Year 10 Yearly 2019

TIME : 65 minutes + 5 minutes reading time

Name:

Teacher:

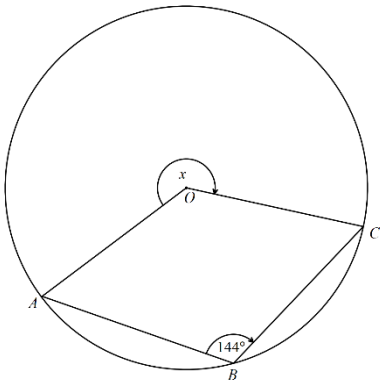
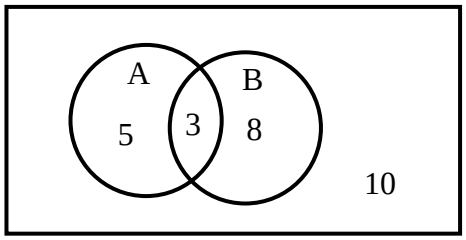
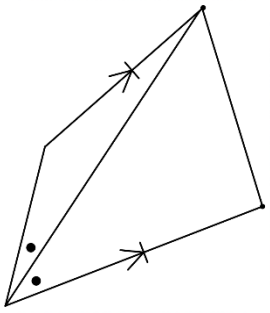
Directions

- Full working should be shown in every question.
  Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only (not pencils) to write your solutions.
- No liquid paper/correction tape is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.
- The diagrams are not to scale.

	Section A		Section B		Section C			
Polynomials						Q5 /4		Q7 /4
Probability		Q5		Q5 – 7 /4				
Trigonometry		Q2						Q8 /5
Similarity								Q6 /3
Surface Area and Volume				Q8 /2				
Coordinate Geometry		Q1						
Sequences and Series		Q3						Q1 /3
Sufficiency Conditions		Q6						Q4 /5
Circle Geometry		Q4						Q3 /3
Quadratic Equations, Inequations and Identities				Q1 - 4 /10				
Non Linear Relationships		Q7						Q2 /4
Sub-Total	/7		/16		/31			
Total					/54			

Section A – 1 mark each

Question and Working	Answer
1. Find the perpendicular distance from the point (-3, 8) to the line $4x - 7y + 6 = 0$. Leave your answer in exact form.	
2. Solve for θ, given $0 \leq \theta \leq 360$ $sin\,\theta = \frac{\sqrt{3}}{2}$	
3. Find the 40th term of the series 5, 8, 11, 14, …	

4. O is the centre of a circle. A, B and C are points on the circumference of the circle such that $\angle ABC = 144^o$.  Give a reason why $x = 288^o$.	
5. Find P $(A \cup B')$  	
6. What shape best describes this quadrilateral 	
7. An exponential curve has a *y*-intercept of 1 and passes through the point $\left(4, \frac{1}{625}\right)$. Find the equation of the curve, if the *x*-axis is the asymptote.	

Section B

Marks shown to the right of the question. Show all necessary working.

1. Solve the inequality **3**

$$3x^2 - 7x - 6 > 0$$

2. Expand and simplify $(1 + 5x)^5$ **2**

3. For what values of k will the equation $2x^2 - kx + 2 = 0$ have equal roots? **2**

4. Let the equation $8x^2 + 7x - 5 = 0$ have roots α and β. **3**

Find

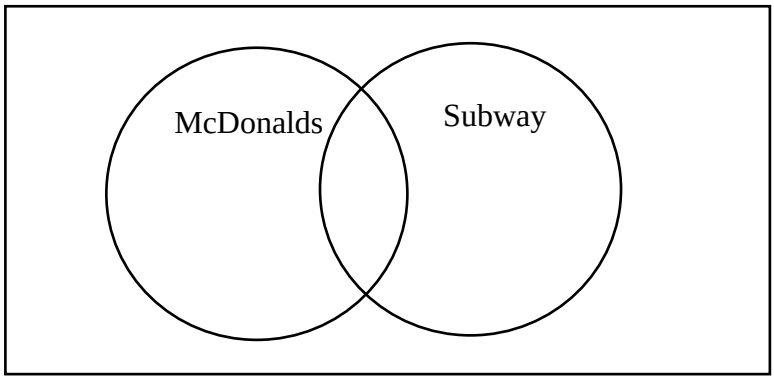
a) $\alpha + \beta$

b) $\alpha\beta$

c) $\frac{1}{\alpha} + \frac{1}{\beta}$

5. A group of 60 students are surveyed about their likes for take away food. 36 students like Subway and 30 like McDonalds while 11 dislike both Subway and McDonalds.

a) Show this information on the Venn Diagram below. **2**



b) What is the probability that a student who likes McDonalds, dislikes Subway? **1**

6. How many four letter arrangements can be made using the letters of the word CHILDREN? **1**

7. How many different ways can 7 people be seated around a circular table? **1**

8. A company makes cylindrical medallions in two sizes. Both medallions have the same thickness. The ratio of the radii of the medallions is 5:2.

   400 large medallions can be made from a quantity of metal.

