Sydney Technical High School

Name: .....

Maths Class: .....

Year 10 Mathematics

June 2022

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided

Section 1 Multiple Choice

Questions 1-10

10 Marks

Section II Questions 11-15

50 Marks

Total = 60 marks

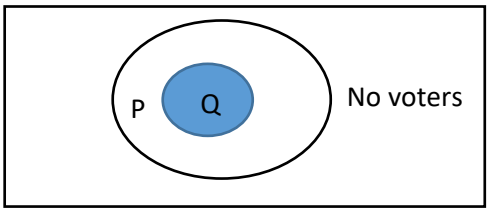
SECTION I – MULTIPLE CHOICE

Circle the letter of your choice on the Multiple Choice Answer sheet provided

You should spend about 15 minutes on this section

1	The GST is currently 10% A shirt is shown as selling for $45 plus GST. What is the cost to the customer of buying it? A. $40 B. $ 40.50 C. $49.50 D. $50
2	100 balls of the same size, with the numbers 1 to 100 written on them are placed in a barrel for a raffle draw. The first ball is drawn, and its value noted, but the person holding that winning number says she doesn't want the prize and to put the ball back and draw again. What is the chance that on the second draw, the same number comes out? A. $\frac{1}{100} \times \frac{1}{99}$ B. $\frac{1}{100} \times \frac{1}{100}$ C. $\frac{2}{100}$ D. $\frac{1}{100}$
3	Bobby normally works for a rate of $45 per hour. This week he worked 4 extra hours at time and a half. How much <u>extra</u> did he earn during this week? A. $22.50 B. $90 C. $180 D. $270
4	George is to choose one meal from <u>each</u> of the following categories to form his 3-course meal: ENTRÉES: Pasta, Bread, Cheese MAIN MEAL: Chicken, Beef, vegetarian, fish DESSERT: Ice cream, cake How many different meals can George have? A. 3 B. 6 C. 9 D. 24

5	Calculate the amount that a fridge, costing $1200 new is worth after 10 years if it depreciates at the rate of 5% p.a., giving your answer to the nearest dollar. A. $1955 B. $718 C. $114 D. Not worth anything
6	The reason why the equation $x^2 - 3x + 3 = 0$ has no solutions is because: A. Δ = 0 B. Δ > 0 C. Δ < 0 D. the coefficient of x is negative
7	Which of the following statements is correct? A. Everyone earning an income pays taxation B. Taxable Income is annual income less deductions C. Income tax is charged at the same rate to all persons earning an income D. At the end of a financial year, all taxpayers receive money back from the Taxation Department
8	A company which manafactures lightglobes has found, over time, that in any batch of globes, 1% are faulty. From a large random sample of lightglobes, 2 are selected. What is the probability that BOTH are faulty? A. $(\frac{1}{100})^2$ B. $\frac{1}{100} \times \frac{1}{99}$ C. $\frac{2}{100} \times \frac{1}{99}$ D. 0.01

9	The value of $\frac{8!}{5!}$ is given by: A. 3! B. 8 X 7 X 6 C. (8 – 5)! D. 8! – 5!
10	 The Venn Diagram above represents the voting preferences of a sample of people from the town of Undecided. The two political parties are P and Q. The Venn Diagram shows that A. All who vote for party P will also vote for party Q B. All who vote for party Q will also vote for party P C. Nobody will vote for both parties D. Everybody will vote for both parties

SECTION 2

All answers and working are to be done in the answer booklet provided.

You must show all necessary working.

Start each new question on a new page.

You should spend about 75 minutes on this section

QUESTION 11: (10 Marks)

Marks

(a) Solve the equation below giving your answer in exact terms: **2**

$$2x^2 - 3x - 5 = 0$$

(b) A normal, unbiased coin is tossed twice. What is the probability that the 2 throws result in a Head and a Tail (in any order)? **1**

(c) Calculate the Simple Interest earned on an investment of $660 at 9% per annum over 8 months. **1**

(d) (i) From the letters of the word WORONORA, a letter is chosen at random. **1**

What is the probability that it is an "O" ?

(ii) In how many arrangements can 5 people stand in a straight line? **1**

QUESTION 11 continues overleaf…..)

QUESTION 11 continued…..)

(e) A DVD was bought by a dealer for \$10 and then sold to the public for \$25. **2**

(i) What is the profit made as a percentage of the cost price?

(ii) What is the profit made as a percentage of the selling price?

