MEREWETHER
HIGH SCHOOL

Name: _____

Class: _____

Teacher: _____

2022 Year 10
Semester 2 Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Total Working Time - 80 minutes.
- Write using blue or black pen.
- NESA approved calculators may be used.
- Show relevant mathematical working, reasoning, and calculations.
- Submit ALL work at the end of the examination.
- Marks will be deducted for poorly arranged work.
- Diagrams are NOT drawn to scale.

Total marks: 72

- Attempt **ALL** Questions.
- Answer in the spaces provided.

Section	Topic	Marks
A	Probability	**/17**
B	Statistics	**/16**
C	Graphing Parabolas	**/20**
D	Non-Linear Functions	**/19**
	Total Marks Gained	**/72**

Part A: Probability (17 Marks)

Circle the letter for the correct response

1. A card is drawn from a well-shuffled standard deck of 52 playing cards. What is the theoretical probability of not selecting a diamond?

(A) $\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{13}$

2. A jar contains five red lollies and three green lollies. Maya selects two lollies at random. Calculate the probability that they are of different colours.

(A) $\frac{13}{28}$

(B) $\frac{15}{32}$

(C) $\frac{15}{28}$

(D) $\frac{15}{56}$

3. Find $P(B)$ if $P(A \cup B) = 0.8,\ P(A) = 0.68$ and $P(A \cap B) = 0.42.$

(A) 0.49
(B) 0.54
(C) 0.59
(D) 0.65

Answer in the space provided. Show all necessary working.

4. A pair of dice is rolled, and the **sum** of the numbers shown is noted.

(a) Complete the two-way table below showing the sample space. **1**

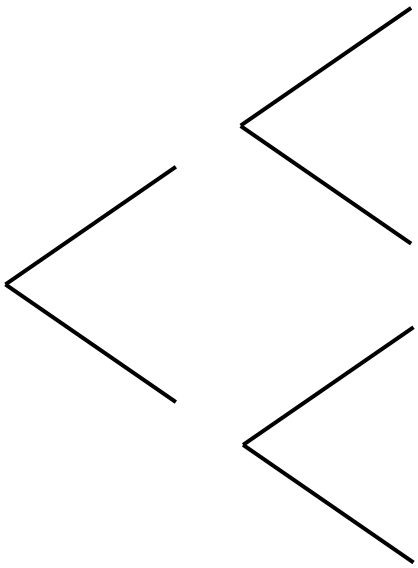
		1st die					
		1	2	3	4	5	6
2nd die	1						
	2						
	3						
	4						
	5						
	6						

(b) What is the probability of obtaining a sum of 7? **1**

(c) If a pair of dice is rolled 300 times, how many times do you expect to obtain the sum of 7? **1**

5. There is a 70% chance that Henry will be chosen for the boys' debating team, and an 80% chance that Grace will be chosen for the girls' team.

(a) Complete the probability tree diagram to represent this information. **2**



(b) Find the probability that neither Henry nor Grace is chosen. **1**

(c) Find the probability that at least one of them is chosen. **1**

6. In a group of 60 tourists to Tasmania, 31 visited Queenstown (Q), 33 visited Strahan (S) and 14 visited both places.

(a) Draw a Venn diagram using the above information. **2**

(b) Find:

(i) $P\left(Q \cap \overline{S}\right)$ **1**

(ii) $P(S \mid Q)$ **1**

(iii) $P\left(\overline{Q \cup S}\right)$ **1**

7. The probability that a student is well and misses a work shift is 0.045, and the probability that a student misses work is 0.05. What is the probability that a student is well, given that they miss a work shift? Let W be the event the student is well and M be the event the student misses a work shift. **2**

Part B: Statistics (16 Marks)

Circle the letter for the correct response.

1. Consider the following dataset

13 16 17 17 21 24

Which row of the table shows how the median and mean are affected when a score of 5 is added to the dataset?

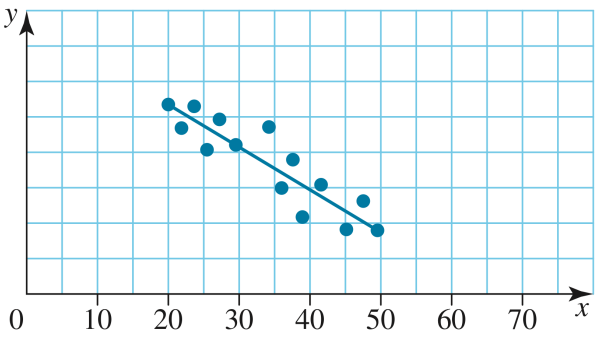
	Median	*Mean*
(A)	Changes	Changes
(B)	Stays the same	Stays the same
(C)	Changes	Stays the same
(D)	Stays the same	Changes

2. The population standard deviation for the following data set, correct to 1 decimal place, is:

5.2, 4.7, 5.1, 12.6, 4.8

(A) 3.1
(B) 9.4
(C) 6.5
(D) 4.2

3. The line of best fit on the scatter plot below is used to predict the values of y when $x = 15$, $x = 40$ and $x = 60$.



