Name:

Class:

Multiple Choice (10 marks)

Attempt Questions 1–10. Use the multiple-choice answer sheet for Questions 1–10.

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1 In a large group, 60% of the people are either 170 cm tall or taller than 170 cm. If two people will be selected randomly, what is the probability of selecting at least one person shorter than 170 cm?

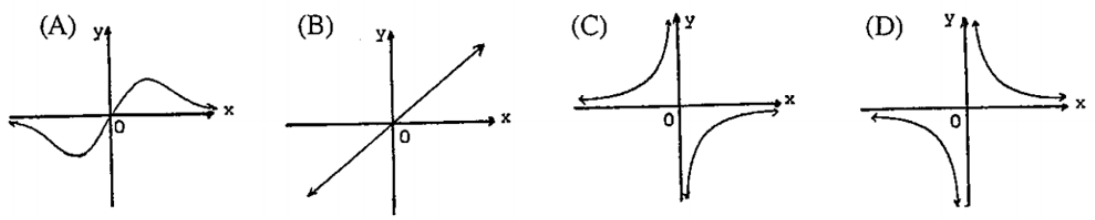
A. 0.36

B. 0.48

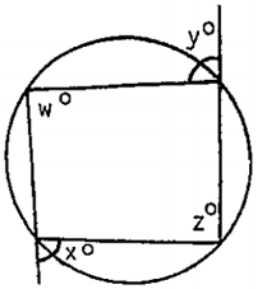
C. 0.84

D. 0.64

2 The equation $xy - 1 = 0$ is best represented by the graph:



3 The diagram shows a cyclic quadrilateral. Which statement is always true?



A. $z = w$

B. $x = w$

C. $x + y = 180^\circ$

D. $x + w = 180^\circ$

4 The equation $2x^3 - x^2 - 4x + 9 = 0$ has roots α, β and γ. The value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\gamma\alpha}$ is:

A. $-\dfrac{1}{2}$

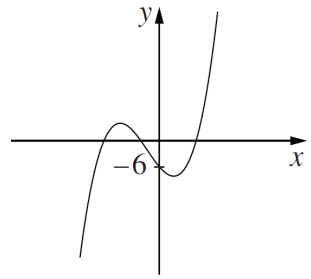
B. $-\dfrac{1}{9}$

C. $\dfrac{1}{2}$

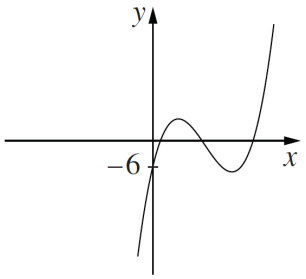
D. $\dfrac{1}{9}$

5 Consider the polynomial $P(x) = ax^3 + bx^2 + cx - 6$, where $a, b > 0$. Which of the following is a potential graph of $y = P(x)$?

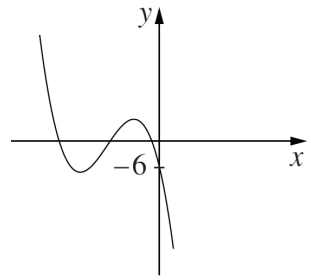
(A)



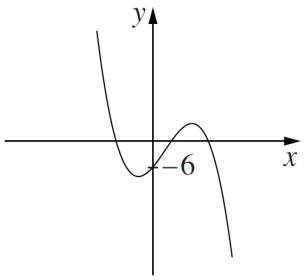
(B)



(C)



(D)



6 The minimum value of $x^2 - 4x - 5$ is

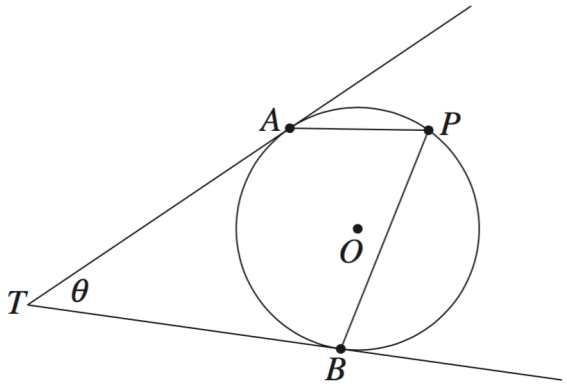
A. 7

B. 9

C. -9

D. -7

7 The points A, B and P lie on a circle centred at O. The tangents to the circle at A and B meet at the point T, and $\angle ATB = \theta$.



What is $\angle APB$ in terms of θ?

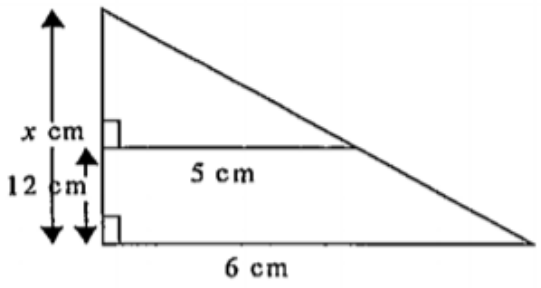
A. $\frac{\theta}{2}$

B. $90^\circ - \frac{\theta}{2}$

C. θ

D. $180^\circ - \theta$

8 The value of x is



(Diagram not to scale)

A. 10cm

B. 60cm

C. 72cm

D. 6cm

9 A group of 485 people was surveyed. The people were asked whether or not they smoke. The results are recorded in the table.

	Smokers	*Non-smokers*	*Total*
Male	88	176	264
Female	68	153	221
	156	329	485

A person is selected at random from the group. What is the approximate probability that the person selected is a smoker OR is male?

A. 33%

B. 18%

C. 68%

D. 87%

10 In a class of 30 students, 5 study no languages at all, 9 study French, 14 study Japanese and 15 study Korean. If 5 students study French and Japanese, and 3 students study French and Korean, what is the probability that a student chosen at random studies all 3 languages.

A. $\frac{4}{15}$

B. $\frac{8}{25}$

C. $\frac{1}{15}$

D. $\frac{2}{25}$

Name:

Class:

Sections A - E

Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses. Your responses should include relevant mathematical reasoning and/or calculations. Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Section A (20 Marks)

(a) Simplify $\frac{x+2y}{3} - \frac{2x-y}{4}$ **2**

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(b) Solve the equation $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = 2(x^{\frac{1}{2}} - x^{-\frac{1}{2}})$ **3**

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(c) Show that $\frac{4}{2+\sqrt{5}} - \frac{1}{9-4\sqrt{5}}$ is a rational number. **3**

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(d) An athlete records his times in seconds running 100m over a number of events and trials.