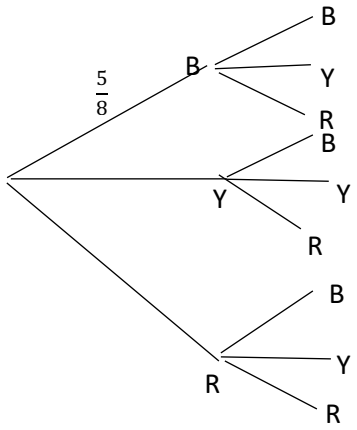
(f) Solve the equation $x^2 - 5x + 1 = 0$, giving your answer correct to 2 dec. places **2**

QUESTION 12: (10 Marks) Start a New Page

Marks

(a) A bag of coloured marbles of the same size, contains 5 Blue, 2 Yellow and 1 Red marble. 2 Random draws are to be made from the bag, with the marble colour noted and the marble replaced before the next draw.

(i) Copy the tree diagram below into your answer booklet, and fill in the probabilities onto each branch. **1**



(ii) What is the probability of drawing a red marble followed by a blue marble? **1**

(iii) What is the probability of yellow marble and a blue marble in any order? **1**

(iv) What is the probability of drawing 2 marbles of the same colour? **1**

(v) What is the probability of drawing at least one red marble? **1**

(b) Solve the equation $(3x - 4)^2 = 5$, giving your answer in exact form **2**

(c) Two successive draws are made from a standard 52-card deck of cards, with the first card NOT replaced after drawing. **1**
Find the probability of drawing 2 red cards.

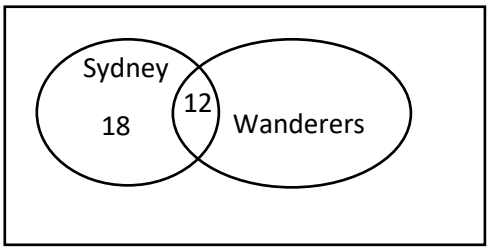
(d) Harry works for a rate of $22.50 per hour for a normal 40-hour week. **2**
At Christmas he gets paid holiday-loading of 17.5% for 4 weeks.

What does he earn <u>in total</u> over those 4 weeks?

QUESTION 13: (10 Marks) Start a new page

(a) Allen pays $180,000 into a Superannuation account. Assuming that the account consistently earns 5% per annum compound interest, What is the total value of Allen's investment after 20 years? (to the nearest dollar) **2**

(b) 50 people were surveyed to show which of 2 football teams, Sydney FC or West Sydney Wanderers, they supported. The results were displayed in the following Venn Diagram. (Note that all the 50 people supported one of the teams)



(i) How many people supported just Western Sydney Wanderers? **1**

(ii) Copy the following 2-way table into your answer booklet and then fill it in using the information from the Venn Diagram **2**

	Support Wanderers	Do not support Wanderers	Total
Support Sydney FC			
Do not support Sydney FC			
Totals			

(iii) Using the information supplied above, or otherwise, if a person was randomly selected from the 50 people, find the probability that:

(a) They supported both Sydney and the Wanderers **1**

(b) They only supported the Wanderers **1**

(c) Arthur invests $25 000 into an account earning him 5% p.a. interest compounded every 6 months over a term of 20 years. **3**

William, on the other hand, places $25 000 into an account offering him 7% per annum simple interest for the same period of 20 years.

Setting your work out in an organised fashion, determine who has the greater amount after the 20 years, and by how much, to the nearest dollar.

QUESTION 14: (10 Marks) Start a new page

(a) Two draws are made from a standard 52-card deck of cards. **2**
What is the probability of drawing 2 Kings if:

(i) The first card is replaced after the first draw

(ii) The first card is NOT replaced after the first draw.

(b) The tax table for the Australian Tax Office for 2021 is shown below:

Taxable Income	**Rate**
$0 to $18,200	Nil
$18,201 to $45,000	19 cents for every dollar in excess of $18,200
$45,001 to $120,000	$5,092 plus 32.5 cents for each dollar in excess of $45,000
$120,001 to $180,000	$29,467 plus 37 cents for each dollar in excess of $120,000
Greater than $180,001	$51,667 plus 45 cents for each dollar in excess of $180,000

Over the 2021 financial year, Shirley earned $86,250 and when it came time to complete her taxation return she found she had deductible expenses of $400 for Educational Expenses, $350 for travel incurred while working and the cost of a new work-based computer of $650.

