NBCS
NORTHERN BEACHES CHRISTIAN SCHOOL

Love Learning

Student Name: _____

Mathematics

2023 Year 10 5.3/Year 9 Accelerated

Yearly Examination

General Instructions

- Reading time – 5 minutes
- Writing time – 90 minutes
- Write using black pen.
- NESA approved calculators may be used.
- A two-page handwritten A4 summary sheet is permitted.

Total Marks: 65

Additional Instructions:

Section I – 20 marks (pages 2 – 9)

- Attempt Questions 1 – 20
- Allow about 30 minutes for this section

Section II – 45 marks (pages 10 – 19)

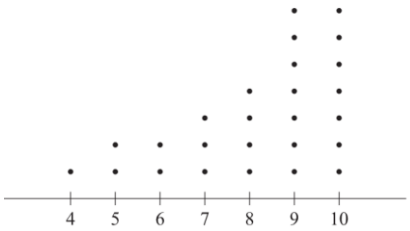
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Section I

20 marks
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Use the multiple-choice answer sheet for Questions 1 – 20

1. Which of the following best describes the distribution of the data in this graph?



(A) Positively skewed

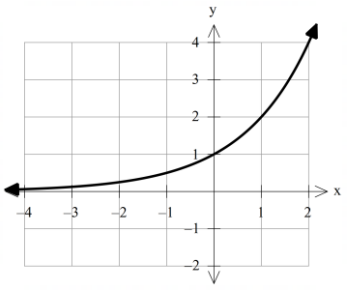
(B) Negatively skewed

(C) Normally distributed

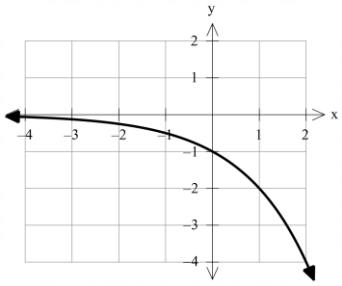
(D) Symmetrical

2. Which graph best represents $y = 2^x - 1$?

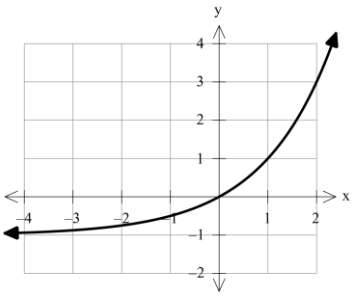
(A)



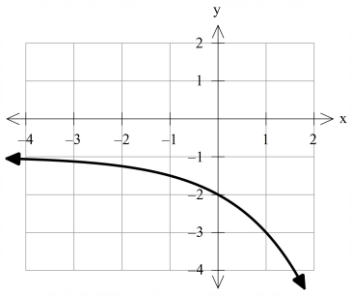
(B)



(C)



(D)



3. Consider the equation $x^2 - 5x + 3 = 0$.

What are the solutions to the equation?

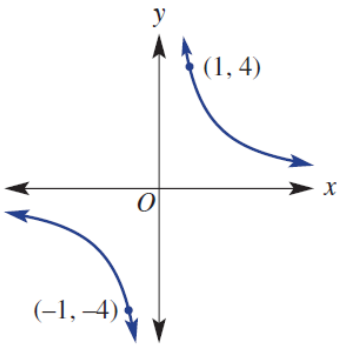
(A) $x = \dfrac{-5 \pm \sqrt{13}}{2}$

(B) $x = \dfrac{-5 \pm \sqrt{37}}{2}$

(C) $x = \dfrac{5 \pm \sqrt{13}}{2}$

(D) $x = \dfrac{5 \pm \sqrt{37}}{2}$

4. What is the equation of the graph below?



(A) $y = 4^x$

(B) $y = -\dfrac{4}{x}$

(C) $y = \dfrac{4}{x}$

(D) $y = 4x + 1$

5. What is the equation of the axis of symmetry for a parabola with the equation given below?

$$y = 3x^2 - 6x + 7$$

(A) $x = 1$

(B) $x = -1$

(C) $x = 6$

(D) $x = 2$

6. What are the solutions to the equation $x^3 = 9x$?

(A) $x = 0$ only

(B) $x = 3$ only

(C) $x = 0$, and $x = 3$ only

(D) $x = 0, x = 3$ and $x = -3$

7. For θ between 0° and 360°, what are the values of θ for which $\sin\theta = -\frac{1}{2}$?

(A) $\theta = 210°$ and $\theta = 330°$

(B) $\theta = 30°$ and $\theta = 150°$

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8. A cubic function has the equation $y = 8x^3 + 5$.

What are the coordinates of the point of inflection?