   How many small medallions can be made from the same quantity of metal? **2**

Section C

Question 1

a) Emma is saving for a car. She decides to deposit $100 per month into a savings account at an interest rate of 3% p.a compounding monthly for 2 years. Will she be able to afford a $3 000 car at the end of the two years? Justify your answer by showing all working. **3**

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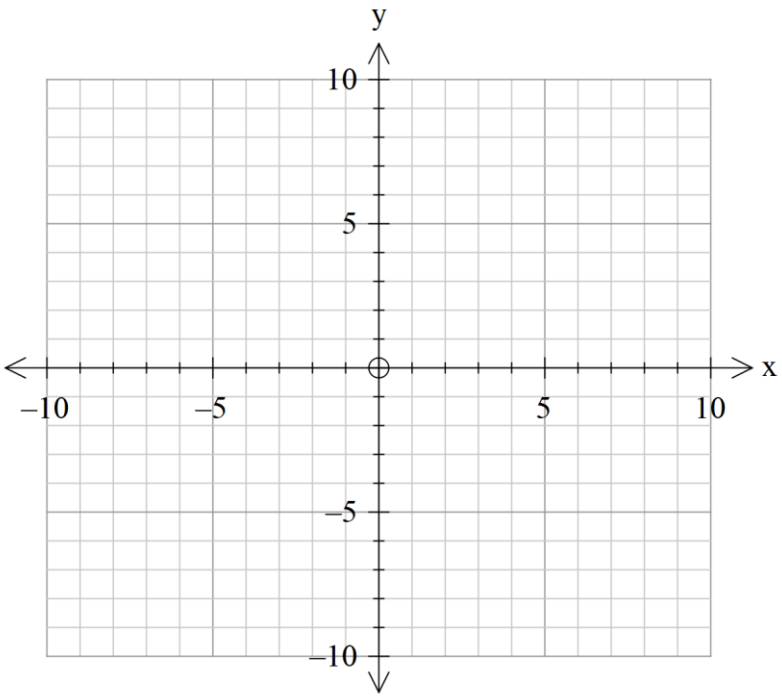
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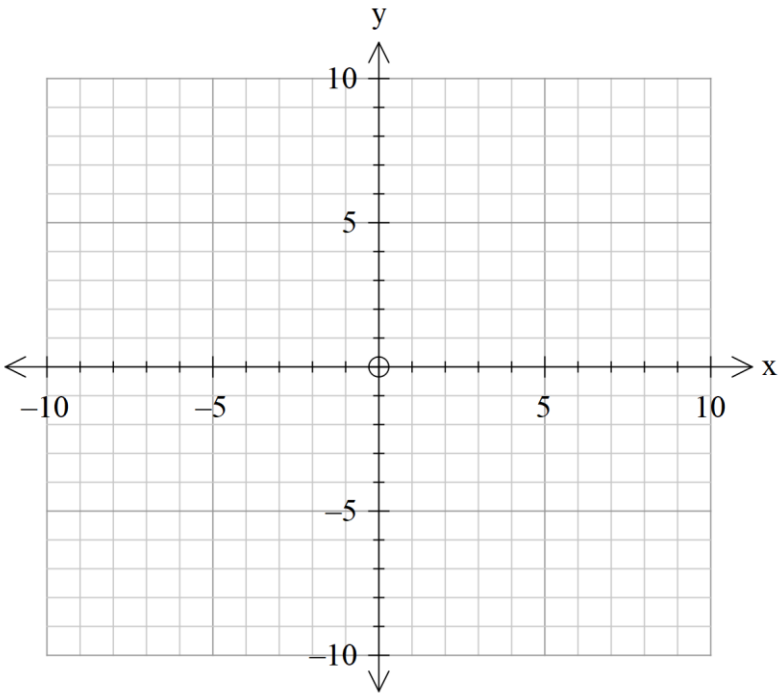
Question 2

Graph the following equations

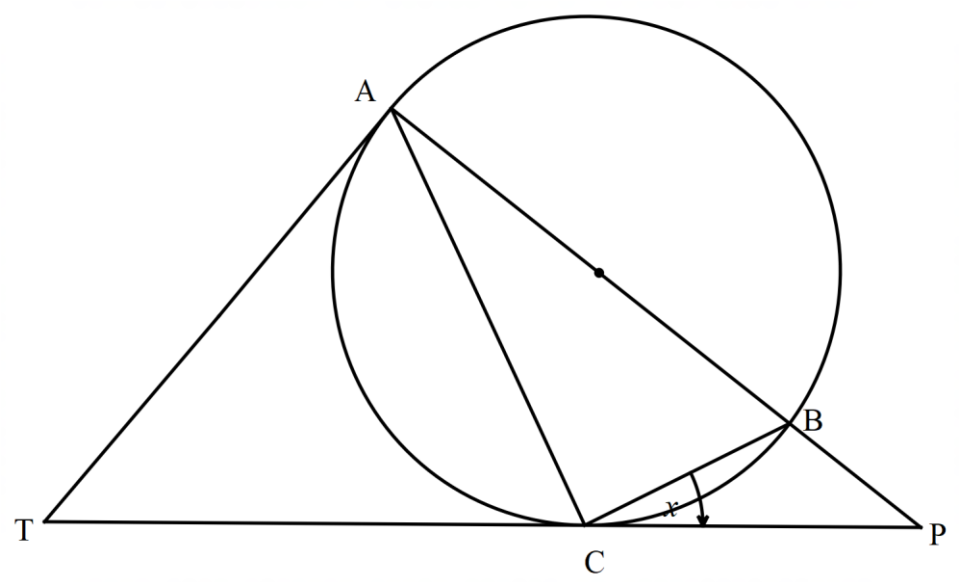
a) $y = \sqrt{x - 4}$ **2**



b) $x = -\sqrt{9 - (y - 1)^2}$ **2**



Question 3



AB is a diameter of a circle ABC. The tangents at A and C meet at T. The lines TC and AB are produced to meet at P. Let ∠BCP = x

