NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL



2019 Half-Yearly Examination

FORM IV

MATHEMATICS

Tuesday 28th May 2019

General Instructions

- Writing time — 2 hours.
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 108 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

4A: RR	4B: REJ	4C: RCF	4D: OMD
4E: BR	4F: LYL	4G: PKH	4H: SDP
4I: LL	4J: NJL		

Checklist

- Writing paper — approx. 18 sheets
- Candidature — 188 boys

Examiner

RR

QUESTION ONE (12 marks) Start a new page. **Marks**

(a) Simplify:

(i) $\sqrt{28}$ **1**

(ii) $2\sqrt{2} \times \sqrt{13}$ **1**

(b) Expand $3a(5 - 2a)$. **1**

(c) Solve:

(i) $x^2 = 36$ **1**

(ii) $(x - 2)(x + 1) = 0$ **1**

(d) What is the gradient of the line $y = \frac{1}{2}x + 3$? **1**

(e) Factorise $x^2 + 3x + 2$. **1**

(f) Rationalise the denominator of $\frac{5}{\sqrt{2}}$. **1**

(g) Solve $2x + 6 < 24$. **1**

(h) A cube has side length 5 cm. What is its volume? **1**

(i) What is the y-intercept of the line with equation $2x + 5y - 10 = 0$? **1**

(j) A particular parabola is concave up and has its vertex at $(0, -3)$. **1**
How many x-intercepts does it have?

Examination continues next page ...

QUESTION TWO (12 marks) Start a new page. **Marks**

(a) Expand and simplify $(x-2)(3x+5)$. **2**

(b) Find the exact distance between the points $P(-2,4)$ and $Q(4,2)$. **2**

(c) Find the height of a cylinder which has a volume of $60\,\text{cm}^3$ and a radius of $2\,\text{cm}$. Give your answer correct to one decimal place. **2**

(d) A rectangular prism has side lengths $3\,\text{cm}$, $5\,\text{cm}$ and $6\,\text{cm}$. What is its surface area? **2**

(e) Simplify $\sqrt{50}-\sqrt{8}$. **2**

(f) Solve $x^2-3x-40=0$ using any appropriate method. **2**

QUESTION THREE (12 marks) Start a new page. **Marks**

(a) Find the midpoint of the points $A(-3,2)$ and $B(-1,-8)$. **2**

(b) A particular parabola has a single x-intercept $A(3,0)$ and a y-intercept $B(0,-2)$.

(i) Sketch this parabola, showing the points A and B. **1**

(ii) State the concavity of this parabola. **1**

(iii) What is the equation of the axis of symmetry? **1**

(iv) Use symmetry to find a third point on the parabola. **1**

(c) Solve $x^2+8x-4=0$ by completing the square. Leave your answers in simplest exact form. **2**

(d) Use the quadratic formula to solve $2x^2-6x+3=0$. Express your answer in simplest exact form. **2**

(e) A scale model of a bus has a length of $6\,\text{cm}$, and a surface area of $40\,\text{cm}^2$. If the actual bus has a length of $12\,\text{m}$, what is its surface area? **2**

Examination continues overleaf ...

QUESTION FOUR (12 marks) Start a new page. **Marks**

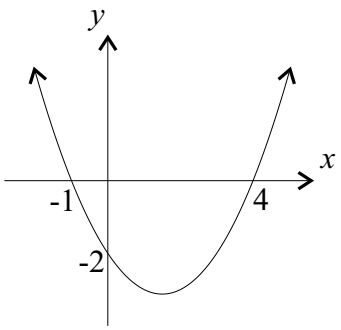
(a) Rationalise the denominator of $\dfrac{6}{3-\sqrt{5}}$. **2**

(b) Casey chooses a number. When she squares the number it is 110 more than the original number. What could the original number have been? **2**

