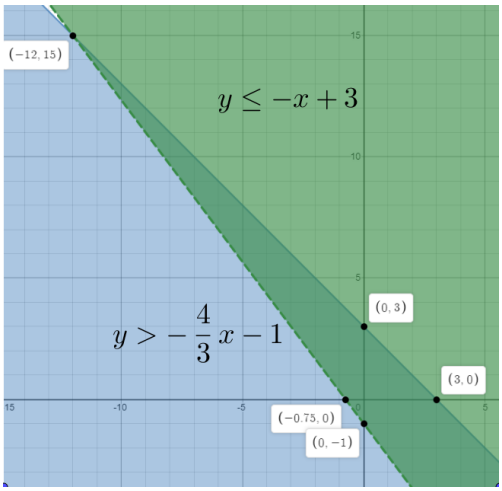
(b) i. (1 mark)

$$ratio\ of\ sides = 1:300$$
$$ratio\ of\ surface\ area = 1:90\ 000$$
$$\therefore 1m^2$$

ii. (1 mark)

$$ratio\ of\ Volume = 1:27\times10^6$$
$$\therefore Volume\ of\ model = \frac{605000}{27000000}$$
$$= 0.0224\ m^3(4\ d.p)$$

(c) (3 marks)



(d) (2 marks)

$$PA = PB \Rightarrow PA^2 = PB^2$$
$$(x-4)^2+(y-3)^2 = (x+2)^2+(y+1)^2$$
$$x^2-8x+16+y^2-6y+9$$
$$= x^2+4x+4+y^2+2y+1$$
$$12x+8y-20=0$$
$$3x+2y-5=0$$

Question 9

(a) (1 mark)

$$\log_2\left(\frac{3\times5}{7}\right) = \log_2\frac{15}{7}$$

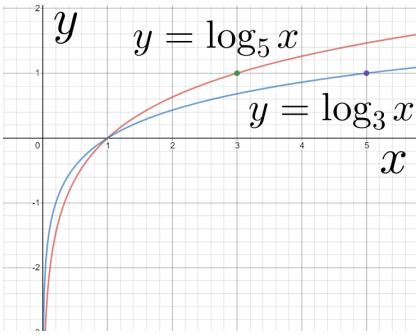
(b) i. (1 mark)

$$\log_x 30 = \log_x 2\times3\times5$$
$$= \log_x 2+\log_x 3+\log_x 5$$
$$= a+b+c$$

ii. (2 marks)

$$\log_x 1875 = \log_x 3\times5^4$$
$$= \log_x 3+\log_x 5^4$$
$$= \log_x 3+4\log_x 5$$
$$= b+4c$$

(c) (3 marks)



(d) i. (2 marks)

$$5^x = 7$$
$$x\ln5 = \ln7$$
$$x = \frac{\ln7}{\ln5}$$
$$= 1.21$$

ii. (2 marks)

$$2^{2-x} = 13^{3x+1}$$
$$(2-x)\ln2 = (3x+1)\ln13$$
$$x(3\ln13+\ln2) = 2ln2-\ln13$$
$$x = \frac{2\ln2-\ln13}{3\ln13+\ln2}$$
$$= -0.14$$

iii. (2 marks)

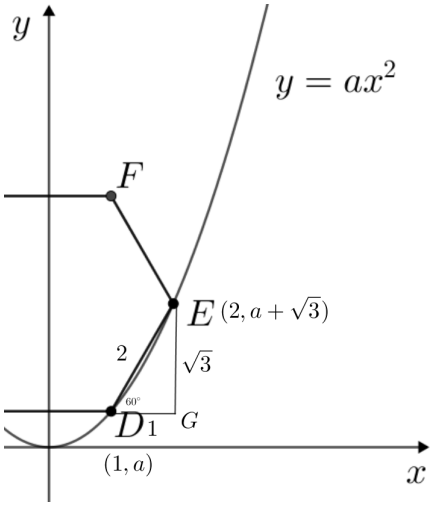
$$Let\ \log_2 x = a$$
$$a^2-8 = 2a$$
$$a^2-2a-8=0$$
$$(a-4)(a+2)=0$$
$$\therefore \log_2 x = 4, \log_2 x = -2$$
$$x = 16\ or\ \frac{1}{4}$$

Question 10

(a) (2 marks)

$$\frac{1}{\sqrt{2}-\sqrt{1}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}-\sqrt{3}}$$
$$- \cdots - \frac{1}{\sqrt{99}-\sqrt{98}} + \frac{1}{\sqrt{100}-\sqrt{99}}$$
$$= \sqrt{2} + \sqrt{1} - \sqrt{3} - \sqrt{2} + \sqrt{4} + \sqrt{3}$$
$$- \cdots - \sqrt{99} - \sqrt{98} + \sqrt{100} + \sqrt{99}$$
$$= \sqrt{1} + \sqrt{100}$$
$$= 11$$

(b) i. (1 mark)



D=(1,a) as the regular hexagon is symmetrical about y-axis
ED = 2 (side of a regular hexagon)
extend CD until perpendicular to E and let the point be G
DG=1 (exact value of trig ratio)
EG=$\sqrt{3}$
$\therefore E = (2, a+\sqrt{3})$
a+$\sqrt{3} = a(2)^2$
$\therefore a = \frac{\sqrt{3}}{3}$

ii. (2 marks) To bisect $ABCF$, need to bisect $\triangle ABP$
$\therefore P = (0, \frac{4\sqrt{3}}{3})$
$n = \frac{4\sqrt{3}}{3}$ & gradient is 150°
$m = -\frac{1}{\sqrt{3}}$