a) Prove that ∠CAT = $(90 - x)^o$ **2**

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b) Hence or otherwise, find ∠ATC in terms of x **2**

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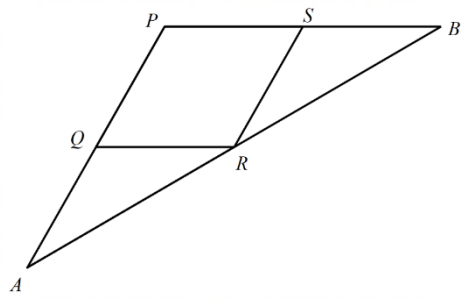
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Question 4



The parallelogram *PQRS* is inscribed in ΔPBA where *R* lies on *AB*. *QA* = *QR* and *PS* = *SB*.

a) Explain why *PS* = *QR*, *PQ* = *RS*, *PS* || *QR and PQ* || *SR* **1**

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b) Prove that $\Delta BSR \equiv \Delta RQA$ **3**

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c) Hence or otherwise, prove that PQRS is a rhombus **2**

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Question 5

a) Show that the polynomial $P(x) = -x^3 - 2x^2 + 5x + 6$ has a root at $x = -3$ **1**

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b) Hence or otherwise sketch the graph of $y = P(x)$ on the axes below. **3**

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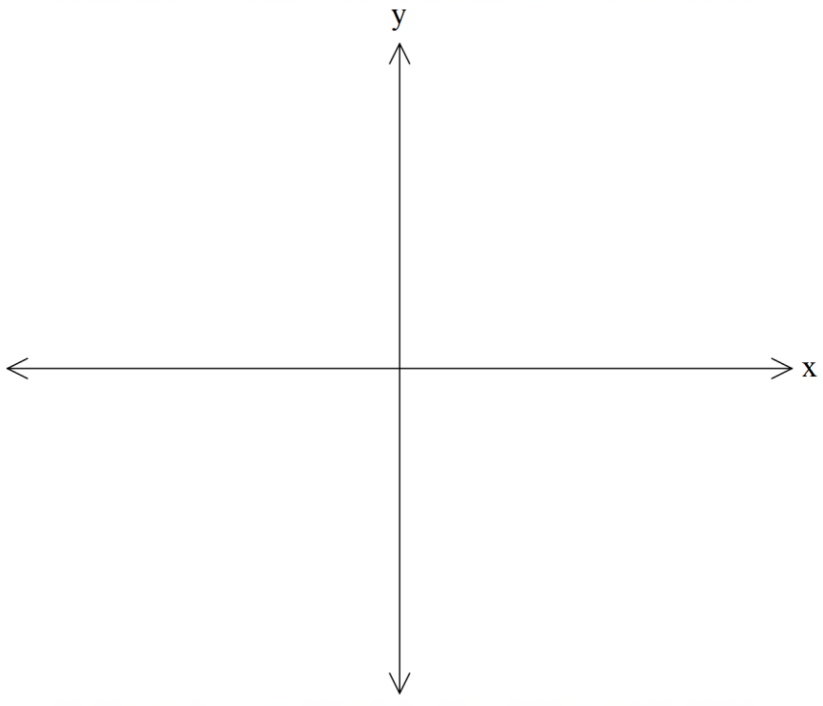
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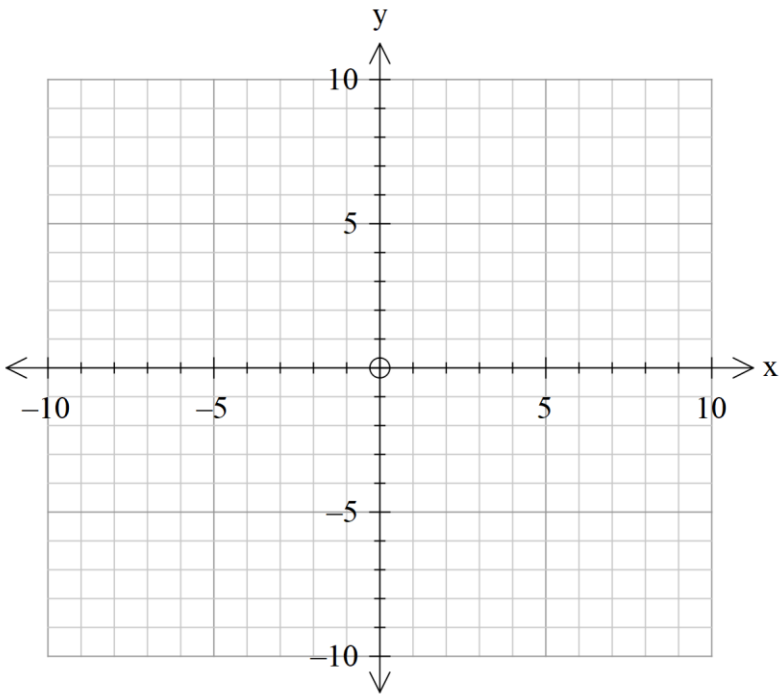
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Question 6

a) On the number plane below, graph $y = 2^{3-x} + 1$ in the domain $0 \leq x \leq 5$ **2**



b) Hence or otherwise state the range of $y = 2^{3-x} + 1$ in the domain $0 \leq x \leq 5$ **1**

