BAULKHAM HILLS HIGH SCHOOL

2020

YEAR 10 YEARLY EXAMINATION

Mathematics

General Instructions

- Working time – 70 minutes
- Write using black pen
- NESA approved calculators may be used
- In Questions 11 – 28, show relevant mathematical reasoning and/or calculations

Total marks: 62

Section I – 10 marks (pages 2–6)

- Attempt Questions 1 – 10
- Allow about 10 minutes for this section

Section II – 52 marks (Answer Booklet pages 4–14)

- Attempt Questions 11 – 28
- Allow about 60 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Which expression is a factorisation of $8x^3 - 27$?

A. $(2x - 3)(4x^2 + 6x + 9)$

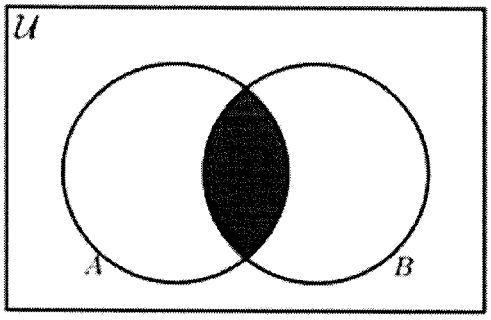
B. $(2x + 3)(4x^2 - 6x + 9)$

C. $(2x - 3)(4x^2 + 12x + 9)$

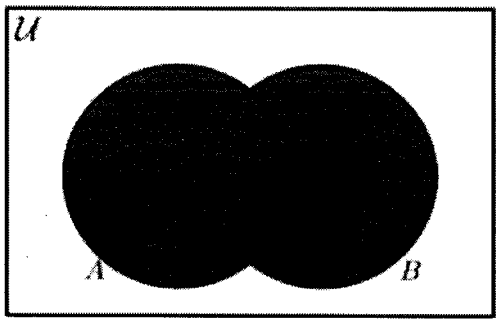
D. $(2x + 3)(4x^2 - 12x + 9)$

2 Which of the following Venn diagrams illustrate $A \cap \bar{B}$?

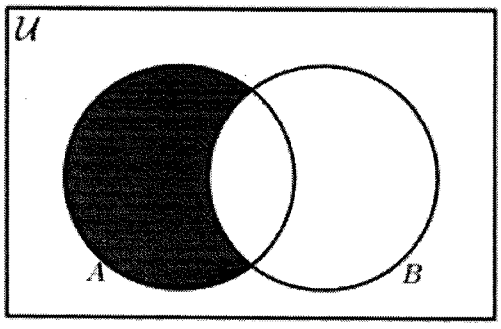
A.



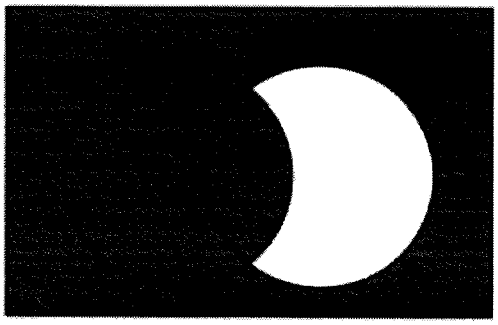
B.



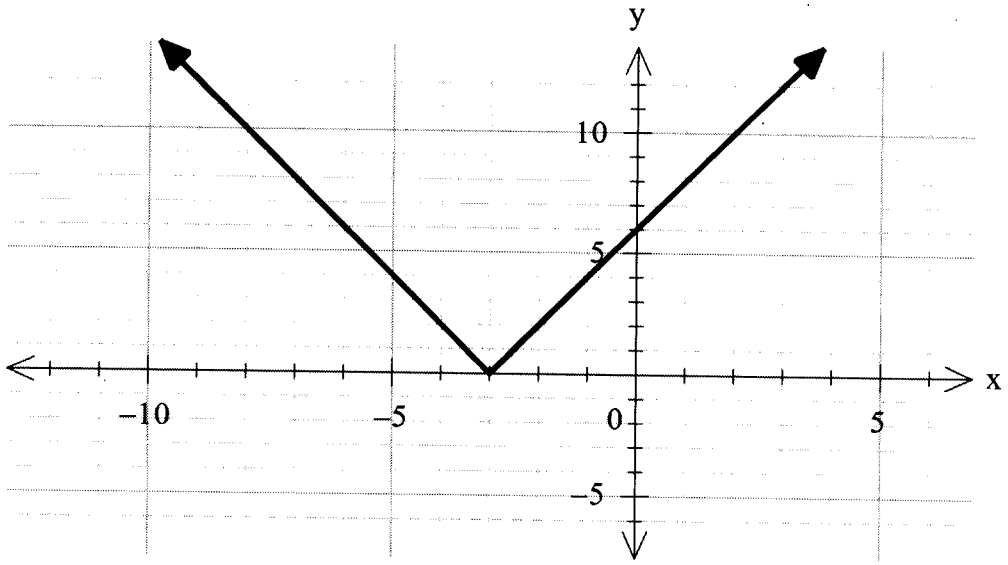
C.



D.



