STUDENT'S NAME……

CLASS TEACHER……

Year 10
Common Task 2 2021

MATHEMATICS

Instructions

- Time allowed 90 minutes
- All questions may be attempted on the examination paper.
- Calculators may be used.
- All necessary working should be shown where indicated. Full marks may not be awarded for careless or badly arranged work.

Topics Covered	*Syllabus Ref.*	*Outcome Description*	*Section*
Quadratics and Quadratics Equations	MA5.2-6NA MA5.3-5NA MA5.3-7NA	Simplifies algebraic fractions, and expands and factorises quadratic expressions Selects and applies appropriate algebraic techniques to operate with algebraic expressions. Solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations.	A
Trigonometry	MA5.1-10MG MA5.2-13MG MA5.3-15MG	Applies trigonometry, given diagrams, to solve problems, including problems involving angles of elevation and depression Applies trigonometry to solve problems, including problems involving bearings Applies Pythagoras' theorem, trigonometric relationships, the sine rule, the cosine rule and the area rule to solve problems, including problems involving three dimensions	B
Non-Linear Graphs	MA5.2-8NA MA5.3-9NA	Solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques Sketches and interprets a variety of non-linear relationships	C
Working Mathematically	MA5.3-1WM MA5.3-2WM MA5.3-3WM	Uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures Generalises mathematical ideas and techniques to analyse and solve problems efficiently Uses deductive reasoning in presenting arguments and formal proofs	D

Part	A	B	C	D	Total	%
Marks	/20	/27	/31	/22	/100	

Part A: Quadratic expressions and quadratic equations (20 marks)

Question 1

Factorise fully the following expressions:

a) $ap^2 - apq$ [1]

b) $4x^2 - 28x + 9x - 63$ [2]

c) $x^2 - 12x + 36$ [1]

d) $(y+3)^2 - 4$ [2]

Question 2

Simplify the expression $\dfrac{4x}{3x^2 + x - 2} \div \dfrac{4x^2 + 12x}{3x - 2}$ [2]

Marker's Use only

Question 3

Solve the following equations by factorising:

a) $x^2 - 5x - 14 = 0$ [2]

b) $6x^2 + 11x - 10 = 0$ [2]

c) $3y^2 = -15y$ [2]

Question 4

Solve the equation $x^2 - 6x + 3 = 0$ by completing the square. Leave your answer in surd form. [2]

Marker's Use only

Question 5

Solve the equation $2x^2 + 4x - 5 = 0$ by using the quadratic formula. Leave your answer in simplest exact form. [2]

Question 6

a) Find the discriminant of the quadratic expression $3x^2 - 6x + 2$. [1]

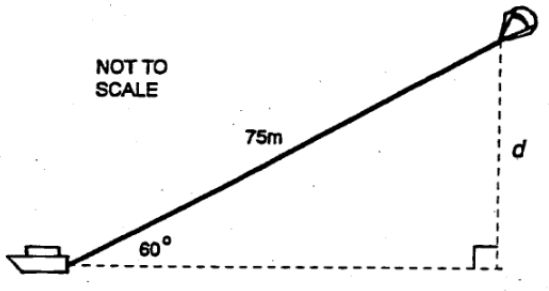
b) Explain what this tells you about the graph of $y = 3x^2 - 6x + 2$. [1]

Marker's Use only

Part B Trigonometry (27 marks)

Question 1

Sue is parasailing. She is joined to a boat by a rope which is 75 m long.



a) If the angle between the rope and the water is $60°$, find the distance d between Sue and the water (answer to the nearest metre).

[2]

b) When Sue is $30\ m$ above the water, calculate the angle θ between the rope and the water (answer to the nearest degree).

[2]

Question 2

Suppose that θ is an acute angle such that $\cos\theta = \frac{2}{5}$. Find the exact value of $\tan\theta$.

[2]

Marker's Use only

Question 3

Find the exact value of the trigonometric ratios:

a) $\cos 330°$ [1]

b) $\sin 225°$ [1]

Question 4

Find the value of θ in the following equations, where $0° \le \theta \le 360°$. Answer to the nearest degree.

a) $\cos\theta = -0.4$ [2]

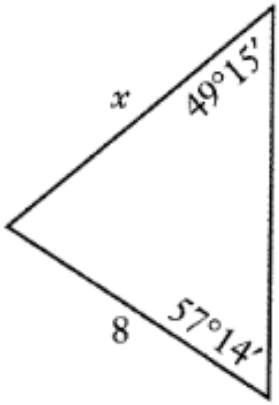
b) $1 + \sqrt{3}\tan\theta = 0$ [2]

Marker's Use only

Question 5

Find the length x in the diagram below, correct to 2 decimal places.

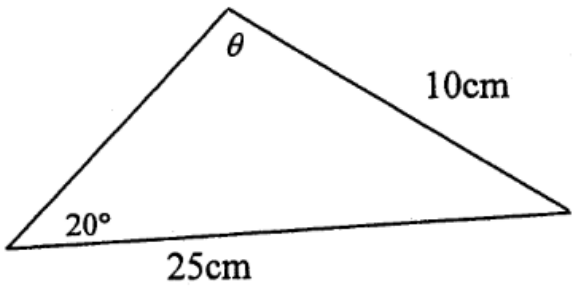
[2]



Question 6

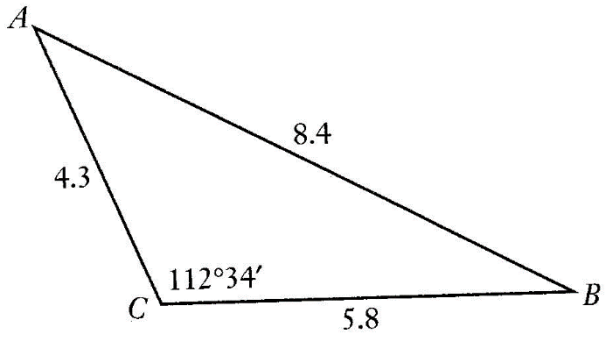
Find the value(s) of angle θ to the nearest degree.

[3]



Marker's Use only

Question 7



Find the area of ΔABC correct to 2 decimal places. [2]

Question 8

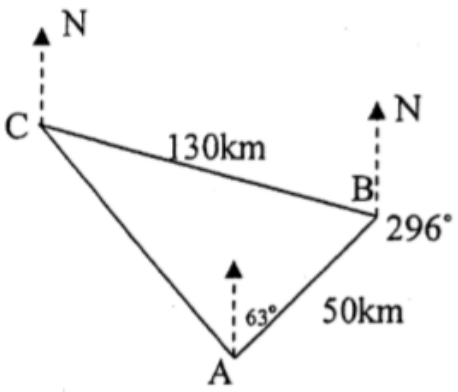
Simplify the expression [2]

$$\frac{\sin(180° + \theta)}{\cos(360° - \theta)}$$

Marker's Use only

Question 9

A ship sails 50 km from port A to Port B on a bearing of $63°$, then sails 130 km from Port B to Port C on a bearing of $296°$.



a) Show that $\angle$ ABC = $53°$. [1]

b) Find to the nearest km, the distance of Port A to Port C. [2]

c) Using the cosine rule, find $\angle$ACB , correct to the nearest degree and hence the bearing of Port A from Port C, correct to the nearest degree. [3]

Marker's Use only

Part C Non-linear relationships (31 marks)