(i) Explain how the figure of $5,092 ***(circled in the table above***) is calculated **1**
You do not have to include any calculations

(ii) Calculate Shirley's taxable income for the 2021 financial year **1**

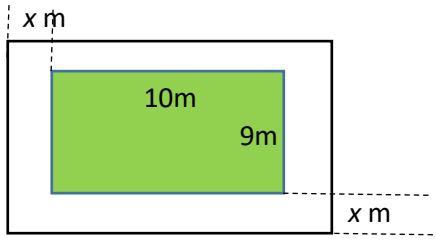
(iii) Using the table, calculate the amount of tax Shirley should have paid over the 2021 financial year **2**
(**YOU MUST SET THIS OUT)**

(iv) Shirley found she had paid taxation of $22,145 for the year. State the amount owing or returned to her. **1**

QUESTION 14 continues overleaf…….)

QUESTION 14 continued……

(c) A garden in the shape of a rectangle measures 10 m by 9 m.
A path of constant width, x m, is to be laid around the entire perimeter of the garden.



The entire garden plus path is to have an area of 182 m^2

(i) Construct an equation to show the formula for the whole area of garden and paths. **1**

(ii) Solve the equation to find the width of the path. **2**

QUESTION 15: (12 Marks) Start a new page

(a) An amount of \$5 000 is invested at 8% pa compounded annually over 10 years. **3**

At what rate of Simple interest would the \$5 000 have needed to be invested to have earned the same interest over the 10 years?

Give your answer to the nearest whole percentage

(b) 8 cards with the numbers 1 to 8 written on them are placed on a table face down. Each card has a different digit.

Three cards are chosen at random and placed face-up side by side on the table

(i) How many 3-digit numbers can be formed? **1**

(ii) What is the probability that the 3-digit number formed is even? **2**

(iii) What is the probability that the number formed is greater than 600? **1**

(c) Showing all necessary working, solve the equation **3**

$x^4 - 5x^2 - 6 = 0,$ giving your answer in exact terms

BREAKDOWN OF MARKS

TOPIC	Easy Marks	Moderate	Difficult	% Breakdown
FINANCE	10	8	4	**37%**
PROBABILITY	10	9	6	**41%**
QUADRATICS	5	5	3	**22%**
TOTALS	**26**	**22**	**13**	**60**
	42%	**37%**	**21%**	

TOTAL: 60 Marks

YEAR 10 WORKED SOLUTIONS

M/C 1/ C 2/ D 3/ D 4/ D 5/ B
6/ C 7/ B 8/ A 9/ B 10/ B

SECTION 2:

QUESTION 11:

(a) $(2x-5)(x+1)=0$

$\therefore x = 5/2$ or $x=-1$

(b) $P(E) = 1/2$

(c) $SI = 668 \times 0.09 \times 2/3$

$\approx \$39.60$

(d) $3/8$

(e) (i) $15/10 \times 100/1 = 150\%$

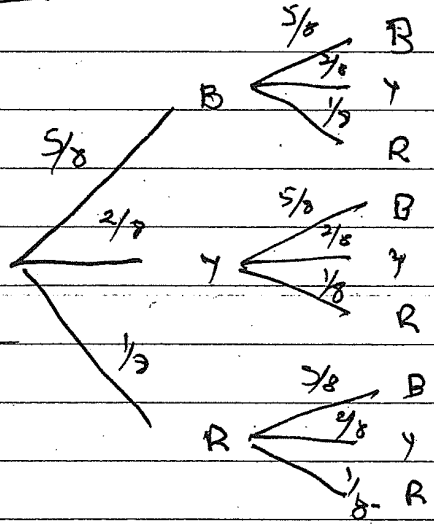
(ii) $15/25 \times 100/1 = 60\%$

(f) $x = \dfrac{5 \pm \sqrt{25-4}}{2}$

≈ 4.79 or 0.21

QUESTION 12:

(a) (i)



(ii) $5/64$

(iii) $2/8 \times 5/8 + 5/8 \times 2/8$

$= 20/64$ or $5/16$

(iv) $25/64 + 4/64 + 1/64$

$= 30/64$ or $15/32$

(v) $P(\text{at least 1 red}) = 1 - P(\text{no reds})$

$= 1 - 49/64$

$= 15/64$

(b) EITHER $3x - 4 = \pm\sqrt{5}$

$3x = 4 \pm \sqrt{5}$

$x = \dfrac{4 \pm \sqrt{5}}{3}$

OR $9x^2 - 24x + 11 = 0$

$x = \dfrac{24 \pm \sqrt{180}}{18}$

$= \dfrac{4 \pm \sqrt{5}}{3}$

(d) NORMAL EARNINGS $= 22.50 \times 40 \times 4 = 3600$

EXTRA EARNINGS $= 12.50 \times 40 \times 4 \times 0.175 = \$140$

c) $= 26/52 \times 25/51$
$= 650/2652$
$= 25/102$

QUESTION 13:

(a) $180\,000(1.05)^{20}$

$= \$477,594$

(b) (i) 20

(ii)

	s-sport wsw	Do not s-sport wsw	TOTAL	
s-sport SFC	12	18	30	
Do not SFC	20	0	20	
TOTAL	32	18	50	

(iii) (a) $12/50$ OR $6/25$

(b) $20/50 = 2/5$

(c) ARTHUR:

$25\,000(1.025)^{40}$

$\approx \$67,126.60$

WILLIAM:

$25000 \times 7/100 \times 20/1 + 25000$

$= \$10\,000$

$\therefore$ ARTHUR earns $\$7,126.60$ extra

QUESTION 14:

(a) $-1/13 \times 1/13 = 1/169$

(b) (i) $\$84,850$

(ii) $1/13 \times 3/5 = 1/221$

(iii) TAX owed $= (84,550 - 45000) \times 0.325 + 5092$

$= \$18043.25$

(iv) Tax paid $= 22145$

$\therefore$ Refund of $\$4,102$

(iv) 5092

$= (45001 - 18200) \times 0.19$

(c) (i) $(10 - 2x)/(9 + 2x) = 1/2$

(ii) $4x^2 + 38x - 92 = 0$

$\Rightarrow x = -23/2$ OR $x = 2$

$\therefore$ Path is 2m wide

QUESTION 15:

(a) $A = \$10,794.62$

Interest $= \$5795$

$\therefore R = 5795/10 \times 5000$

$\approx 12\%$

(b) $8 * 7 \times 6 = 336$

(i) $P(\text{Ace}) = 4/8$

$= 1/2$

(ii) $P(>600) = P(1^{\text{st}} \text{ card is } 6, 7, 8)$

$= 3/8$

(c) $(x^2 - 6)(x^2 + 1) = 0$

$\therefore x = \pm\sqrt{6}$

OR

Let $u = x^2$

$\therefore u^2 - 5u - 6 = 0$

$(u - 6)(u + 1) = 0$

YEAR 10 WORKED SOLUTIONS

M/C

1/ $49.50 (C)

2/ (D)

3/ $4 \times 45 \times 1\frac{1}{2} = \$270$ ∴ (D)

4/ (D)

5/ (D) $A = 1200(0.95)^{10} =$ ∴

6/ (C) $b^2 - 4ac = -3$

7/ (B)

8/ (A)

9/ (B) $\frac{8 \times 7 \times (\times 3 \ldots \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$

10/ (B)

SECTION 2:

QUESTION 11:

(a) $(2x-5)(x+1)=0$

$\therefore x = 5/2$ or $x = -1$ — 1 MARK each answer

(b) $P(E) = 1/2$ — 1 MARK

(c) $SI = 660 \times 0.09 \times 3/8 = \$39.60$ — 1 MARK

(d) (i) $3/8$ — 1 MARK

(ii) $5! = 120$ — 1 MARK each

(e) (i) $15/10 \times 100/1 = 150\%$ — 1 MARK each

(ii) $15/25 \times 100/1 = 60\%$

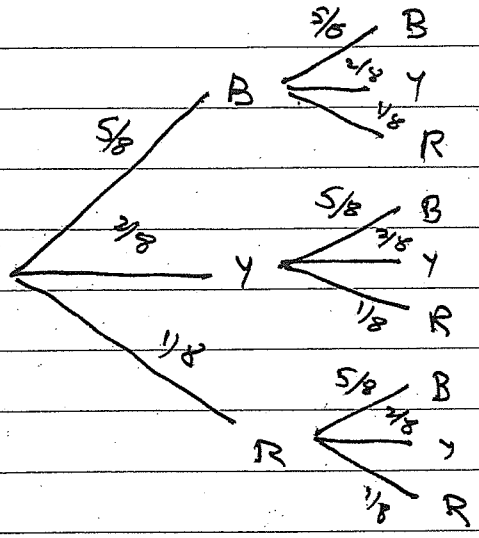
(f) $x = \frac{5 \pm \sqrt{25-4}}{2}$

≈ 4.79 or 0.21 — 1 MARK each answer (ignore the 2 decimal places)

MARKING

QUESTION 12:

(a)(i)



