Name:
Maths Class:

YEARLY EXAMINATION

YEAR 10 2017

MATHEMATICS

Time Allowed – 120 minutes
(Plus 5 minutes Reading time)

Question	Section A	Section B	Section C	Section D	Section E	MC	Total
Algebra	Q1-4 /11	Q1-2 /5	Q1 /6	Q1-2 /7	Q1 /3		/32
Probability & Statistics	Q5 /6		Q2 /4		Q2-3 /8		/18
Trigonometry	Q6 /3	Q3 /3	Q3 /4		Q4 /9		/19
Coord Geom & Graphing		Q4 /12	Q4 /2	Q3-4 /9			/23
Geometry			Q5 /4	Q5 /4			/7
Other						/10	/10
Total	/20	/20	/20	/20	/20	/10	/110

INSTRUCTIONS:

- All questions may be attempted.
- Write your **name** and **Maths class** at the top of each page.
- **Write** in **Pen** and draw diagrams in **Pencil.**
- Answers to Multiple choice Questions 1-10 are to be entered onto the answer sheet provided.
- Answers to Questions 11-15 are to be returned in separate bundles.
- Department of Education approved calculators and templates are permitted.
- Show all necessary working.
- Marks may not be awarded for untidy or carelessly arranged work.
- No grid paper is to be used unless provided with the examination paper.

Multiple Choice – Answer on the Multiple-Choice Answer Sheet Provided

Question One: $3(x+2)^2 + 7(x+2) - 4$ is equivalent to:

a) $3x^2 + 7x - 4$

b) $3x^2 + 7x - 2$

c) $3x^2 - 5x - 6$

d) $3x^2 + 19x + 22$

Question Two: A set of 10 scores are recorded. The mean is 20, median is 25 and the standard deviation is 3. A new score of 23 is added. Which the following statement is NOT true?

a) The mean will increase

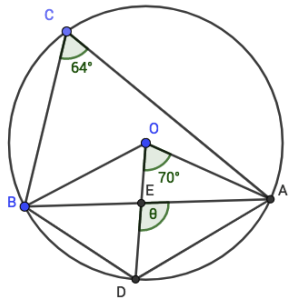
b) The new median will either stay the same or decrease

c) The standard deviation will stay the same

d) None of the above

Question Three: What is the value of $\theta°$?

NOT TO SCALE



a) $96°$

b) $102°$

c) $106°$

d) $112°$

Question Four: What is the value of $cos210°$?

a) $\frac{\sqrt{3}}{2}$

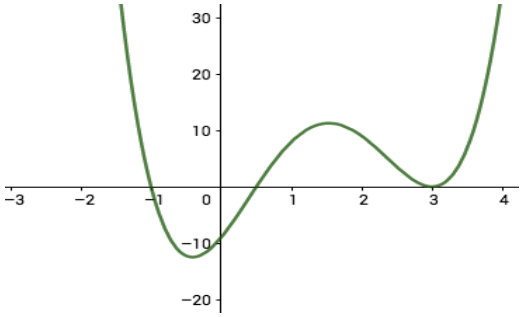
b) $-\frac{\sqrt{3}}{2}$

c) $\frac{1}{2}$

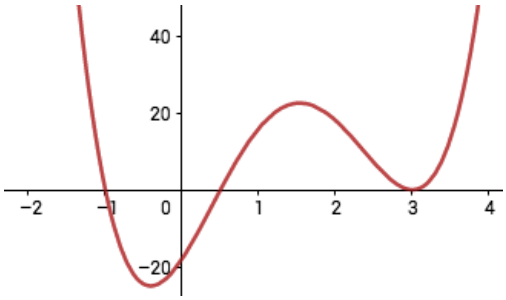
d) $-\frac{1}{2}$

Question Five: Which if the following is the graph of $y = 2(x+1)(x-3)^2(2x-1)$?

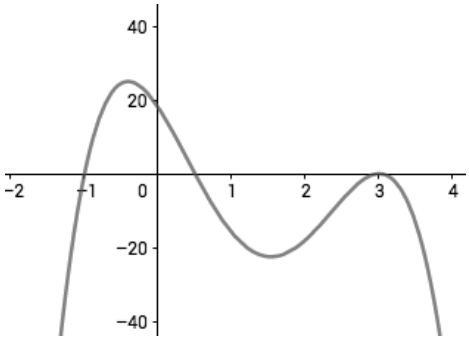
a)



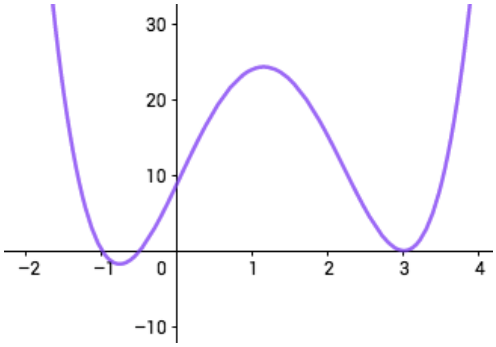
b)



c)



d)



Question Six: The equation of a circle with radius 6 and centre (2, -5) is given by:

a) $x^2 + y^2 - 4x + 10y + 23 = 0$

b) $x^2 + y^2 - 4x + 10y - 7 = 0$

c) $x^2 + y^2 + 4x - 10y + 23 = 0$

d) $x^2 + y^2 + 4x - 10y - 7 = 0$

Question Seven: Sue bought 2 raffle tickets for a fundraiser. 3 tickets are drawn out of 100 for 3 different prizes. What is the probability of Sue winning exactly 1 prize?

a) 5.94%

b) 1.96%

c) 5.88%

d) 94.06%

Question Eight: What is the remainder when $x^4 - 3x^3 + 2x^2 - x + 7$ is divided by $2x + 3$?

a) $4\frac{15}{16}$

b) $6\frac{43}{81}$

c) $9\frac{52}{81}$

d) $28\frac{3}{16}$

Question Nine: If you increase the dimensions of a cylinder by 40%. How much will its volume increase by?

a) 274%

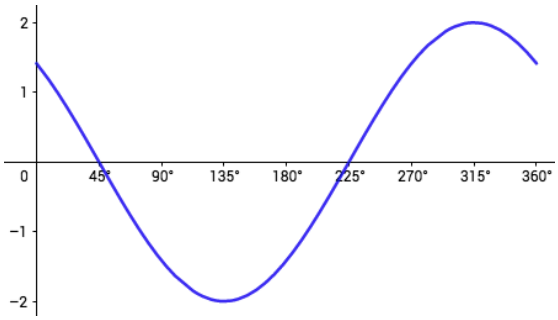
b) 196%

c) 96%

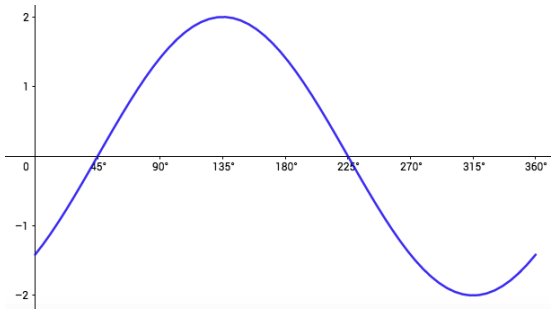
d) 174%

Question Ten: The graph of $y = 2\sin(x + 45)$ for $0° \leq x \leq 360°$ is:

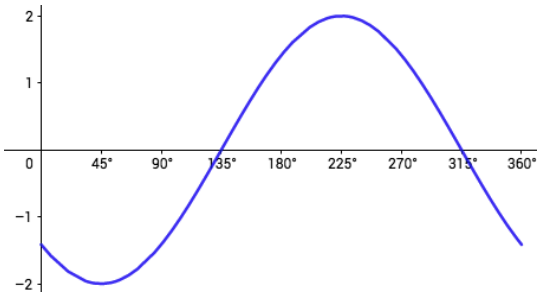
a)



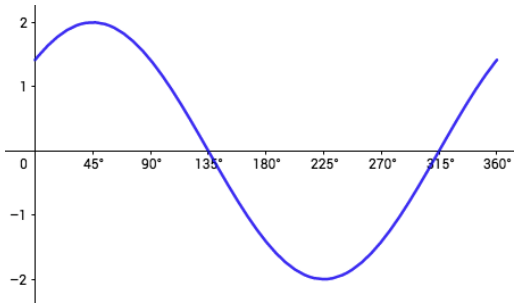
b)



c)



d)



END OF MULTIPLE CHOICE

Section A: (20 Marks) Answer on your own writing paper

1. Expand and simplify:

   a) $(3x - 4y)(2x + y)$ 1

   b) $(2a + 3)^3$ 2

2. Factorise Fully:

   a) $64 + 27y^3$ 1

   b) $4ax - 5yb - 4bx + 5ay$ 2

3. Rewrite $\frac{2-\sqrt{3}}{1+\sqrt{3}}$ in the form of $a + c\sqrt{b}$ where a, b and c are real numbers. 2

4. Is the function $f(x) = -x^2 + 2(4 - k)x + 4$ positive definite, negative definite or neither? Explain your answer mathematically. 3

5. A group of 8 students took a test and got the following results:

   4, 5, 7, 8, 9, 10, 17, 20

   a) What is the mean of the scores? 1

   b) What is the standard deviation of the score? 1

   c) Draw a box and whisker plot for the set of data above. 3

   d) What score needs to be added so that the mean becomes 9? 1

6. A triangle has sides 4cm, 10cm and 8cm respectively. Find the area of the triangle in cm^2 correct to 3 significant figures. 3

PLEASE TURN OVER

Section B: (20 Marks) Start a new sheet of paper

1. Simplify:

   a) $\log_9 27$ 1

   b) $\log_6 16 + \log_6 13.5$ 1

2. Solve for x:

$$2\log_2(x-3) = 4$$ 3

3. Show that:

$$\frac{cot^2x\ +\ 2}{cosec^2x - 1} = tan^2x + sec^2x$$ 3

4. a) On a number plane, clearly label the points A(2, 1) and B(6,-1). (You may add to this diagram as the question progresses if necessary). 1

   b) Show that M, the midpoint of AB has coordinates (4, 0). 1

   c) Show that l_1, the perpendicular bisector of AB is given by $2x - y - 8 = 0$ 2

   d) Find the coordinates of the point C that is equidistant from A and B, such that it is above the interval AB and has perpendicular distance $4\sqrt{5}$ units from it. 3

   e) If AC was extended. What angle will it make with the $x - axis$ in the positive direction? Give your answer to the nearest minute. 2

   f) Find the coordinates of D such that ACBD is a kite and $\angle CAD = 90°$. 3

PLEASE TURN OVER

Section C: (20 Marks) Start a new sheet of paper

1. a) Rewrite $f(x) = 2x^2 + 3x - 9$ in the form of $f(x) = A(x)Q(x) + R(x)$ where $A(x) = x - 1$ 2

   b) Hence write down the equation of all the asymptotes of $y = \frac{2x^2+3x-9}{x-1}$ 1

   c) Sketch the graph of $y = \frac{2x^2+3x-9}{x-1}$ labelling all important features and asymptotes. 3

2. 60 students were asked to pick from a list of sporting brands they would purchase: Nike, Adidas or Puma. They could choose 1, 2 or all 3 options. Out of the students, Nike had 32 votes, Adidas had 29 votes and Puma had 20 votes. There were 11 students who voted for both Nike and Adidas, 9 students voted for both Adidas and Puma, 8 students voted for all three brands.

   a) Draw a Venn Diagram to illustrate all the information given above. 3

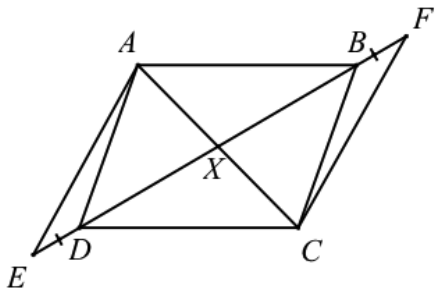
   b) If 2 students were selected at random, what is the probability that both had selected exactly 2 brands? 1

3. Solve for θ to the nearest degree: 4

   $5cos^2\theta sin\theta - sin^3\theta = cos\theta sin\theta + sin\theta$ for $0° \leq \theta \leq 360°$

4. Sketch $y = x + \log_2 x$ by addition of ordinates. 2

5. ABCD is a parallelogram. Diagonal BD is produced so that $BF = ED$.



   Prove that $\Delta AED \equiv \Delta CFB$ 4

PLEASE TURN OVER

Section D: (20 Marks) Start a new sheet of paper

1. Factorise $f(x) = 12x^3 - 4x^2 - 3x + 1 = 0$ to its linear factors given the sum of two of its roots is 0. 3

2. The time T (in hours) taken for W workers to be build N number of cupboards are correlated.

   a) Write an equation to illustrate the direct and indirect variations of the three quantities using T as the subject. 1

   b) If it takes 18 hours for 6 workers to build 3 cupboards, how many complete cupboards can 10 workers build in 27 hours? 2

   c) Using T as the $y - axis$ and N as the $x - axis$. Draw a graph illustrating the relationship between T and N for a project that involves 12 workers. 1