3 By using the graph below for $y = |2x + 6|$, what is the solution to $|2x + 6| < 2$?



A. $x > -4, x > -2$

B. $x < -4, x > -2$

C. $-4 > x > -2$

D. $-4 < x < -2$

4 The first four terms of a geometric sequence are 6000, t_2, 8640, -10368.
What is the value of t_2?

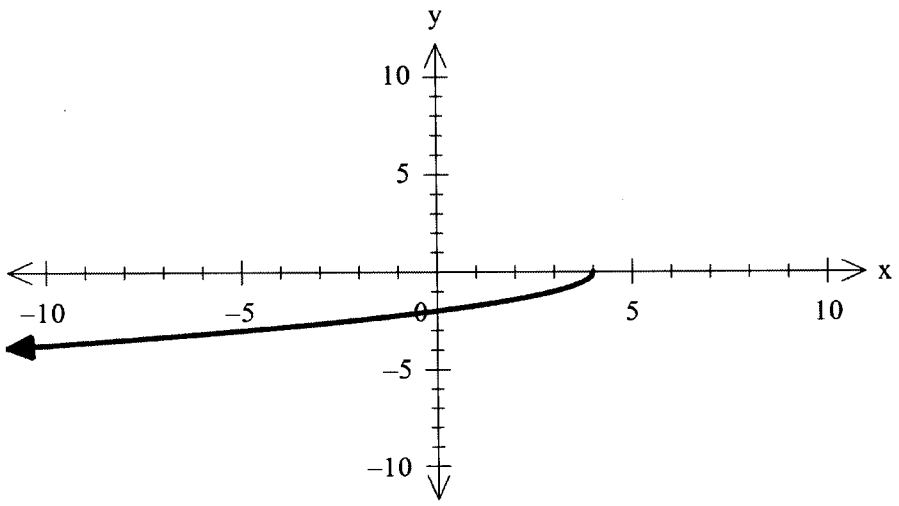
A. -7320

B. -7200

C. 7200

D. 7320

5 Which equation best represents the following graph?



A. $y = -\sqrt{4-x}$

B. $y = -\sqrt{x-4}$

C. $x = -\sqrt{y-4}$

D. $x = -\sqrt{4-y}$

6 A family of 6 is seated randomly around a circular table. In how many ways can the two youngest members of the family sit together?

A. 4!

B. 4! 2!

C. 5!

D. 5! 2!

7 What is the coefficient of x^2 in the expansion of $(1-x)^4$?

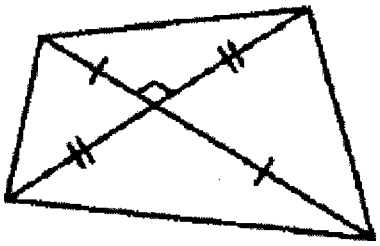
A. -1

B. 1

C. -6

D. 6

8 Which of the following best describes the given quadrilateral?



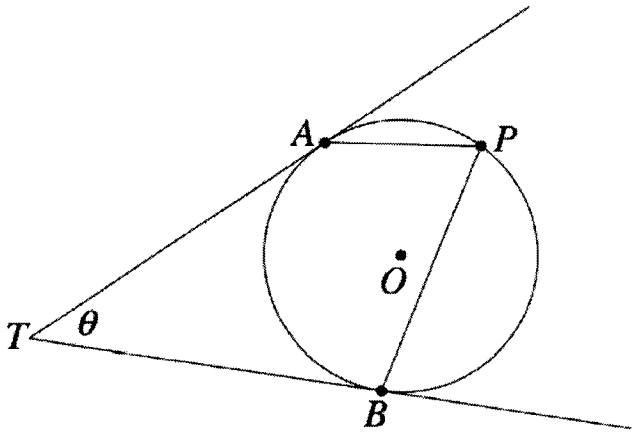
A. Parallelogram

B. Rhombus

C. Rectangle

D. Square

9. The points A, B and P lie on a circle centred at O. The tangents to the circle at A and B meet at the point T, and $\angle ATB = \theta$.



What is $\angle APB$ in terms of θ?

