Name:
Class:

SEMESTER 2 EXAMINATION

YEAR 10 2024

MATHEMATICS

Time allowed – 100 minutes
(Plus 5 minutes reading time)

INSTRUCTIONS:

- All questions may be attempted.
- Write your **name** and **Maths class** at the top of each question.
- **Write** in **Pen** and draw diagrams in **Pencil.**
- Answers to Section I Multiple choice Questions 1-10 are to be entered onto the answer sheet provided at the back of this cover page.
- Answers to Section II Questions 11 - 15 are to be written on the space provided.
- NESA approved calculators are permitted.
- Show all necessary working.
- Marks may not be awarded for untidy or carelessly arranged work.
- No grid paper is to be used unless provided with the examination paper.

Multiple Choice 10 Marks

Attempt Question 1 – 10 (1 mark each)
Allow approximately 15 minutes for this section.

Use the multiple choice answer sheet below to record your answers to Question 1 – 10.

Select the alternative: A, B, C or D that best answers the question.

Colour in the response oval completely.

Sample:

2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ⬭ B ⬬ C ⬭ D ⬭

If you think you have made a mistake, draw a cross through the incorrect answer and colour in the new answer

ie A ⬬ B ⬬ (crossed) C ⬭ D ⬭

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word "*correct*" and draw an arrow as follows:

correct ↓

A ⬬ B ⬬ (crossed) C ⬭ D ⬭

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Year 10 Semester 2 Examination 2024

Multiple Choice Answer Sheet

Completely colour in the response oval representing the most correct answer.

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

Marks: / 10

Name:
Class:
Multiple Choice (10 marks)
Attempt Questions 1 – 10. Use the multiple choice answer sheet for Questions 1 – 10.

1. Which of the following most accurately describes the roots to the equation $3x^2 - 7x + 2 = 0$?

A. Two equal roots
B. Two unequal irrational roots
C. Two unequal rational roots
D. None of the above

2. Which of the following is **not** a sufficiency condition for a square?

I. Quadrilateral with diagonals equal and bisect each other at right angles
II. Quadrilateral with 4 equal sides

A. I only
B. II only
C. Both I and II
D. Neither I nor II

3. Which of the following is a factor of $2x^3 - 5x^2 - 4x + 3$?

A. $x + 2$
B. $x - 1$
C. $2x - 1$
D. $x + 3$

4. Which of the following is false?

A. $1.5 \in \mathbb{R}$
B. $1.5 \in \mathbb{Q}$
C. $1.5 \in \mathbb{Z}$
D. None of the above.

5. Given $n(A) = 10$ and $n(B) = 15$ and $n(U) = 30$, which of the following must be false?

A. $A \cap B = \emptyset$
B. $n(A \cup B) \leq 25$
C. $n\left(\widetilde{A \cup B}\right) = 15$
D. None of the above.

6. Twenty-seven students sit a test in which the result had a mean of 60, standard deviation 3 and median 58. If two extra students sit the test after and obtained scores of 57 and 63 respectively, which of the following may change?

A. Mean
B. Median
C. Standard Deviation
D. None of the above

7. Which of the following is not equal to $\log_{\sqrt{2}} 8$?

A. 6
B. $\frac{\log_{99} 8}{\log_{99} \sqrt{2}}$
C. $\log_{\sqrt{2}} 32 - \log_{\sqrt{2}} 4$
D. $\log_{\sqrt{2}} 32 \div \log_{\sqrt{2}} 4$

8. A set of data is normally distributed with mean 85 and standard deviation 1. What percentage of scores lie between 84 and 87?

A. 47.5%
B. 63.5%
C. 81.5%
D. 83.85%

9. Which of the following quantities are not direct variations of each other?

A. The area of a circle and the square of its radius.
B. The average speed of a trip and the time it takes to reach its destination.
C. The mean of 20 positive numbers and the sum of the same 20 numbers.
D. The length of two corresponding sides in a pair of similar triangles.

10. Let D and R be the domain and range of a relation F respectively. Given that $n(D) > n(R)$, what can be concluded about F and its inverse?