Interpolation would be used to predict the value of y when the value of x is:

(A) 15 and 40

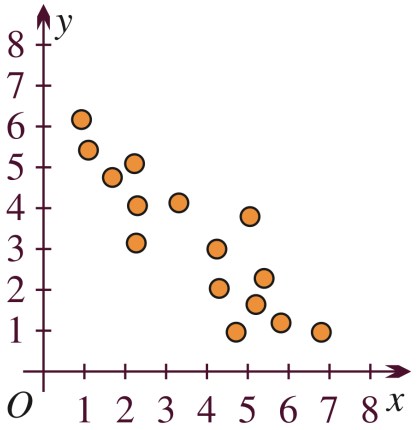
(B) 40 only

(C) 15 and 60

(D) 15 only

Answer in the space provided. Show all necessary working.

4. Describe the correlation, regarding strength and direction, for the x and y variables in this scatterplot. **2**



5. The table below records the number of sales made in a shoe shop per day.

(a) Complete the table. **1**

Sales	Class Centre	Frequency
0 – 4		6
5 – 9		2
10 – 14		3
15 – 19		5
20 – 24		8
25 – 29		9
30 – 34		5

(b) Find the mean number of sales. **1**
(correct to 1 decimal place)

(c) What is the modal class? **1**

6. A set of 51 scores has a mean of 81. Find the new mean, correct to 1 decimal place, after a score of 36 is added to the set. **2**

7. The five-number summary of a data set is given.

Lowest score = 1

Lower Quartile $(Q_1) = 4$

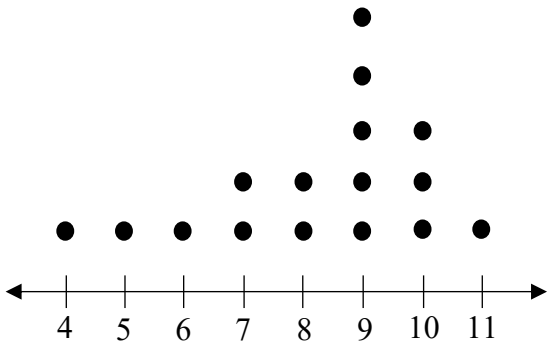
Median $(Q_2) = 7$

Upper quartile $(Q_3) = 10$

Highest score = 20

Is 20 an outlier? Justify your answer with calculations. **2**

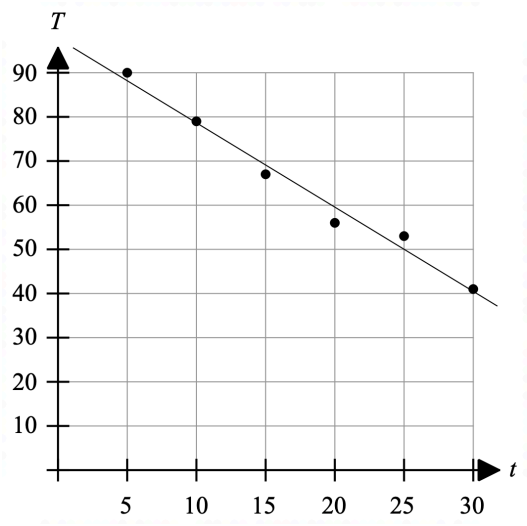
8.



Describe the shape of the data in the dot plot above. **1**

9. The following table and graph show the results in an experiment testing the temperature of a liquid as it cools down.

t (min)	5	10	15	20	25	30
T (°C)	90	79	67	56	53	41



(a) Find the equation for the line of best fit. **2**

(b) Use the equation to find the temperature after 35 minutes. **1**

Part C: Graphing Parabolas (20 Marks)

Circle the letter for the correct response.

1. Which of the following is true for the graph of $y = 2(x-3)^2 + 4$?

(A) Vertex $(3,\ 4)$, y – intercept -14

(B) Vertex $(-3,\ 4)$, y – intercept 22

(C) Vertex $(3,\ -4)$, y – intercept -14

(D) Vertex $(3,\ 4)$, y – intercept 22

2. The parabola $y = x^2 - 5x + 2$ and the line $y = -x - 2$ have how many points of intersection?

(A) 0

(B) 1

(C) 2

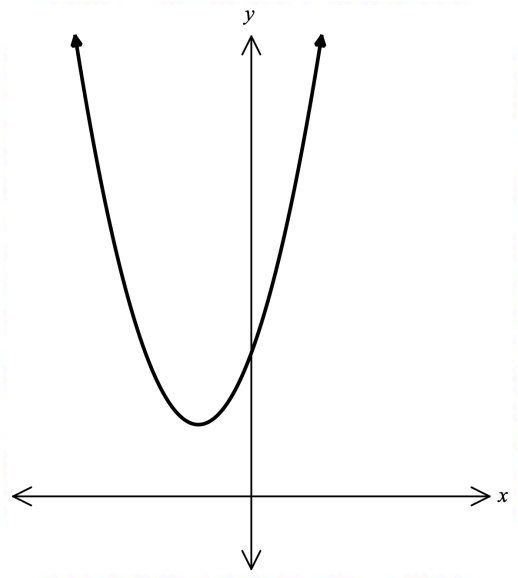
(D) 3

3. The height of the water level in a cave is determined by the tides. At any time, t, in hours after 9 am, the height, $h(t)$, in metres, can be modelled by the function $h(t) = t^2 - 12t + 32,\ 0 \le t \le 12$. At what time does the water level in the cave first reach 0 metres?

(A) 9 am
(B) 4 am
(C) 1 pm
(D) 5 pm

Answer in the space provided. Show all necessary working.

4. For the graph below state whether the discriminant is zero, positive or negative. **1**



5. For the quadratic $y = x^2 - 2x - 8$:

(a) Find the y – intercept. **1**

(b) Find the x – intercept(s). **2**

(c) Find the turning point. **2**

6. 400 m of fencing is to be used to form a rectangular paddock. Let x m be the breadth of the paddock.