Question 1 For the parabola $y = -x^2 + x + 6$

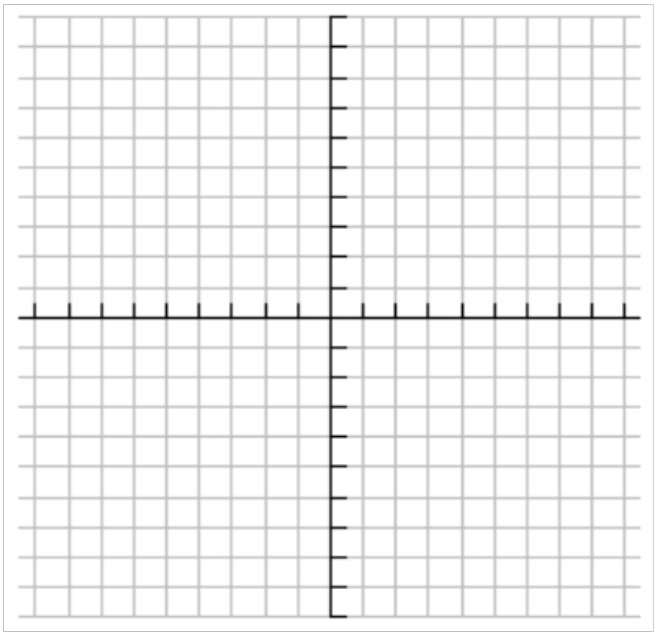
a) Find the y - intercept. [1]

b) Find the x - intercepts. [2]

c) Find the equation of the axis of symmetry. [1]

d) Find the co-ordinates of the vertex. [1]

e) Graph this parabola showing all the above features. [2]



Marker's Use only

Question 2 Consider the parabola $y = \frac{1}{2}(x+2)^2 + 3$.

a) State the coordinates of its turning point and whether it is a maximum or minimum. [2]

b) Without graphing, explain why this parabola has no x - intercepts. [2]

Question 3

State the domain and range of the following relations and whether they are functions

a) $x^2 + y^2 = 25$ [3]

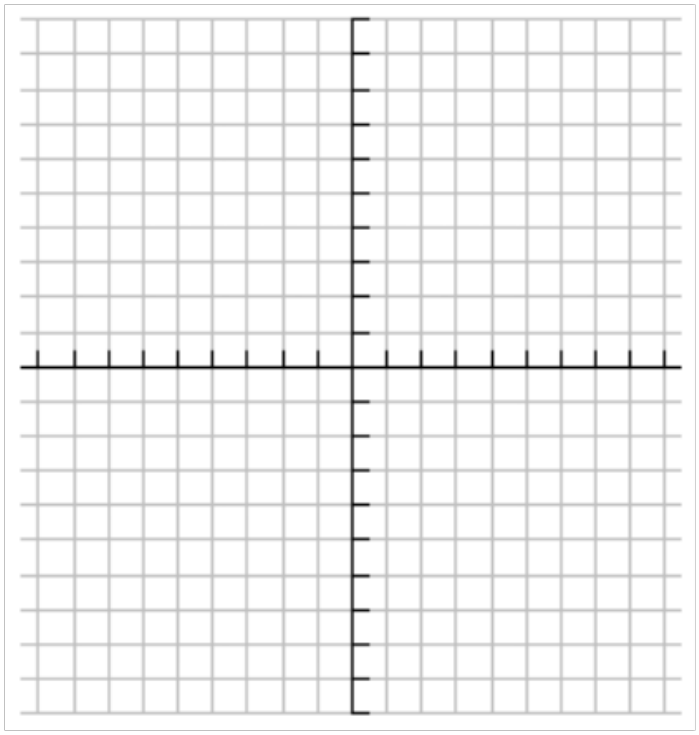
b) $y = \frac{2}{x+3}$ [3]

Marker's Use only

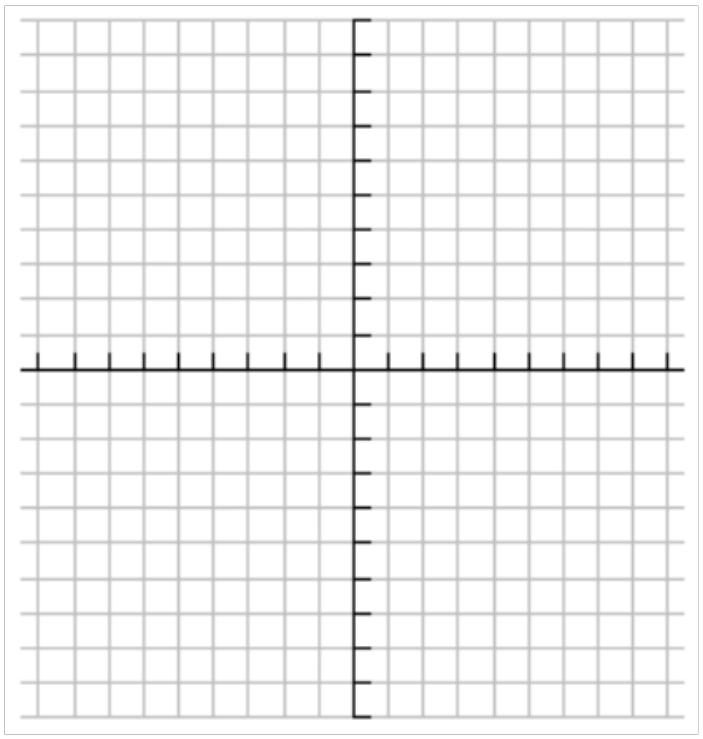
Question 4

On the number planes below, sketch the following graphs, showing intercepts with axes and asymptotes (if any).

a) $y = 2^{-x} - 1$ [2]



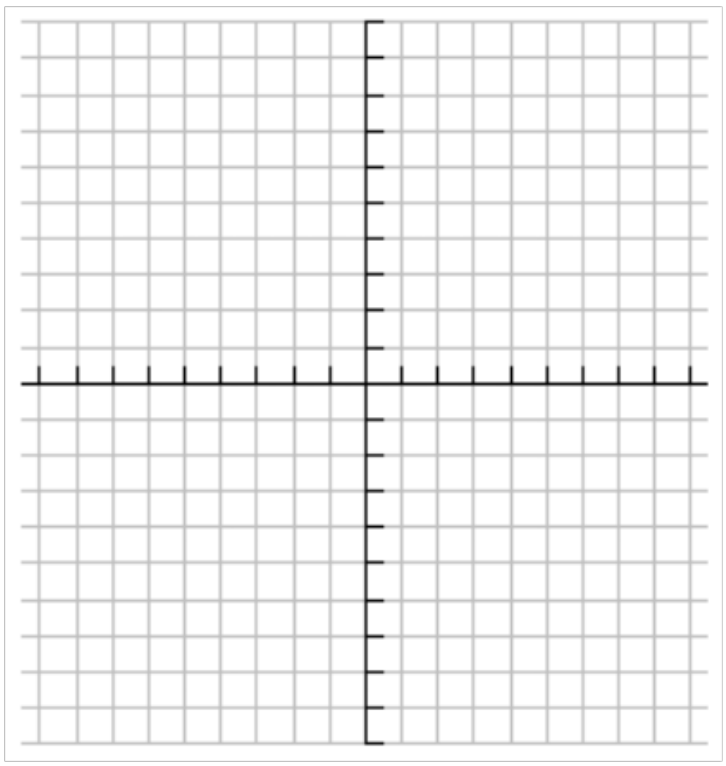
b) $(x+3)^2 + (y-4)^2 = 9$ [2]