11.2 11.4 11.1 11.5 11.3
11.2 11.6 11.3 11.7 11.5

(i) Calculate the mean and standard deviation for the data using your calculator. **2**

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(ii) Construct a box-and-whisker plot for the data. **3**

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(e) If $\sin\theta = -\frac{4}{9}$ and $\cos\theta > 0$ find the exact value of $\tan\theta$. **2**

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(f) A line makes an angle of 120° with the positive direction of the x axis. Find the gradient of the line. **2**

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(g) Find the value of $|\,7-|\,2-11\,||$ **1**

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(h) Sketch the graph of $y=|\,2x-3\,|$ **2**

Name:

Class:

Section B (20 Marks)

(a) Solve the following pair of simultaneous equations: **3**

$$xy + x = -3$$
$$2x + 5y = 8$$

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(b) Solve $(x^2 + 2x)^2 - 4(x^2 + 2x) + 3 = 0$ **4**

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(c) An army bunker currently has 60 soldiers and enough food to last them 20 days. They are joined by 40 more soldiers. Assuming inverse variation, how long will their food supply last? **2**

(d) Let $f(x) = \dfrac{x}{x^2 - 1}$. Show that $f(x) + f(\dfrac{1}{x}) = 0$, where $x \neq 0$. **3**

(e) When a polynomial $P(x)$ is divided by $(x-2)(x+5)$, the remainder is $x+3$. Find the remainder when $P(x)$ is divided by $(x+5)$. **2**

(f) Show that the roots of $p(1+x) = q(1-x-x^2)$ are always real. **3**

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(g) Three employees, David, Eliza and Frank, work for the same company. They respectively handle 40%, 35% and 25% of the company's orders. It has been found that they make mistakes in 5%, 4% and 3% of their orders respectively.

(i) Use a tree diagram to display this information. **2**

(ii) Calculate the probability that an order handled by the company has a mistake. **1**

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Name:

Class:

Section C (20 Marks)

(a) Given $P(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

(i) Factorise $P(x)$ **1**

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(ii) Hence, sketch the graph of $y = f(x)$ **2**

(b) Simplify $\dfrac{\sin^3 x}{\cos x} + \sin x \cos x$ **2**

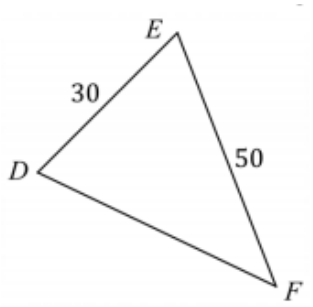
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(c) Three girls are standing in the park at points D, E and F. D is 30 metres from E and E is 50 metres from F. The bearing of E from D is 045°and the bearing of F from E is 158°.



(i) Show all information given above on the diagram and find the size of angle DEF giving reasons. **2**

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(ii) Show that the distance of F from D is approximately 47 metres. **1**

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(iii) Hence, or otherwise, find the bearing of D from F. Answer to the nearest degree. **3**

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(d) Solve $2\sin^2 x - 5\sin x - 3 = 0$ for $0° \leq x \leq 360°$ **3**

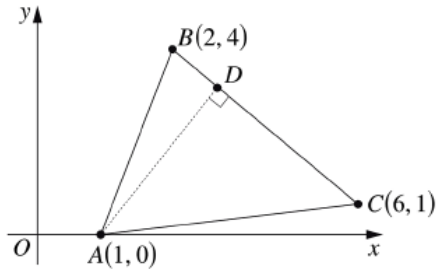
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(e) The diagram shows points $A(1,0)$, $B(2,4)$ and $C(6,1)$. The point D lies on BC such that $BC \perp AD$.



NOT TO SCALE

(i) Show that the equation of BC is $3x + 4y - 22 = 0$. **2**

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(ii) Find the length of AD **2**

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(iii) Hence, or otherwise, find the area of $\triangle ABC$. **2**

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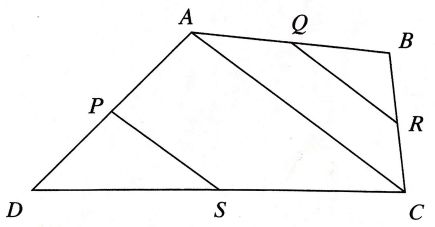
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Name:

Class:

Section D (20 Marks)

(a) The sides of a quadrilateral $ABCD$ have midpoints P, Q, R and S, as shown below.



(i) Show that $\triangle DPS$ is similar to $\triangle DAC$. **3**

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(ii) Show that $PS \parallel QR$. **1**

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(iii) Show that $PQRS$ is a parallelogram. **2**

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(b) Solve the equation $3^{2x} = 4^{2-x}$, giving your answer to 3 significant figures. **3**

(c) If $\log_a b = 0.3$ and $\log_a c = 0.4$, find the value of $\log_b(\frac{a}{c})$ **3**

(d) Let $f(x) = \sqrt{4x-3}$

(i) Find the domain of $f(x)$. **1**

(ii) Find an expression for the inverse function $f^{-1}(x)$. **2**

(iii) Find the coordinates of the points where the graphs $y = f(x)$ and $y = x$ intersect. **2**

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(iv) On the same set of axes, sketch the graphs $y = f(x)$ and $y = f^{-1}(x)$ showing the information found in part (iii). **3**

Name:

Class:

Section E (20 Marks)

(a) Two cards are drawn at random without replacement from a pack of 7 cards numbered 1, 2, 3, 4, 5, 6, 7, and their product is noted.

(i) Draw a table or dot diagram to represent this situation. **2**

(ii) Find the probability that a prime number is included in at least one of the two cards. **1**

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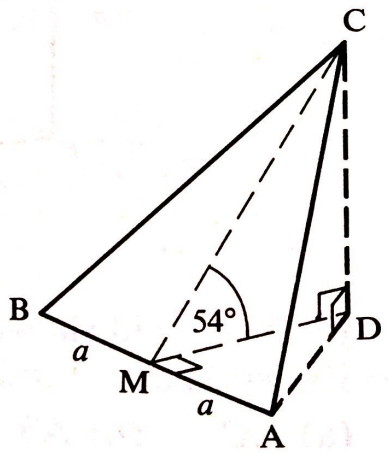
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(iii) Find the probability that the product of the numbers on the two cards is even given that exactly one of the numbers is prime. **1**

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(b) The base AB of an isosceles triangle, ABC, with $BC = AC$, is horizontal. The plane containing the triangle is inclined to the horizontal at $54°$. If the angle ACB is $48°$, find the angle between AC and the horizontal plane. **4**



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(c) A triangle ABC has side lengths $AB = 3$, $BC = x + 1$, $AC = x$ and $\angle ACB = \theta$.