A. $\frac{\theta}{2}$

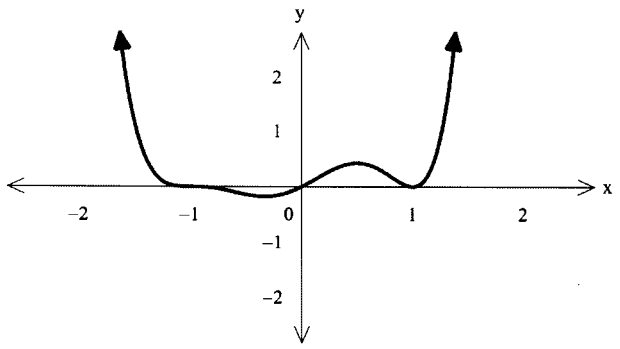
B. $90^o - \frac{\theta}{2}$

C. θ

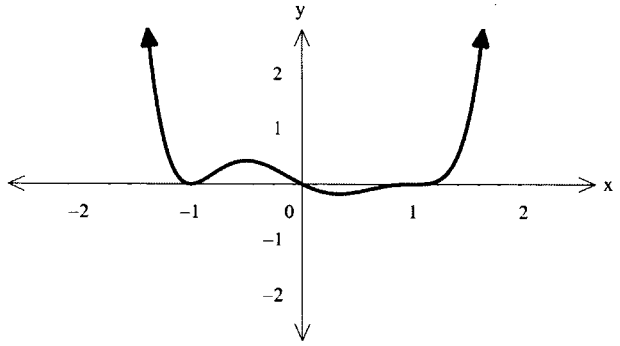
D. $180^o - \theta$

10 Which diagram best represents the graph $y = -x(1-x)^2(x+1)^3$?

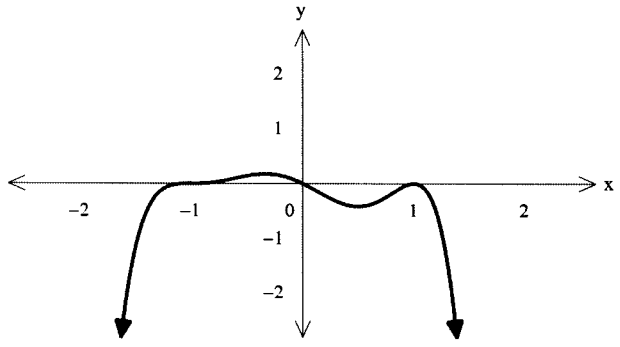
A.



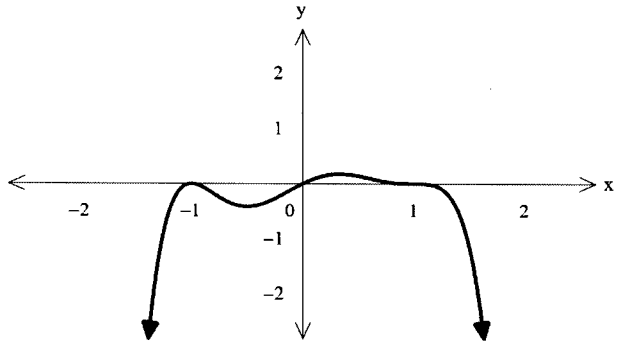
B.



C.



D.



End of Section I

2020 Year 10 Yearly Examination

Mathematics

Section II Answer Booklet

52 marks
Attempt Questions 11 – 28
Allow about 60 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Solve $|2m + 3| = 9$. **2**

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Question 12 (2 marks)

Find all possible values of x for $\sqrt{3}\tan x = -1$ for $0 \leq x \leq 360°$. **2**

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Question 13 (2 marks)

Simplify $\frac{25^{3n} \times 5^{n-1}}{5^{2n+1}}$. **2**

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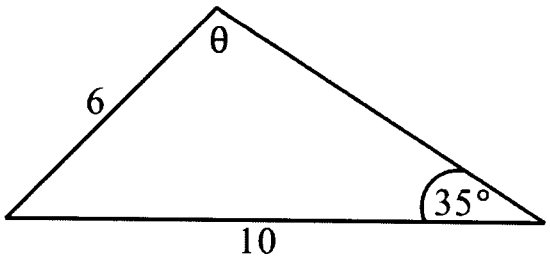
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Question 14 (3 marks)

Find the value(s) of θ in the diagram. Give your answer to the nearest degree. **3**



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Question 15 (3 marks)

Two marbles are selected at random from a bag containing 4 red and 7 yellow marbles.

(a) What is the probability that the two marbles will be of the same colour if the marbles are drawn without replacement? **2**

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(b) What is the probability that the two marbles will not be of the same colour if the marbles are drawn without replacement? **1**

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Question 16 (2 marks)

Find the sum of the first 50 terms of the arithmetic series $3 + 7 + 11 + \ldots$ **2**

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Question 17 (2 marks)

Find the limiting sum of the geometric series $\frac{3}{4} + \frac{3}{10} + \frac{3}{25} + \ldots$ **2**

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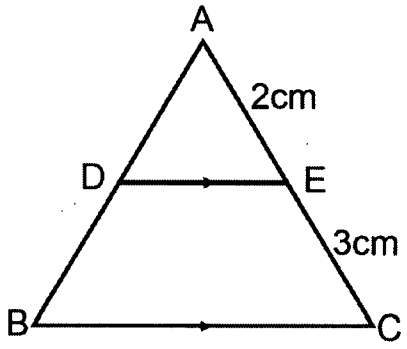
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Question 18 (2 marks)

Triangle ADE is similar to triangle ABC. AE is 2cm and EC is 3cm. Triangle ADE has an area of $16cm^2$. Find the area of the triangle ABC. **2**



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Question 19 (3 marks)

If α and β are the roots of the quadratic equation $3x^2 - 3x + 1 = 0$, find the values of:

(a) $\alpha + \beta$ **1**

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(b) $\alpha^2 + \beta^2$ **2**

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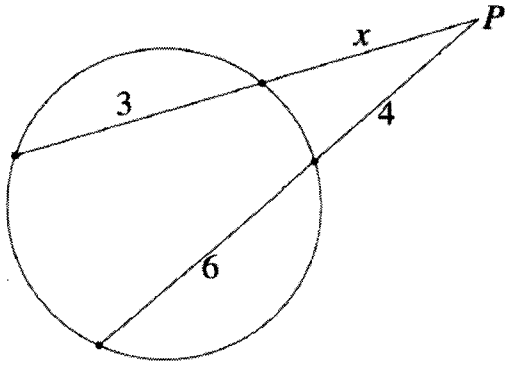
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Question 20 (3 marks)

Two secants from the point P intersect a circle as shown in the diagram. **3**



NOT TO SCALE

What is the value of x? Give reasons for your answer.