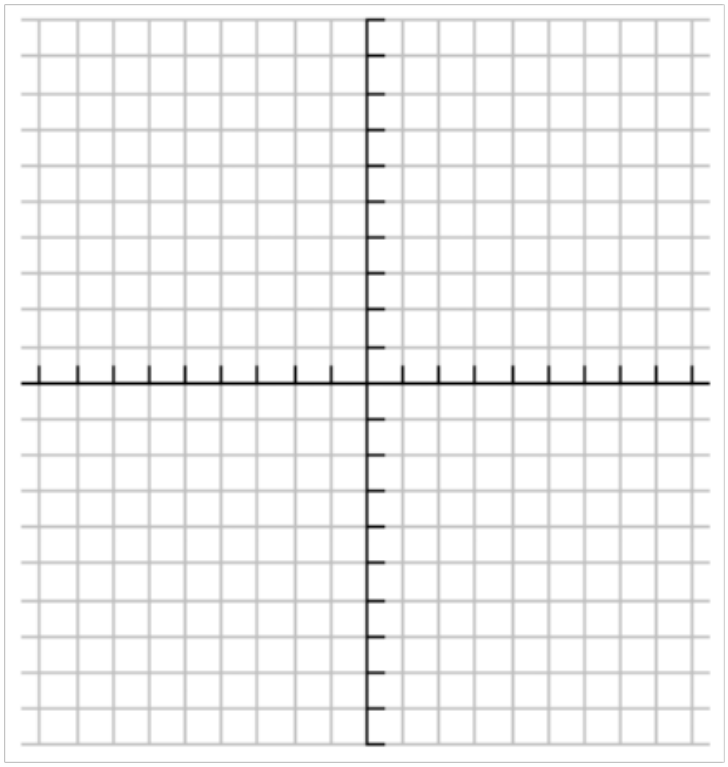
Marker's Use only

Question 4 continued

c) $y = (x-2)^3 + 1$ [2]



d) $y = \frac{-3}{x}$ [2]



Marker's Use only

Question 5

Given the functions $f(x) = x^2 + 8x$ and $g(x) = x + 1$, evaluate:

a) $f(2) + f(-6)$ [2]

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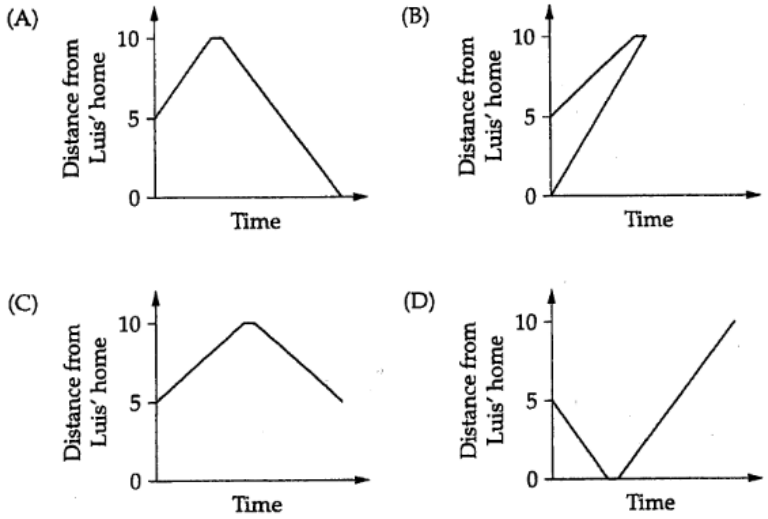
b) $f(g(2))$ [1]

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Question 6

Luis starts his journey 5 km from his home and travels to Bella's place. Bella's place is 10 km from Luis' home. Luis then returns to his home. Which graph best represents Luis' journey? [1]



Question 7

Water is poured, at a constant rate, into the container shown below. Sketch the relationship between the **water level** and **time** as the container fills. Make sure to label the axes appropriately. [2]





Marker's Use only

Part D Working Mathematically (22 marks)

Question 1

Find the points where the circle $x^2 + y^2 = 5$ meets the hyperbola $xy = -2$. [4]

Marker's Use only

Question 2

A gift box is made from a rectangular piece of cardboard that is three times as long as it is wide. From each corner of the cardboard, 5cm squares are cut and the ends are then folded up to make the box.

a) Show the volume of the box is given by the formula $V = 15x^2 - 200x + 500$. [2]

b) Find the length and width of the box if it's volume is $4340\,cm^3$. [3]

Marker's Use only

Question 3

An enclosure is to be built adjoining a barn, as shown below. The walls of the barn meet at $135°$ and cannot be moved. The farmer has 117 metres of fencing available to build the enclosure so that $x + y = 117$, where x and y are shown in the diagram.

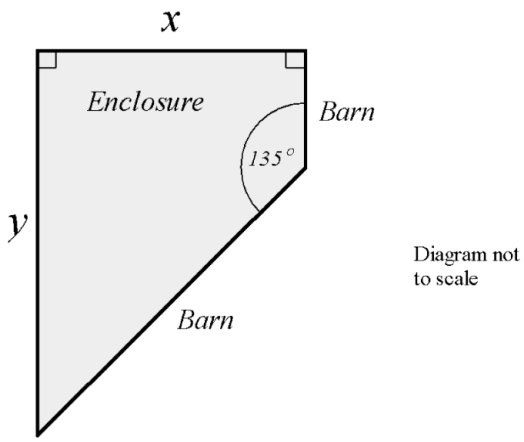


Diagram not to scale

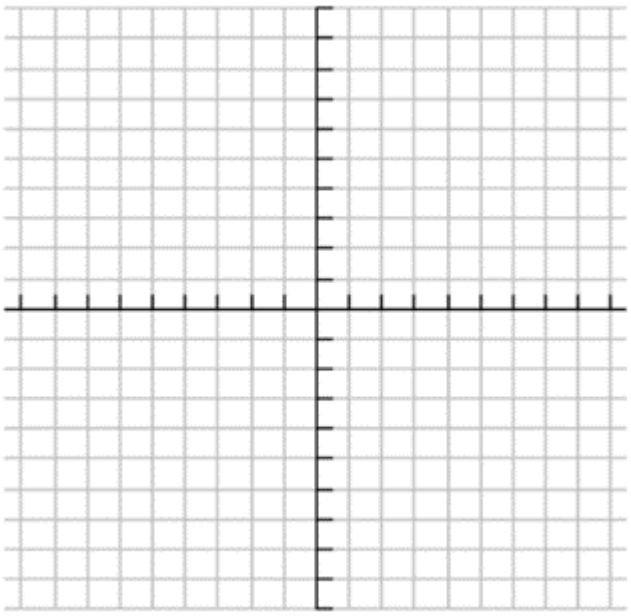
a) Show the shaded area in square metres is given by $A = \frac{-3}{2}x^2 + 117x$ [2]

b) Find the area of the largest possible enclosure. [2]

Marker's Use only

Question 4

a) Draw a neat graph of $f(x) = x^2 - 4$ for $x \leq 0$, on the axes below, showing any intercepts. [1]



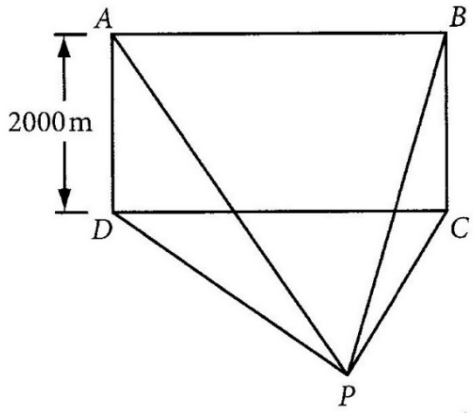
b) On the same axes sketch the graph of the inverse function $f^{-1}(x)$, showing any intercepts [1]

c) Find the equation of the inverse function $f^{-1}(x)$. [3]

Marker's Use only

Question 5

A and B represent successive positions of an aircraft flying horizontally at an altitude of 2000 metres. From a point P the angles of elevation of A and B are 15° and 25° respectively.
If $\angle DPC = 60°$, calculate the distance AB correct to the nearest metre. [4]



Marker's Use only

END OF EXAMINATION

STUDENT'S NAME.....SOLUTIONS.....