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Question 7

When the polynomial P(x) is divided by (x – 2)(x + 3), the quotient is Q(x) and the remainder is R(x).

a) Explain why the most general form of R(x) = $ax + b$. **1**

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b) When P(x) is divided by (x + 3) the remainder is -1. When P(x) is divided by (x – 2) the remainder is 9. Find R(x). **3**

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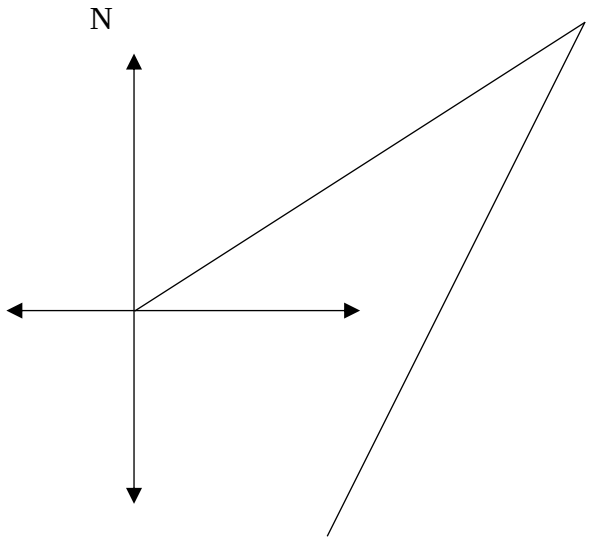
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Question 8

Caitlin sets sail from Sydney on a bearing of 053^{o}T. After travelling for 560km, she changes course at P and travels on a bearing of 190^{o}T for a further 750km until she reaches her destination at Q

a) Show this information on the diagram below **1**



b) Find the final bearing of Caitlin, from Q to Sydney **4**

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End of Exam

Year 10 Yearly 2019

TIME : 65 minutes + 5 minutes reading time

Name:

Teacher:

Directions

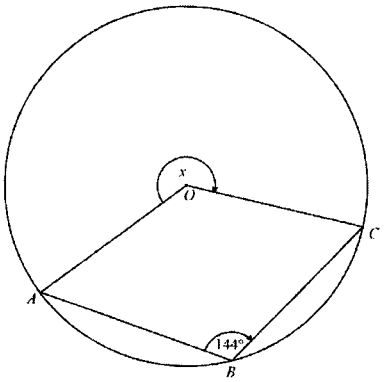
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	Section A		Section B		Section C			
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Probability		Q5		Q5 – 7 /4				
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Non Linear Relationships		Q7						Q2 /4
Sub-Total		/7		/16				/31
Total						/54		

Section A – 1 mark each

Question and Working	Answer
1. Find the perpendicular distance from the point (-3, 8) to the line $4x - 7y + 6 = 0$. Leave your answer in exact form. $\frac{4(-3)-7(8)+6}{\sqrt{4^2+7^2}}$	$\frac{38}{\sqrt{65}}$ OR $\frac{38\sqrt{65}}{65}$ must be in exact form.
2. Solve for θ, given $0 \leq \theta \leq 360$ $\sin\theta = \frac{\sqrt{3}}{2}$	$\theta = 60$ OR $120°$ must have both answers
3. Find the 40th term of the series 5, 8, 11, 14, … $5 + (40-1)3$	122

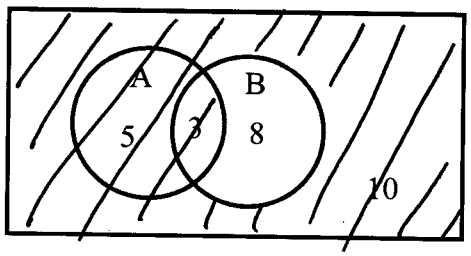
4. O is the centre of a circle. A, B and C are points on the circumference of the circle such that $\angle ABC = 144^o$.



Give a reason why $x = 288^o$.

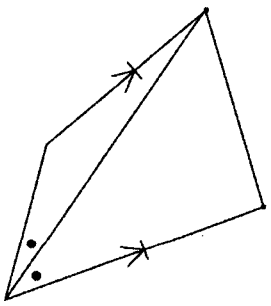
Angle at the centre is twice the angle at the circumference

5. Find P $(A \cup B')$



$\frac{9}{13}$.

6. What shape best describes this quadrilateral



Rhombus.

7. An exponential curve has a y-intercept of 1 and passes through the point $\left(4, \frac{1}{625}\right)$. Find the equation of the curve, if the x-axis is the asymptote.