(c) (i) Find the gradient of the line through $A(2,-2)$ and $B(-1,3)$. **1**

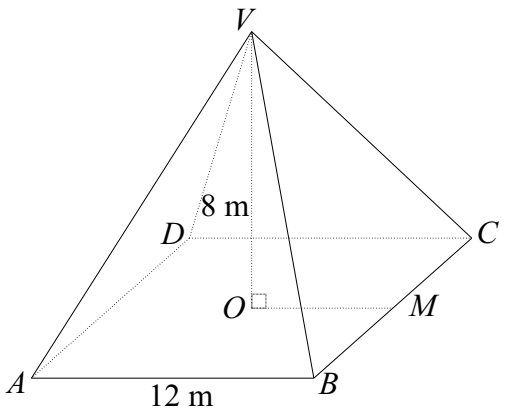
(ii) Find the equation of the line parallel to AB through the point $(1,-1)$. Give your answer in general form. **2**

(d) **2**



Determine the equation of the parabola graphed above. You may leave your answer in factored form.

(e)



A right square-based pyramid has a perpendicular height of 8 m and a base length of 12 m as shown above.

(i) Calculate the volume of the pyramid. **1**

(ii) Calculate the surface area of the pyramid. **2**

Examination continues next page ...

QUESTION FIVE (12 marks) Start a new page. **Marks**

(a) Consider the equation $6x^2 + x - 2 = 0$.

(i) Calculate the value of the discriminant, $b^2 - 4ac$. **1**

(ii) What does this tell us about the number of the solutions to the equation? If solutions exist, are they rational or irrational? **2**

(b) Expand and simplify $\left(3\sqrt{2} + 5\right)\left(3 - \sqrt{2}\right)$. **2**

(c) Simplify $\dfrac{2x^2 - 18}{x^2 + 3x - 18}$. **2**

(d) Consider the parabola given by $y = x^2 + 3x + 6$.

(i) With reference to the value of $b^2 - 4ac$, explain why there are no x-intercepts. **1**

(ii) What is the minimum value that this parabola attains? **2**

(e) A cylinder of modelling clay with radius 6 cm and height 20 cm is molded completely into a sphere. Find the radius of the sphere correct to two decimal places. **2**

QUESTION SIX (12 marks) Start a new page. **Marks**

(a) Factorise fully $a^3 + a^2b - ab^2 - b^3$ by grouping in pairs. **2**

(b) Make t the subject of the following equation. **2**

$$m = \frac{t}{1 + t}$$

(c) Consider the parabola given by the equation $y = x^2 - 5x - 6$.

(i) State the y-intercept. **1**

(ii) Find the two x-intercepts. **1**

(iii) Determine the coordinates of the vertex. **2**

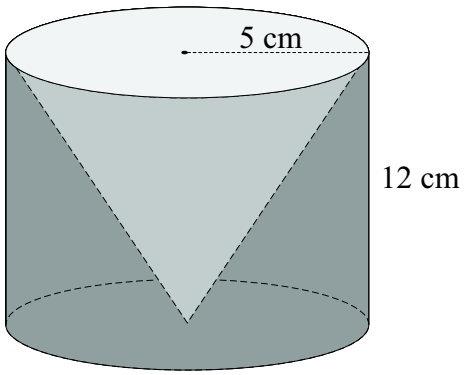
(iv) Sketch the parabola, clearly showing the information in parts (i) to (iii). **2**

(d) Arjun discovers that he has a large collection of coins in his piggy bank. **2**
Of the 80 coins inside, some of them are 20-cent pieces, and some are 50-cent pieces.
The total value of these coins is \$31.90.
By forming a pair of simultaneous equations, calculate how many of each coin he has.

Examination continues overleaf ...

QUESTION SEVEN (12 marks) Start a new page. **Marks**

(a) **2**



A cylinder with radius 5 cm and height 12 cm has a conical hole drilled out, as shown in the diagram above. What is the surface area of the resulting solid?
Give your answer in terms of π.