CLASS TEACHER.....

Year 10
Common Task 2 2021

MATHEMATICS

Instructions

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Non-Linear Graphs	MA5.2-8NA MA5.3-9NA	Solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques Sketches and interprets a variety of non-linear relationships	C
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Part	A	B	C	D	Total	%
Marks	/20	/27	/31	/22	/100	

Part A: Quadratic expressions and quadratic equations (20 marks)

Question 1

Factorise fully the following expressions:

a) $ap^2 - apq$ [1]

$= ap(p - q)$ 1 for factorising fully

b) $4x^2 - 28x + 9x - 63$ [2]

OR $4x^2 - 19x - 63$

$(4x + 9)(x - 7)$

$4x \times 9$, $x \times -7$

$= (4x^2 - 28x) + (9x - 63)$

$= 4x(x - 7) + 9(x - 7)$ 1 for factorising pairs

$= (x - 7)(4x + 9)$ 1 for factorising

c) $x^2 - 12x + 36$ [1]

$= (x - 6)^2$ 1 for factorising

d) $(y+3)^2 - 4$ [2]

$= (y + 3 + 2)(y + 3 - 2)$ 1. for factorising correctly

$= (y + 5)(y + 1)$ 1 for simplifying factors

Question 2

Simplify the expression $\dfrac{4x}{3x^2 + x - 2} \div \dfrac{4x^2 + 12x}{3x - 2}$ [2]

$= \dfrac{4x^{1}}{{}^{1}(3x-2)(x+1)} \times \dfrac{(3x-2)^{1}}{{}^{1}4x(x+3)}$ 1 for factorising

$= \dfrac{1}{(x+1)(x+3)}$ 1 for correct answer

Marker's Use only

/8

Question 3

Solve the following equations by factorising:

a) $x^2 - 5x - 14 = 0$ [2]

$(x-7)(x+2)=0$ 1 for factorising

$x-7=0$ or $x+2=0$

$x=7$ or $x=-2$ 1 for both solutions

b) $6x^2 + 11x - 10 = 0$ [2]

$(3x-2)(2x+5)=0$ 1 for factorising

$3x-2=0$ or $2x+5=0$

$x=\frac{2}{3}$ or $x=\frac{-5}{2}$ 1 for both solutions

c) $3y^2 = -15y$ [2]

$3y^2+15y=0$

$3y(y+5)=0$ 1 for factorising

$3y=0$ or $y+5=0$ $\therefore y=0$ or $y=-5$ 1 for both solutions

Question 4

Solve the equation $x^2 - 6x + 3 = 0$ by completing the square. Leave your answer in surd form. [2]

$x^2-6x=-3$

$x^2-6x+(-3)^2=-3+9$ 1 for completing the square

$(x-3)^2=6$

$x-3=\pm\sqrt{6}$

$x=3\pm\sqrt{6}$ 1 for both solutions

Marker's Use only

/8

Question 5

Solve the equation $2x^2+4x-5=0$ by using the quadratic formula. Leave your answer in simplest exact form. [2]

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-4 \pm \sqrt{(4)^2-4(2)(-5)}}{2(2)}$ ← 1 for correct substitution

$= \frac{-4 \pm \sqrt{56}}{4}$ $= \frac{-4 \pm 2\sqrt{14}}{4}$ $= \frac{-2 \pm \sqrt{14}}{2}$

1 for simplified exact answer

Question 6

a) Find the discriminant of the quadratic expression $3x^2-6x+2$. [1]

$\Delta = b^2-4ac$

$\Delta = (-6)^2-4(3)(2) = 12$ 1 for discriminant

b) Explain what this tells you about the graph of $y=3x^2-6x+2$. [1]

As $\Delta > 0$, the parabola has 2 x-intercepts.

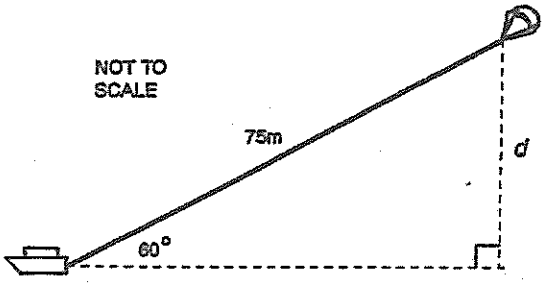
1 for explanation. If $\Delta > 0$ is not written, make a comment but do not deduct marks.

Marker's Use only 1/4

Part B Trigonometry (27 marks)

Question 1

Sue is parasailing. She is joined to a boat by a rope which is 75 m long.



a) If the angle between the rope and the water is $60°$, find the distance d between Sue and the water (answer to the nearest metre). [2]

$\frac{d}{75} = \sin 60°$ 1 for correct ratio

$d = 75 \sin 60°$

$d \doteq 64.95 = 65$ m (nearest m) 1 for correct 'd'. (ignore incorrect rounding)

b) When Sue is 30 m above the water, calculate the angle θ between the rope and the water (answer to the nearest degree). [2]

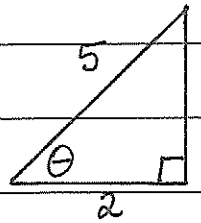
$\sin\theta = \frac{30}{75}$ 1 for correct ratio

$\theta \doteq 23°35'$

$\theta = 24°$ (nearest degree) 1 for correct θ. (Ignore rounding error)

Question 2

Suppose that θ is an acute angle such that $\cos\theta = \frac{2}{5}$. Find the exact value of $\tan\theta$. [2]



$x^2 = 5^2 - 2^2$ 1 for working

$x^2 = 21$

$x = \sqrt{21}$

$(x > 0)$

$\therefore \tan\theta = \frac{\sqrt{21}}{2}$ 1 for answer

Marker's Use only 6

Question 3

Find the exact value of the trigonometric ratios:

a) $\cos 330°$ [1]

$= \cos 30°$

$= \frac{\sqrt{3}}{2}$ 1 for ratio

b) $\sin 225°$ [1]

$= -\sin 45°$

$= -\frac{1}{\sqrt{2}}$ 1 for ratio

Question 4

Find the value of θ in the following equations, where $0° \le \theta \le 360°$. Answer to the nearest degree.

a) $\cos\theta = -0.4$ [2]

related $\angle \doteq 66°25'$

$\theta = 180° - 66°25', 180° + 66°25'$

$\theta \doteq 113°35', 246°25'$

i.e. $\theta = 114°, 246°$ (nearest degree) ← 2 marks i.e. 1 for each angle

b) $1 + \sqrt{3}\tan\theta = 0$ [2]

$\tan\theta = \frac{-1}{\sqrt{3}}$

related $\angle = 30°$

$\therefore \theta = 180° - 30°, 360° - 30°$

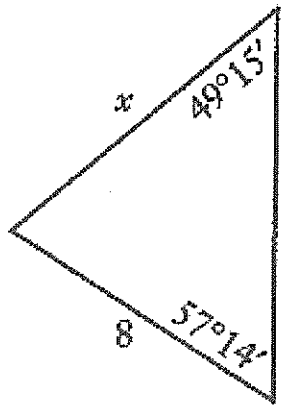
$\theta = 150°, 330°$. ← 2 marks i.e. 1 for each angle.