(i) Draw a diagram showing the information above. **1**

(ii) Show that $\cos\theta = 1 - \dfrac{4}{x(x+1)}$. **2**

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(iii) Hence find $\sin^2\theta$ in terms of x. Give your answer in factored form. **3**

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(iv) Show that the area of the triangle is $\sqrt{2(x+2)(x-1)}$. **2**

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(d) Find the equation of the circle that goes through the points $A(0,1)$, $B(4,7)$ and $C(4,-1)$. **4**

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End of Section E
End of paper

Name:
Class:
Extra working pages. Number your questions with Section Number and part.

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Name:

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Multiple Choice (10 marks)

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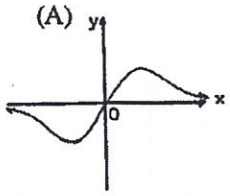
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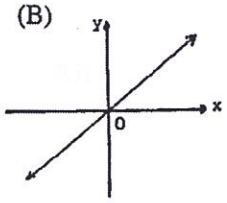
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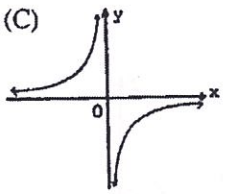
(A)



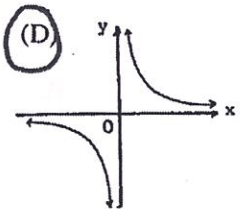
(B)



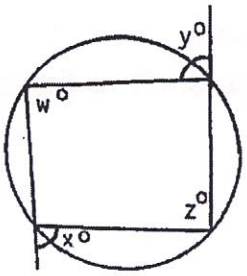
(C)



(D)



3 The diagram shows a cyclic quadrilateral. Which statement is always true?



A. $z = w$

B. $x = w$

C. $x + y = 180°$

D. $x + w = 180°$

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$\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{1}{2} \div \frac{-9}{2}$

$= -\frac{1}{9}$

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B. $-\frac{1}{9}$

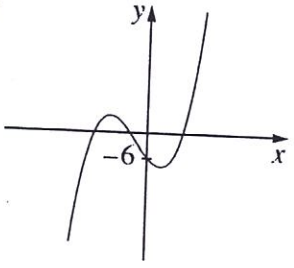
C. $\frac{1}{2}$

D. $\frac{1}{9}$

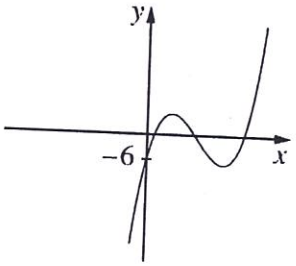
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Sum of roots $= -\frac{b}{a} < 0$, since $a, b > 0$

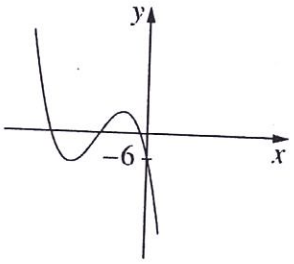
(A)



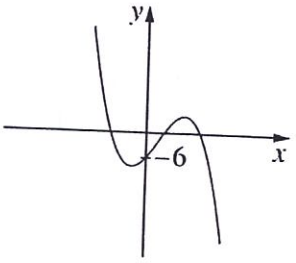
(B)



(C)



(D)



6 The minimum value of $x^2 - 4x - 5$ is

$x^2 - 4x + 4 - 9 = (x-2)^2 - 9$

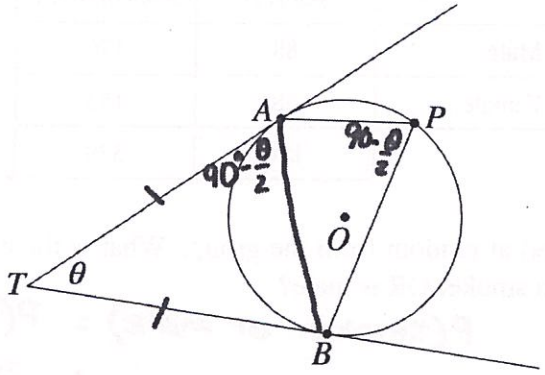
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B. 9

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What is $\angle APB$ in terms of θ?

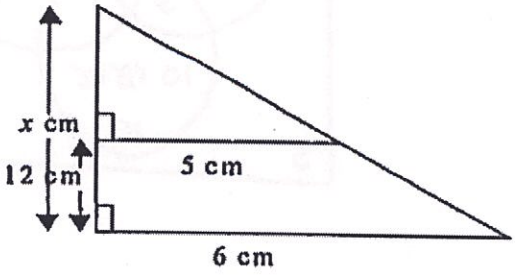
A. $\frac{\theta}{2}$

B. $90° - \frac{\theta}{2}$

C. θ

D. $180° - \theta$

8 The value of x is



(Diagram not to scale)

$\frac{x-12}{x} = \frac{5}{6}$

$6x - 72 = 5x$

$x = 72$

A. 10cm

B. 60cm

C. 72cm

D. 6cm

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A person is selected at random from the group. What is the approximate probability that the person selected is a smoker OR is male?

A. 33%

B. 18%

C. 68%

D. 87%

P(smoker or male) = P(male) + P(female smoker)

$= \frac{264}{485} + \frac{68}{485}$

$= \frac{332}{485}$

$= 0.68$

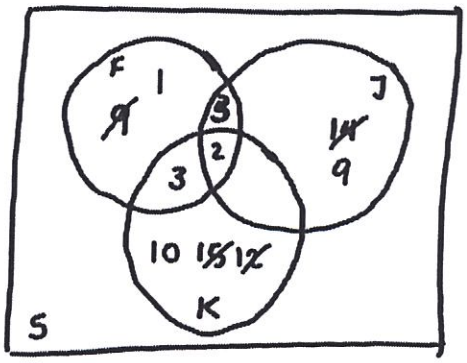
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A. $\frac{4}{15}$

B. $\frac{8}{25}$

C. $\frac{1}{15}$

D. $\frac{2}{25}$



14+15+9

= 38

8

22+8+5 = 35

$P(\text{all 3}) = \frac{2}{30}$

$= \frac{1}{15}$

Name:

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Sections A - E

Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses. Your responses should include relevant mathematical reasoning and/or calculations. Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Section A (20 Marks)

(a) Simplify $\frac{x+2y}{3} - \frac{2x-y}{4}$ **2**

$= \frac{4x+8y-6x+3y}{12}$ (1)

$= \frac{-2x+11y}{12}$ (1)

This question was well done. Most of the students got it correct.