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Question 21 (4 marks)

(a) Show that $\frac{2x-3}{x+1} = 2 - \frac{5}{x+1}$. **1**

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(b) Hence, or otherwise, sketch $y = \frac{2x-3}{x+1}$. Show all intercepts and asymptotes. **3**

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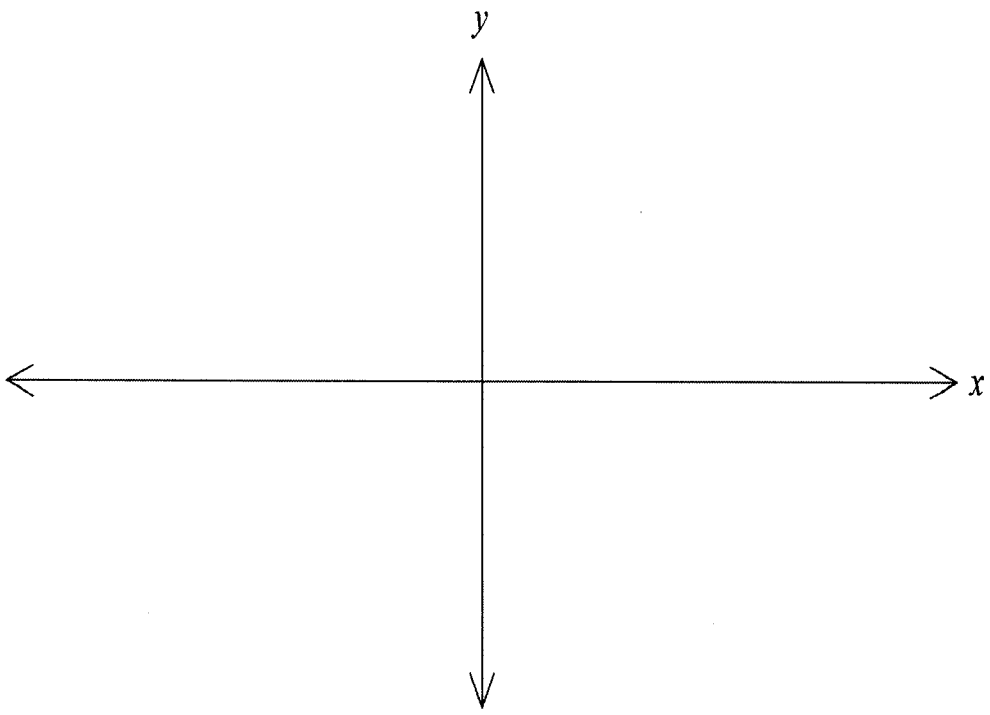
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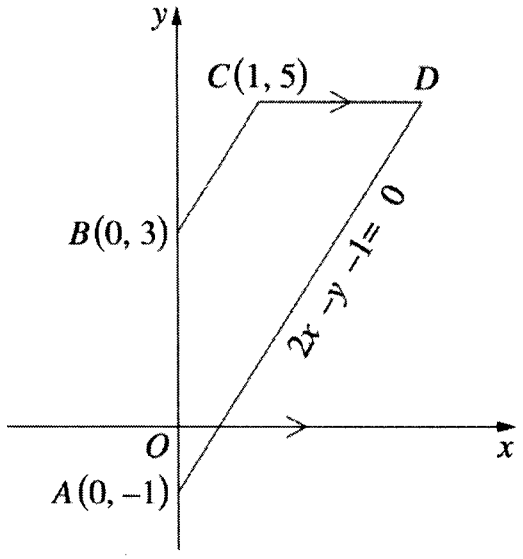


Question 22 (3 marks)

Solve $x^3 + 2x^2 - 5x - 6 = 0$ **3**

Question 23 (8 marks)

In the following diagram, $ABCD$ is a quadrilateral. The equation of the line AD is $2x - y - 1 = 0$.



NOT TO SCALE

(a) Show that $ABCD$ is a trapezium. **2**

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(b) The line CD is parallel to the x-axis. Find the coordinates of D. **1**

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(c) Find the length of BC. **1**

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(d) Show that the perpendicular distance from B to AD is $\frac{4}{\sqrt{5}}$. **2**

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(e) Hence, or otherwise, find the area of the trapezium $ABCD$. **2**

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Question 24 (3 marks)

Reuben needs to arrange 4 English books, 3 History books and 2 Geography books on a shelf.

(a) How many arrangements are there if there are no restrictions? **1**

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(b) How many arrangements are there if the books of each subject are kept together? **2**

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Question 25 (2 marks)

Solve $2^{2x+1} - 5(2^x) + 2 = 0$ **2**

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Question 26 (2 marks)

State the range of the relation $x = 2 - \sqrt{-y^2 + 4y}$. **2**

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Question 27 (3 marks)

A sum of \$9500 is invested at the beginning of each year in a superannuation fund. If interest is paid at 6% p.a., how much money will be available at the end of 40 years? **3**

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Question 28 (3 marks)

Let $P(x) = (x + 2)(x - 1)Q(x) + a(x + 2) + b$, where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x + 2)$ the remainder is -11.

When $P(x)$ is divided by $(x - 1)$ the remainder is 1.