(A) $(8,0)$

(B) $(8,5)$

(C) $(0,8)$

(D) $(0,5)$

9. Two events A and B are such that $P(A) = 0.5$ and $P(B) = 0.4$ and $P(A \cup B) = 0.54$.

What is $P(A \cap B)$?

(A) 0.9

(B) 0.2

(C) 0.36

(D) 0.74

10. What is the simplest form of the expression below?

$$\frac{x^2 - 2x + 1}{x - 1}$$

(A) $x^2 - x - 1$

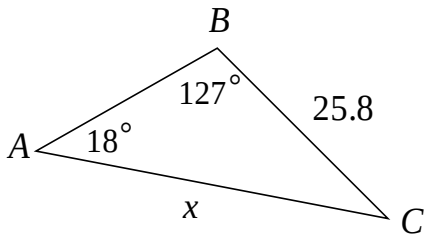
(B) $x - 1$

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11. What is the length of the side marked with the letter x (rounded to 1 decimal place)?

Side measurements are in cm.



(A) 10.0 cm

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12. Consider the following data set:

89 91 92 89 91 88

What is the standard deviation for the data (correct to 2 decimal places)?

(A) 1.41

(B) 4.00

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13. Given that $2^{7x-3} = 16$, what is the solution for x?

(A) $\frac{1}{7}$

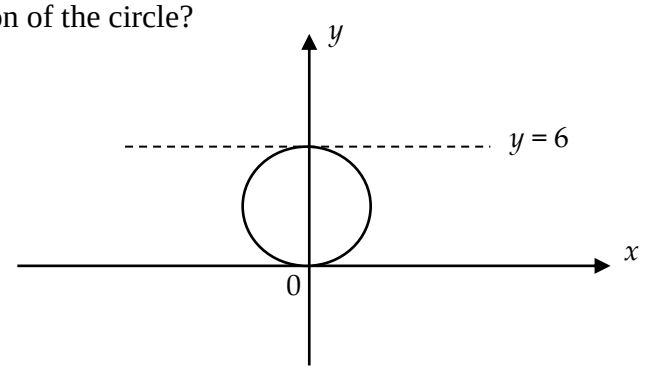
(B) $\frac{11}{7}$

(C) $\frac{19}{7}$

(D) 1

14. The points $(0,0)$ and $(0,6)$ lie on the circumference of a circle. The y-axis passes through the centre of the circle.

What is the equation of the circle?



(A) $x^2 + (y-3)^2 = 36$

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(C) $(x-3)^2 + y^2 = 9$

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15. The point $M(2,4)$ is the midpoint of the interval PQ where P is the point $(-2,6)$.

What are the coordinates of Q?

(A) $(0,5)$

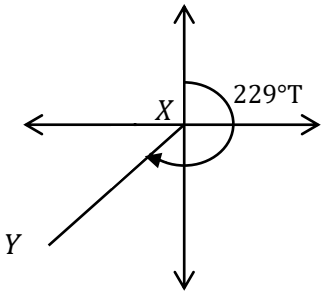
(B) $(6,2)$

(C) $(-6,8)$

(D) $(6,5)$

16. The true bearing of Y from X is $229°T$.

What is the true bearing of X from Y?



(A) $131°\text{T}$

(B) $041°\text{T}$

(C) $049°\text{T}$

(D) $311°\text{T}$

17. What is the equation of the line through the points $(8,0)$ and $(0,6)$?

(A) $4x + 3y + 24 = 0$

(B) $3x + 4y + 24 = 0$

(C) $4x + 3y - 24 = 0$

(D) $3x + 4y - 24 = 0$

18. What is the solution to the inequality $4 - \frac{x}{3} \leq 6$?

(A) $x \geq -6$

(B) $x \leq -18$

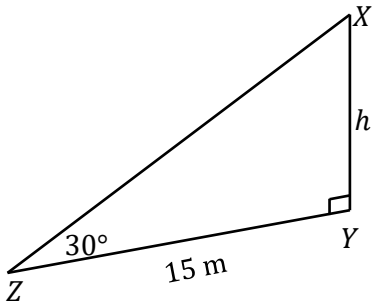
(C) $x \leq -6$

(D) $x \geq 6$

19. A point Z is 15 m from the base of a building XY. The angle of elevation to the top of the building is 30°.

What is the exact height of the building?