(a) Write an expression for the length of the paddock in terms of x. **1**

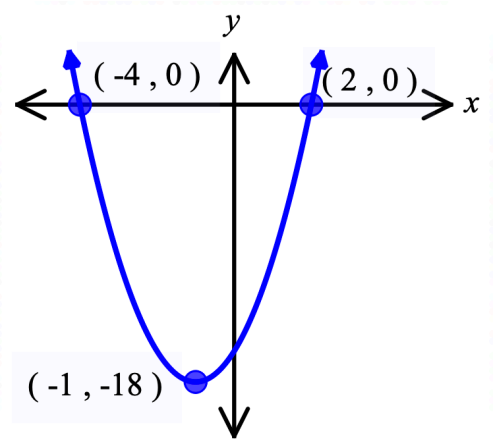
(b) Write an equation for the area of the paddock (A m^2) in terms of x. **1**

(c) Identify the permissible values of x given $A \geq 0$. **2**

(d) What is the maximum area that can be formed? **2**

(e) Sketch the graph of A versus x. **2**

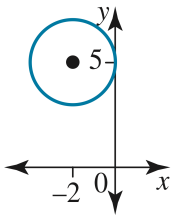
7. Determine the equation of the parabola below in the form $y = k(x-a)(x-b)$. **3**

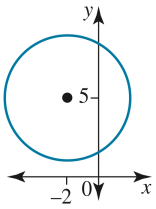


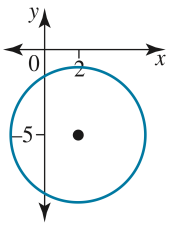
Part D: Non-Linear Functions (19 Marks)

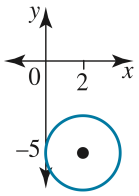
Circle the letter for the correct response.

1. The graph of $(x-2)^2 + (y+5)^2 = 4$ is:

(A) 

(B) 

(C) 

(D) 

2. The graph of $y = \dfrac{1}{x+3} - 2$ has asymptote(s) at:

(A) $x = -3, y = -2$

(B) $x = -2, y = 3$

(C) $x = 3, y = -2$

(D) $x = -2, y = -3$

3. The graph of $y = -2^x$ has a y – intercept with coordinates:

(A) $(0,2)$

(B) $(2,0)$

(C) $(-1,0)$

(D) $(0,-1)$

Answer in the space provided. Show all necessary working.

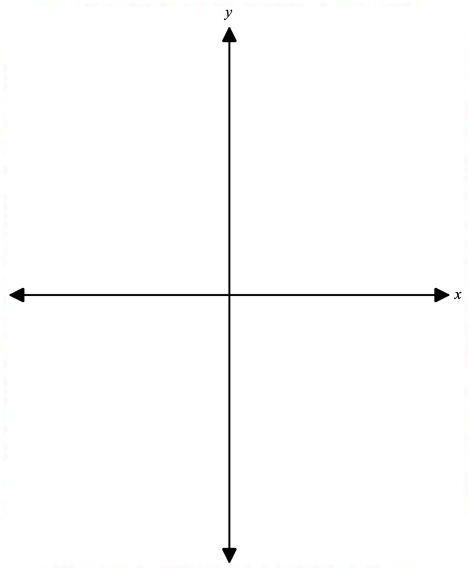
4. State the domain and range for $y = \sqrt{x+2}$. **2**

5. If $f(x) = -2x^3 + x^2 - 6$, find $f(-3)$. **2**

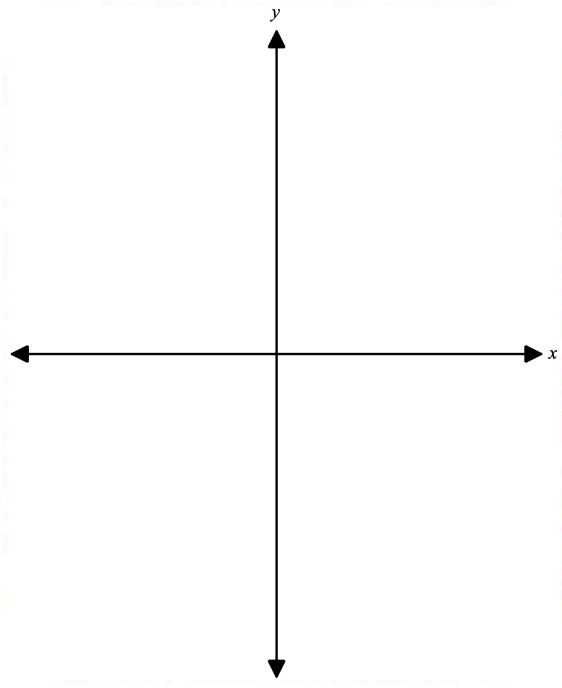
6. Find the inverse function $f^{-1}(x)$ of $y = 4x - 1$. **2**