3. Find the coordinates of a point $C(a, b)$ that divides $A(-4, 8)$ and $B(1, 3)$ externally in the ratio of $1 : 3$. 2

4. For the function $f(x) = \sqrt{x^2 - 9}$:

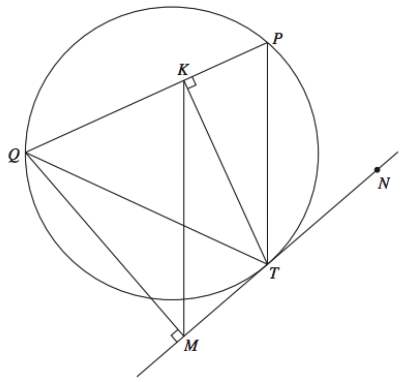
   a) Find the natural domain and range of $f(x)$. 1

   b) Write down the largest positive domain such that $f(x)$ has an inverse. 1

   c) Let $f^{-1}(x)$ be defined as the inverse of $f(x)$ with the largest positive domain. Sketch $y = f^{-1}(x)$ and the corresponding branch of $y = f(x)$ on the same plane. 2

   d) Find the expression for $f^{-1}(x)$. 3

5. The points P, Q and T lie on a circle. The line MN is tangent to the circle at T with M chosen so that QM is perpendicular to MN. The point K on PQ is chosen so that TK is perpendicular to PQ as shown in the diagram.



   Copy the diagram into your answer sheet and prove that $KM \parallel PT$. 4

PLEASE TURN OVER

Section E: (20 Marks) Start a new sheet of paper

1. When a polynomial $P(x)$ is divided by $(x-p)$ the remainder is p^3, when $P(x)$ is divided by $(x-q)$ the remainder is q^3. What is the remainder when $P(x)$ is divided by $(x^2-(p+q)x+pq)$? $(p \neq q)$. 3

2. Aimee is choosing her subjects for next year. She needs to select 2 more subjects to complete her form. Aimee decides to allocate a card to each subject starting from French, Modern History, Biology, Japanese, PE, Food Technology, and Music. She puts them into a box and draws 2 out at random.

   a) Draw a diagram to illustrate the sample space. 1

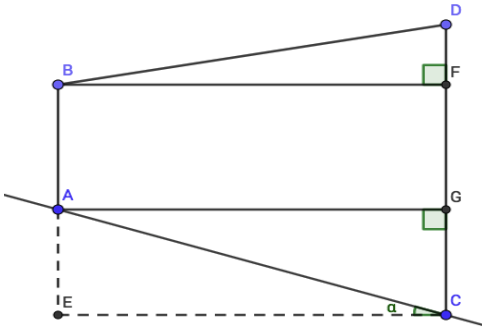
   b) What is the probability that Aimee will be doing at least one of the language subjects? 1

   c) After Aimee takes both cards out, she opens one of them and sees that it is Food Technology. What is the probability that the other card is one of the languages? 2

3. A bag contains only red and blue marbles. The probability of selecting two red marbles in a row without replacement is $\frac{1}{3}$. The probability of selecting two red marbles followed by a blue marble is $\frac{1}{6}$. How many red marbles and blue marbles are there? 4

4. Sketch the graph $y = secx$ for $-180° \leq x \leq 180°$ 3

5. AB and CD are two vertical towers of different height standing d metres apart (perpendicular distance) on a slope as shown on the diagram.



From A and B, the angles of elevation of D are $52°$ and $35°$ respectively. From B, the angle of depression of C is $50°$. α is the angle the slope makes with the horizontal.

   a) Draw the diagram into your answers sheet marking in all the relevant angles given. 1

   b) Show that $DF = d \times tan35°$. 1

   c) Hence or otherwise, find the value of α, to the nearest minute. 4

END OF TEST

Student Number _____

Multiple Choice

1. D
2. C
3. A
4. B
5. B
6. B
7. C
8. ~~B~~ D
9. D
10. D

Section A.

1 a) $(3x-4y)(2x+y)$

$= 6x^2 + 3xy - 8xy - 4y^2$

$= 6x^2 - 5xy - 4y^2$.

① for correct answer

b) $(2a+3)^3 = (4a^2 + 12a + 9)(2a+3)$ 1 Correct expansion, 1 for one error

$= 8a^3 + 12a^2 + 24a^2 + 36a + 18a + 27.$

$= 8a^3 + 36a^2 + 54a + 27.$ ② Correct answer

or $(2a)^3 + 3(2a)^2 \cdot 3 + 3(2a)3^2 + 3^3$ 1 Correct expansion, 1 for one error

$= 8a^3 + 36a^2 + 54a + 27.$ ② Correct answer

2. a) $4^3 + (3y)^3 = (4+3y)(16 - 12y + 9y^2)$ ① Correct answer

b) $4ax - 4bx - 5yb + 5ay.$ 1 Correct rearrangement

$4x(a-b) + 5ay - 5yb$

$4x(a-b) + 5y(a-b)$

$(a-b)(4x+5y).$ ② Correct answer

3. $\frac{2-\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{2-2\sqrt{3}-\sqrt{3}+3}{1-3}$ 1 multiplying by conjugate

$= \frac{5-3\sqrt{3}}{-2}.$

$= -\frac{5}{2} + \frac{3}{2}\sqrt{3}.$ ② correct answer

E 4 Function is - positive definite if $a > 0$, $\Delta < 0$

- negative definite if $a < 0$, $\Delta < 0$
- neither positive definite or negative definite if $\Delta > 0$

3 Methods

1. For $f(x)$, $\Delta = 4(4-k)^2 - 4(-1)4$

$= 4(4-k)^2 + 16$ 1 for Δ

Now $(4-k)^2 > 0$ for all real values of k

$\therefore \Delta = 4(4-k)^2 + 16 > 0$ +1 for $\Delta > 0$

$\therefore$ Function is neither positive definite or negative definite +1 for conclusio

2 For $f(x)$, $\Delta = 4(4-k)^2 - 4(-1)4$ 1 for Δ

$= 4(4-k)^2 + 16$

The coefficient of $k^2 = 4 > 0$

$\therefore$ The graph of $4(4-k)^2 + 16$ is concave up and has a minimum value of 16 when $k = 4$

$\therefore \Delta > 0$ for all real values of k } +1

$\therefore$ Function is neither positive definite or negative definite } +1

OR 3. $\Delta_1 = 4(4-k)^2 + 16$ 1 for Δ_1

$= 80 - 32k + 4k^2$

The coefficient of $k^2 = 1 > 0$

For $80 - 32k + 4k^2$

$\Delta_2 = 32^2 - 4 \times 80 \times 4$

$= 1024 - 1280$

< 0

$\therefore$ Since $\Delta_2 < 0$ and the coefficient of $k^2 = 4 > 0$,

$\Delta = 80 - 32k + 4k^2 > 0$ ... +1 for proving

$\therefore$ The function is neither positive definite or negative definite. +1 for conclusion