NOT TO SCALE



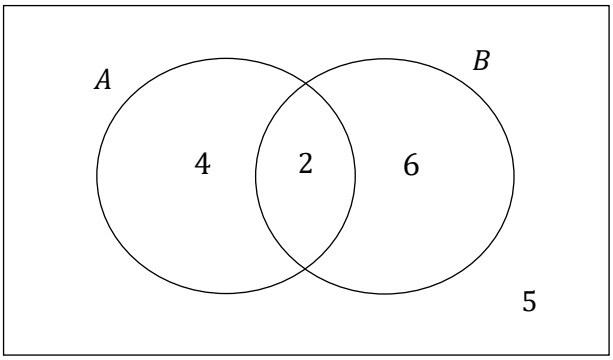
(A) $15\sqrt{3}$

(B) $5\sqrt{3}$

(C) $\dfrac{5}{\sqrt{3}}$

(D) $7.5\sqrt{3}$

20. For the events A and B given below, what is $P(A|B)$?



(A) $\dfrac{6}{17}$

(B) $\dfrac{4}{17}$

(C) $\dfrac{1}{4}$

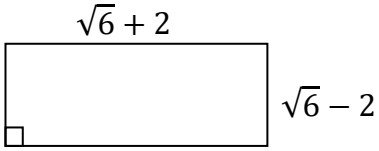
(D) $\dfrac{1}{6}$

END OF SECTION I

Section II

45 marks
Attempt Questions 21 – 31
Allow about 60 minutes for this section.

21. a) Determine the area of this rectangle in exact form (side measurements are in cm). **1**



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b) Factorise fully: $4a^2 - 12a - 16$ **2**

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c) Given that $c - a = 12$ and $c + 2a = 15$, find the value of $c + a$. **2**

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d) Simplify the following and express your answer using positive indices. **2**

$$\left(\frac{m^4n^{-2}}{r^3}\right)^2 \div \left(\frac{m^{-3}n^2}{r^3}\right)^2$$

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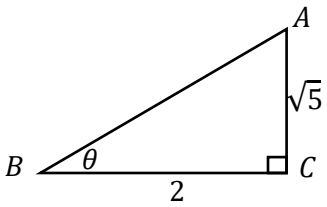
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22. Given the triangle ABC below with side lengths as shown:



a) Determine $\sin\theta$ in exact form. **1**

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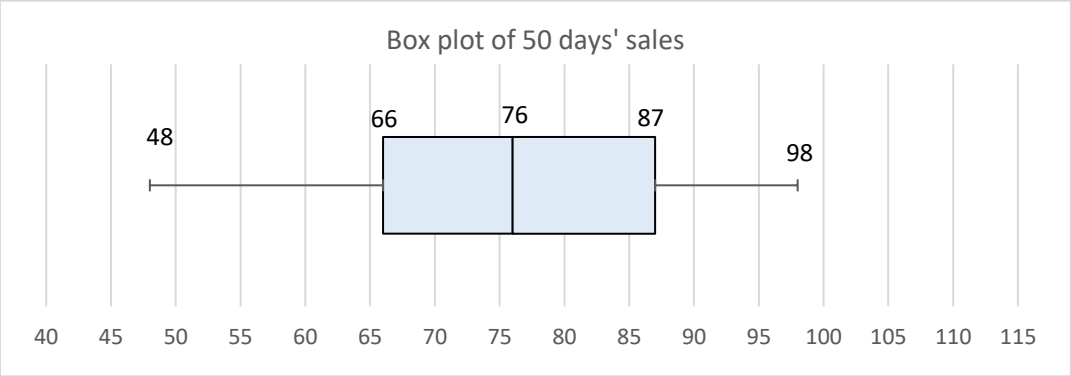
b) Calculate the size of θ to the nearest degree. **1**

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23. The daily number of doughnut sales at a coffee shop was recorded for 50 days.



a) What was the median number of doughnuts sold during this period? **1**

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b) Determine the interquartile range for the data. **1**

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c) On how many days did the coffee shop sell more than 76 doughnuts? **1**

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d) If the coffee shop had sold 109 doughnuts in one day, would that be an outlier? Justify your answer by showing relevant calculations. **2**

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24. Consider the data below:

	Station A	Station B	Total
Train left on time		45	
Train left late	80		110
Total			250

a) Complete the two-way table by calculating the missing values. **1**

b) What is the probability that a randomly selected train left on time and from Station A? **1**

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c) Determine the probability that a train left late given it departed from Station B. **1**

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25. A parabola has the equation $y = x^2 - 6x + 2$

a) Rewrite the equation in the form $y = (x - h)^2 + k$ **2**

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b) Give the coordinates of the turning point of the parabola. **1**

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26. The probability of an archer hitting a target is 0.65. The archer shoots 3 arrows in a competition.

a) What is the probability that all 3 arrows hit the target? (Give your answer correct to 3 decimal places) **1**

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b) What is the probability that at least one arrow hit the target? (Correct to 3 decimal places) **2**

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27. The equations of two parabolas are given as

$$y = x^2 + 4x + 2 \text{ and } y = -x^2 - 2x - 2$$

a) Determine the coordinates of the points of intersection of the parabolas. **3**

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b) By using your answer from (a), and without sketching the graphs, write down the equation of the line through the point(s) of intersection. **1**