7. Sketch each of the following functions, showing all important features.

(a) $y = 3^{-x}$ **2**



(b) $y = (x-1)^3$ **2**

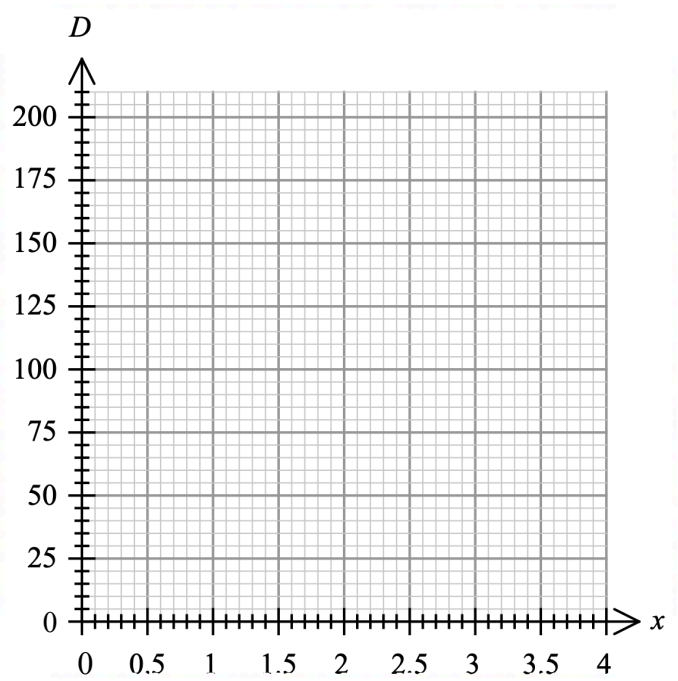


8. The diameter of a balloon (D mm) varies inversely with the thickness of the rubber (x mm). The diameter of the balloon is 80 mm when the rubber is 2 mm thick

(a) Find the equation relating D and x. **2**

(b) By first completing this table of values, graph the relationship between diameter and thickness from $x = 0.8$ to $x = 3.2$. **2**

x	0.8	2	3.2
D			



9. Find the x – coordinate(s) for the point(s) of intersection for the hyperbola $y = \frac{3}{x}$ and the line $y = 2x + 1$. **2**

Extra Writing Space

End of Examination

MEREWETHER
HIGH SCHOOL

Name: Answers

Class: _____

Teacher: _____

2022 Year 10
Semester 2 Examination

Mathematics

General Instructions

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- Total Working Time - 80 minutes.
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Total marks: 72

- Attempt **ALL** Questions.
- Answer in the spaces provided.

Section	Topic	Marks
A	Probability	**/17**
B	Statistics	**/16**
C	Graphing Parabolas	**/20**
D	Non-Linear Functions	**/19**
	Total Marks Gained	**/72**

Part A: Probability (17 Marks)

Circle the letter for the correct response

1. A card is drawn from a well-shuffled standard deck of 52 playing cards. What is the theoretical probability of not selecting a diamond?

$1 - \frac{1}{4} = \frac{3}{4}$

(A) $\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{13}$

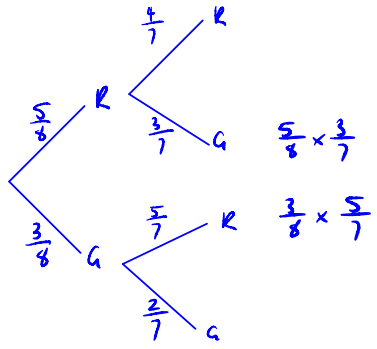
2. A jar contains five red lollies and three green lollies. Maya selects two lollies at random. Calculate the probability that they are of different colours.

(A) $\frac{13}{28}$

(B) $\frac{15}{32}$

(C) $\frac{15}{28}$

(D) $\frac{15}{56}$



$P(RG \text{ or } GR) = 2 \times \frac{3}{8} \times \frac{5}{7}$

3. Find $P(B)$ if $P(A \cup B) = 0.8$, $P(A) = 0.68$ and $P(A \cap B) = 0.42$.

(A) 0.49

(B) 0.54

(C) 0.59

(D) 0.65

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.68 + P(B) - 0.42$

$P(B) = 0.54$

Answer in the space provided. Show all necessary working.

4. A pair of dice is rolled, and the **sum** of the numbers shown is noted.

(a) Complete the two-way table below showing the sample space. **1**

		1st die					
		1	2	3	4	5	6
2nd die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(b) What is the probability of obtaining a sum of 7? **1**

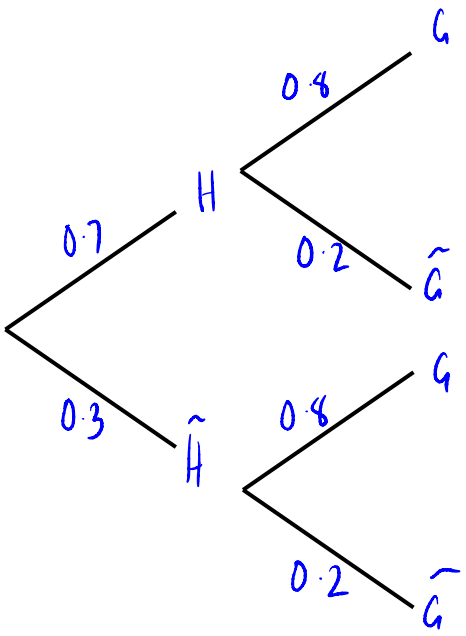
$P(7) = \frac{6}{36} = \frac{1}{6}$

(c) If a pair of dice is rolled 300 times, how many times do you expect to obtain the sum of 7? **1**

$\frac{1}{6} \times 300 = 50$

5. There is a 70% chance that Henry will be chosen for the boys' debating team, and an 80% chance that Grace will be chosen for the girls' team.