(b) Solve the equation $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = 2(x^{\frac{1}{2}} - x^{-\frac{1}{2}})$ **3**

$x^{1/2} + x^{-1/2} = 2x^{1/2} - 2x^{-1/2}$

$3x^{-1/2} = x^{1/2}$ (1)

$\frac{3}{\sqrt{x}} = \sqrt{x}$ (1)

$x = 3$ (1)

Some students followed a complicated way of getting quardatic equation

(c) Show that $\frac{4}{2+\sqrt{5}} - \frac{1}{9-4\sqrt{5}}$ is a rational number. **3**

$= \frac{4}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} - \frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}}$ (1)

$= \frac{8-4\sqrt{5}}{4-5} - \frac{9+4\sqrt{5}}{81-80}$ (1)

$= 4\sqrt{5} - 8 - 9 - 4\sqrt{5}$

$= -17$ which is of the form $\frac{a}{b}$ (1)

$\therefore$ rational

Some were confused about rationalising the individual terms and so could not get the final answer.

(d) An athlete records his times in seconds running 100m over a number of events and trials.

11.2 11.4 11.1 11.5 11.3
11.2 11.6 11.3 11.7 11.5

(i) Calculate the mean and standard deviation for the data using your calculator. **2**

$\bar{x} = 11.38$ ①
$\sigma = 0.19$ ①

Most of the students got it correct.

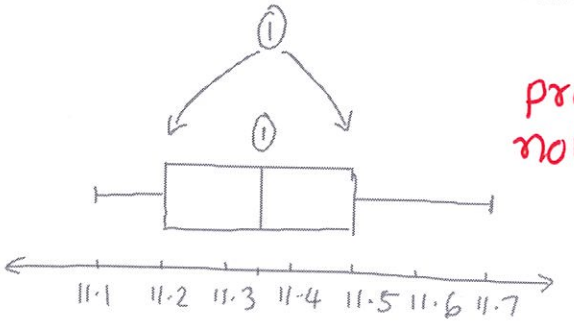
(ii) Construct a box-and-whisker plot for the data. **3**

11.1, 11.2, 11.2, 11.3, 11.3, 11.4, 11.5, 11.5, 11.6, 11.7

median = 11.35

$Q_1 = 11.2$

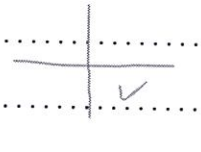
$Q_3 = 11.5$



① ① ①

Some students did not provide scale and some did not provide end points, so they ① scale and end points were penalised

(e) If $\sin\theta = -\frac{4}{9}$ and $\cos\theta > 0$ find the exact value of $\tan\theta$. **2**



$\tan\theta = \frac{-4}{\sqrt{65}}$ ① ①

Some students were confused about the sign of final answer and some calculated θ which was unnecessary

(f) A line makes an angle of 120° with the positive direction of the x axis. Find the gradient of the line. **2**

$m = \tan\theta$
$= \tan 120°$ ①
$= -\sqrt{3}$ ①

A considerable number of students were not sure about the sign of the final answer, so were penalised

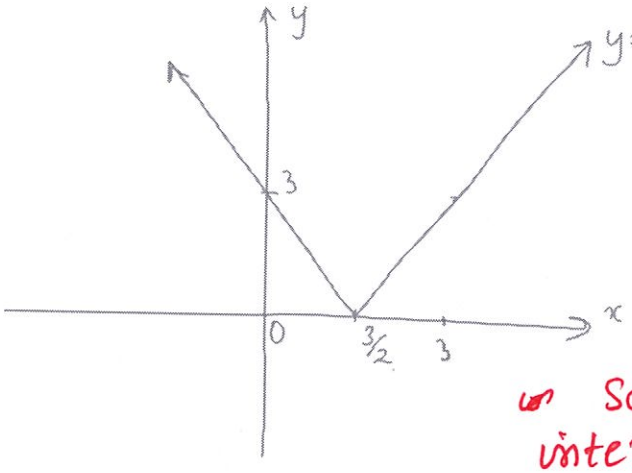
(g) Find the value of $|7-|2-11||$ **1**

$= |7-|-9|| = |7-9|$

$= 2$ (1)

most students got it correct.

(h) Sketch the graph of $y = |2x-3|$ **2**



(1) intercepts
(1) shape

some did not provide intercepts, so were penalised.

Name:

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Section B (20 Marks)

(a) Solve the following pair of simultaneous equations: **3**

$$xy+x=-3$$
$$2x+5y=8$$

$x(y+1)=-3$ $\qquad 2x=8-5y$

$x=\frac{8-5y}{2}$

$(8-5y)(y+1)=-6$ (1)

$8y-5y^2-5y+8=-6$

$5y^2-3y-14=0$

$(5y+7)(y-2)=0$

$y=-\frac{7}{5}, 2$ (1)

when $y=-\frac{7}{5}$, $x=\frac{8+7}{2}=7.5$

$y=2$, $x=-1$ } (1)

Note:
students lost a mark if they did not write solution as an ordered pair.
ie when $x=-1, y=2$
and $x=7.5$ $y=-6.4$

4

(b) Solve $(x^2+2x)^2-4(x^2+2x)+3=0$

Let $u=x^2+2x$

$u^2-4u+3=0$

$(u-3)(u-1)=0$

$u=3$ or 1 (1)

$x^2+2x=3$ or $x^2+2x=1$

$x^2+2x-3=0$ $\qquad x^2+2x-1=0$ (1)

$(x+3)(x-1)=0$ $\qquad x=\frac{-2\pm\sqrt{4+4}}{2}$

$x=-3$ or 1 $\qquad =\frac{-2\pm 2\sqrt{2}}{2}$

(1) $\qquad =-1\pm\sqrt{2}$ (1)

$\therefore x=-3, 1, -1+\sqrt{2}$ or $-1-\sqrt{2}$

(c) An army bunker currently has 60 soldiers and enough food to last them 20 days. They are joined by 40 more soldiers. Assuming inverse variation, how long will their food supply last? **2**

Let no. of soldiers = S, let no. of days = d

$S \propto \frac{1}{d} \Rightarrow S = \frac{k}{d}$

$60 = \frac{k}{20}$

$k = 1200$ ✓1

$S = \frac{1200}{d}$

$100 = \frac{1200}{d}$ ✓1

$d = 12$

$\therefore$ food will last 12 days

No marks if bold answer 12 only

(d) Let $f(x) = \frac{x}{x^2-1}$. Show that $f(x) + f(\frac{1}{x}) = 0$, where $x \neq 0$. **3**

$f(\frac{1}{x}) = \frac{1}{x} \div \left((\frac{1}{x})^2 - 1\right)$ ✓1 Sub $\frac{1}{x}$ for x