Marker's Use only

6

Question 5

Find the length x in the diagram below, correct to 2 decimal places. [2]



$\frac{x}{\sin 57°14'} = \frac{8}{\sin 49°15'}$ 1 for correct statement of sine rule

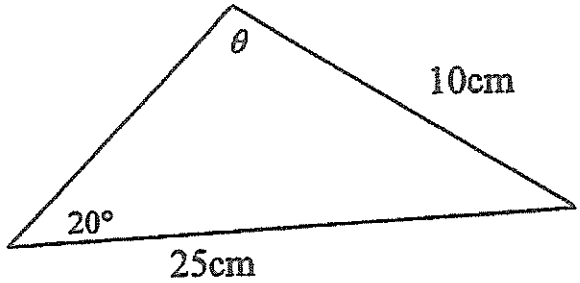
$x = \frac{8 \sin 57°14'}{\sin 49°15'}$

$x \doteq 8.879836124$

$x = 8.88$ units 1 for correct response (ignore rounding error)

Question 6

Find the value(s) of angle θ to the nearest degree. [3]



$\frac{\sin\theta}{25} = \frac{\sin 20°}{10}$ 1 for correct statement of sine rule

$\sin\theta = \frac{25 \sin 20°}{10}$

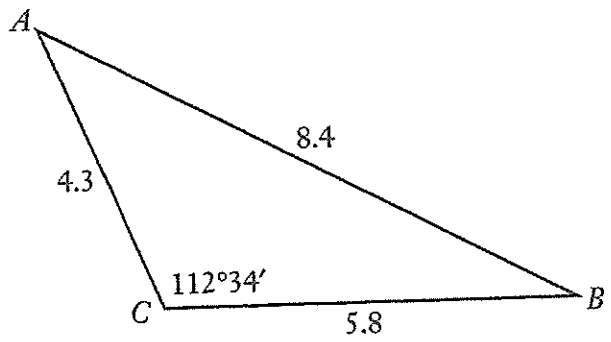
$\sin\theta \doteq 0.85505036$

$\therefore \theta = 58°46', 180° - 58°46'$

$\theta = 59°$ or $121°$ 1 mark for EACH angle (nearest degree)

Marker's Use only 5

Question 7



Find the area of ΔABC correct to 2 decimal places. [2]

$A = \frac{1}{2} \times 4.3 \times 5.8 \times \sin 112°34'$ 1 mark for correct statement of area rule

$\fallingdotseq 11.515217 43$

$= 11.52 \text{ units}^2$ 1 mark for correct area (ignore rounding error)

Question 8

Simplify the expression [2]

$$\frac{\sin(180° + \theta)}{\cos(360° - \theta)}$$

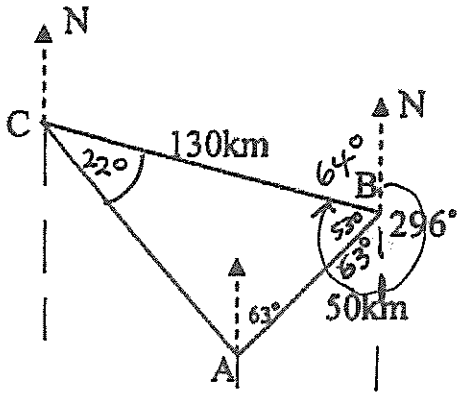
$= \frac{-\sin\theta}{\cos\theta}$ ← 1 mark

$= -\tan\theta$ ← 1 mark

Marker's Use only
/4

Question 9

A ship sails 50 km from port A to Port B on a bearing of $63°$, then sails 130 km from Port B to Port C on a bearing of $296°$.



a) Show that $\angle ABC = 53°$. [1]

1 mark for working

$\angle ABC = 180° - 63° - (360 - 296)°$

$= 53°$

b) Find to the nearest km, the distance of Port A to Port C. [2]

$AC^2 = 130^2 + 50^2 - 2(130)(50)\cos 53°$ ← 1 for correct statement of cosine rule

$AC^2 \div 11576.4047$

$AC = 108$ km (nearest km) ← 1 for distance.

c) Using the cosine rule, find $\angle ACB$, correct to the nearest degree and hence the bearing of Port A from Port C, correct to the nearest degree. [3]

$\cos \angle ACB = \dfrac{108^2 + 130^2 - 50^2}{2(108)(130)}$ ← 1 for correct statement of cosine rule

$\cos \angle ACB = 0.928205$

$\angle ACB = 22°$ (nearest degree) ← 1 for $\angle ACB$

Bearing $= (180 - 64) + 22$

$= 138°$ T. ← 1 for bearing.

Marker's Use only 6

Part C Non-linear relationships (31 marks)

Question 1 For the parabola $y = -x^2 + x + 6$

a) Find the y - intercept. [1]

$y = 6$ 1 mark

b) Find the x - intercepts. [2]

$-x^2 + x + 6 = 0$ ← 1 mark for equating to zero

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0 \quad \therefore x = 3, \; x = -2$ ← 1 mark for intercepts

c) Find the equation of the axis of symmetry. [1]

$x = \frac{-b}{2a} = \frac{-1}{2(-1)}$

$x = \frac{1}{2}$ ← 1 mark

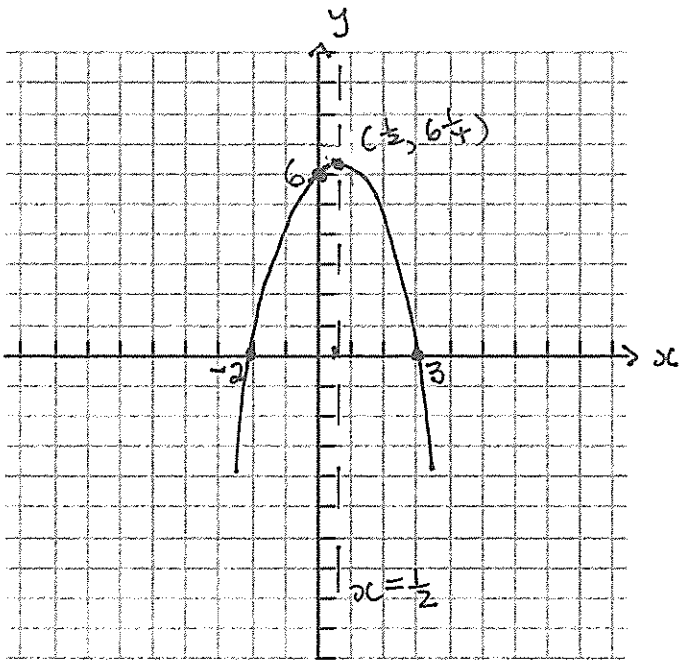
d) Find the co-ordinates of the vertex. [1]