1 MARK for completely correct

(ii) $P(R \text{ then } B) = 5/64$ — 1 MARK

(iii) $P(Y \text{ and } B) = 2/8 \times 5/8 + 5/8 \times 2/8$

$= 20/64$ OR $5/16$ — 1 MARK for either answer

(iv) $P(2 \text{ same}) = 25/64 + 4/64 + 1/64$

$= 30/64$ OR $15/32$ — 1 MARK for either answer

(v) $P(\text{at least 1 red}) = 1 - P(\text{no reds})$

$= 1 - 49/64$

$= 15/64$ — 1 MARK

(b) EITHER: $3x - 4 = \pm\sqrt{5}$

$3x = 4 \pm \sqrt{5}$

$x = \frac{4 \pm \sqrt{5}}{3}$

OR, $9x^2 - 24x + 11 = 0$

$x = \frac{24 \pm \sqrt{576 - 396}}{18}$

$\approx \frac{24 \pm \sqrt{180}}{18}$

$= \frac{4 \pm \sqrt{5}}{3}$

1 MARK for each answer

← this line is sufficient

(c) $P(\bar{K}) = 1/2 \times 25/51$

$= 25/102$ — 1 MARK

(d) NORMAL earnings $= 22.50 \times 40 \times 4 = €3600$

Extra loadings $= 22.50 \times 40 \times 4 \times 0.175$

$= €630$

∴ TOTAL $= €4230$

2 MARKS

1 MARK only for €630

QUESTION 13:

(a) $180{,}000(1.05)^{20}$

$\doteq \$477{,}594$

1 for formula
1 for answer
(ignore repeat $)

(b) (i) 20

← 1 MARK

(ii)

	Support WSW	do NOT support WSW	TOTAL
SFC	12	18	30
do not support SFC	20	0	20
TOTAL	32	18	50

2 MARKS
(deduct 1 for incorrect entry or omission)

(iii) (a) $12/50$ OR $6/25$

1 MARK

(b) $20/50$ OR $2/5$

1 MARK

(c) ARTHUR: $25{,}000(1.025)^{40}$

$\doteq \$67{,}126.60$

1 for 40
1 for 0.025

WILLIAM: $25000 \times \frac{7}{100} \times \frac{20}{1} + 25000$

1 MARK

$\doteq \$60{,}000$

∴ ARTHUR earns $7,126.60 more

no marks.

QUESTION 14:

(a) (i) $P(2K) = \frac{1}{13} \times \frac{1}{13}$ — 1 MARK
$= \frac{1}{169}$

(ii) $P(2K) = \frac{1}{13} \times \frac{3}{51}$ — 1 MARK
$= \frac{1}{221}$

(b) (i) $5092 = 0.19 \times (45001 - 18200)$ — 1 for a reasonable explanation
$= 0.19 \times 26801$

(ii) TAXABLE I/C $= 86250 - (1410)$ — 1 MARK
$= \$84850$

(iii) TAX owed $= (84850 - 45000) \times 0.325 + 5092$ — 2 MARKS (look at subsequent error from part (ii))
≈ 18043.25

(iv) $\$22115 - 18043.00$ — 1 MARK (ignore cents)
$= \$4102$
OWING to stingy

(c) (i) $(10+2x)(9+2x) = 182$ — 1 MARK.

(ii) $4x^2 + 38x - 92 = 0$
$\therefore 2x^2 + 19x - 46 = 0$
$(2x+23)(x-2) = 0$
$\therefore x = -\frac{23}{2}$ or $x = 2$
NOT A SOLN $\therefore$ Path is 2m wide
— 2 MARKS (Pay $x = 2$)

QUESTION 15:

(a). $A = 5000(1.08)^{10}$ — 1 MARK

$\approx \$10,794.62$

$PRN = 10,795 - 5000$

$= \$5795$ — 1 MARK

$\therefore R \approx \frac{5795}{10 \times 5000}$

≈ 0.116 OR 12% — 1 MARK (Acc 11%)

(b) (i) $8 \times 7 \times 6 = 336$ — 1 MARK

(ii) $P(\text{even}) = P(\text{last cod is even})$ — 1 MARK

$= 4/8$ — 1 MARK

$= 1/2$

(iii) $P(>600) = P(\text{first cod is } 6, 7, 8)$ — 1 MARK

$= 3/8$

(c) EITHER:

$(x^2 - 6)(x^2 + 1) = 0$ — 3 MARKS

$\therefore x = \pm\sqrt{6}$

OR/

Let $u = x^2$

$\therefore u^2 - 5u - 6 = 0$

$(u - 6)(u + 1) = 0$ — 1 MARK

$\therefore u = 6$ OR $u = -1$

$\therefore x^2 = 6$ OR $x^2 = -1$ — 1 for dismissing $x^2 = -1$

↑ NO SOLN

$\therefore x = \pm\sqrt{6}$ — 1 MARK