(a) What is the value of b? **1**

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(b) What is the remainder when $P(x)$ is divided by $(x + 2)(x - 1)$? **2**

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End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 10 Yearly Examination, 2020
MATHEMATICS

FINAL MARK

Name: Solutions

Teacher: _____

Question	Algebra	Measurement	Geometry	Probability	Sequence & Series	Total
MC	1, 3, 5, 7, 10 /5		8, 9 /2	2, 6 /2	4 /1	**/10**
Q.11-15 p 4-5	11, 13 /4	12, 14 /5		15 /3		**/12**
Q.16-21 p 6-8	19, 21 /7	18 /2	20 /3		16, 17 /4	**/16**
Q.22-23 p 9-11	22, 23 /11					**/11**
Q.24-28 p 12-14	25, 26, 28 /7			24 /3	27 /3	**/13**
Total	**/34**	**/7**	**/5**	**/8**	**/8**	**/62**

2020 Year 10 Yearly Examination

Mathematics

Section II Answer Booklet

52 marks
Attempt Questions 11 – 28
Allow about 60 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Solve $|2m + 3| = 9$. **2**

$2m+3=9$, $2m+3=-9$

$2m=6$ $\quad 2m=-12$ 2Mks: Provides 2 correct answers

$m=3$ $\quad m=-6$ 1Mk: Provides 1 correct answer

$\therefore m=3$ or -6

Question 12 (2 marks)

Find all possible values of x for $\sqrt{3}\tan x = -1$ for $0 \leq x \leq 360°$. **2**

$\tan x = -\frac{1}{\sqrt{3}}$

Related angle $= 30°$ $\quad$ S | A / T | C ✓

$\therefore x = 150°, 330°$ 2Mk: Provides 2 correct answers.

1Mk: Obtains $x = 30°$ or $-30°$

Question 13 (2 marks)

Simplify $\frac{25^{3n} \times 5^{n-1}}{5^{2n+1}}$. **2**

$= \frac{5^{6n} \times 5^{n-1}}{5^{2n+1}}$

$= 5^{6n+(n-1)-(2n+1)}$

$= 5^{6n+n-1-2n-1}$

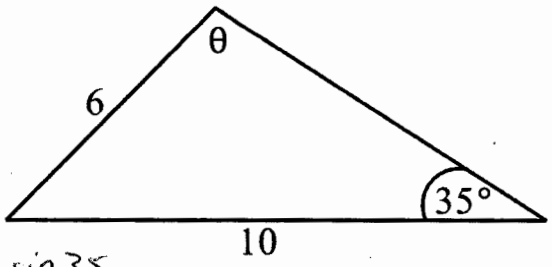
$= 5^{5n-2}$

2Mk: Provides correct solution.

1Mk: Obtains 5^{6n}

Question 14 (3 marks)

Find the value(s) of θ in the diagram. Give your answer to the nearest degree. **3**



$\frac{\sin\theta}{10} = \frac{\sin 35}{6}$

$\sin\theta = \frac{10\sin 35}{6}$

$\sin\theta = 0.95596...$

$\theta = \sin^{-1}(0.95596...)$

$= 72.93...$

$\hat{=} 73°$ or $180 - 73°$

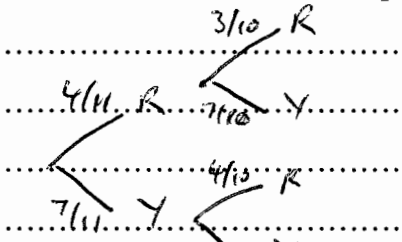
$\therefore \theta = 73°$ or $107°$

3MK: Provides correct solution.
2MK: Obtains $\theta = 73°$
1MK: Correctly substitutes into the sine rule.

Question 15 (3 marks)

Two marbles are selected at random from a bag containing 4 red and 7 yellow marbles.

(a) What is the probability that the two marbles will be of the same colour if the marbles are drawn without replacement? **2**



$\therefore P(RR \text{ or } YY) = \frac{4}{11} \times \frac{3}{10} + \frac{7}{11} \times \frac{6}{10}$

$= \frac{27}{55}$

2MK: Provides correct solution.
1MK: Obtains $P(RR)$ or $P(YY)$ or equivalent merit

(b) What is the probability that the two marbles will not be of the same colour if the marbles are drawn without replacement? **1**

$\therefore 1 - P(\text{same colour}) = 1 - \frac{27}{55}$

$= \frac{28}{55}$

1MK: Provides correct solution - 1- answer to (a)!

Question 16 (2 marks)

Find the sum of the first 50 terms of the arithmetic series $3 + 7 + 11 + \ldots$ **2**

$a = 3, \quad d = 4$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{50}{2}[2(3) + (50-1)(4)]$

$= 5050$

2mk: Provides correct solution.

1mk: Attempts to use S_n formula for AP.

Question 17 (2 marks)

Find the limiting sum of the geometric series $\frac{3}{4} + \frac{3}{10} + \frac{3}{25} + \ldots$ **2**

$a = \frac{3}{4}, \quad r = \frac{2}{5}$

$\therefore S_\infty = \frac{a}{1-r}$

$= \frac{\frac{3}{4}}{(1-\frac{2}{5})}$

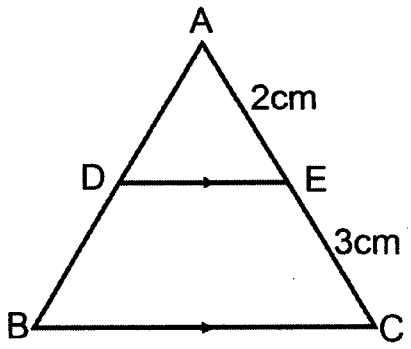
$= \frac{5}{4}$ or 1.25

2mk: Provides correct solution.