A. F is a function but its inverse is not.
B. Neither F nor its inverse are functions.
C. Both F and its inverse may be functions.
D. F may be a function but its inverse is not.

Name:
Class:
Questions 11 - 15
Answer the questions in the space provided. Sufficient space is provided for typical responses. Your responses should include relevant mathematical reasoning and/or calculations. Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (16 marks)

a) Simplify the following:

i. $\frac{x}{x^2-4}+\frac{2}{x-2}$ 2

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ii. $\frac{x^3-27}{x^2+3x-18}\div\frac{2x^2+6x+18}{x^2-36}$ 2

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iii. $5\sqrt{45}-\sqrt{27}+8\sqrt{20}-7\sqrt{243}$ 2

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b) Solve the following:

i. $2x^2 - 8x + 3 = 0$ by completing the square 2

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ii. $4^{x+y} = 64$
$7^{2x-y} = 49\sqrt{7}$ 3

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c) Rewrite $\frac{7}{\sqrt{3}-\sqrt{5}}$ with a rational denominator. 1

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d) Given that the universal set $U = \{a, b, c, d, e, f\}$, and $A = \{a, b, c\}, B = \{c, d, e\}$ and $C = \{e, f\}$.

i. List all the elements in $(A \cup C) \cap B$. 1

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ii. List all the elements in $\tilde{C}$. 1

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iii. How many elements are there in the power set of U? 1

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iv. Write down the power set of C. 1

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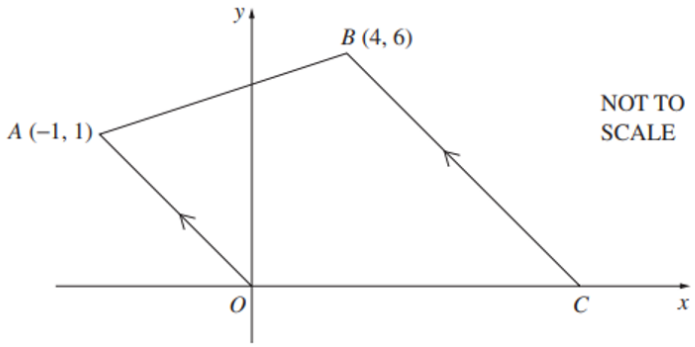
End of Question 11

Spare working paper. Clearly label the question number for each response.

Name:
Class:

Question 12 (16 marks)

a)



In the diagram, $OABC$ is a trapezium with $OA \parallel CB$. The coordinates of O, A and B are $(0,0)$, $(-1,1)$ and $(4,6)$ respectively.

i. Calculate the length of the interval OA. 1

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ii. Write down the gradient of the line OA. 1

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iii. Find the equation of the line BC, and hence find the coordinates of C. 2

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iv. Find the perpendicular distance from O to the line BC. 2

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v. Hence, or otherwise, calculate the area of the trapezium $OABC$. 2

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vi. Find $P(x, y)$ such that P divides the interval AB internally in the ratio $5:3$. 2

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b) i. Show that $\cos\theta \tan\theta = \sin\theta$. 1

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ii. Hence, solve $8\sin\theta\cos\theta\tan\theta = \operatorname{cosec}\theta$ for $0 \leq \theta \leq 360°$. 2

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c) Five candidates, A, B, C, D and E, are standing for an election. Their names are written on pieces of cardboard that are placed in a barrel and are drawn out randomly to determine their positions on the ballot paper.

i. What is the probability that A is drawn first? 1

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ii. What is the probability that the names are drawn out in the order of A, B, C, D, E given that A was drawn first? 1

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iii. Is drawing a vowel first dependent on drawing a vowel last? Justify your answer mathematically. 1

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End of Question 12

Spare working paper. Clearly label the question number for each response.

Name:
Class:

Question 13 (16 marks)

a) It is known that a quantity Q is inversely proportional to the **square root** of another quantity x.