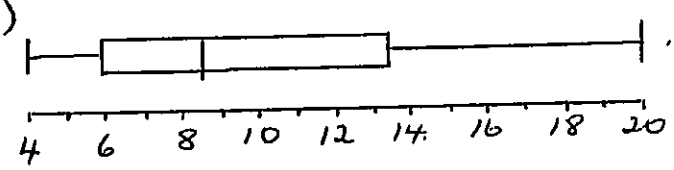
5 a) mean = 10

b) standard deviation = 5.29

① Correct mean

① Correct standard deviation

c)

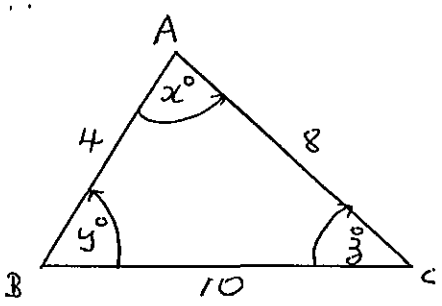


max (3) { 1 Correct whiskers, 1 Correct box, 1 Correct median

d) If there are 9 scores the total would need to be 81 for an averge of 9

$\therefore$ The extra score would be 1. ① for correct answer

6



• To find the value of x

$$\text{Cos } A = \frac{b^2+c^2-a^2}{2bc}$$

$$\therefore \text{Cos } x^\circ = \frac{4^2+8^2-10^2}{2\times4\times8}$$

$$= \frac{16+64-100}{64}$$

$$= \frac{80-100}{64}$$

$$= -\frac{20}{64}$$

$$\therefore x = \text{Cos}^{-1}\left(-\frac{20}{64}\right)$$

$$\approx 108^\circ 13'$$

1 correct use of cosine rule

+1 for correct angle

$$\therefore \text{Area} = \frac{1}{2}bc \sin A$$

$$= \frac{1}{2}\times4\times8\times\sin x^\circ$$

$$= 15.2 \text{ cm}^2.$$

(3) for correct answer

• To find the value of y

$$\text{Cos } y = \frac{4^2+10^2-8^2}{2\times4\times10}$$

$$= \frac{116-64}{80}$$

$$= \frac{52}{80} = \frac{13}{20}$$

$$y = \text{Cos}^{-1}\left(\frac{52}{80}\right) = \text{Cos}^{-1}\left(\frac{13}{20}\right)$$

$$\approx 49^\circ 28'$$

$$\therefore \text{Area} = \frac{1}{2}\times4\times10\times\sin y$$

$$= 15.2 \text{ cm}^2.$$

• To find the value of z

$$\text{Cos } z = \frac{8^2+10^2-4^2}{2\times8\times10}$$

$$= \frac{164-16}{160}$$

$$= \frac{148}{160} = \frac{37}{40}$$

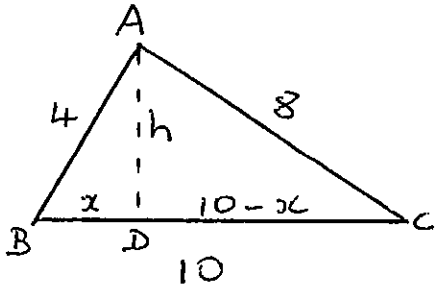
$$z = \text{Cos}^{-1}\left(\frac{37}{40}\right)$$

$$\approx 22^\circ 20'$$

$$\therefore \text{Area} = \frac{1}{2}\times8\times10\times\sin z$$

$$= 15.2 \text{ cm}^2.$$

ALTERNATE SOLUTION FOR Q6



In ΔABD, $x^2 + h^2 = 4^2 \Rightarrow x^2 + h^2 = 16$ ..... ①

In ΔACD $(10-x)^2 + h^2 = 64 \Rightarrow 100 - 20x + x^2 + h^2 = 64$ ... ②

② − ①

$$100 - 20x = 48$$
$$-20x = -52$$
$$x = \frac{52}{20}$$
$$= \frac{13}{5}$$

1 for correct x

Substitute $x = \frac{13}{5}$ into ①

$$\therefore \left(\frac{13}{5}\right)^2 + h^2 = 16$$
$$\frac{169}{25} + h^2 = 16$$
$$169 + 25h^2 = 400$$
$$25h^2 = 231$$
$$h^2 = 9.24$$
$$h \approx 3.0397$$

+1 for h.

$$\text{Area} = \frac{1}{2} \times 10 \times 3.0397$$
$$= 15.2 \text{ cm}^2 \left(\begin{array}{c}\text{3 significant}\\ \text{figures}\end{array}\right)$$

+1 for correct answer.

YR 10

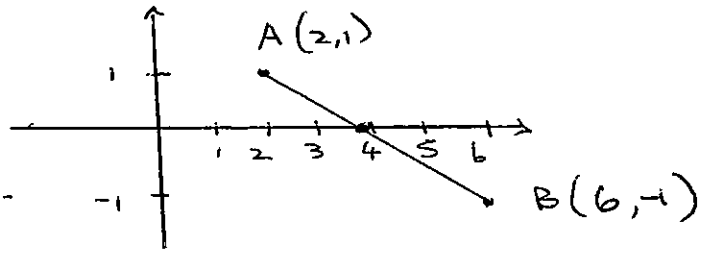
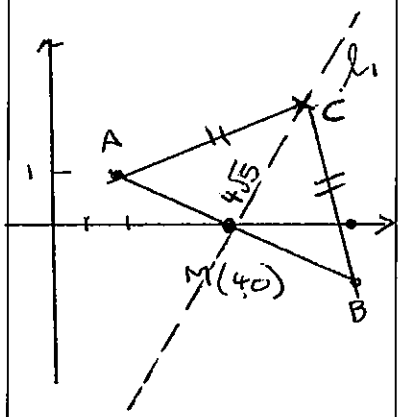
MATHEMATICS: Question..B...