(b) Solve the following equation, expressing your answer with a rational denominator. **2**

$$\frac{4}{x\sqrt{3}} - 1 = \sqrt{3}$$

(c) Simplify $\dfrac{4-x^2}{x^2+x-6} \div \dfrac{x^2-x-12}{x^2+6x+9}$. **3**

(d) By completing the square, solve the following equation for x in terms of t. **2**

$$x^2 - \left(2\sqrt{t+3}\right) x = 3$$

(e) The points $A(-1,0)$, $B(0,3)$, $C(6,1)$, and $D(5,-2)$ form a quadrilateral. Verify that this is a rectangle by showing:

(i) the diagonals AC and BD are equal in length, and **1**

(ii) the diagonals bisect each other. **2**

Examination continues next page ...

QUESTION EIGHT (12 marks) Start a new page. **Marks**

(a) Consider the line l with equation $2x + 3y - 5 = 0$ and the point $P(3, 2)$.

(i) Find the equation of the line perpendicular to l through the point P. Give your answer in general form. **2**

(ii) Find the point of intersection between l and this perpendicular line. **2**

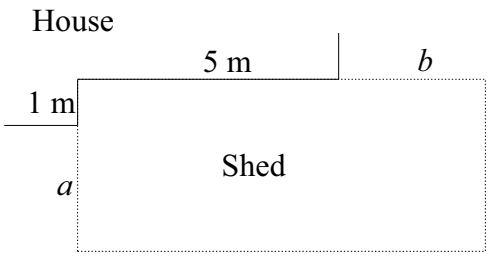
(iii) Hence find the shortest distance between the line l and the point P. Give your answer in simplest exact form with a rational denominator. **1**

(b) Consider the equation $4x - \dfrac{4}{x-1} = 4 + 7\sqrt{2}$.

(i) By letting $x = u + 1$, show that $4u^2 - 7\sqrt{2}\,u - 4 = 0$. **1**

(ii) Hence find the values of x that satisfy the original equation. **2**

(c)



Shannon wishes to attach a shed with a rectangular base to his house. He has 28 metal panels to use which are all 1 m wide. Two existing walls, of length 5 m and 1 m, will form part of the structure as shown in the diagram above. Let the number of panels extending from the 1 m wall be given by a, and the number of panels extending from the 5 m wall be given by b.

(i) Show that $b = 11 - a$. **1**

(ii) Write an expression for the area of the base in terms of a only. **1**

(iii) What is the maximum possible area of the base of the shed given that the metal panels cannot be divided into smaller parts? **1**

(iv) What are the dimensions of the base that produce the largest area? **1**

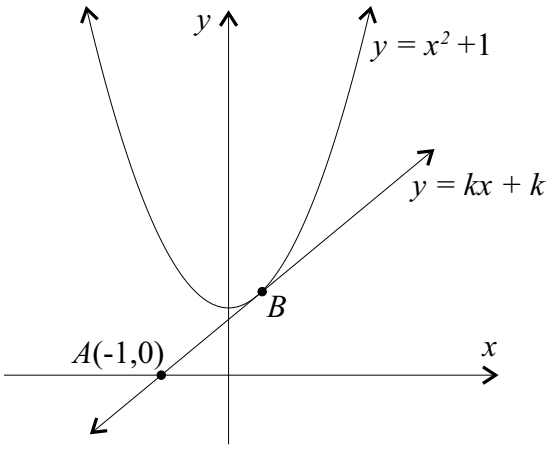
Examination continues overleaf ...

QUESTION NINE (12 marks) Start a new page. **Marks**

(a) Expand and simplify the expression $(a-b)^3$. **1**

(b) Solve the equation $\sqrt{1-x}-\sqrt{1+x}=1$. **3**

(c)



Consider the parabola given by the equation $y=x^2+1$ as shown above. The line l through $A(-1,0)$ has equation $y=kx+k$ for some value of k and is tangent to the parabola at some point B in the first quadrant. That is to say, there is only one intersection point between the line and the parabola.