(a) Complete the probability tree diagram to represent this information. **2**



(b) Find the probability that neither Henry nor Grace is chosen. **1**

$P(\hat{H}\hat{G}) = 0.3 \times 0.2$

$= 0.06$

(c) Find the probability that at least one of them is chosen. **1**

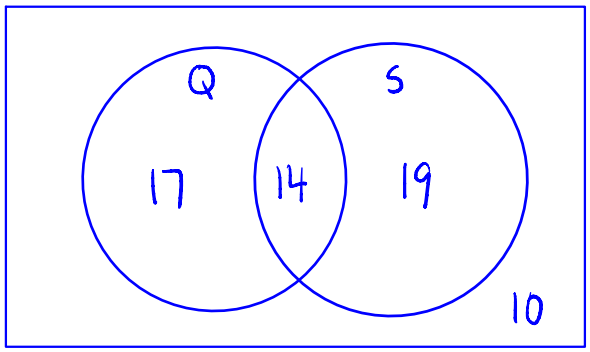
$P(\text{at least 1}) = 1 - P(\hat{H}\hat{G})$

$= 1 - 0.06$

$= 0.94$

6. In a group of 60 tourists to Tasmania, 31 visited Queenstown (Q), 33 visited Strahan (S) and 14 visited both places.

(a) Draw a Venn diagram using the above information. **2**



(b) Find:

(i) $P(Q \cap \overline{S}) = \frac{17}{60}$ **1**

(ii) $P(S \mid Q) = \frac{14}{31}$ **1**

(iii) $P(\overline{Q \cup S}) = \frac{10}{60}$ **1**

$= \frac{1}{6}$

7. The probability that a student is well and misses a work shift is 0.045, and the probability that a student misses work is 0.05. What is the probability that a student is well, given that they miss a work shift? Let W be the event the student is well and M be the event the student misses a work shift. **2**

$$P(W|M) = \frac{P(W \cap M)}{P(M)}$$

$$= \frac{0.045}{0.05}$$

$$= 0.9$$

Part B: Statistics (16 Marks)

Circle the letter for the correct response.

1. Consider the following dataset

13 16 17 17 21 24

Which row of the table shows how the median and mean are affected when a score of 5 is added to the dataset?

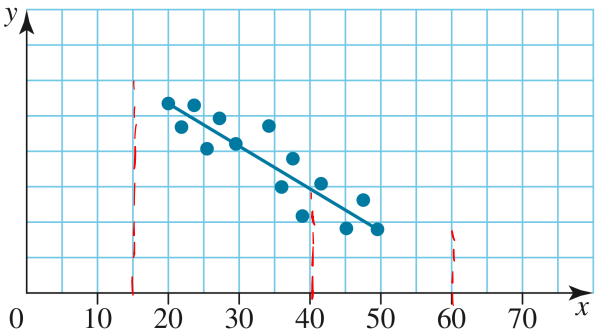
	Median	*Mean*
(A)	Changes	Changes
(B)	Stays the same	Stays the same
(C)	Changes	Stays the same
(D)	Stays the same	Changes

2. The population standard deviation for the following data set, correct to 1 decimal place, is:

5.2, 4.7, 5.1, 12.6, 4.8

(A) 3.1
(B) 9.4
(C) 6.5
(D) 4.2

3. The line of best fit on the scatter plot below is used to predict the values of y when $x = 15$, $x = 40$ and $x = 60$.



Interpolation would be used to predict the value of y when the value of x is:

(A) 15 and 40

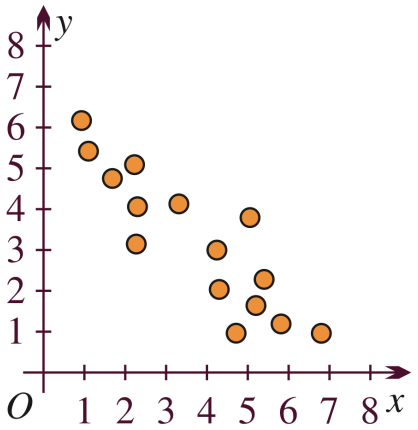
(B) 40 only

(C) 15 and 60

(D) 15 only

Answer in the space provided. Show all necessary working.

4. Describe the correlation, regarding strength and direction, for the x and y variables in this scatterplot. **2**



strong, negative

5. The table below records the number of sales made in a shoe shop per day.

(a) Complete the table. **1**

Sales	Class Centre	Frequency	f×cc
0 – 4	2	6	12
5 – 9	7	2	14
10 – 14	12	3	36
15 – 19	17	5	85
20 – 24	22	8	176
25 – 29	27	9	243
30 – 34	32	5	160
		$\Sigma f = 38$	$\Sigma = 726$

(b) Find the mean number of sales. **1**

(correct to 1 decimal place)

$\bar{x} = \frac{726}{38}$

$= 19.1$

(c) What is the modal class? **1**

25 – 29

6. A set of 51 scores has a mean of 81. Find the new mean, correct to 1 decimal place, after a score of 36 is added to the set. **2**

$$\bar{x} = \frac{51 \times 81 + 36}{51 + 1}$$

$$= \frac{4167}{52}$$

$$= 80.134\ldots$$

$$= 80.1$$

7. The five-number summary of a data set is given.

Lowest score = 1

Lower Quartile $(Q_1) = 4$

Median $(Q_2) = 7$

Upper quartile $(Q_3) = 10$

Highest score = 20

Is 20 an outlier? Justify your answer with calculations. **2**

$$IQR = 10 - 4$$

$$= 6$$

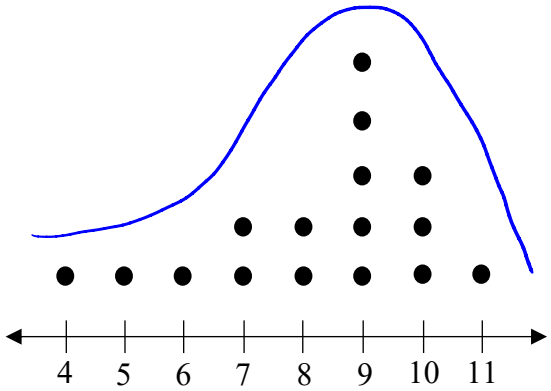
$$Q_3 + 1.5 \times IQR = 10 + 1.5 \times 6$$

$$= 19$$

$$20 > 19$$

$\therefore$ 20 is an outlier

8.