When $x = 144$, $Q = 3$.

i. Find Q in terms of x. 1

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ii. What is the percentage increase/decrease to x when Q is increased by 25%? 1

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b) Solve for x if $\log_2(2x - 3) - \log_2(x + 1) = 2$. 3

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c) Given $P(x) = x^3 - 4x^2 - x + 4$.

i. Show that $x - 4$ is a factor of $P(x)$. 1

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ii. By long division, factorise $P(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic. 2

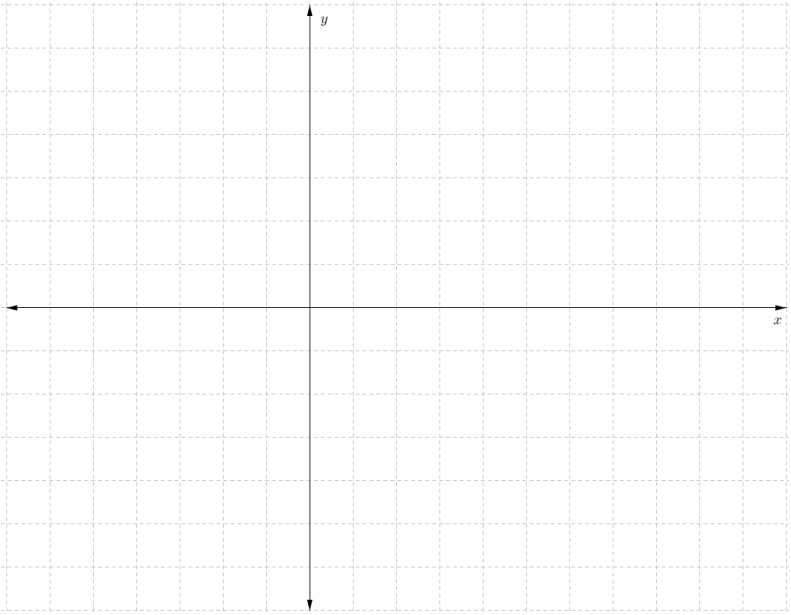
iii. Hence or otherwise, factorise $P(x)$ fully. 1

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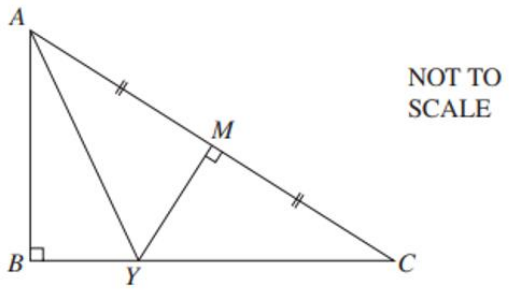
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iv. Hence otherwise, sketch the graph $y = P(x)$ on the graph below, labelling all intercepts. 2



d)



The diagram shows a right-angled triangle ABC with $\angle ABC = 90°$. The point M is the midpoint of AC, and Y is the point where the perpendicular to AC at M meets BC.

i. Prove that $\Delta AYM \equiv \Delta CYM$. 2

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ii. Suppose that it is also given that AY bisects $\angle BAC$. Find the size of $\angle YCM$ and hence find the ratio $MY : AC$. 3

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End of Question 13

Spare working paper. Clearly label the question number for each response.

Name:
Class:

Question 14 (16 marks)

a) In the swimming carnival, Thorpie swam the 100m freestyle 52.14 seconds while Klimmie swam the 100m butterfly in 54.76 seconds. The statistics for the two races are as follows:

	100m freestyle	100m butterfly
Average	58.00 seconds	59.50 seconds
Standard Deviation	1.5 seconds	2.00 seconds

Who did better in their respective race? Justify your answer with reference to their z-scores. 2

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b) The winning margin of an undefeated netball team after 11 rounds are recorded below:

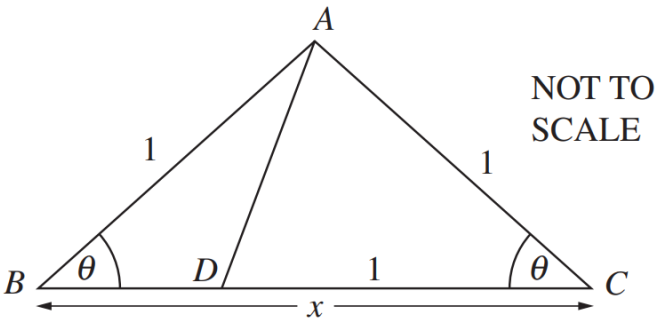
4, 12, 8, 9, 2, 1, 15, 15, 19, 1, 13.

i. Write down the mean and the standard deviation. 2

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ii. Write down the 5 number summary and hence draw a box and whisker plot in the space below. 3

c)



In the diagram, ABC is an isosceles triangle where $\angle BAC = 108°$ and $AB = AC = 1$. The point D is chosen on BC such that $CD = 1$.