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28. The heights in centimetres of four people are $ax, 2x, ay$ and $2y$.

a) Write down the expression for their total height. **1**

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b) Fully factorise the expression in part a) **1**

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c) Using the factorised expression, or otherwise, find the mean height of the four people when $x = 65, y = 75$ and $a = 3$ **2**

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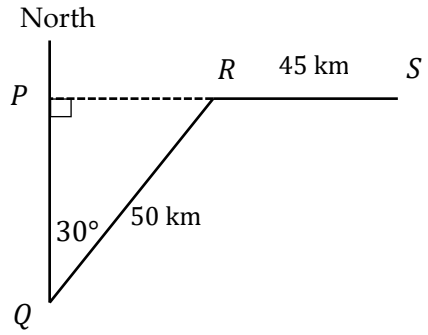
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29. In the diagram, the point R is on a bearing of $030°$T and a distance of 50 km from Q.

R is due east of P and 45 km due west of S.



NOT TO SCALE

a) Calculate the distance PS. **2**

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b) Show that $\angle QRS = 120°$. **1**

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c) Calculate the distance QS, correct to 1 decimal place. **2**

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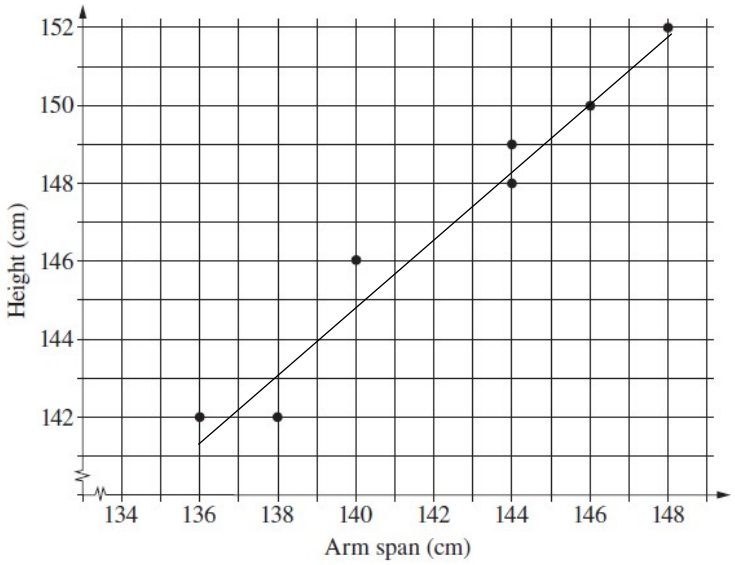
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30. A set of bivariate data is collected by measuring the height and arm span of seven children. The graph shows a scatter plot of the measurements.

A regression line has been fitted to the data.



a) Determine the equation of the least squares regression line. **2**
(Round to 3 decimal places)

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b) A child has an arm span of 143 cm. Calculate the predicted height for this child using the equation above, to the nearest cm. **1**

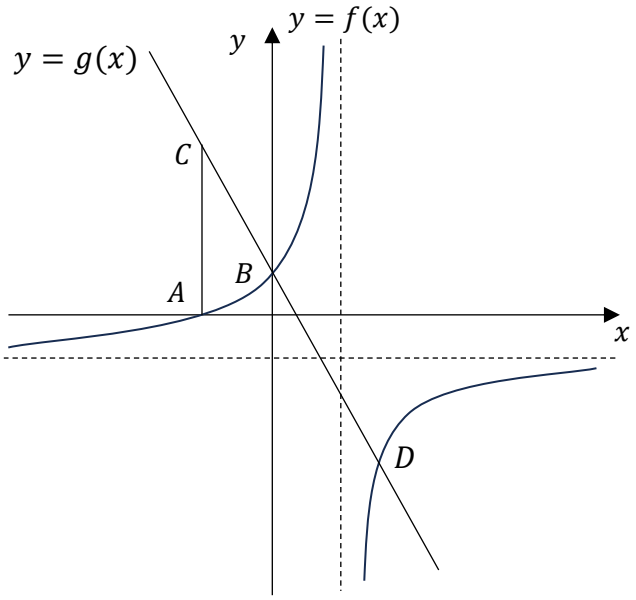
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31. The functions f and g are defined as $f(x) = -\frac{4}{x-2} - 1$ and $g(x) = mx + c$.
The graphs of $y = f(x)$ and $y = g(x)$ are shown below.