$y = 5^{-x}$

OR $y = \left(\frac{1}{5}\right)^x$.

$y = a^x$ when $x = 4, y = \frac{1}{625}$

$\frac{1}{625} = a^4$

$5^{-4} = a^4$

Section B

Marks shown to the right of the question. Show all necessary working.

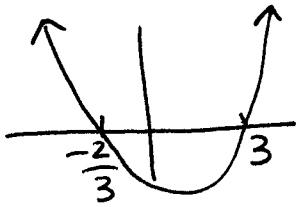
1. Solve the inequality $3x^2 - 7x - 6 > 0$ **3**

1 - Factorising correctly
1 - Sketch
1 - Answer.

$3x^2 - 9x + 2x - 6$

$= 3x(x-3) + 2(x-3)$

$= (3x+2)(x-3)$ — (1)



$y > 3$ OR $y < -\frac{2}{3}$ — (1)

2. Expand and simplify $(1 + 5x)^5$ **2**

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

(1) $1 + 5(5x) + 10(5x)^2 + 10(5x^3) + 5(5x^4) + (5x)^5$

(1) $1 + 25x + 250x^2 + 1250x^3 + 3125x^4 + 3125x^5$

3. For what values of k will the equation $2x^2 - kx + 2 = 0$ have equal roots? **2**

$k^2 - 4 \times 2 \times 2 = 0$ — (1)

$k^2 = 16$

$k = \pm 4$ — (1)

4. Let the equation $8x^2 + 7x - 5 = 0$ have roots α and β. **3**

Find

a) $\alpha + \beta$

$\frac{-7}{8}$ (1)

b) $\alpha\beta$

$\frac{-5}{8}$ (1)

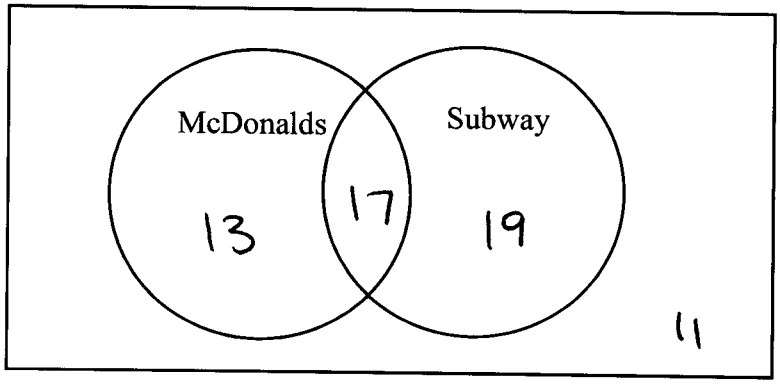
c) $\frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{-7/8}{-5/8} = \frac{7}{5}$ (1)

5. A group of 60 students are surveyed about their likes for take away food. 36 students like Subway and 30 like McDonalds while 11 dislike both Subway and McDonalds.

a) Show this information on the Venn Diagram below. **2**



b) What is the probability that a student who likes McDonalds, dislikes Subway? **1**

$\frac{13}{30}$

6. How many four letter arrangements can be made using the letters of the word CHILDREN? **1**

$8 \times 7 \times 6 \times 5 = 1680$

7. How many different ways can 7 people be seated around a circular table? **1**

$6! = 720$.

8. A company makes cylindrical medallions in two sizes. Both medallions have the same thickness. The ratio of the radii of the medallions is 5:2.

   400 large medallions can be made from a quantity of metal.

   How many small medallions can be made from the same quantity of metal? **2**

Ratio of areas = 25 : 4 (1)

$400 \times 6.25 = 2500$ (1)

Section C

Question 1

a) Emma is saving for a car. She decides to deposit $100 per month into a savings account at an interest rate of 3% p.a compounding monthly for 2 years. Will she be able to afford a $3 000 car at the end of the two years? Justify your answer by showing all working. **3**

Let A_n be the amount the nth deposit grows to

$A_1 = 100(1.0025)^{24}$
$A_2 = 100(1.0025)^{23}$
$A_3 = 100(1.0025)^{22}$ ①
$\vdots$
$A_{24} = 100(1.0025)$

Total $= 100(1.0025) + 100(1.0025)^2 + 100(1.0025)^3 + \ldots + 100(1.0025)^{24}$

∴ GP $a = 100(1.0025)$, $r = 1.0025$ $n = 24$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{100(1.0025)(1.0025^{24} - 1)}{1.0025 - 1} \quad ①$$