Suggested Solutions	Marks	Marker's Comments
Question 1		
(a) $\log_9 27 = \frac{\log_3 27}{\log_3 9}$ (change of base) $= \frac{\log_3 3^3}{\log_3 3^2} = \frac{3\log_3 3}{2\log_3 3} = \frac{3}{2}$ ✓	1	
(b) $\log_6 16 + \log_6 13.5 = \log_6 (16 \times 13.5)$ $= \log_6 (216)$ $= 3$ ✓	1	
Question 2		
$2\log_2(x-3) = 4 \quad (x>3)$ $\log_2(x-3)^2 = \log_2 16$ $(x-3)^2 = 16$ $x-3 = \pm 4$ $x = 3 \pm 4$ $x = 7$ or -1 Since $x>3$, the only solution is $x=7$ OR Simply $\log_2(x-3)^2 = 4$ change it to base index form $2^4 = (x-3)^2$ solve as above	3	✓ converting 4 as $\log_2 16$ and simplifying LHS. ✓ $(x-3)^2 = 16$ and solving to get $x = 7$ or -1 ✓ rejecting $x = -1$

MATHEMATICS: Question 3

Suggested Solutions	Marks	Marker's Comments
Question 3	3	Lots of alternate methods
$\frac{\cot^2 x + 2}{\operatorname{cosec}^2 x - 1} = \tan^2 x + \sec^2 x$		- One had to show all the steps to get to the answer and get full marks
LHS		• $\cot^2 x = \operatorname{cosec}^2 - 1$
$\frac{\operatorname{cosec}^2 x - 1 + 2}{\operatorname{cosec}^2 x - 1}$ Since $\cot^2 x = \operatorname{cosec}^2 x - 1$		$\cos^2 x = 1 - \sin^2 x$
$\frac{\operatorname{cosec}^2 x + 1}{\operatorname{cosec}^2 x - 1}$		$\frac{1}{\sin^2 x} = \operatorname{cosec}^2 x$
$\frac{\frac{1}{\sin^2 x} + 1}{\frac{1}{\sin^2 x} - 1} = \frac{1 + \sin^2 x}{\sin^2 x}$		$\frac{1}{\cos^2 x} = \sec^2 x$
$\frac{1 - \sin^2 x}{\sin^2 x}$		$\cot^2 x = \frac{1}{\tan^2 x}$
$= \frac{1 + \sin^2 x}{1 - \sin^2 x}$		$\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$
Since $1 - \sin^2 x = \cos^2 x$		etc
$= \frac{1 + \sin^2 x}{\cos^2 x}$		
$= \frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$		
$= \sec^2 x + \tan^2 x$		
$=$ RHS		

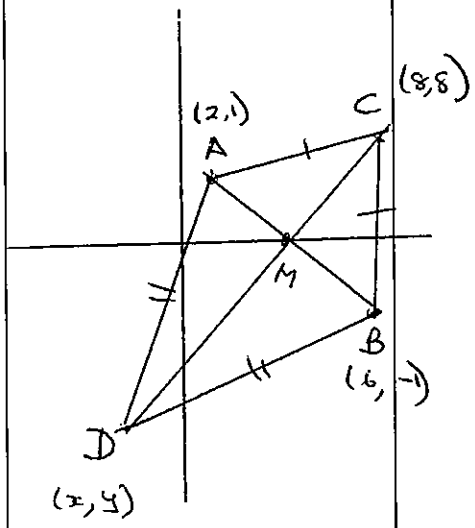
MATHEMATICS: Question.3.....

Suggested Solutions	Marks	Marker's Comments
Question 4 4 (a)  	1	Most plotted correctly Both y and x axes scale is important.
(b) Midpoint of AB $\left(\frac{2+6}{2}, \frac{1+(-1)}{2}\right) = \left(\frac{8}{2}, \frac{0}{2}\right)$ ✓ $= (4,0)$	1	Show question so marks were given for working
* (c) $M_{AB} = \frac{-1-1}{6-2} = \frac{-2}{4} = -\frac{1}{2}$ ∴ gradient of perpendicular line is 2 ✓ ∴ Equation of line l_1 = $y-y_1 = m(x-x_1)$ $y-0 = 2(x-4)$ $y = 2x-8$ or $2x-y-8=0$ ✓	2.	Show question again. 1 mark for showing gradient is 2 1 mark for getting the equation
* (d) Point C equidistant from A and B C must lie on l_1 $l_1 = 2x-y-8=0$ $y = 2x-8$ —— eq ① Method 1 Intersection of Circle with centre (4,0) and line l_1. $(x-4)^2 + (y-0)^2 = (4\sqrt{5})^2$ or $(\sqrt{80})^2$ $(x-4)^2 + y^2 = 80$ —— eq ② ✓ Sub eq ① into eq ② $(x-4)^2 + (2x-8)^2 = 80$ $(x^2-8x+16) + (4x^2-32x+64) = 8$ $5x^2 - 40x = 0$ $5x(x-8) = 0$		

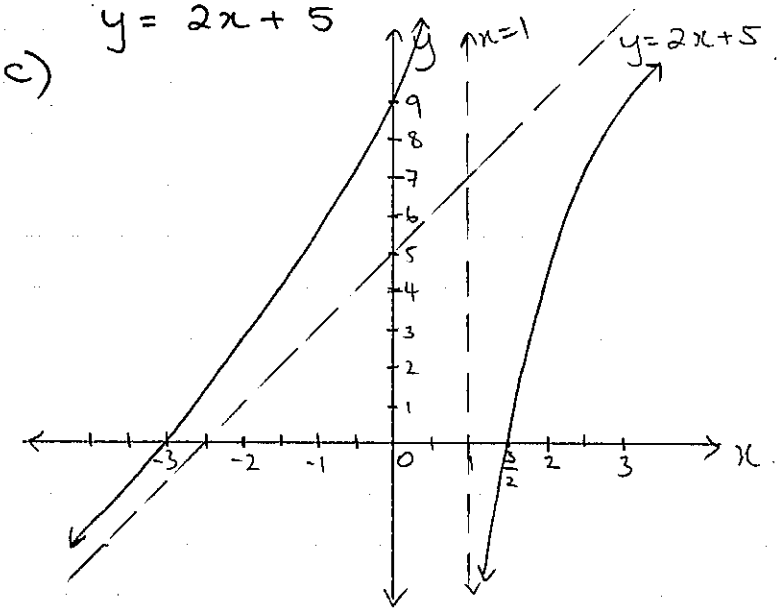
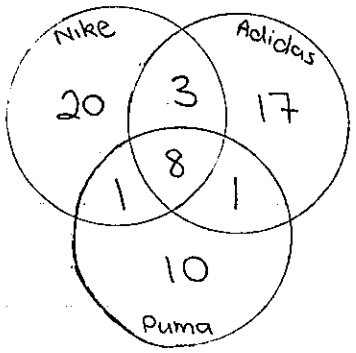
MATHEMATICS: Question……..