The graphs of the functions intersect each other at points B and D.



a) Write down the equations of the asymptotes of the graph of $y = f(x)$. **2**

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b) Determine the coordinates of A and B (the intercepts of the graph of $y = f(x)$ with the x and y-axes). **2**

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c) Given that CA is parallel to the y-axis and $CA = 5$, determine the equation of $g(x)$. **2**

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END OF EXAMINATION

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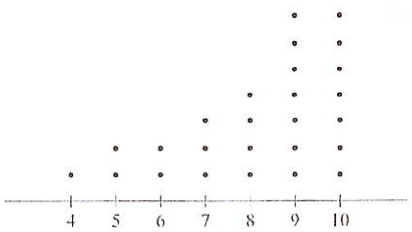
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20 marks
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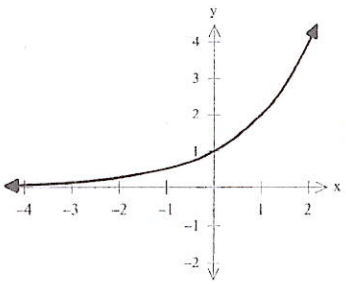
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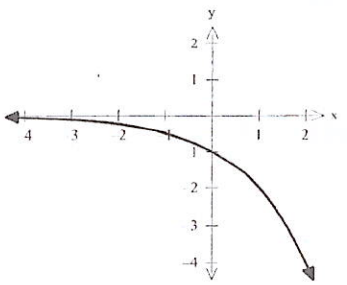
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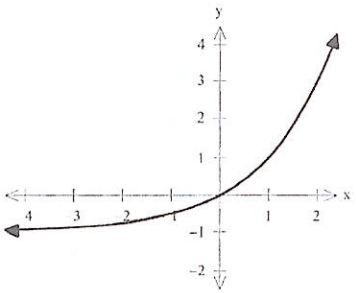
(A)



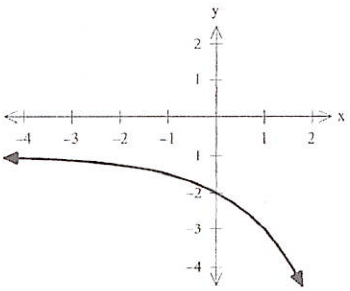
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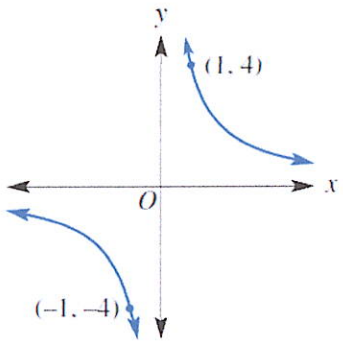
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4. What is the equation of the graph below?



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$x = \frac{-b}{2a}$

$= \frac{6}{6}$

$= 1$

6. What are the solutions to the equation $x^3 = 9x$?

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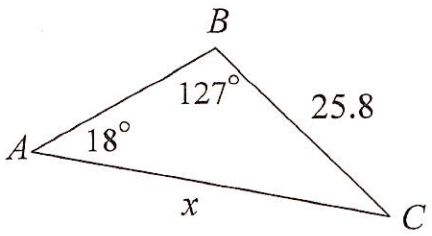
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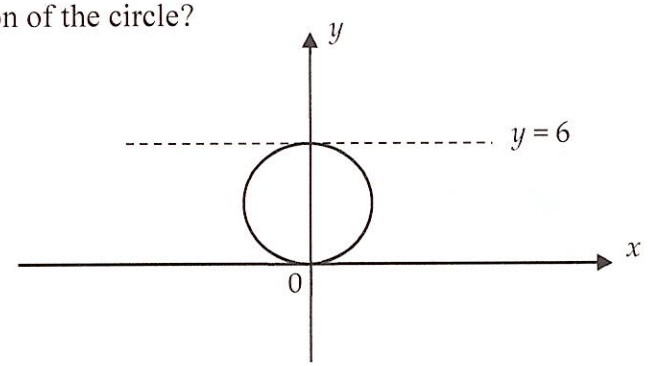
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(C) $\frac{19}{7}$

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r = 3

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What are the coordinates of Q?

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(B) (6,2)

(C) (−6,8)

(D) (6,5)

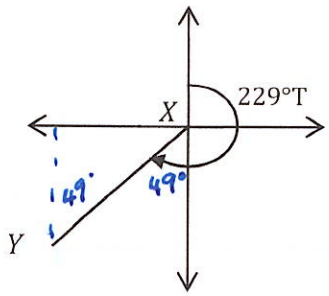
P(-2,6) M(2,4) Q(x,y)

$\frac{-2+x}{2} = 2$ $x = 6$

$\frac{6+y}{2} = 4$ $y = 2$

16. The true bearing of Y from X is $229°T$.

What is the true bearing of X from Y?