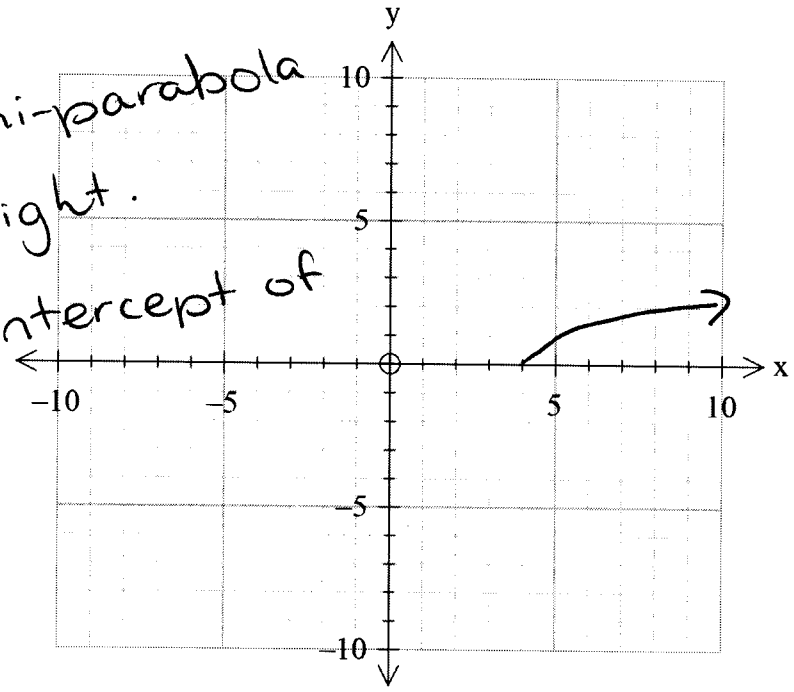
$$= \$2476.46$$

∴ She will not have enough money ①

Question 2

Graph the following equations

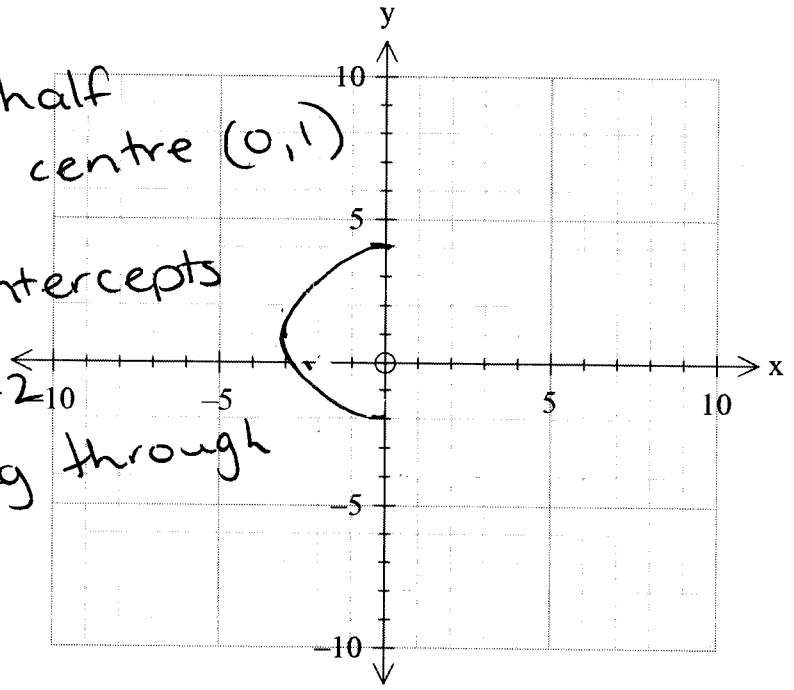
a) $y = \sqrt{x-4}$ **2**



1 for semi-parabola facing right.

1 for x intercept of 4.

b) $x = -\sqrt{9-(y-1)^2}$ **2**

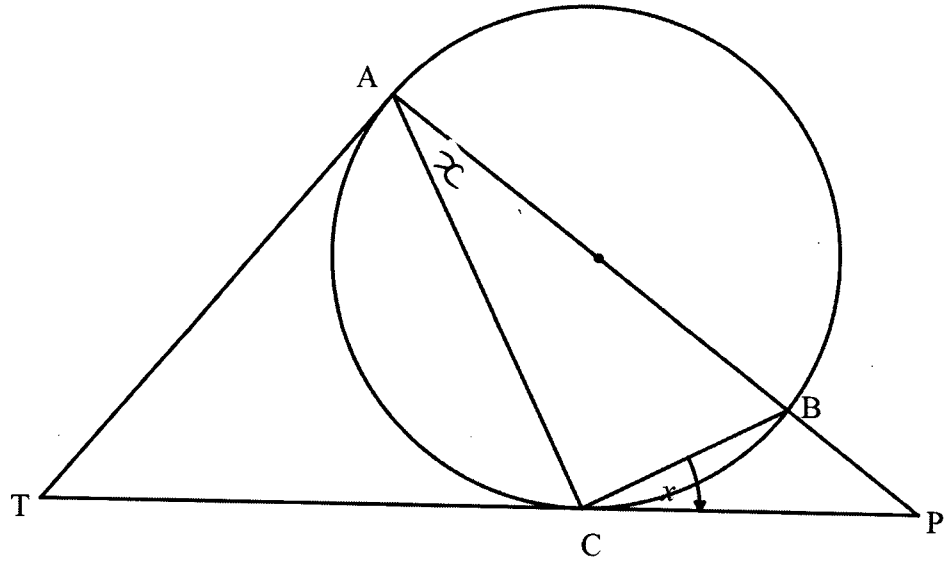


1 for left half Semi circle centre (0,1)

1 for y intercepts of 4 and -2 and passing through (-3,1).