Suggested Solutions	Marks	Marker's Comments
$x = 0$ or $x = 8$ Since C lies above AB accept ✓ $x = 8$ only when $x = 8 \Rightarrow y = 2x - 8$ $= 2(8) - 8$ $= 8$ ✓ $\therefore$ Point C is $(8, 8)$ Method 2 Perpendicular distance from AB.	3	* 1 mark for using correct perpendicular distance formula equal $4\sqrt{5}$ * 1 mark for working * 1 mark for correct equation for getting coordinates
$M_{AB} = \frac{-1-1}{6-2} = -\frac{1}{2}$. $y - 1 = -\frac{1}{2}(x-2)$ $y - 1 = -\frac{x}{2} + 1$ $2y - 2 = -x + 2$ $2y + x - 4 = 0$ $d = \left\lvert \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right\rvert = 4\sqrt{5}$ $\left\lvert \frac{x + 2y - 4}{\sqrt{4^2+1^2}} \right\rvert = 4\sqrt{5}$ $\left\lvert \frac{x+2y-4}{\sqrt{5}} \right\rvert = 4\sqrt{5}$. $x + 2y - 4 = 20$ or $x + 2y - 4 = -20$. — eq (1) $y = 2x - 8$ → eq (2) $x + 2(2x-8) - 4 = 20$ $x + 4x - 16 - 4 = 20$ $x + 4x - 20 = 20$ $5x = 40$ $\therefore x = 8$ $y = 8$ Only accept $(8, 8)$.		
or $x + 2(2x-8) - 4 = -20$ $x + 4x - 16 - 4 = -20$ $5x = 0$ $x = 0$ $y = 8$		

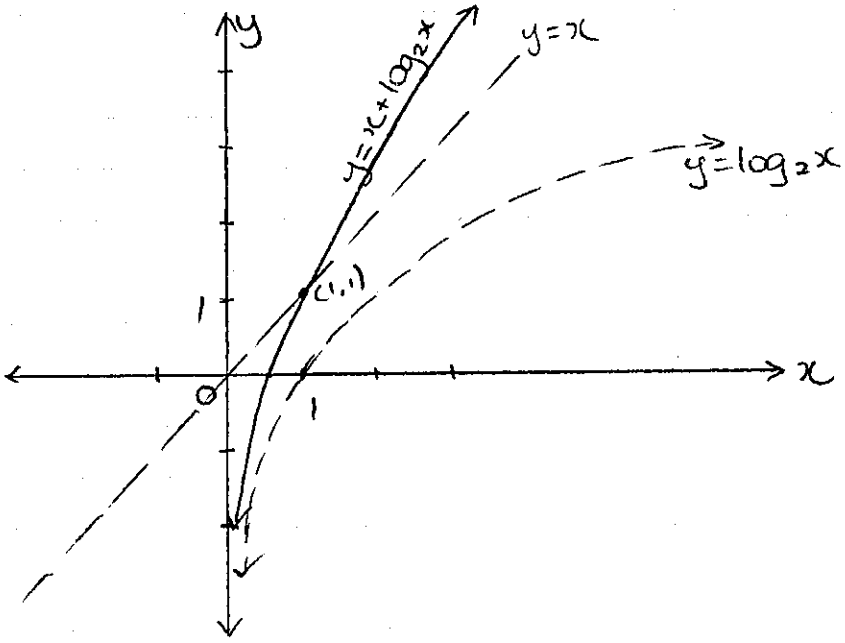
MATHEMATICS: Question 3....

Suggested Solutions	Marks	Marker's Comments
* (e) $A(2,1)$ $C=(8,8)$ $M_{AC} = \frac{8-1}{8-2} = \frac{7}{6}$ ✓ $\tan\theta = \frac{7}{6}$ $\theta = \tan^{-1}\left(\frac{7}{6}\right)$ $\theta = 49^\circ 24'$ ✓	2	1 mark for showing gradient of AC 1 mark for finding the angle CFE only if you show working
* (f) Coordinates of D let D have coordinates (x,y) D also lies on l_1. $l_1 = 2x-y-8=0$ —— eq ① we also know $\angle CAD = 90$ $\therefore AC \perp AD$ From (e) we know $M_{AC} = \frac{7}{6}$ $\therefore M_{AD} = -\frac{6}{7} = \frac{y-1}{x-2}$ ✓ $-6x+12 = 7y-7$ $7y+6x-19=0$ — eq ② Solve simultaneously eq ① ×3 ⇒ $6x-3y-24=0$ eq ② $6x+7y-19=0$ ✓ ② − ① $-10y-5=0$ $-10y=5$ $y=-\frac{1}{2}$. To get x sub in eq ① $2x-\left(\frac{-1}{2}\right)-8=0$ $2x+\frac{1}{2}-8=0$ $2x-7\frac{1}{2}=0$ $2x=\frac{15}{2}$ $\therefore x=\frac{15}{4}$ ✓ ∴ Coordinates of D is $\left(\frac{15}{4}, -\frac{1}{2}\right)$		 Method 2 AD = DB and solve simultaneously

MATHEMATICS: Question C

Suggested Solutions	Marks	Marker's Comments
1. a) $x-1 \overline{)2x^2+3x-9}$ $\quad 2x+5$ $-(2x^2-2x)$ $5x-9$ $-(5x-5)$ -4	1	
$\therefore f(x) = (x-1)(2x+5) - 4$	1	
b) $x = 1$ $y = 2x + 5$	1	
c) 	1	x & y intercepts
	1	asymptotes
	1	correct graph and scale!!!
*2 a) 	3	
b) $\frac{5}{60} \times \frac{4}{59} = \frac{1}{177}$	1	

MATHEMATICS: Question C

Suggested Solutions	Marks	Marker's Comments
3. $5\cos^2\theta\sin\theta - \sin^3\theta = \cos\theta\sin\theta + \sin\theta$ $5\cos^2\theta\sin\theta - \sin^3\theta - \cos\theta\sin\theta - \sin\theta = 0$ $\sin\theta(5\cos^2\theta - \sin^2\theta - \cos\theta - 1) = 0$ $\sin\theta(5\cos^2\theta - (1-\cos^2\theta) - \cos\theta - 1) = 0$	1	
$\sin\theta(5\cos^2\theta + \cos^2\theta - \cos\theta - 2) = 0$ $\sin\theta(6\cos^2\theta - \cos\theta - 2) = 0$ $\sin\theta(3\cos\theta - 2)(2\cos\theta + 1) = 0$	1	
$\therefore \sin\theta = 0, \cos\theta = \frac{2}{3}, \cos\theta = -\frac{1}{2}$	1	
$\therefore \theta = 0, 180, 360, 48°, 312°, 120°, 240°$	1	
*4  	1 1	must have $y=x$ & $y=\log_2 x$ correct graph.
5. AD//BC (opposite sides of parallelogram are parallel)	1	
$\angle ADB = \angle CBD$ (alternate angles, AD//BC) $\angle ADE = 180 - \angle ADB$ (angle sum of straight angle) similarly, $\angle FBC = 180 - \angle CBD$ $\therefore \angle FBC = 180 - \angle ADB$ $= \angle ADE$	1	
In $\triangle AED$ & $\triangle CFB$, AD = CB (opposite sides of parallelogram) ED = FB (given)	1	