1mk: Identifies 'r' correctly

Question 18 (2 marks)

Triangle ADE is similar to triangle ABC. AE is 2cm and EC is 3cm. Triangle ADE has an area of 16cm^2. Find the area of the triangle ABC. **2**



Length ratio: $2 : 2+3$

$2 : 5$

Area ratio: $2^2 : 5^2$

$4 : 25$

$\therefore$ Area $16\text{cm}^2 = x$

$\therefore x = 25 \times 4 = 100\text{cm}^2$

2mk: Provides correct solution.

1mk: Obtains correct ratio for area

Question 19 (3 marks)

If α and β are the roots of the quadratic equation $3x^2 - 3x + 1 = 0$, find the values of:

(a) $\alpha + \beta$ **1**

$\alpha + \beta = \frac{3}{3} = 1$

1MK: Provides correct solution.

(b) $\alpha^2 + \beta^2$ **2**

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \quad \left(\alpha\beta = \frac{1}{3}\right)$

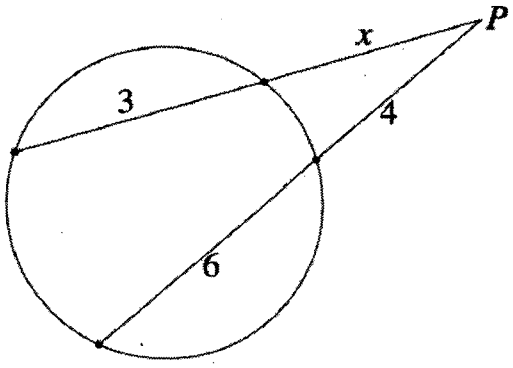
$= (1)^2 - 2\left(\frac{1}{3}\right)$

$= 1 - \frac{2}{3}$ 2MKs: Provides correct solution.

$= \frac{1}{3}$ 1MK: Obtains correct $\alpha\beta = \frac{1}{3}$

Question 20 (3 marks)

Two secants from the point P intersect a circle as shown in the diagram. **3**



NOT TO SCALE

What is the value of x? Give reasons for your answer.

$x(x+3) = 4(4+6)$ (The product of the intercepts of two intersecting secants from an external point are equal).

$x^2 + 3x = 40$

$x^2 + 3x - 40 = 0$

$(x+8)(x-5) = 0$

$\therefore x = 5$ or -8 3MK: Provides correct solution.

$\therefore x = 5$ (since $x > 0$) 2MK: Obtain equation with reasoning

1MK: Obtains correct equation for product of secants.

Question 21 (4 marks)

(a) Show that $\frac{2x-3}{x+1} = 2 - \frac{5}{x+1}$. **1**

RHS $= \frac{2x+2}{x+1} - \frac{5}{x+1}$

$= \frac{2x-3}{x+1}$

$=$ LHS as required

1MK: Provides correct proof.

(b) Hence, or otherwise, sketch $y = \frac{2x-3}{x+1}$. Show all intercepts and asymptotes. **3**