(A) 131°T

(B) 041°T

(C) 049°T

(D) 311°T

17. What is the equation of the line through the points $(8,0)$ and $(0,6)$?

(A) $4x + 3y + 24 = 0$

(B) $3x + 4y + 24 = 0$

(C) $4x + 3y - 24 = 0$

(D) $3x + 4y - 24 = 0$

$m = \frac{0-6}{8-0} = -\frac{3}{4}$

$y - 0 = -\frac{3}{4}(x-8)$

$y = -\frac{3}{4}x + 6$

$3x + 4y - 24 = 0$

18. What is the solution to the inequality $4 - \frac{x}{3} \leq 6$?

(A) $x \geq -6$

(B) $x \leq -18$

(C) $x \leq -6$

(D) $x \geq 6$

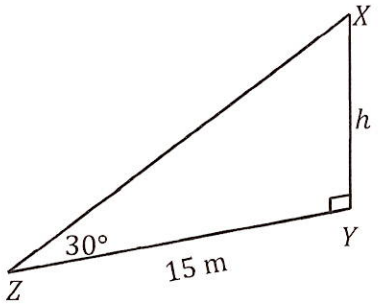
$-\frac{x}{3} \leq 2$

$x \geq -6$

19. A point Z is 15 m from the base of a building XY. The angle of elevation to the top of the building is 30°.

What is the exact height of the building?

NOT TO SCALE



$\tan 30° = \frac{h}{15}$

$h = 15 \tan 30°$

$= \frac{15\sqrt{3}}{3}$

$= 5\sqrt{3}$

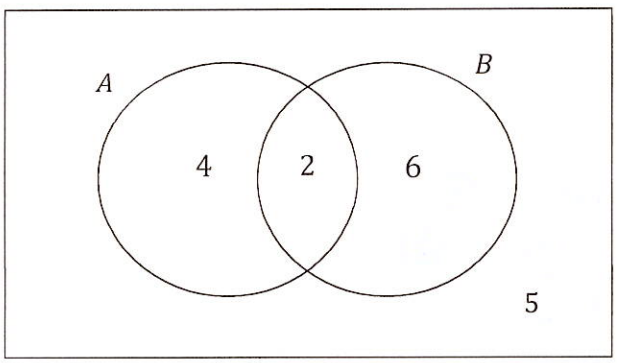
(A) $15\sqrt{3}$

(B) $5\sqrt{3}$

(C) $\frac{5}{\sqrt{3}}$

(D) $7.5\sqrt{3}$

20. For the events A and B given below, what is $P(A|B)$?



(A) $\frac{6}{17}$

(B) $\frac{4}{17}$

(C) $\frac{1}{4}$

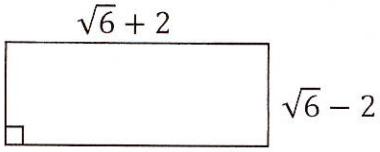
(D) $\frac{1}{6}$

END OF SECTION I

Section II

45 marks
Attempt Questions 21 – 31
Allow about 60 minutes for this section.

21. a) Determine the area of this rectangle in exact form (side measurements are in cm). **1**



$A = l \times b$

$= (\sqrt{6}+2)(\sqrt{6}-2)$

$= 2$

b) Factorise fully: $4a^2 - 12a - 16$ **2**

$= 4(a^2 - 3a - 4)$

$= 4(a-4)(a+1)$

c) Given that $c - a = 12$ and $c + 2a = 15$, find the value of $c + a$. **2**

$c - a = 12 \quad -①$

$c + 2a = 15 \quad -②$

②−① $\quad 3a = 3$

$a = 1$

$c - 1 = 12$

$c = 13$

$c + a = 1 + 13$

$= 14$

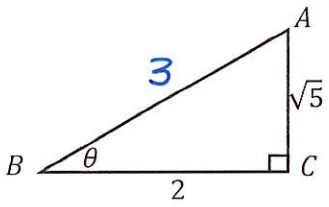
d) Simplify the following and express your answers using positive indices. **2**

$$\left(\frac{m^4n^{-2}}{r^3}\right)^2 \div \left(\frac{m^{-3}n^2}{r^3}\right)^2$$