MATHEMATICS: Question C

Suggested Solutions	Marks	Marker's Comments
$\angle FBC = \angle ADE$ (proven above) $\therefore \triangle AED \equiv \triangle CFB$ (SAS)	1	

YR 10 Yearly Exam 2017

Section D

*1. $f(x) = 12x^3 - 4x^2 - 3x + 1$

If two roots sum to 0 then let the roots be $\alpha, -\alpha, \beta$.

sum of roots $= \alpha + (-\alpha) + \beta = \frac{4}{12}$

$\therefore \beta = \frac{1}{3}$

product of roots $= -\alpha^2\beta = -\frac{1}{12}$

$-\alpha^2 \times \frac{1}{3} = -\frac{1}{12}$

$\alpha^2 = \frac{1}{4}$

$\alpha = \frac{1}{2}, -\alpha = -\frac{1}{2}$

$\therefore f(x) = 12(x - \frac{1}{2})(x + \frac{1}{2})(x - \frac{1}{3})$

$= (2x - 1)(2x + 1)(3x - 1)$

1 - α, β values

1 - writing $f(x)$ as linear factors

1 - Correct factorisation of $f(x)$ including 12.

*2. a) $T = \frac{kN}{W}$

1 - must be an equation: $T \propto \frac{N}{W}$ is not sufficient.

b) $18 = \frac{3K}{6}$

$= \frac{K}{2}$

$\therefore K = 36$

1 - Correct k based on your a).

$27 = \frac{36N}{10}$

$270 = 36N$

$N = 270 \div 36$

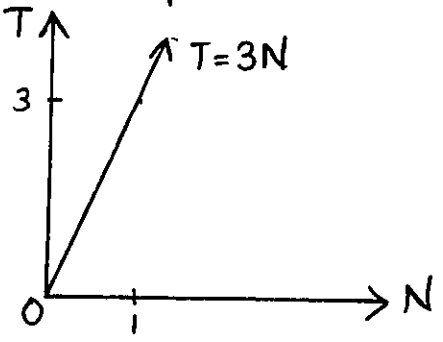
$= 7.5$

$\therefore$ 10 workers can complete 7 cupboards in 27 hours.

1 - number of complete cupboards.

c) $T = \frac{36N}{12}$

$T = 3N$



1 - Correct graph based on your equation including one correct point shown. Graph must not continue for -ve values.

3. A(-4,8) B(1,3)

$-1 : 3$

$a = \frac{-1-12}{-1+3}$ $\quad b = \frac{-3+24}{-1+3}$

$= \frac{-13}{2}$ $\quad = \frac{21}{2}$

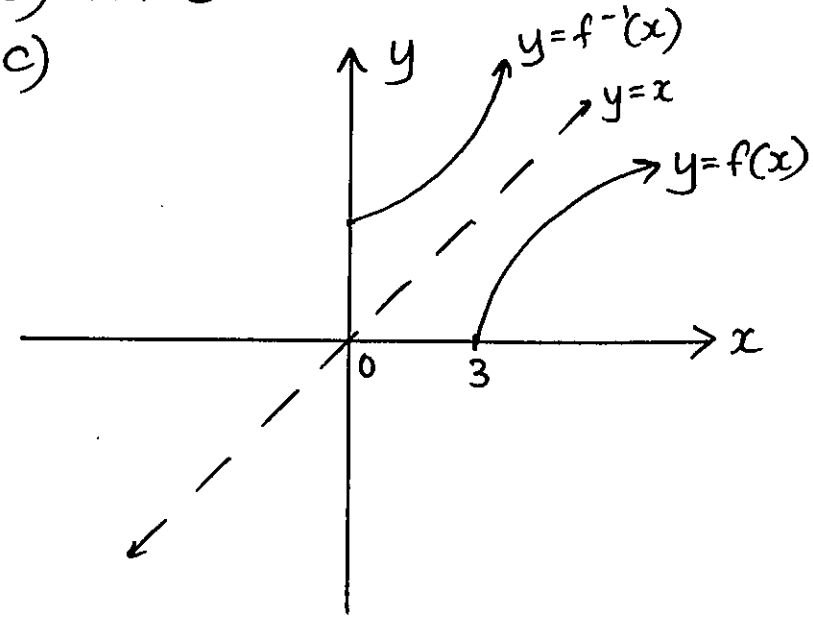
$\therefore$ C is $(\frac{-13}{2}, \frac{21}{2})$

1 - x-value
1 - y-value
Must use correct formula for any marks.

4. a) D: $x \leq -3, x \geq 3$
R: $y \geq 0$

b) $x \geq 3$

c)



1 - f(x) graph according to domain in b)
1 - $f^{-1}(x)$ graph according to f(x).
1/2 if graph was a semi-circle as made easier.

d) $y = \sqrt{x^2-9}$

$x = \sqrt{y^2-9}$

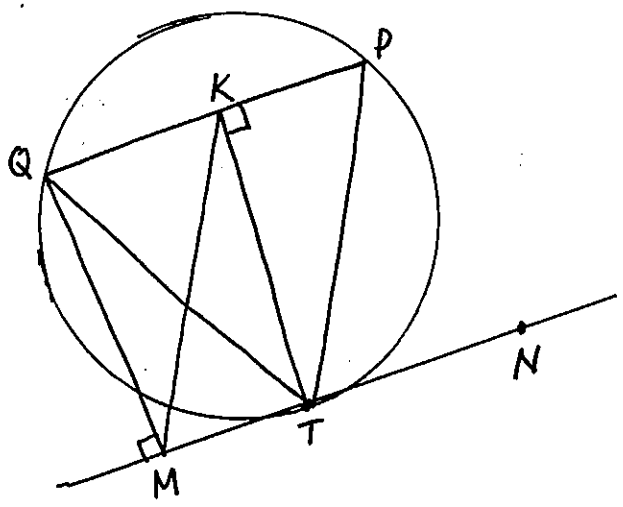
$x^2 = y^2 - 9$

$y^2 = x^2 + 9$

$y = \sqrt{x^2+9}$ as $y \geq 3$.