$= \frac{1}{x} \div \frac{1-x^2}{x^2}$

$= \frac{1}{x} \times \frac{x^2}{1-x^2}$

$= \frac{x}{1-x^2}$ ✓1

$f(x) + f(\frac{1}{x}) = \frac{x}{x^2-1} + \frac{x}{1-x^2}$

$= \frac{x}{x^2-1} - \frac{x}{x^2-1}$ ✓1 writing $\frac{x}{1-x^2}$ as $\frac{-x}{x^2-1}$

$= 0$

No marks if $* f(\frac{1}{x}) = \frac{x^2-1}{x}$

(e) When a polynomial $P(x)$ is divided by $(x-2)(x+5)$, the remainder is $x+3$. Find the remainder when $P(x)$ is divided by $(x+5)$. **2**

Let $P(x) = (x-2)(x+5)Q(x) + x + 3$ ✓1

$P(-5) = (-5-2)(-5-5)Q(-5) - 5 + 3$

$= -2$

$\therefore$ remainder $= -2$ ✓1

another method

$ax + b = x + 3$

$\therefore a = 1$

$b = 3$

then sub in $x = -5$ in $ax + b$

(f) Show that the roots of $p(1+x) = q(1-x-x^2)$ are always real. **3**

$qx^2 + (p+q)x + p - q = 0$

For real roots $\Delta \geq 0$ — (1)

$\Delta = (p+q)^2 - 4q(p-q)$ (1) Finding Δ ✓

$= p^2 + 2pq + q^2 - 4pq + 4q^2$

$= p^2 - 2pq + q^2 + 4q^2$

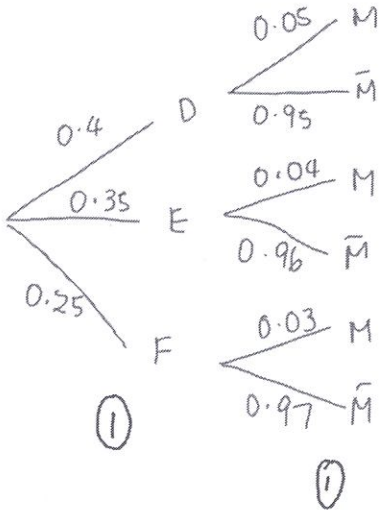
$= (p-q)^2 + 4q^2 \geq 0$ since $(p-q)^2 \geq 0$ (1) proving both $(p-q)^2$ and $4q^2$ positive

and $4q^2 \geq 0$

$\therefore$ roots are always real

(g) Three employees, David, Eliza and Frank, work for the same company. They respectively handle 40%, 35% and 25% of the company's orders. It has been found that they make mistakes in 5%, 4% and 3% of their orders respectively.

(i) Use a tree diagram to display this information. **2**



(1) (1)

(ii) Calculate the probability that an order handled by the company has a mistake. **1**

$P(\text{mistake}) = 0.4 \times 0.05 + 0.35 \times 0.04 + 0.25 \times 0.03$

$= 0.415$ (1)

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Section C (20 Marks)

(a) Given $P(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

(i) Factorise $P(x)$ **1**

$P(1) = 1 - 5 + 5 + 5 - 6$

$= 0$

$P(-1) = 1 + 5 + 5 - 5 - 6$

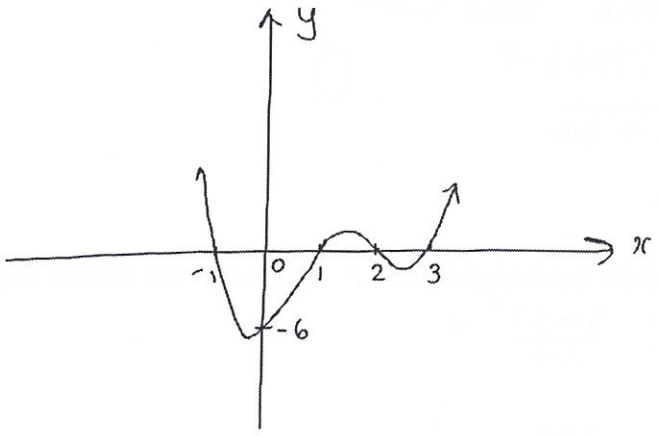
$= 0$

$\therefore (x^2 - 1)$ is a factor

$P(x) = (x^2 - 1)(x^2 - 5x + 6)$ by inspection

$= (x-1)(x+1)(x-3)(x-2)$ (1)

(ii) Hence, sketch the graph of $y = f(x)$ **2**



when $x = \frac{1}{2}$

$P(\frac{1}{2}) = \frac{1}{16} - \frac{5}{8} + \frac{5}{4} + \frac{5}{2} - 6$

$= -2.8125$

$P(-\frac{1}{2}) = \frac{1}{16} + \frac{5}{8} + \frac{5}{4} - \frac{5}{2} - 6$

$= -6.5625$

(1) intercepts

(1) shape and scale

(b) Simplify $\dfrac{\sin^3 x}{\cos x} + \sin x \cos x$ **2**

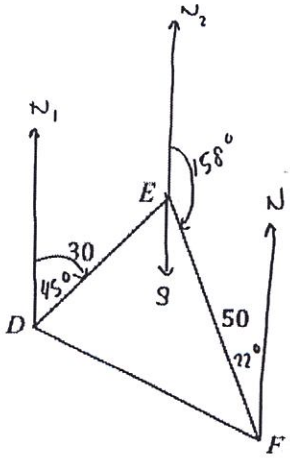
$= \dfrac{\sin^3 x + \sin x \cos^2 x}{\cos x}$

$= \dfrac{\sin x (\sin^2 x + \cos^2 x)}{\cos x}$ (1)

$= \dfrac{\sin x \times 1}{\cos x}$

$= \tan x$ (1)

(c) Three girls are standing in the park at points D, E and F. D is 30 metres from E and E is 50 metres from F. The bearing of E from D is 045°and the bearing of F from E is 158°.