x-int: $0 = \frac{2x-3}{x+1}$

$0 = 2x - 3$

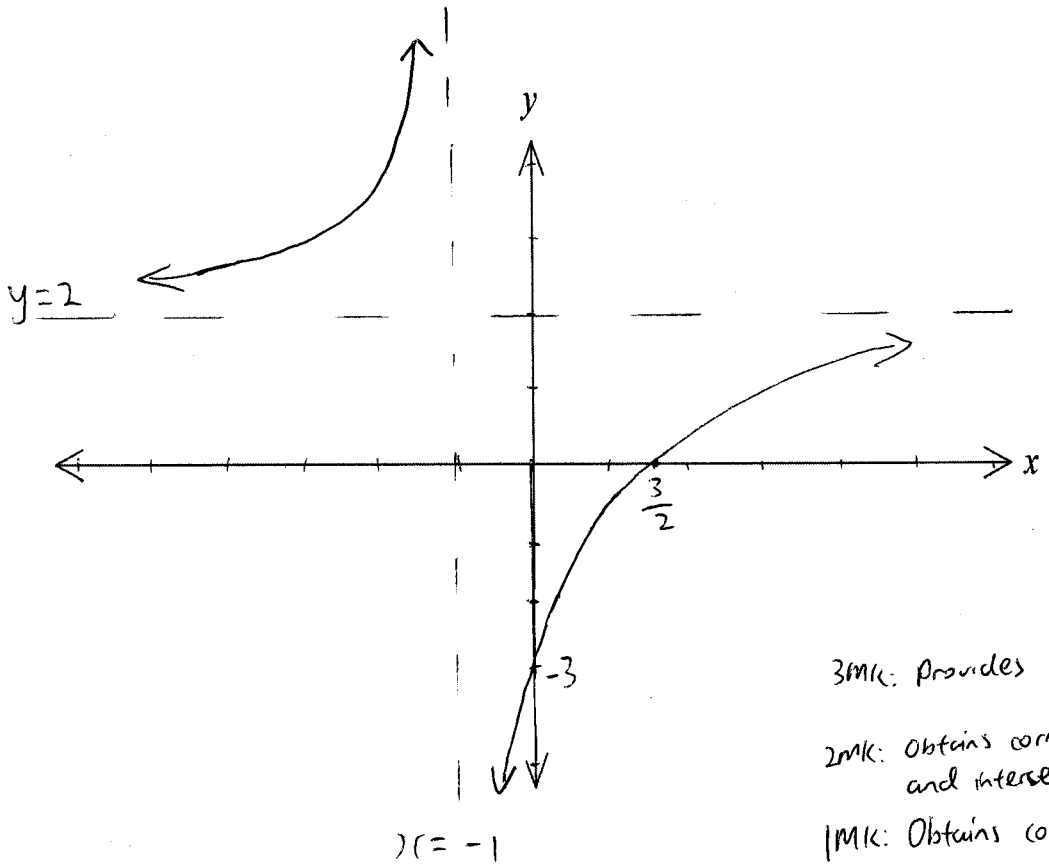
$\therefore x = \frac{3}{2}$

y-int: $y = \frac{2(0)-3}{0+1}$

$\therefore y = -3$

Asymptotes: $x \neq -1$

$y \neq 2$



3MK: Provides correct sketch.

2MK: Obtains correct asymptotes and intercepts

1MK: Obtains correct asymptotes or intercepts.

Question 22 (3 marks)

Solve $x^3 + 2x^2 - 5x - 6 = 0$ **3**

When $x = 1$: $(1)^3 + 2(1)^2 - 5(1) - 6 = -8 \neq 0$

When $x = -1$: $(-1)^3 + 2(-1)^2 - 5(-1) - 6 = 0$

$\therefore (x+1)$ is a factor

$$x+1 \overline{)\, x^3 + 2x^2 - 5x - 6}$$

$$x^2 + x - 6$$

$$-(x^3 + x^2)$$

$$x^2 - 5x$$

$$-(x^2 + x)$$

$$-6x - 6$$

$$-6x - 6$$

$$0$$

$\therefore (x+1)(x^2 + x - 6) = 0$

$(x+1)(x+3)(x-2) = 0$

$\therefore x = -1, -3, 2$

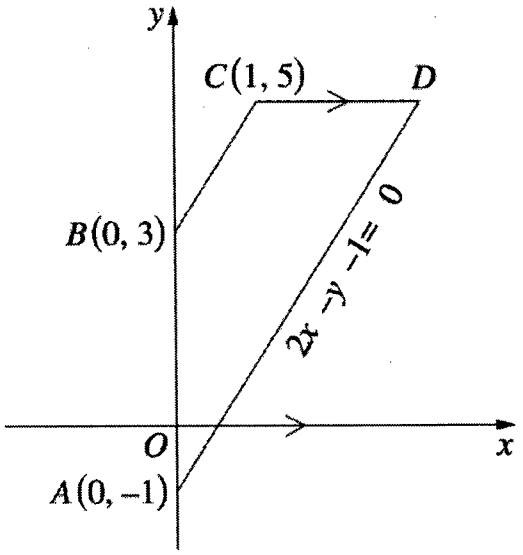
3MK: Provides correct solution.

2MK: Obtains 2 correct factors

1MK: Obtains 1 correct factor

Question 23 (8 marks)

In the following diagram, $ABCD$ is a quadrilateral. The equation of the line AD is $2x - y - 1 = 0$.



NOT TO SCALE

(a) Show that $ABCD$ is a trapezium. **2**

$m_{BC} = \frac{5-3}{1-0}$

$= \frac{2}{1}$ 2MK: Provides correct solution.

$= 2$ 1MK: Obtains 1 correct gradient

$m_{AD}: 2x - y - 1 = 0$

$y = 2x - 1$

$\therefore m_{AD} = 2$

$\therefore m_{BC} = m_{AD}$

$\therefore$ ABCD is a parallelogram (1 pair of parallel sides).

(b) The line CD is parallel to the x-axis. Find the coordinates of D. **1**

D has y-coordinate of 5 (parallel to x-axis.)

$\therefore 2x - (5) - 1 = 0$

$2x = 6$

$x = 3$

$\therefore D(3, 5)$ 1MK: Provides correct solution.

(c) Find the length of BC. **1**

$d = \sqrt{(5-3)^2 + (1-0)^2}$

$= \sqrt{4+1}$

$= \sqrt{5}$ units

1MK: Provides correct solution.

(d) Show that the perpendicular distance from B to AD is $\frac{4}{\sqrt{5}}$. **2**

$B(0,3)$ $AD: 2x - y - 1 = 0$

$\therefore d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

$= \frac{|2(0) + (-1)(3) - 1|}{\sqrt{(2)^2 + (-1)^2}}$

$= \frac{|-4|}{\sqrt{5}}$

$= \frac{4}{\sqrt{5}}$ as required.

2MK: Provides correct proof.

1MK: Attempts to use perpendicular distance formula.

(e) Hence, or otherwise, find the area of the trapezium $ABCD$. **2**

First find AD: $A(0,-1)$, $D(3,5)$

$AD = \sqrt{(5+1)^2 + (3-0)^2}$

$= \sqrt{36+9}$

$= \sqrt{45}$

$= 3\sqrt{5}$

$\therefore$ Area $= \frac{h}{2}(a+b)$

$= \frac{1}{2} \times \frac{4}{\sqrt{5}}(\sqrt{5} + 3\sqrt{5})$

$= \frac{2}{\sqrt{5}}(4\sqrt{5})$

$= 8$ units //

2MK: Provides correct solution

1MK: Obtains $AD = 3\sqrt{5}$

Question 24 (3 marks)

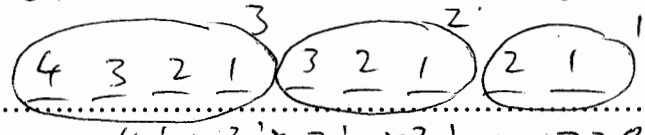
Reuben needs to arrange 4 English books, 3 History books and 2 Geography books on a shelf.