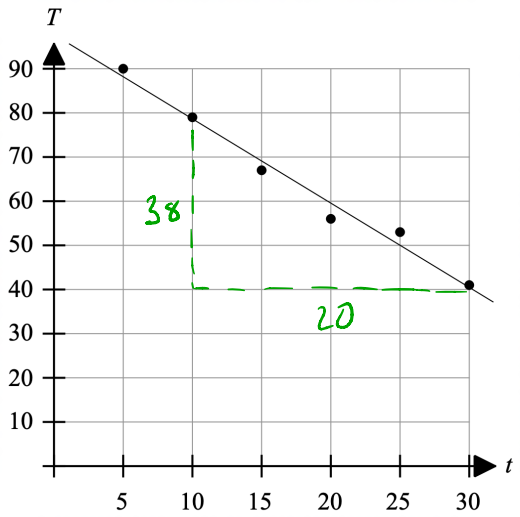


Describe the shape of the data in the dot plot above. **1**

Negatively skewed

9. The following table and graph show the results in an experiment testing the temperature of a liquid as it cools down.

t (min)	5	10	15	20	25	30
T (°C)	90	79	67	56	53	41



(a) Find the equation for the line of best fit. **2**

$m = \frac{41-79}{80-10}$

$= -1.9$

$T - 79 = -1.9(t - 10)$

$T - 79 = -1.9t + 19$

$T = -1.9t + 98$

Using calculator

$T = A + Bt$

$T = 97.7 - 1.9t$

(b) Use the equation to find the temperature after 35 minutes. **1**

$T = -1.9 \times 35 + 98$

$= 31.5°$

$T = 97.7 - 1.9 \times 35$

$= 31.2°$

Part C: Graphing Parabolas (20 Marks)

Circle the letter for the correct response.

1. Which of the following is true for the graph of $y = 2(x-3)^2 + 4$?

(A) Vertex $(3,\ 4)$, y – intercept -14

(B) Vertex $(-3,\ 4)$, y – intercept 22

(C) Vertex $(3,\ -4)$, y – intercept -14

(D) Vertex $(3,\ 4)$, y – intercept 22

$y = 2(0-3)^2 + 4$

$= 18 + 4$

$= 22$

2. The parabola $y = x^2 - 5x + 2$ and the line $y = -x - 2$ have how many points of intersection?

(A) 0

(B) 1

(C) 2

(D) 3

$x^2 - 5x + 2 = -x - 2$

$x^2 - 4x + 4 = 0$

$\Delta = b^2 - 4ac$

$= (-4)^2 - 4(1)(4)$

$= 0$

3. The height of the water level in a cave is determined by the tides. At any time, t, in hours after 9 am, the height, $h(t)$, in metres, can be modelled by the function $h(t) = t^2 - 12t + 32,\ 0 \le t \le 12$. At what time does the water level in the cave first reach 0 metres?

$t^2 - 12t + 32 = 0$

$(t-4)(t-8) = 0$

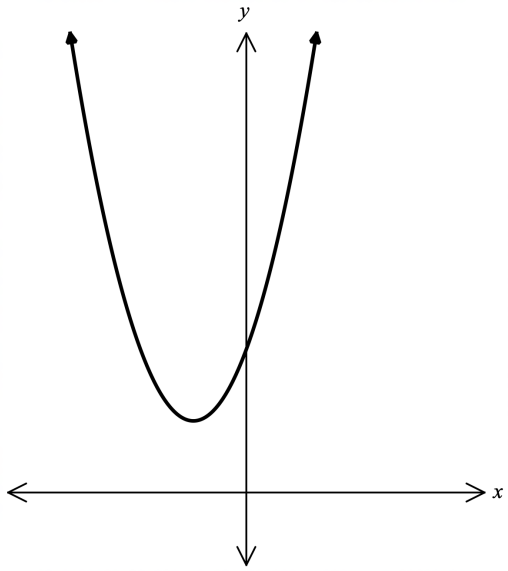
$t = 4, 8$

9am + 4 = 1pm

(A) 9 am
(B) 4 am
(C) 1 pm
(D) 5 pm

Answer in the space provided. Show all necessary working.

4. For the graph below state whether the discriminant is zero, positive or negative. **1**



Negative

5. For the quadratic $y = x^2 - 2x - 8$:

(a) Find the y – intercept. **1**

$y = 0^2 - 2(0) - 8$

$= -8$

(b) Find the x – intercept(s). **2**

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

(c) Find the turning point. **2**

$x = \frac{4 + (-2)}{2}$

$= 1$

$y = 1^2 - 2(1) - 8$

$= -9$

$\therefore$ turning point at $(1, -9)$

6. 400 m of fencing is to be used to form a rectangular paddock. Let x m be the breadth of the paddock.