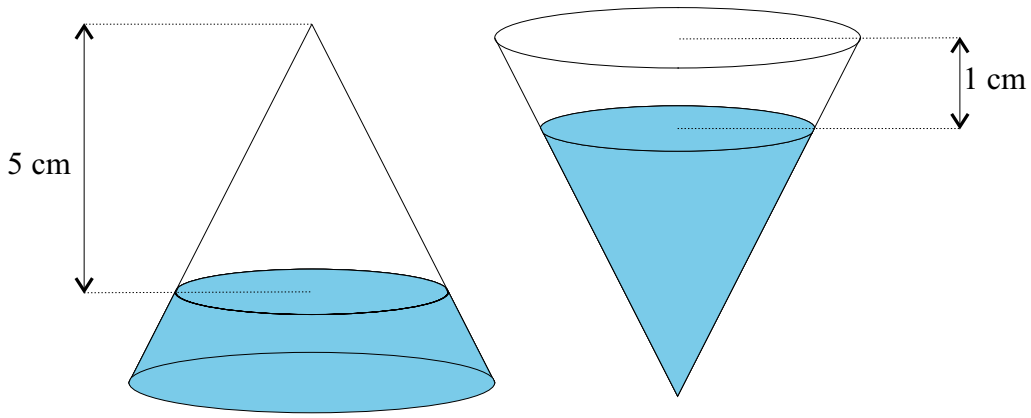
(i) Show that k must satisfy the equation $k^2+4k-4=0$. **1**

(ii) Hence find the value of k, and the coordinates of the point B. **2**

(iii) The point C, which is distinct from B, also lies on the parabola such that the line AC is a tangent to the parabola. What are the coordinates of C? **1**

(iv) By considering the midpoint of BC, or otherwise, find the area of $\triangle ABC$. **1**

(d) **3**



A right-conical container is partially filled with water. When it rests on its base the water level sits 5 cm below the vertex. When it is upside down, the water level is 1 cm below the base. By considering similar figures, or otherwise, determine the height of the cone correct to two decimal places.

END OF EXAMINATION

Form IV Half-Yearly 2019 Solutions

Question One

ai) $\sqrt{28} = 2\sqrt{7}$ ✓

ii) $2\sqrt{2} \times \sqrt{13} = 2\sqrt{26}$ ✓

b) $3a(5-2a) = 15a - 6a^2$ ✓

ci) $x^2 = 36$

$\Rightarrow x = \pm 6$ ✓

ii) $(x-2)(x+1) = 0$

$\Rightarrow x = -1$ or $x = 2$ ✓

d) $m = \frac{1}{2}$ ✓

e) $x^2 + 3x + 2 = (x+1)(x+2)$ ✓

f) $\frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$ ✓

g) $2x + 6 < 24$

$\Rightarrow x < 9$ ✓

h) $\ell = 5\,cm$

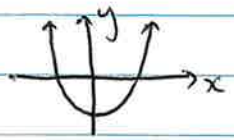
$\Rightarrow V = 125\,cm^3$ ✓ (Do not penalise units here)

i) When $x = 0$

$2x + 5y - 10 = 0$

$\Rightarrow 5y - 10 = 0$

$\Rightarrow y = 2 \rightarrow (0,2)$ ✓ (Allow $y = 2$)

j) 

two x-intercepts ✓

Question Two

a) $(x-2)(3x+5) = 3x^2 + 5x - 6x - 10$ ✓ FOIL present

$= 3x^2 - x - 10$ ✓

b) $d = \sqrt{(-2-4)^2 + (4-2)^2}$ ✓ correct formula

$= \sqrt{36+4}$

$= \sqrt{40}$

$= 2\sqrt{10}$ units ✓

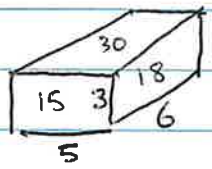
c) ~~c)~~ $V = \pi r^2 h$

If $V = 60$ & $r = 2$

$60 = \pi(2)^2 \times h$ ✓ correct substitution

$h = \frac{60}{4\pi}$

$\div 4.8$ cm ✓ penalise rounding here only.

d) 

$SA = 2 \times (3 \times 5) + 3 \times 6 + 5 \times 6)$ ✓

$= 2 \times (15 + 18 + 30)$

$= 126\ cm^2$ ✓ do not penalise units here.