8

Question 3



AB is a diameter of a circle ABC. The tangents at A and C meet at T. The lines TC and AB are produced to meet at P. Let $\angle BCP = x$

a) Prove that $\angle CAT = (90 - x)^\circ$ **2**

$\angle BAC = x$ (angle between a tangent and chord at point of contact is equal to the angle in the alternate segment) (1)

$\angle TAP = 90^\circ$ (angle between a diameter and tangent is 90° at the point of contact)

$\therefore \angle CAT = 90 - x$ (1)

b) Hence or otherwise, find $\angle ATC$ in terms of x **2**

$AT = TC$ (tangents from an external point to point of contact are equal) (1)

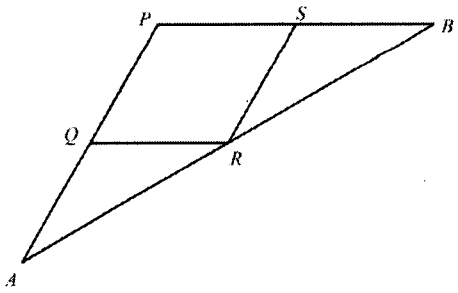
$\angle ACT = 90 - x$ (sides opposite equal angles in a triangle)

$\angle ACT + \angle CAT + \angle ATC = 180^\circ$ (angle sum of a triangle)

$\angle ATC + 90 - x + 90 - x = 180$ (1)

$\angle ATC = 2x$

Question 4



The parallelogram $PQRS$ is inscribed in ΔPBA where R lies on AB. $QA = QR$ and $PS = SB$.

a) Explain why $PS = QR$, $PQ = RS$, $PS \parallel QR$ and $PQ \parallel SR$ **1**

Opposite sides of a parallelogram are equal and parallel

b) Prove that $\Delta BSR \equiv \Delta RQA$ **3**

In ΔBSR and ΔRQA (1)

$PB \parallel QR$ (P,S,B are collinear and proven $PS \parallel QR$)

$\angle SBR = \angle QRA$ (corresponding angles, $PQ \parallel QR$)

$\angle QAR$ is common

$QR = PS$ (proven)

$PS = SB$ (given)

$\therefore QR = SB$

$\Delta BSR \equiv \Delta RQA$ (AAS) (1)

c) Hence or otherwise, prove that PQRS is a rhombus **2**

$QA = SR$ (matching sides of congruent triangles are equal) (1)

$QA = QR$ (given)

PQRS is a rhombus since PQRS is a parallelogram with one pair of adjacent sides equal (1)

Question 5

a) Show that the polynomial $P(x) = -x^3 - 2x^2 + 5x + 6$ has a root at $x = -3$ **1**

$P(-3) = -(-3)^3 - 2(-3)^2 + 5(-3) + 6$

$= 0$.

b) Hence or otherwise sketch the graph of $y = P(x)$ on the axes below. **3**

$-x^2 + x + 2$

$x+3)\overline{-x^3 - 2x^2 + 5x + 6}$

$-x^3 - 3x^2$

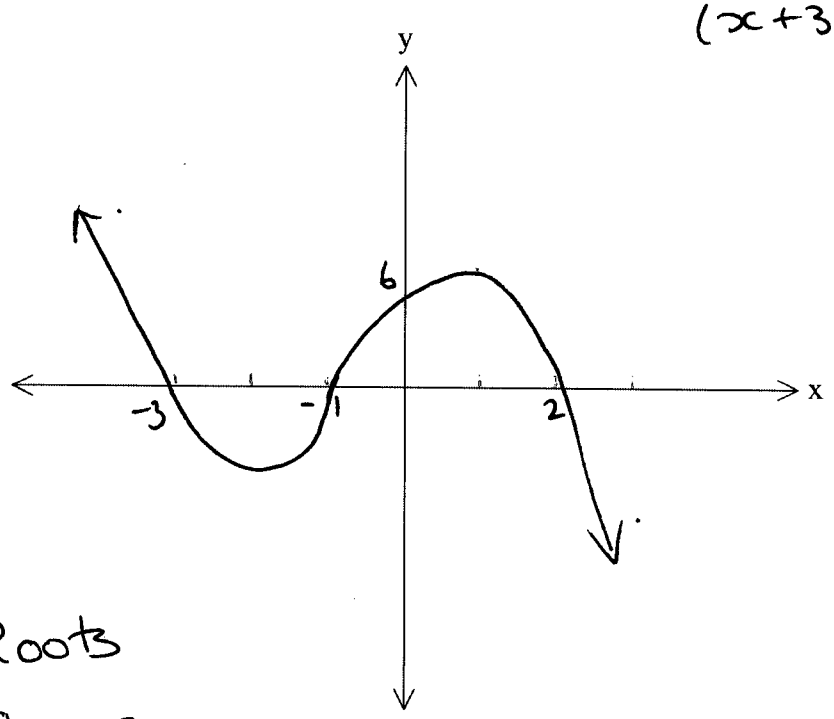
$x^2 + 5x$

$x^2 + 3x$

$2x + 6$

$2x + 6$

$(x+3)(2-x)(x+1)$.