(a) Write an expression for the length of the paddock in terms of x. **1**

$2l + 2x = 400$

$2l = 400 - 2x$

$l = 200 - x$

(b) Write an equation for the area of the paddock (A m^2) in terms of x. **1**

$A = l \times b$

$= x(200 - x)$

$= 200x - x^2$

(c) Identify the permissible values of x given $A \geq 0$. **2**

$x \geq 0 \qquad 200 - x \geq 0$

$200 \geq x$

$\therefore \quad 0 \leq x \leq 200$

(d) What is the maximum area that can be formed? **2**

Coefficient of x^2 is negative

$\therefore$ Maximum area at vertex

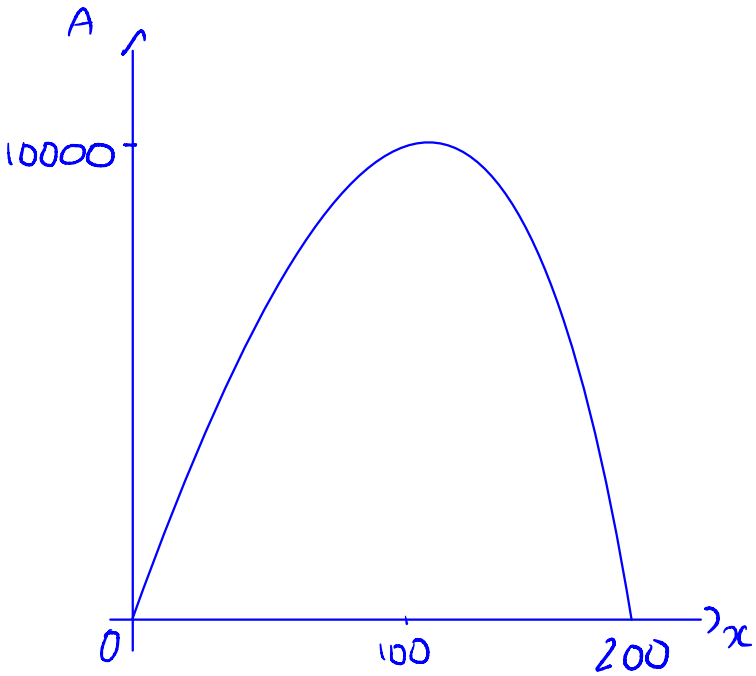
axis of symmetry $= \frac{0 + 200}{2}$

$= 100$

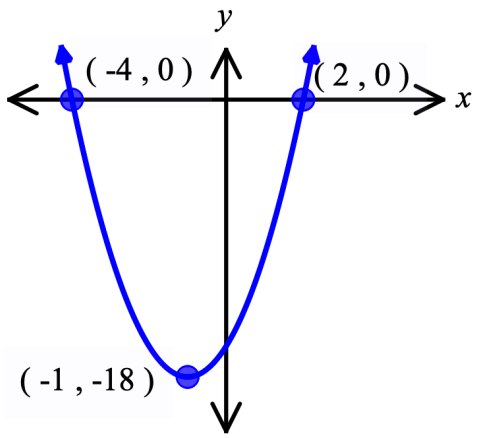
$A = 200(100) - 100^2$

$= 10000 \text{ m}^2$

(e) Sketch the graph of A versus x. **2**



7. Determine the equation of the parabola below in the form $y = k(x-a)(x-b)$. **3**



x - intercepts at
$x = -4$ and $x = 2$

$\therefore y = k(x+4)(x-2)$

vertex at $(-1,-18)$

$\therefore$ when $x = -1$, $y = -18$

$-18 = k(-1+4)(-1-2)$

$-18 = k(3)(-3)$

$-18 = -9k$

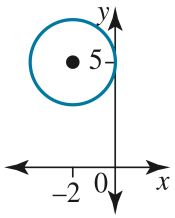
$\therefore k = 2$

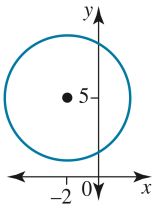
$\therefore y = 2(x+4)(x-2)$

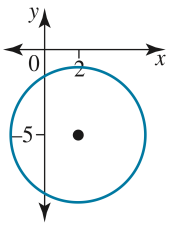
Part D: Non-Linear Functions (19 Marks)

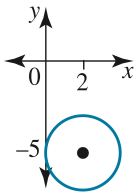
Circle the letter for the correct response.

1. The graph of $(x-2)^2 + (y+5)^2 = 4$ is:

(A) 

(B) 

(C) 

(D) 

centre = $(2,-5)$ radius = 2

2. The graph of $y = \dfrac{1}{x+3} - 2$ has asymptote(s) at:

(A) $x = -3, y = -2$
(B) $x = -2, y = 3$
(C) $x = 3, y = -2$
(D) $x = -2, y = -3$

3. The graph of $y = -2^x$ has a y – intercept with coordinates:

(A) $(0,2)$
(B) $(2,0)$
(C) $(-1,0)$
(D) $(0,-1)$

$y = -2^0$
$= -1$

Answer in the space provided. Show all necessary working.