(a) How many arrangements are there if there are no restrictions? **1**

$9! = 362880$

1MK: Provides correct solution.

(b) How many arrangements are there if the books of each subject are kept together? **2**



$\therefore 4! \times 3! \times 2! \times 3! = 1728$

2MK: Provides correct solution.

1MK: Obtains '$4! \times 3! \times 2!$' or equivalent merit.

Question 25 (2 marks)

Solve $2^{2x+1} - 5(2^x) + 2 = 0$ **2**

Let $u = 2^x$

$\therefore 2^{2x} \times 2 - 5(2^x) + 2 = 0$

$\therefore 2u^2 - 5u + 2 = 0$

$2u^2 - 4u - u + 2 = 0$

$2u(u-2) - 1(u-2) = 0$

$(2u-1)(u-2) = 0$

$\therefore u = \frac{1}{2}$ or 2

$\therefore 2^x = 2^{-1}$ or 2^1

$\therefore x = -1$ or 1

2MK: Provides correct solution.

1MK: Obtains correct factorisation

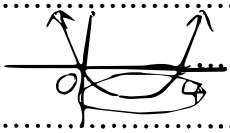
Question 26 (2 marks)

State the range of the relation $x = 2 - \sqrt{-y^2 + 4y}$. **2**

Range: $-y^2 + 4y \geq 0$

$y^2 - 4y \leq 0$

$y(y-4) \leq 0$



$\therefore 0 \leq y \leq 4$

2MK: Provides correct solution.

1MK: Identifies $-y^2 + 4y \geq 0$

Question 27 (3 marks)

A sum of \$9500 is invested at the beginning of each year in a superannuation fund. If interest is paid at 6% p.a., how much money will be available at the end of 40 years? **3**

Let A_n be the amount at the end of n years.

$\therefore A_1 = 9500 \times 1.06$

$A_2 = (9500 \times 1.06 + 9500) \times 1.06$

$= 9500 \times 1.06^2 + 9500 \times 1.06$

$A_3 = (9500 \times 1.06^2 + 9500 \times 1.06 + 9500) \times 1.06$

$= 9500 \times 1.06^3 + 9500 \times 1.06^2 + 9500 \times 1.06$

$= 9500(1.06^3 + 1.06^2 + 1.06)$

$= 9500(1.06 + 1.06^2 + 1.06^3)$

$\therefore A_n = 9500(1.06 + 1.06^2 + 1.06^3 \ldots + 1.06^n)$

$a = 1.06, \ r = 1.06, \ n = n$

$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1.06(1.06^n - 1)}{0.06}$

$\therefore A_{40} = 9500\left(\frac{1.06(1.06^{40} - 1)}{0.06}\right)$

$= \$1558452.994$

$\approx \$1558452.99$

3MK: Provides correct solution.

2MK: Obtains correct GP formula.

1MK: Obtains correct values for 'P = 9500', '1+r = 1.06' and '40 periods'.

Note: Formula without deriving gets 2/3.

Question 28 (3 marks)

Let $P(x) = (x + 2)(x - 1)Q(x) + a(x + 2) + b$, where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x + 2)$ the remainder is -11.

When $P(x)$ is divided by $(x - 1)$ the remainder is 1.

(a) What is the value of b? **1**

$P(-2)$: $(0)(x-1)Q(x) + a(0) + b = -11$

$\therefore b = -11$

1mk: Provides correct solution.

(b) What is the remainder when $P(x)$ is divided by $(x + 2)(x - 1)$? **2**

$P(1)$: $(1+2)(0)Q(x) + a(1+2) - 11 = 1$

$3a - 11 = 1$

$3a = 12$

$a = 4$

$\therefore$ When divided by $(x+2)(x-1)$:

$R(x) = a(x+2) + b$

$= 4(x+2) + (-11)$

$= 4x + 8 - 11$

$= 4x - 3$

2mk: Provides correct solution.

1mk: Obtains $a = 4$

End of paper

BAULKHAM HILLS HIGH SCHOOL

YEAR 10 Yearly Examination 2020

MATHEMATICS

Mathematics
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$

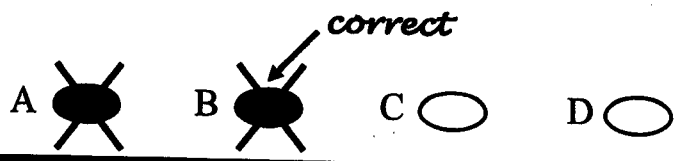
(A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



Start Here →

1.	A ●	B ○	C ○	D ○	**6.**	A ○	B ●	C ○	D ○
2.	A ○	B ○	C ●	D ○	**7.**	A ○	B ○	C ○	D ●
3.	A ○	B ○	C ○	D ●	**8.**	A ○	B ●	C ○	D ○
4.	A ○	B ●	C ○	D ○	**9.**	A ○	B ●	C ○	D ○
5.	A ●	B ○	C ○	D ○	**10.**	A ○	B ○	C ●	D ○