1 - Roots

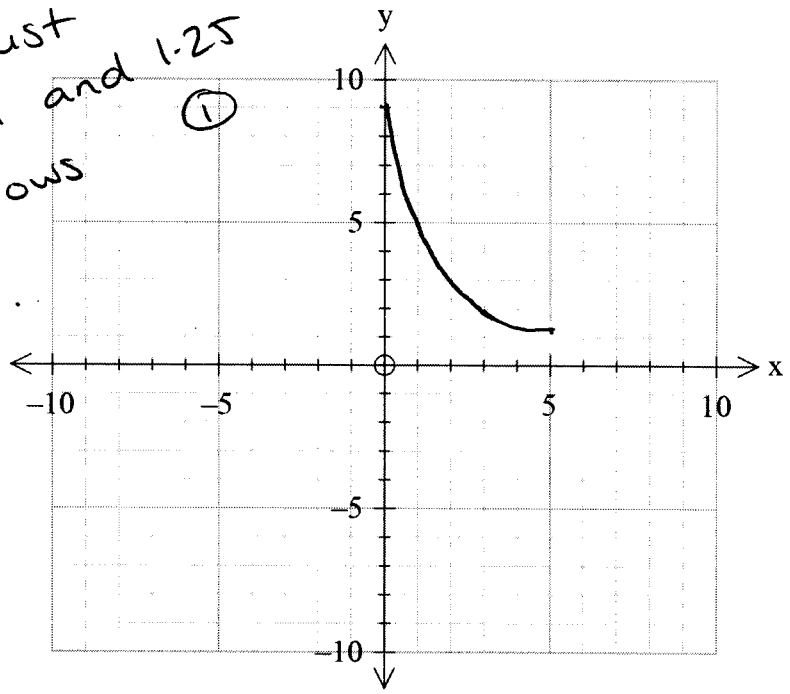
1 - Shape

must have y intercept

Question 6

a) On the number plane below, graph $y = 2^{3-x} + 1$ in the domain $0 \leq x \leq 5$ **2**

Endpoints must clearly be 9 and 1.25 (1)
with no arrows
1 for shape.



b) Hence or otherwise state the range of $y = 2^{3-x} + 1$ in the domain $0 \leq x \leq 5$ **1**

$[1.25, 9]$ or $1.25 \leq y \leq 9$

Question 7

When the polynomial P(x) is divided by $(x-2)(x+3)$, the quotient is Q(x) and the remainder is R(x).

a) Explain why the most general form of $R(x) = ax + b$. **1**

The degree of the remainder has to be less than the degree of divisor.

must use "degree"

b) When P(x) is divided by $(x+3)$ the remainder is -1. When P(x) is divided by $(x-2)$ the remainder is 9. Find R(x). **3**

$P(x) = (x-2)(x+3)Q(x) + ax+b$

$P(-3) = (-3-2)(-3+3)Q(-3) - 3a + b = -1$

$b - 3a = -1$

$P(2) = (2-2)(2+3)Q(2) + 2a + b = 9$

$2a + b = 9$ } ①

$b - 3a = -1$ }

$5a = 10$

$a = 2$.

$b - 6 = -1$.

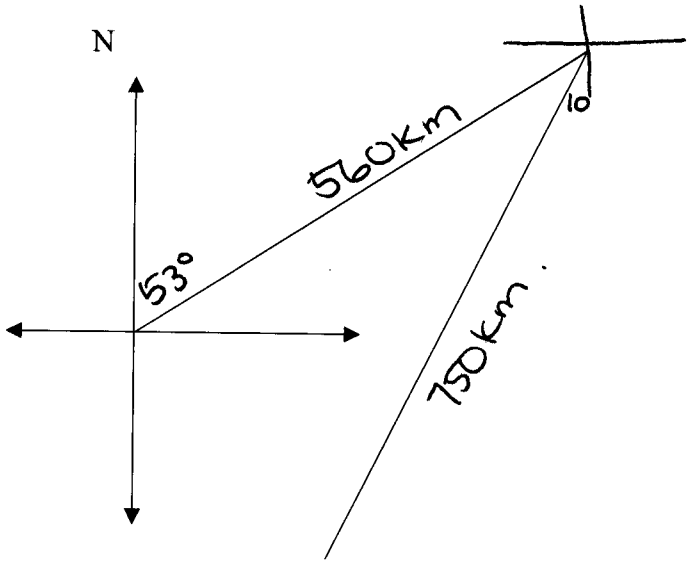
$b = 5$ ① for solving

$R(x) = 2x + 5$ ① for writing in form.

Question 8

Caitlin sets sail from Sydney on a bearing of 053°T. After travelling for 560km, she changes course at P and travels on a bearing of 190°T for a further 750km until she reaches her destination at Q

a) Show this information on the diagram below **1**



b) Find the final bearing of Caitlin, from Q to Sydney **4**

$SQ^2 = 560^2 + 750^2 - 2\times560\times750\cos 43$

$= 409804.8267$ — (1)

$SQ = 640.1600009$

$= 640.16$ km (1)

$\cos \angle PQS = \dfrac{640.16^2 + 750^2 - 560^2}{2\times640.16\times750}$

$\angle PQS = 47°$ — (1)

Bearing is $360 - 37 = 314°$ T (1)

End of Exam

Must acknowledge ambiguous case if Sine rule is used.