(1)

(i) Show all information given above on the diagram and find the size of angle DEF giving reasons. **2**

$\angle DES = \angle N_1DE = 45°$ (alternate angles $N_1D // N_2E$)

$\angle SEF = 180° - 158°$ (straight angle)

$= 22°$

$\therefore \angle DEF = 67°$ (1)

(ii) Show that the distance of F from D is approximately 47 metres. **1**

$FD^2 = 30^2 + 50^2 - 2 \times 30 \times 50 \cos 67°$

$= 900 + 2500 - 3000 \cos 67°$

$= 2227.806615$ (1)

$FD = \sqrt{2227.807}$

$= 47\text{ m}$

(iii) Hence, or otherwise, find the bearing of D from F. Answer to the nearest degree. **3**

$\frac{\sin \angle DFE}{30} = \frac{\sin 67°}{47}$ (1)

$\sin \angle DFE = \frac{30 \sin 67°}{47}$

$\angle DFE = 36°$ (1)

$\therefore$ Bearing $= 360° - 36° - 22°$

$= 302°$ (1)

(d) Solve $2\sin^2 x - 5\sin x - 3 = 0$ for $0° \leq x \leq 360°$ **3**

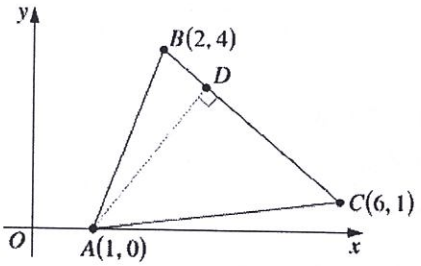
$(2\sin x + 1)(\sin x - 3) = 0$

$\sin x = -\frac{1}{2}$ $\sin x = 3$ (1)

$x = 210°, 330°$ no solution since $-1 \leq \sin x \leq 1$

(1) (1)

(e) The diagram shows points $A(1,0)$, $B(2,4)$ and $C(6,1)$. The point D lies on BC such that $BC \perp AD$.



NOT TO SCALE

(i) Show that the equation of BC is $3x + 4y - 22 = 0$. **2**

$m_{BC} = \frac{4-1}{2-6}$ Eqn is: $y - 4 = -\frac{3}{4}(x-2)$ (1)

$= -\frac{3}{4}$ (1) $4y - 16 = -3x + 6$

$3x + 4y - 22 = 0$

(ii) Find the length of AD **2**

$AD = \frac{|3\times 1 + 4\times 0 - 22|}{\sqrt{3^2+4^2}}$ (1)

$= \frac{19}{5}$ units (1)

(iii) Hence, or otherwise, find the area of $\triangle ABC$. **2**

$BC = \sqrt{(6-2)^2 + (1-4)^2}$ Area $= \frac{1}{2} \times 5 \times \frac{19}{5}$

$= \sqrt{25}$ $= \frac{19}{2}$ square units

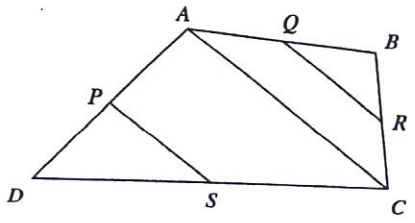
$= 5$ (1) (1)

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Section D (20 Marks)

(a) The sides of a quadrilateral $ABCD$ have midpoints P, Q, R and S, as shown below.



(i) Show that $\triangle DPS$ is similar to $\triangle DAC$. **3**

In $\triangle DPS$ and $\triangle DAC$

$\angle PDS$ is common (1)

$\frac{DP}{DA} = \frac{DS}{DC} = \frac{1}{2}$ (P, S are midpoints of DA and DC) (1)

$\therefore \triangle DPS \,|||\, \triangle DAC$ (2 pairs of sides in proportion and included angle equal) (1)

Other methods possible

(ii) Show that $PS \parallel QR$. **1**

$PS \parallel AC$ (line joining midpts of two sides of triangle is parallel to 3rd side and half its length)

Similarly $QR \parallel AC$ (1)

$\therefore PS \parallel QR$ (both parallel to AC)

(iii) Show that $PQRS$ is a parallelogram. **2**

$PS = \frac{1}{2}AC$ (proven above)

Similarly $QR = \frac{1}{2}AC$

$\therefore PS = QR$ (1)

$\therefore PQRS$ is a parallelogram (one pair of opposite sides equal and parallel) (1)

(b) Solve the equation $3^{2x} = 4^{2-x}$, giving your answer to 3 significant figures. **3**

$2x \ln 3 = (2-x)\ln 4$ (1)

$2x\ln 3 + x\ln 4 = 2\ln 4$

$x(2\ln 3 + \ln 4) = 2\ln 4$

$x = \dfrac{2\ln 4}{2\ln 3 + \ln 4}$ (1)

$= 0.774$ (1)

(c) If $\log_a b = 0.3$ and $\log_a c = 0.4$, find the value of $\log_b\left(\frac{a}{c}\right)$ **3**

$\log_b \frac{a}{c} = \log_b a - \log_b c$

$= \dfrac{\log_a a}{\log_a b} - \dfrac{\log_a c}{\log_a b}$ (1)

$= \dfrac{1}{0.3} - \dfrac{0.4}{0.3}$ (1)

$= \dfrac{0.6}{0.3}$

$= 2$ (1)

(d) Let $f(x) = \sqrt{4x-3}$

(i) Find the domain of $f(x)$. **1**

$4x - 3 \geq 0$

D: $x \geq \frac{3}{4}$ (1)

(ii) Find an expression for the inverse function $f^{-1}(x)$. **2**

$x = \sqrt{4y-3}$

$x^2 = 4y - 3$

$4y = x^2 + 3$

$y = \dfrac{x^2+3}{4}$, $x \geq 0$

(1) (1)

(iii) Find the coordinates of the points where the graphs $y = f(x)$ and $y = x$ intersect. **2**

$\sqrt{4x-3} = x$

$4x - 3 = x^2$

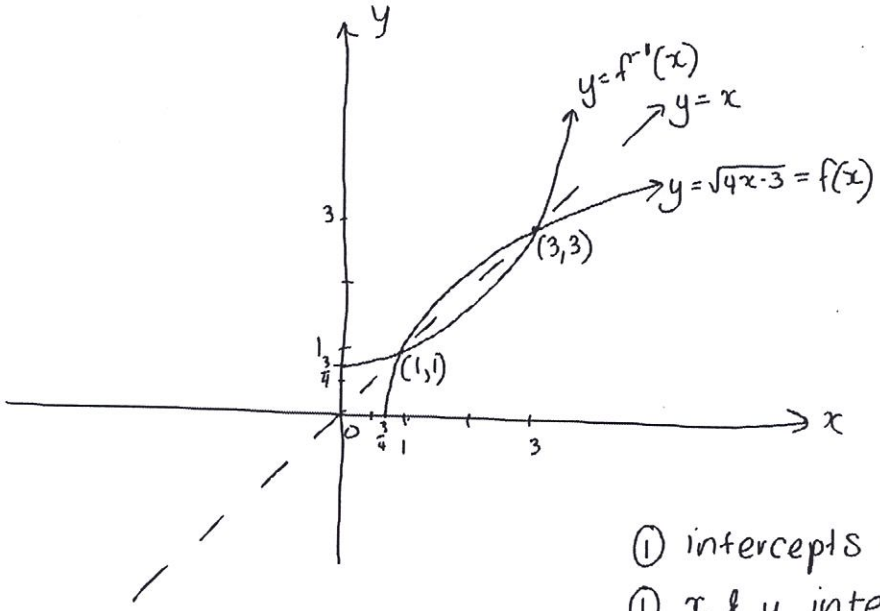
$x^2 - 4x + 3 = 0$ (1)