Let $BC = x$ and $\angle ABC = \theta$, where $\theta = 36°$.

i. Show that $\angle ADC = 2\theta$ and hence prove that ΔDBA is similar to ΔABC. 3

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ii. Hence, deduce that $x^2 - x - 1 = 0$. 1

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iii. Hence, deduce that $\cos 36° = \frac{1+\sqrt{5}}{4}$. 2

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d) Consider the graph $y = \frac{x+1}{x-3}$.

i. Write down its new equation after the graph has been dilated horizontally from the y-axis by a factor of 3. 1

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ii. The graph obtained in part i. is then translated 2 units up before being reflected about the x-axis. Write down its new equation in the form $y = k + \frac{c}{x-h}$. 2

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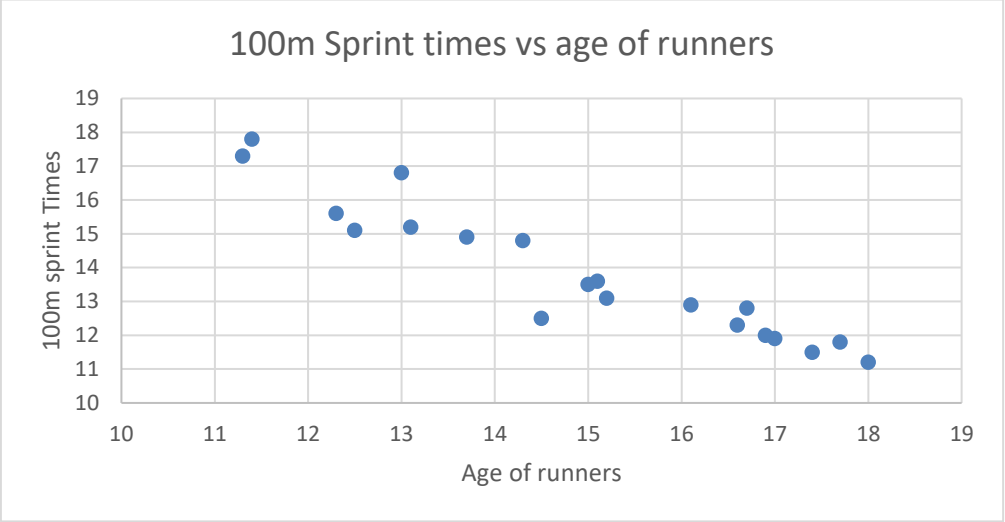
End of Question 14

Spare working paper. Clearly label the question number for each response.

Name:
Class:

Question 15 (16 marks)

a) The 100m sprint time and the age of 20 runners are recorded in the scatter plot below:



i. Describe the relationship between sprint times and age in terms of direction and strength. 2

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ii. On the scatter plot above, draw an appropriate line of best fit. 1

iii. Albus drew a line of best fit that passes through $(13, 15.5)$ and $(17, 12)$. Find its equation in gradient intercept form. 2

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iv. Use your answer in iii. to estimate the age of someone who can sprint 100m in exactly 20 seconds. Is this an example of interpolation or extrapolation? Explain your answer. 2

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b) The equation $2x^2 + ax + (b + 3) = 0$ has 2 real distinct roots. Show that $a^2 + b^2 > 8$. 2

c) Solve $2\cos^2\theta + 7\sin\theta = 5$ for $0° \leq \theta \leq 720°$. 3

d) If $\frac{a+b}{\operatorname{cosec} x} = \frac{a-b}{\cot x}$, show that $\operatorname{cosec} x \cot x = \frac{a^2-b^2}{4ab}$, for $0° < x < 90°$. 4

End of Paper

Spare working paper. Clearly label the question number for each response.