$= \frac{m^8 n^{-4}}{r^6} \times \frac{r^6}{m^{-6} n^4}$

$= \frac{m^{14}}{n^8}$

22. Given the triangle ABC below with side lengths as shown:



$AB^2 = 2^2 + (\sqrt{5})^2$

$= 9$

$AB = 3$

a) Determine $\sin\theta$ in exact form. **1**

$\sin\theta = \frac{\sqrt{5}}{3}$

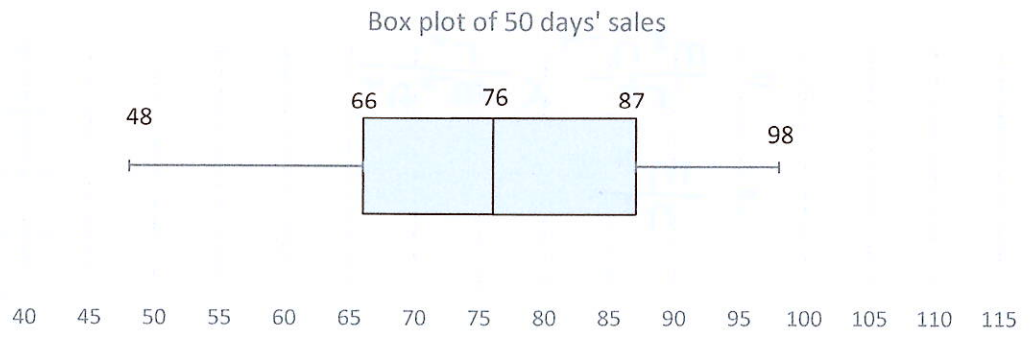
b) Calculate the size of θ to the nearest degree. **1**

$\theta = \sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$

$= 48.19$

$\approx 48°$ (nearest degree)

23. The daily number of doughnut sales at a coffee shop was recorded for 50 days.



a) What was the median number of doughnuts sold during this period? **1**

76

b) Determine the interquartile range for the data. **1**

87 - 66 = 21

c) On how many days did the coffee shop sell more than 76 doughnuts? **1**

50% x 50 = 25 days

d) If the coffee shop had sold 109 doughnuts in one day, would that be an outlier? Justify your answer by showing relevant calculations. **2**

87 + 1.5 x 21
= 118.5
No, 109 doughnuts would not be an outlier.

24. Consider the data below:

	Station A	Station B	Total
Train left on time	95	45	140
Train left late	80	30	110
Total	175	75	250

a) Complete the two-way table by calculating the missing values. **1**

b) What is the probability that a randomly selected train left on time and from Station A? **1**

$\frac{95}{250}$

$= \frac{19}{50}$

c) Determine the probability that a train left late given it departed from Station B. **1**

$\frac{30}{75} = \frac{2}{5}$

25. A parabola has the equation $y = x^2 - 6x + 2$

a) Rewrite the equation in the form $y = (x - h)^2 + k$ **2**

$y = x^2 - 6x + 9 - 9 + 2$

$= (x-3)^2 - 7$

b) Give the coordinates of the turning point of the parabola. **1**

$(3, -7)$

26. The probability of an archer hitting a target is 0.65. The archer shoots 3 arrows in a competition.

a) What is the probability that all 3 arrows hit the target? (Give your answer correct to 3 decimal places) **1**

$(0.65)^3 = 0.275$ (3 dp)

b) What is the probability that at least one arrow hit the target? (Correct to 3 decimal places) **2**

P(at least one) = 1 − P(all miss)

$= 1 - (0.35)^3$

$= 1 - 0.043$

$= 0.957$ (3 dp)

27. The equations of two parabolas are given as

$$y = x^2 + 4x + 2 \text{ and } y = -x^2 - 2x - 2$$

a) Determine the coordinates of the points of intersection of the parabolas. **3**

$x^2 + 4x + 2 = -x^2 - 2x - 2$

$2x^2 + 6x + 4 = 0$

$2(x^2 + 3x + 2) = 0$

$2(x + 2)(x + 1) = 0$

$x = -2, \quad x = -1$

$y = -2, \quad y = -1$

$(-2, -2)$ and $(-1, -1)$

b) By using your answer from (a), and without sketching the graphs, write down the equation of the line through the point(s) of intersection. **1**

$y = x$

28. The heights in centimetres of four people are $ax, 2x, ay$ and $2y$.

a) Write down the expression for their total height. 1

Total $= ax + 2x + ay + 2y$

b) Fully factorise the expression in part a) 1

$x(a+2) + y(a+2)$

$= (a+2)(x+y)$

c) Using the factorised expression, or otherwise, find the mean height of the four people when $x = 65, y = 75$ and $a = 3$ 2

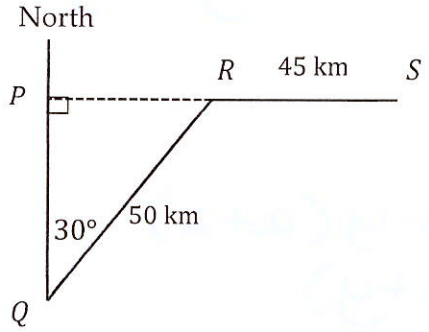
$(3+2)(65+75) = 700$

Mean $= \frac{700}{4}$

$= 175$ cm

29. In the diagram, the point R is on a bearing of 030°T and a distance of 50 km from Q.

R is due east of P and 45 km due west of S.