$(x-3)(x-1) = 0$

$x = 3$ or 1

∴ Graphs intersect at $(3,3)$ and $(1,1)$ (1)

(iv) On the same set of axes, sketch the graphs $y = f(x)$ and $y = f^{-1}(x)$ showing the information found in part (iii). **3**



(1) intercepts on $y = x$

(1) x & y intercepts

(1) shape and scale and labels

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P1

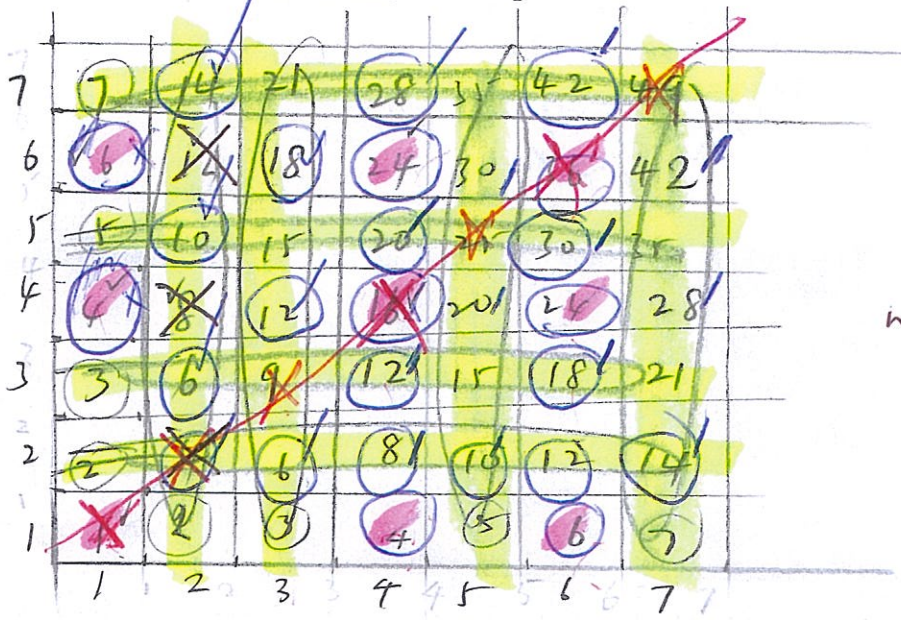
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Class: 10

Section E (20 Marks)

(a) Two cards are drawn at random without replacement from a pack of 7 cards numbered 1, 2, 3, 4, 5, 6, 7, and their product is noted.

(i) Draw a table or dot diagram to represent this situation. **2**

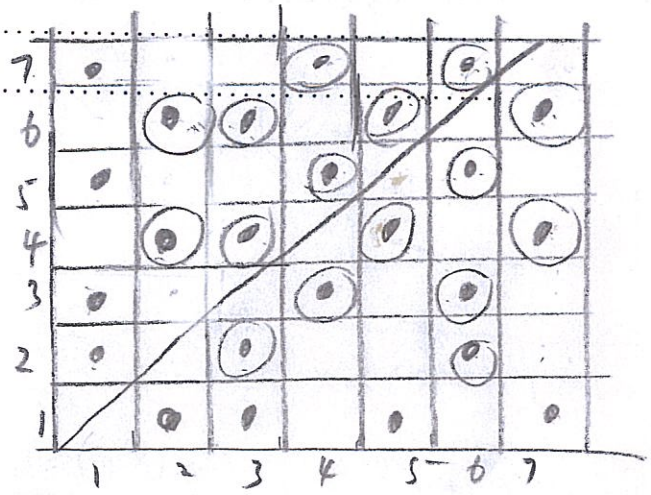


W/o crossing out diagonal only 1 m

(ii) Find the probability that a prime number is included in at least one of the two cards. **1**

$\frac{36}{42} = \frac{6}{7}$

(iii) Find the probability that the product of numbers on the two cards is even given that exactly one of the numbers is prime. **1**

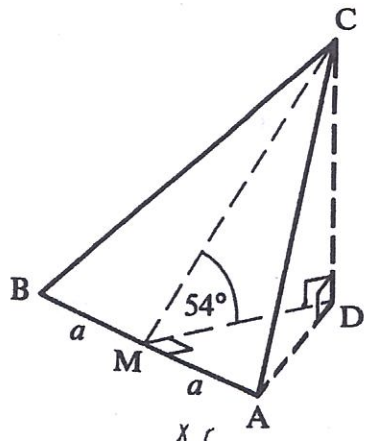


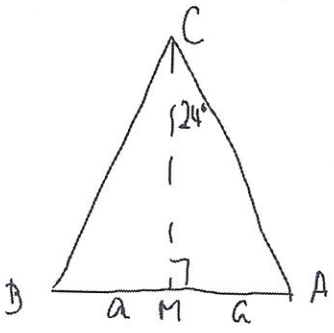
$\frac{18}{24} = \frac{3}{4}$

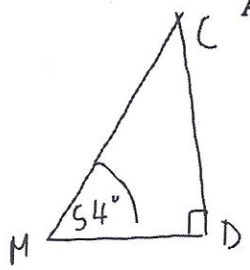
Section E

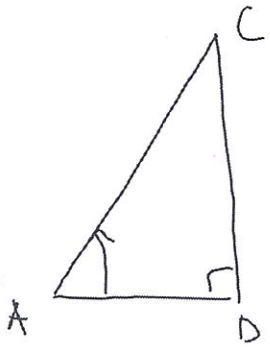
P2

(b) The base AB of an isosceles triangle, ABC, with $BC = AC$, is horizontal. The plane containing the triangle is inclined to the horizontal at 54°. If the angle ACB is 48°, find the angle between AC and the horizontal plane. 4









$MC = a\tan 66°$

$\tan 24° = \frac{a}{CM}$ $\quad \sin 54° = \frac{CD}{CM}$ $\quad \sin\angle DAC = \frac{a\sin 54°}{\tan 24°} \div \frac{a}{\sin 24°}$

$CM = \frac{a}{\tan 24°}$ ① $\quad CD = \frac{a\sin 54°}{\tan 24°}$ ① $\quad = \frac{\sin 54°\sin 24°}{\tan 24°}$

$\sin 24° = \frac{a}{AC}$

$AC = \frac{a}{\sin 24°}$ ①

$\angle DAC = 47°39'$ ① 47,65°

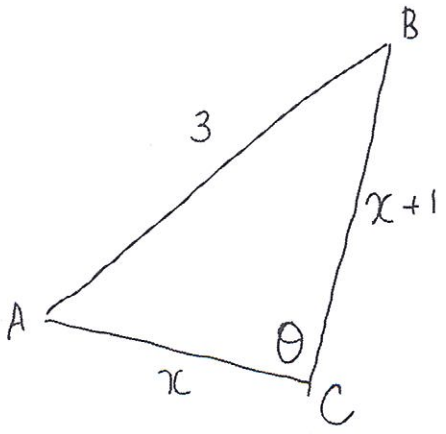
$\sin\theta = 0.7391$

$\frac{\sin 54°\sin 24°}{\cos 24°} \cdot \frac{\cos 24°}{\sin 24°} = \sin 54°$

$\sin\theta = \frac{\sin 54\sin 24\cos 24}{\sin 24}$

(c) A triangle ABC has side lengths $AB = 3$, $BC = x + 1$, $AC = x$ and $\angle ACB = \theta$.