$y = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$

$y = 6\frac{1}{4}$

$\therefore \left(\frac{1}{2}, 6\frac{1}{4}\right)$ ← 1 mark.

e) Graph this parabola showing all the above features. [2]



- 1 mark for intercepts
- 1 mark for shape

Do not penalise if axis is not shown

Marker's Use only

/1

Question 2 Consider the parabola $y=\frac{1}{2}(x+2)^2+3$.

a) State the coordinates of its turning point and whether it is a maximum or minimum. [2]

$(-2, 3)$ ← 1 mark

minimum ← 1 mark

b) Without graphing, explain why this parabola has no x - intercepts. [2]

sub $y=0$: $\frac{1}{2}(x+2)^2+3=0$ ✓ 1 mark for some correct working

$\frac{1}{2}(x+2)^2=-3$

$(x+2)^2=-6$

$x+2=\pm\sqrt{-6}$

Since $\sqrt{-6}$ does not exist in our real number system, $y\neq 0$ ∴ no x-intercepts ✓ 1 mark for reason

* ACCEPT OTHER METHODS !

Question 3

State the domain and range of the following relations and whether they are functions

a) $x^2+y^2=25$ [3]

D: $-5\leq x\leq 5$ ✓ NOT a function ✓

R: $-5\leq y\leq 5$ ✓ (1 mark each)

b) $y=\frac{2}{x+3}$ [3]

D: all real x, $x\neq -3$ ✓ Function ✓

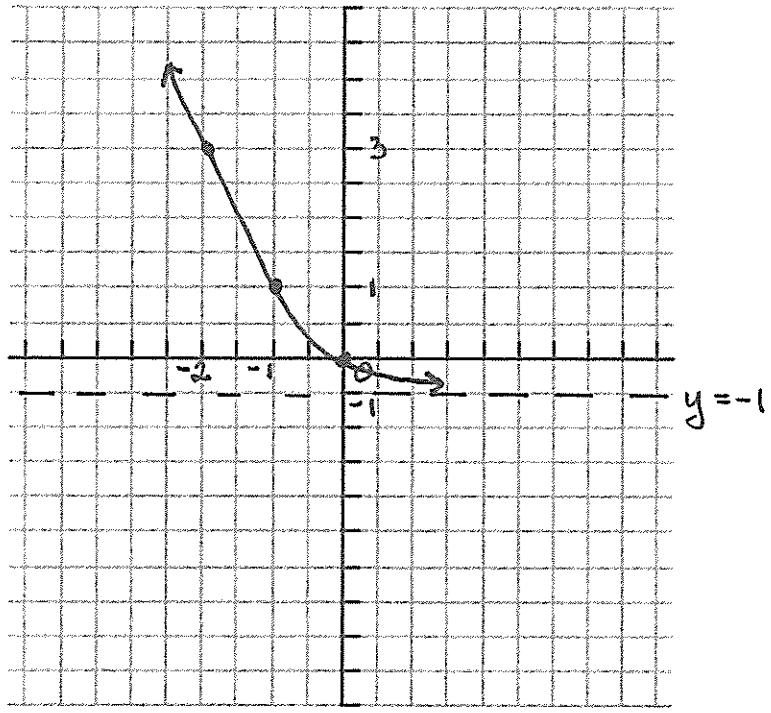
R: all real y, $y\neq 0$ ✓ (1 mark each)

Marker's Use only 10

Question 4

On the number planes below, sketch the following graphs, showing intercepts with axes and asymptotes (if any).

a) $y = 2^{-x} - 1$ [2]



sub $x=0$, $y = 2^0 - 1 = 0$

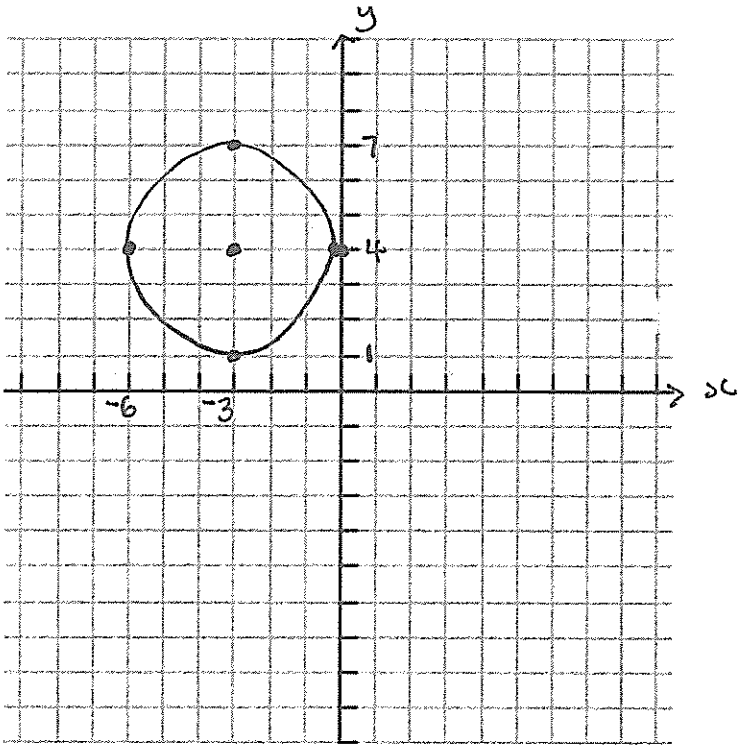
$y=0$, $0 = 2^{-x} - 1$

$1 = 2^{-x}$

$x = 0$

- 1 for asymptote
- 1 for correct shape and intercepts

b) $(x+3)^2 + (y-4)^2 = 9$ [2]



circle centre $(-3, 4)$

$r = 3$

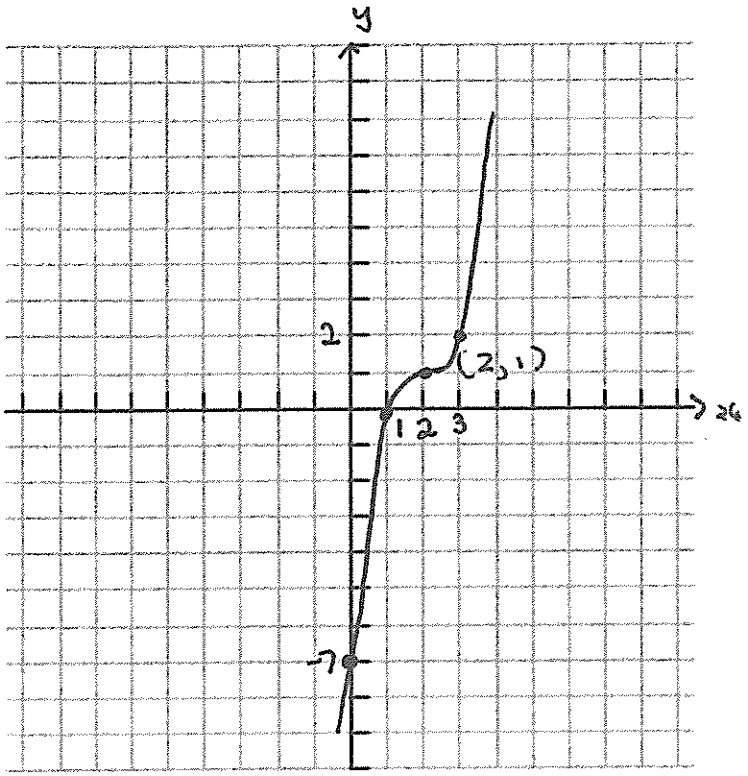
- 1 for intercept
- 1 for shape

Marker's Use only

4

Question 4 continued

c) $y=(x-2)^3+1$ [2]



sub $x=0$: $y=-7$

(2,1) pt. of inflexion

sub $y=0$:

$(x-2)^3+1=0$

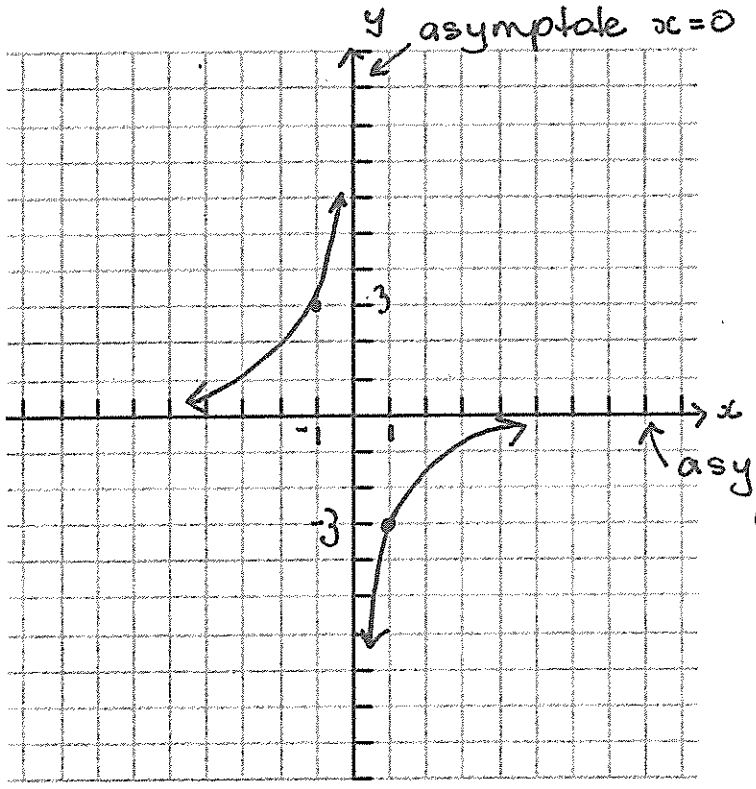
$(x-2)^3=-1$

$x-2=\sqrt[3]{-1}=-1$

$x=1$

- 1 for intercepts
- 1 for pt of inflexion & shape

d) $y=\dfrac{-3}{x}$ [2]



- 1 for asymptotes
- 1 for shape

Marker's Use only

4

Question 5

Given the functions $f(x)=x^2+8x$ and $g(x)=x+1$, evaluate:

a) $f(2)+f(-6)$ [2]

$= (4+16) + (36-48)$ ← 1 mark for substitution

$= 8$ ← 1 mark for answer.

b) $f(g(2))$ [1]

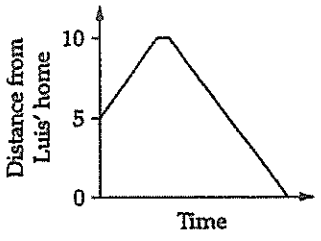
$f(3) = 9 + 24$

$= 33$ 1 mark for answer

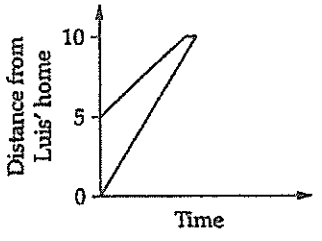
Question 6

Luis starts his journey 5 km from his home and travels to Bella's place. Bella's place is 10 km from Luis' home. Luis then returns to his home. Which graph best represents Luis' journey? [1]

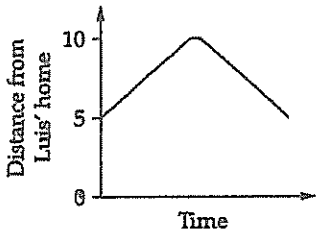
✓ (A)



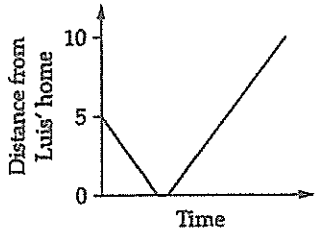
(B)



(C)

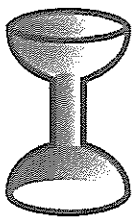


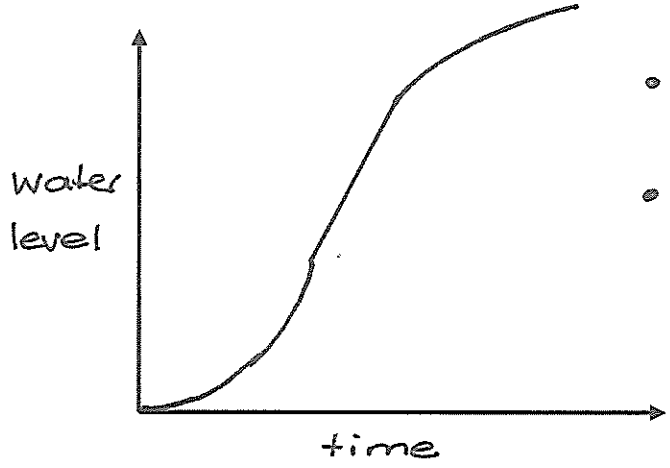
(D)



Question 7

Water is poured, at a constant rate, into the container shown below. Sketch the relationship between the **water level** and **time** as the container fills. Make sure to label the axes appropriately. [2]





- 1 mark if one error
- 2 marks for correct response

Marker's Use only

/6

Part D Working Mathematically **(22 marks)**

Question 1

Find the points where the circle $x^2 + y^2 = 5$ meets the hyperbola $xy = -2$. [4]

$x^2 + y^2 = 5$ (1)

$y = \frac{-2}{x}$ (2)

sub (2) into (1)

$x^2 + \frac{4}{x^2} = 5$ ← 1 mark for substitution

$x^4 + 4 = 5x^2$

$x^4 - 5x^2 + 4 = 0$

$(x^2 - 1)(x^2 - 4) = 0$ ← 1 mark for factorising correctly

$(x+1)(x-1)(x+2)(x-2) = 0$

$\therefore x = -1, \; y = 2$

$x = 1, \; y = -2$

$x = -2, \; y = 1$

$x = 2, \; y = -1$

} 1 mark for correct x values

$\therefore$ point where graphs meet are:

$(-1, 2), (1, -2), (-2, 1), (2, -1)$ ← 1 mark for points

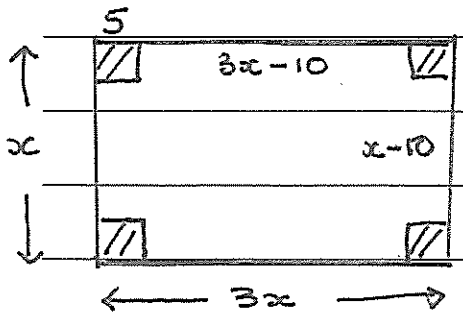
Marker's Use only

4

Question 2

A gift box is made from a rectangular piece of cardboard that is three times as long as it is wide. From each corner of the cardboard, 5cm squares are cut and the ends are then folded up to make the box.