NOT TO SCALE

a) Calculate the distance PS. **2**

$\sin 30° = \frac{PR}{50}$

$PR = 50 \sin 30°$

$= 25 \text{ km}$

$PS = 25 + 45$

$= 70 \text{ km}$

b) Show that $\angle QRS = 120°$. **1**

$\angle QRS = 120° = 90° + 30°$ (Ext. $\angle$ of a $\triangle$)

c) Calculate the distance QS, correct to 1 decimal place. **2**

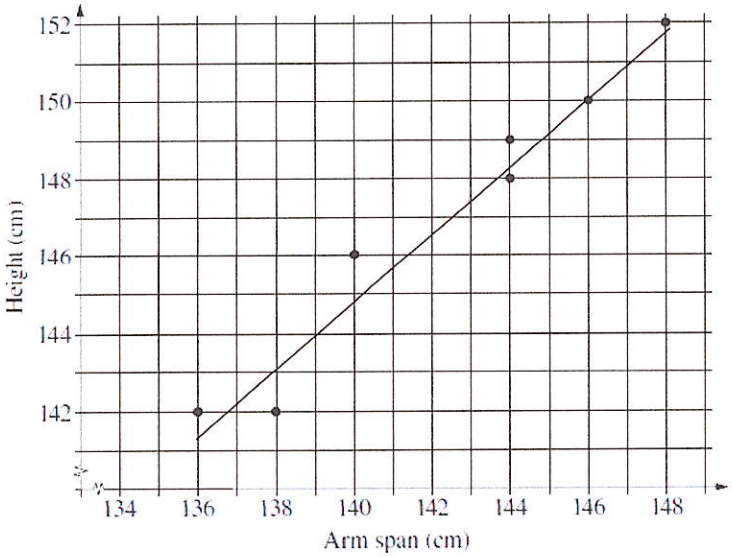
$QS^2 = 50^2 + 45^2 - 2(50)(45)\cos 120°$

$= 6775$

$QS = 82.3 \text{ km}$ (1 d.p.)

30. A set of bivariate data is collected by measuring the height and arm span of seven children. The graph shows a scatter plot of the measurements.

A regression line has been fitted to the data.



a) Determine the equation of the least squares regression line. (Round to 3 decimal places) **2**

A = 23.733
B = 0.866

$y = 0.866x + 23.733$

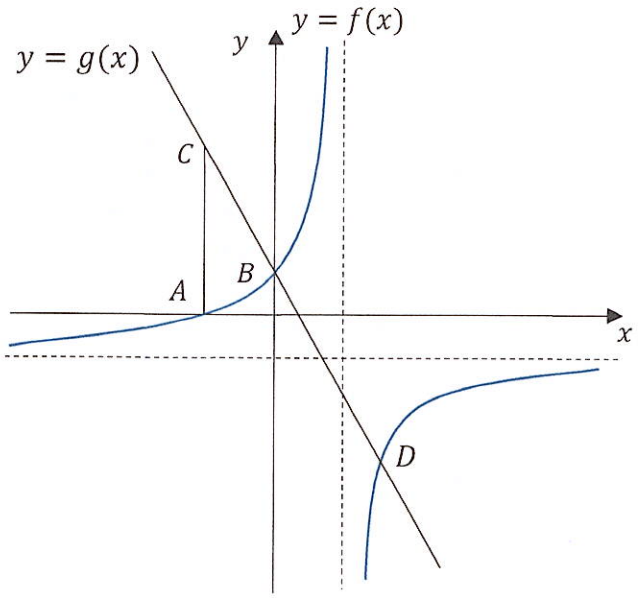
b) A child has an arm span of 143 cm. Calculate the predicted height for this child using the equation above, to the nearest cm. **1**

$y = 0.866(143) + 23.733$

$= 148$ (nearest cm)

31. The functions f and g are defined as $f(x) = -\frac{4}{x-2} - 1$ and $g(x) = mx + c$.
The graphs of $y = f(x)$ and $y = g(x)$ are shown below.

The graphs of the functions intersect each other at points B and D.



a) Write down the equations of the asymptotes of the graph of $y = f(x)$. **2**

$x = 2$ $y = -1$

b) Determine the coordinates of A and B (the intercepts of the graph of $y = f(x)$ with the x and y-axes). **2**

A: $y = 0$ $-\frac{4}{x-2} - 1 = 0$

$\frac{-4}{x-2} = 1$

$-4 = x - 2$

$x = -2$ $(-2, 0)$

B: $x = 0$ $y = \frac{-4}{0-2} - 1$

$= 1$ $(0, 1)$

c) Given that CA is parallel to the y-axis and $CA = 5$, determine the equation of $g(x)$. 2

Point C $(-2, 5)$ B $(0, 1)$

Gradient $m = \frac{5-1}{-2-0}$ y-intercept : $(0, 1)$

$= -2$

$y = -2x + 1$

END OF EXAMINATION