1 - $y^2 = x^2 + 9$
1 - $y = \sqrt{x^2+9}$
1 - $y \geq 3$ as reason for choosing $\sqrt{x^2+9}$

5. M



$\angle QKT = 90°$ ($KT \perp QP$)

$\angle QMT = 90°$ ($OM \perp MN$)

$\angle QKT + \angle QMT = 180°$

$\therefore$ QKTM is a cyclic quadrilateral (opposite angles are supplementary). (1)

Let $\angle PTN = \alpha$

$\angle PQT = \alpha$ (angle between chord and tangent is equal to angle in the alternate segment). (1)

$\angle KMT = \angle PQT = \alpha$ (angles at circumference standing on the same chord KT are equal, since QKTM is cyclic). (1)

$\therefore \angle KMT = \angle PTN = \alpha$

$\therefore$ KM // PT (corresponding angles are equal). (1)

1 - showing cyclic quad
1 - angle between chord & tangent
1 - angles at circumference
1 - reason for KM // PT.

Yr 10 July 2017 Section E

1) $P(x) = A(x)[(x-p)(x-q)] + ax + b$ 1m

$P(p) = ap + b = p^3$ ①

$P(q) = aq + b = q^3$ ②

① − ② $a(p-q) = p^3 - q^3$

$a(p-q) = (p-q)(p^2 + pq + q^2)$

$a = p^2 + pq + q^2$ 1m

① + ② $a(p+q) + 2b = p^3 + q^3$

$a(p+q) + 2b = (p+q)(p^2 - pq + q^2)$

$(p+q)[a - p^2 + pq - q^2] + 2b = 0$ ③

Sub a into ③ $(p+q)[p^2 + pq + q^2 - p^2 + pq - q^2] + 2b = 0$

$(p+q)2pq = -2b$

$b = -(pq)(p+q) = -(p^2q + pq^2)$

Remainder is $(p^2 + pq + q^2)x - (pq)(p+q)$ 1m

* 2a)

	F	MH	B	J	PE	FT	M
F	X					O	
MH		X					
B			X				
J				X		O	
PE					X		
FT	O			O		X	
M							X

1m

All possible outcomes = 42

b) P(at least 1 language) $= \frac{22}{42} = \frac{11}{21}$ 1m

c) $P(L/FT) = \frac{4}{12} = \frac{1}{3}$ 1

* 3) Let R be red marbles, B be blue marbles

EG 1 $\frac{R}{R+B} \times \frac{R-1}{R+B-1} = \frac{1}{3}$ 1m

EG 2 $\frac{1}{3} \times \frac{B}{R+B-2} = \frac{1}{6}$

EG 2 $\frac{B}{R+B-2} = \frac{1}{2}$ $2B = R + B - 2$

$\therefore B = R - 2$ 1m

Sub $B = R-2$ into EG 1 $3R(R-1) = (R+B)(R+B-1)$ 1m

$3R(R-1) = (2R-2)(2R-2-1)$

$3R(R-1) = 2(R-1)(2R-3)$

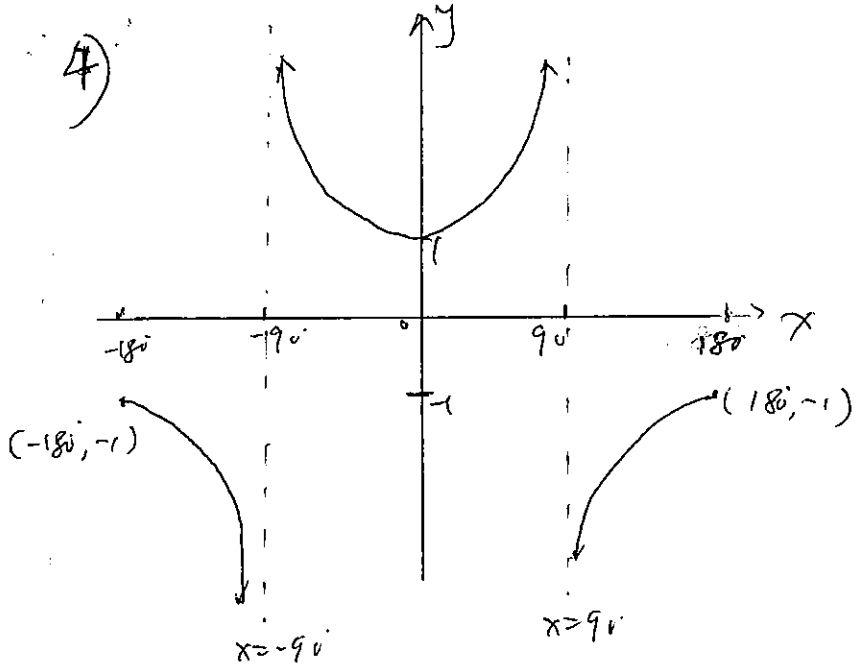
$3R = 4R - 6$

$R = 6$ $\therefore B = 4$ 1m

6 Red marbles
4 Blue marbles

- Need to define variables
- For 3 variables and can't solve only 1m

4)

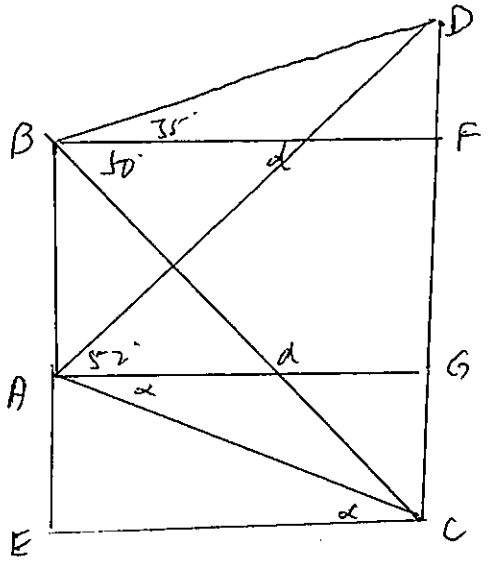


1 m turning pts $y = \pm 1$

1 m Asymptote

1 m branches

X 5a)



1 many student, put angle of depression in the wrong place

In $\triangle DBF$,

b) $\tan 35° = \frac{DF}{d}$ 1

$\therefore DF = d \tan 35°$

c) In $\triangle AGC$ $\frac{GC}{d} = \tan\alpha$ $\therefore GC = d\tan\alpha$

In $\triangle BFC$ $\frac{FG+GC}{d} = \tan 50°$ $FG + GC = d\tan 52°$ 1

In $\triangle DAG$ $\frac{DF+FG}{d} = \tan 52°$ $DF + FG = d\tan 52°$ 1

$\therefore FG = d(\tan 52° - \tan 35°)$

$GC = d\tan 50° - FG$

$GC = d\tan 50° - d(\tan 52° - \tan 35°)$ 1

$\tan\alpha = \frac{GC}{d} = \tan 50° - \tan 52° + \tan 35°$

$\alpha = 31°28'$ (nearest min) 1