a) Show the volume of the box is given by the formula $V = 15x^2 - 200x + 500$. [2]



$V = 5(3x-10)(x-10)$ ← 1 mark

$V = (15x-50)(x-10)$

$= 15x^2 - 150x - 50x + 500$ ← 1 mark for expanding correctly

$= 15x^2 - 200x + 500$

b) Find the length and width of the box if it's volume is $4340\,cm^3$. [3]

$15x^2 - 200x + 500 = 4340$

$5(3x^2 - 40x + 100) = 4340$

$3x^2 - 40x + 100 = 868$

$3x^2 - 40x - 768 = 0$

$x = \dfrac{-(-40) \pm \sqrt{(-40)^2 - 4(3)(-768)}}{2(3)}$ ← 1 mark

$x = \dfrac{40 \pm \sqrt{10816}}{6}$

$x = \dfrac{40 \pm 104}{6}$

$\therefore x = 24$ or $x = -10\frac{2}{3}$ ← 1 mark for solutions for x

But x (length) > 0 $\therefore x = 24$

$\therefore$ length $= 3 \times 24 - 10 = 62$ cm

width $= 24 - 10 = 14$ cm } 1 mark for dimensions

Marker's Use only 1/5

write a comment if they did not say this.

Question 3

An enclosure is to be built adjoining a barn, as shown below. The walls of the barn meet at 135° and cannot be moved. The farmer has 117 metres of fencing available to build the enclosure so that $x + y = 117$, where x and y are shown in the diagram.

$x+y=117$

$y=117-x$

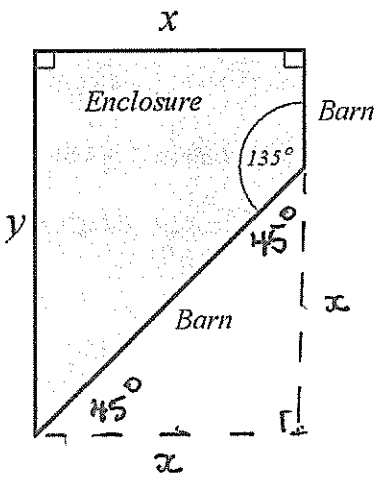


Diagram not to scale

a) Show the shaded area in square metres is given by $A = \frac{-3}{2}x^2 + 117x$ [2]

A = A of rectangle − A of triangle

$= xy - \frac{1}{2}x^2$ ← 1 mark

$= x(117-x) - \frac{1}{2}x^2$

$= 117x - x^2 - \frac{1}{2}x^2$ } ← 1 mark either step

$A = -\frac{3}{2}x^2 + 117x$

b) Find the area of the largest possible enclosure. [2]

Maximum area occurs when $x = \frac{-b}{2a} = \frac{-(-117)}{2\times-\frac{3}{2}}$

i.e. $x = 39$ m ← 1 mark

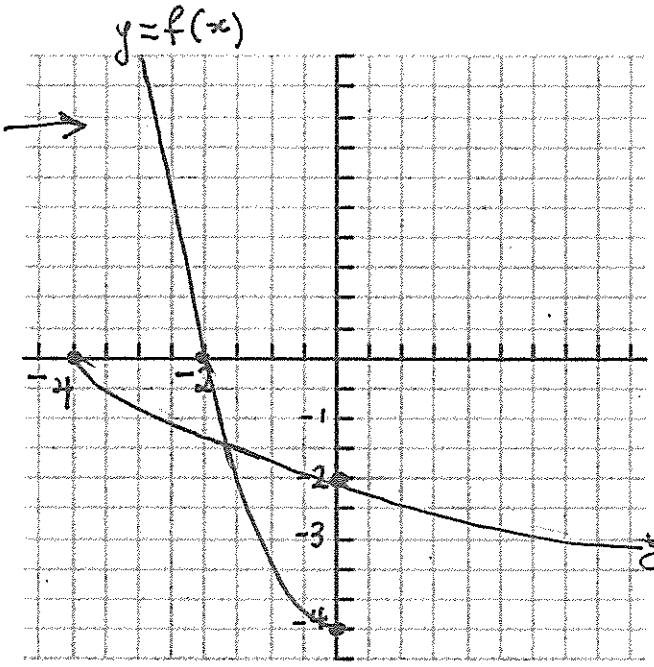
Maximum $A = -\frac{3}{2}(39)^2 + 117(39)$

$= 2281.5\ m^2$ ← 1 for area

Marker's Use only 4

Question 4

a) Draw a neat graph of $f(x) = x^2 - 4$ for $x \leq 0$, on the axes below, showing any intercepts. [1]



1 mark →

← 1 mark

b) On the same axes sketch the graph of the inverse function $f^{-1}(x)$, showing any intercepts [1]

c) Find the equation of the inverse function $f^{-1}(x)$. [3]

let $y = x^2 - 4 \quad x \leq 0$

$x = y^2 - 4$ ← 1 for swapping x & y

$x + 4 = y^2$

$y = \pm\sqrt{x+4}$ ← 1 for making y the subject

Inverse function is:

$y = -\sqrt{x+4}$ as $y \leq 0$

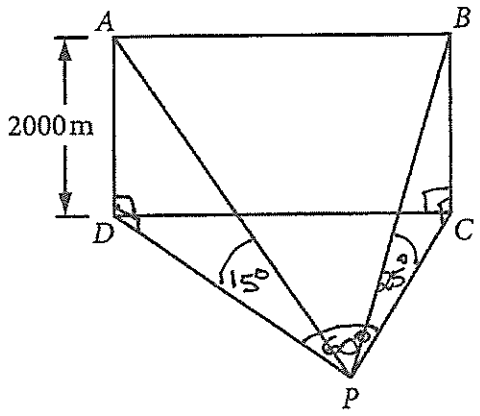
↖ 1 for answer. Ignore if they did not explain 'as $y \leq 0$'

Marker's Use only

5

Question 5

A and B represent successive positions of an aircraft flying horizontally at an altitude of 2000 metres. From a point P the angles of elevation of A and B are 15° and 25° respectively.
If $\angle DPC = 60°$, calculate the distance AB correct to the nearest metre. [4]



In $\triangle ADP$:

$\frac{2000}{DP} = \tan 15°$ } 1 mark for finding DP

$DP = \frac{2000}{\tan 15°}$

In $\triangle BCP$:

$\frac{2000}{PC} = \tan 25°$ } 1 mark for finding PC

$PC = \frac{2000}{\tan 25°}$

In $\triangle DPC$:

$DC^2 = \left(\frac{2000}{\tan 15°}\right)^2 + \left(\frac{2000}{\tan 25°}\right)^2 - 2\left(\frac{2000}{\tan 15°}\right)\left(\frac{2000}{\tan 25°}\right)\cos 60°$

↑ 1 mark for cosine rule

$\therefore DC^2 \doteq 42094817.51$

$\therefore DC \doteq 6488.052$

$DC = AB$

$\therefore AB = 6488$ m (nearest m) ← 1 mark for answer (ignore rounding error)

Marker's Use only 4

END OF EXAMINATION