4. State the domain and range for

$y = \sqrt{x+2}$. **2**

$x + 2 \geq 0$

$x \geq -2$

$\therefore$ Domain: $x \geq -2$ or $[-2, \infty)$

Range: $y \geq 0$ or $[0, \infty)$

5. If $f(x) = -2x^3 + x^2 - 6$, find $f(-3)$. **2**

$f(-3) = -2(-3)^3 + (-3)^2 - 6$

$= 54 + 9 - 6$

$= 57$

6. Find the inverse function $f^{-1}(x)$ of

$y = 4x - 1$. **2**

$x = 4y - 1$

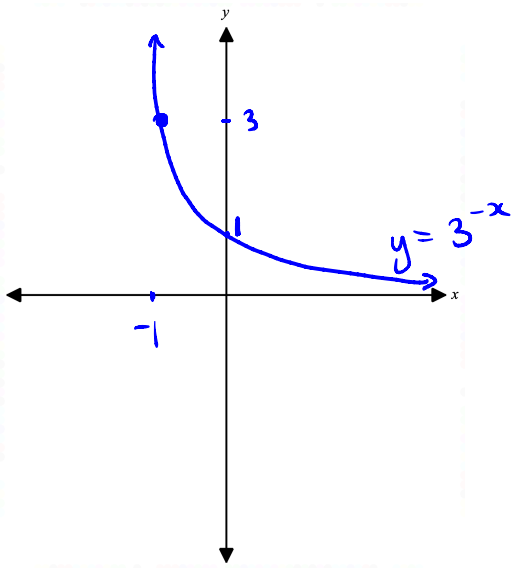
$x + 1 = 4y$

$y = \frac{x+1}{4}$

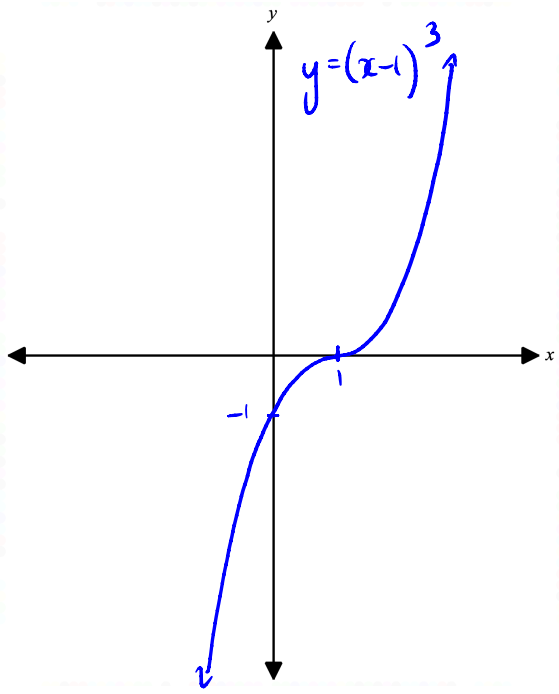
$\therefore f^{-1}(x) = \frac{x+1}{4}$

7. Sketch each of the following functions, showing all important features.

(a) $y = 3^{-x}$ **2**



(b) $y = (x-1)^3$ **2**



8. The diameter of a balloon (D mm) varies inversely with the thickness of the rubber (x mm). The diameter of the balloon is 80 mm when the rubber is 2 mm thick

(a) Find the equation relating D and x. **2**

$D = \frac{k}{x}$

when $x = 2$ $D = 80$

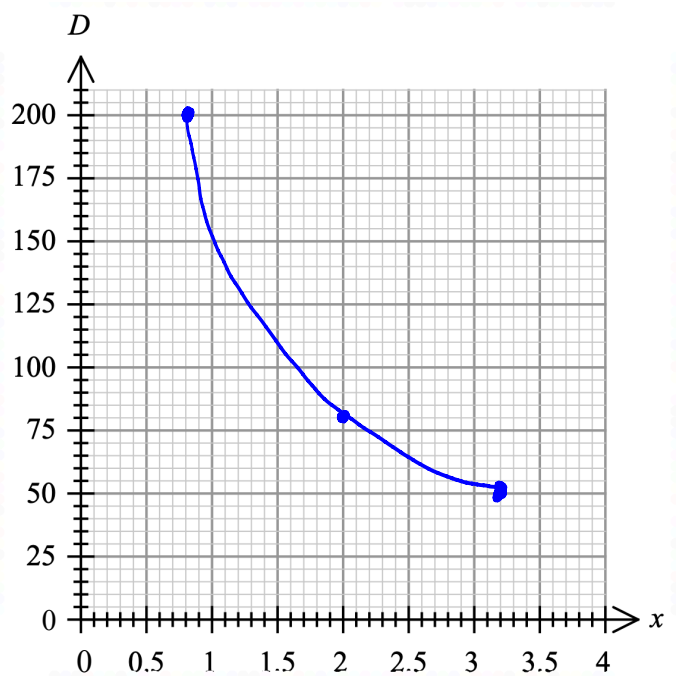
$80 = \frac{k}{2}$

$\therefore k = 160$

$\therefore D = \frac{160}{x}$

(b) By first completing this table of values, graph the relationship between diameter and thickness from $x = 0.8$ to $x = 3.2$. **2**

x	0.8	2	3.2
D	200	80	50



9. Find the x – coordinate(s) for the point(s) of intersection for the hyperbola $y = \frac{3}{x}$ and the line $y = 2x + 1$. **2**

Solve simultaneously

$2x+1 = \frac{3}{x}$

$2x^2 + x = 3$

$2x^2 + x - 3 = 0$

$(2x+3)(x-1) = 0$

$2x+3=0$ $\quad$ $x-1=0$

$2x = -3$ $\quad$ $x = 1$

$x = -\frac{3}{2}$

End of Examination

Extra Writing Space