(i) Draw a diagram showing the information above. **1**



Section E

P3

C

(ii) Show that $\cos\theta = 1 - \dfrac{4}{x(x+1)}$. **2**

$\cos\theta = \dfrac{x^2 + (x+1)^2 - 3^2}{2x(x+1)}$ (1)

$= \dfrac{2x^2 + 2x - 8}{2x(x+1)}$

$= \dfrac{x^2 + x - 4}{x(x+1)}$

$= \dfrac{x(x+1)}{x(x+1)} - \dfrac{4}{x(x+1)}$ (1)

$= 1 - \dfrac{4}{x(x+1)}$

(iii) Hence find $\sin^2\theta$ in terms of x. Give your answer in factored form. **3**

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(1 - \dfrac{4}{x(x+1)}\right)^2$ (1)

$= 1 - \left(1 - \dfrac{8}{x(x+1)} + \dfrac{16}{x^2(x+1)^2}\right)$

$= \dfrac{8}{x(x+1)} - \dfrac{16}{x^2(x+1)^2}$ (1)

$= \dfrac{8x(x+1) - 16}{x^2(x+1)^2}$

$= \dfrac{8x^2 + 8x - 16}{x^2(x+1)^2} = \dfrac{8(x+2)(x-1)}{x^2(x+1)^2}$ (1)

$=$

(iv) Show that the area of the triangle is $\sqrt{2(x+2)(x\mp1)}$. **2**

$A = \frac{1}{2}x(x+1)\sin\theta$

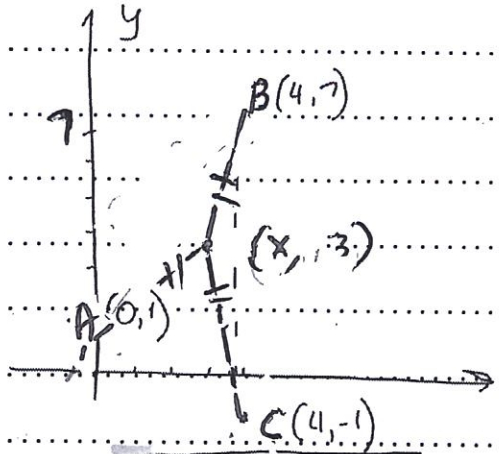
$= \frac{1}{2}x(x+1) \times \sqrt{\dfrac{8(x+2)(x-1)}{x^2(x+1)^2}}$ (1)

$= \frac{1}{2}x(x+1) \times \dfrac{2\sqrt{2(x+2)(x-1)}}{x(x+1)}$ (1)

$= \sqrt{2(x+2)(x-1)}$

No penalty

(d) Find the equation of the circle that goes through the points $A(0,1)$, $B(4,7)$ and $C(4,-1)$. **4**



Perpendicular bisector of BC passes through centre of circle.

ie. $y = 3$

$\therefore$ centre $(x, 3)$

$\sqrt{(x-4)^2 + (3-7)^2} = \sqrt{(x-0)^2 + (3-1)^2}$ = radius

$x^2 - 8x + 16 + 16 = x^2 + 4$ $= \sqrt{\left(\frac{7}{2}\right)^2 + 4}$

$8x = 28$ $= \sqrt{\frac{49+16}{4}}$

$x = 28/8 = 7/2$ $= \sqrt{\frac{65}{4}}$

$\therefore$ centre $(7/2, 3)$

$\therefore$ circle : $(x - 7/2)^2 + (y-3)^2 = \frac{65}{4} = 16.25$

Other methods possible - simultaneous equations

End of Section E

End of paper

Method 2 : 7/10 Yrly Section E

d) Let D(x, y) be the centre

$AD = BD = CD$ $\quad AD^2 = BD^2 = CD^2$

$x^2 + (y-1)^2 = (x-4)^2 + (y-7)^2 = (x-4)^2 + (y+1)^2$ /

$AD^2 = BD^2$ $\quad x^2 + y^2 - 2y + 1 = x^2 - 8x + 16 + y^2 - 14y + 49$

$12y + 8x = 64$

$3y + 2x = 16$ ①

$BD^2 = CD^2$ $\quad (y-7)^2 = (y+1)^2$

$y^2 - 14y + 49 = y^2 + 2y + 1$

$48 = 16y$ $\quad \therefore y = 3$ /

Sub $y = 3$ into ① $\quad 9 + 2x = 16$ $\quad x = 7/2$ /

$AD^2 = (\text{Radius})^2 = (\frac{7}{2})^2 + (3-1)^2$

$= \frac{49}{4} + 4 = \frac{65}{4}$

Eq of circle $(x - 7/2)^2 + (y-3)^2 = \frac{65}{4}$ /

Alternatively ⊥ bisector method

mid AB = (2, 4) $\quad$ mid AC = (2, 0) $\quad$ mid BC = (4, 3)

slope AB = 3/2 $\quad$ slope AC = $-\frac{1}{2}$ $\quad$ slope BC = ∞

$m_{\perp} = -2/3$ $\quad m^{\perp} = 2$ $\quad m^{\perp} = 0$

⊥ bisector through (2, 4) $\quad$ ⊥ bisector from (2, 0) $\quad$ ⊥ bisector through mid BC

$y - 4 = (x-2)\,-2/3$ $\quad y = 2(x-2)$ $\quad y - 3 = 0(x-4)$ $\therefore y = 3$

$3y - 12 = -2x + 4$ /

$2x + 3y = 16$ $\quad y = 2x = 4$

$y = 4 + 2x$ /

$2x + 3(-4 + 2x) = 16$

$2x + -12 + 6x = 16$

$2x + 6x = 28$

$8x = 28$ $\quad 4x = 14$

$x = 7/2$ /

Radius $= \frac{65}{4}$

$(x - \frac{7}{2})^2 + (y-3)^2 = \frac{65}{4}$ /

$= 16.25$