e) $\sqrt{50} - \sqrt{8} = 5\sqrt{2} - 2\sqrt{2}$ ✓

$= 3\sqrt{2}$ ✓

f) $x^2 - 3x - 40 = 0$

$(x-8)(x+5) = 0$

$x = -5$ or $x = 8$ ✓

p	-40
s	-3
f	$5, -8$

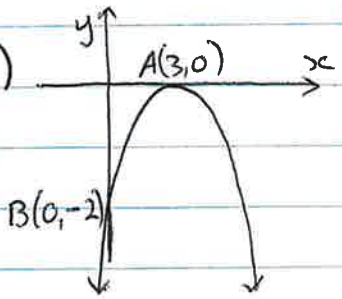
✓ or otherwise

Question Three

a) $M\left(\frac{-3+(-1)}{2}, \frac{2+(-8)}{2}\right)$ ✓ correct formula

$= M(-2,-3)$ ✓

b) i)



✓ correct shape/position

ii) Concave down ✓

iii) $x = 3$ ✓

iv) $(6,-2)$ ✓

c) $x^2 + 8x - 4 = 0$

$x^2 + 8x + 16 = 20$

$(x+4)^2 = 20$ ✓

$x + 4 = \pm\sqrt{20}$

$x = -4 \pm 2\sqrt{5}$ ✓

d) $2x^2 - 6x + 3 = 0$

$x = \frac{6 \pm \sqrt{36-24}}{4}$ ✓

$= \frac{6 \pm 2\sqrt{3}}{4}$

$= \frac{3 \pm \sqrt{3}}{2}$ ✓

e) Scale Ratio is $k = \frac{1200}{6}$

$= 200$ ✓

$SA = 40 \times 200 \times 200$

$= 1600\,000\ cm^2$ } Accept either

$= 160\ m^2$ ✓

Question Four

a) $\frac{6}{3-\sqrt{5}} = \frac{6(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$ ✓

$= \frac{18+6\sqrt{5}}{4}$

$= \frac{9+3\sqrt{5}}{2}$ ✓

b) Let the number be x.

$x^2 = x + 110$

$x^2 - x - 110 = 0$ ✓

$(x-11)(x+10) = 0$

$x = 11$ or $x = -10$ ✓

The number was either -10 or 11.

c) i) $m = \frac{-2-3}{2-(-1)}$

$= \frac{-5}{3}$ ✓

ii) $y-(-1) = \frac{-5}{3}(x-1)$ ✓

$3y + 3 = -5x + 5$

$5x + 3y - 2 = 0$ ✓

d) ~~d) i)~~ $y = a(x+1)(x-4)$ ✓ Use of x-intercepts to write linear factors

Substitute $(0,-2)$

$-2 = a(1)(-4)$

$a = \frac{1}{2}$

$y = \frac{1}{2}(x+1)(x-4)$ ✓

e) i) $V = \frac{1}{3} \times 12^2 \times 8$

$= 384m^3$ ✓

Question Four (cont.)

e) ii) $VM^2 = VO^2 + OM^2$

$= 8^2 + 6^2$

$= 100$

so $VM = 10m$ ✓

hence $S = 12^2 + 4 \times (\frac{1}{2} \times 12 \times 10)$

$= 144 + 240$

$= 384\ m^2$ ✓ penalise units here only.

Question Five

a) i) $6x^2 + x - 2 = 0$

$b^2 - 4ac = 1^2 - 4(6)(-2)$

$= 1 + 48$

$= 49$ ✓

ii) Since 49 is greater than zero there are <u>two</u> ✓ real solutions. Furthermore, as 49 is square, they are both <u>rational</u>. ✓

b) $(3\sqrt{2} + 5)(3 - \sqrt{2}) = 9\sqrt{2} - 6 + 15 - 5\sqrt{2}$ ✓

$= 9 + 4\sqrt{2}$ ✓

c) $\frac{2x^2 - 18}{x^2 + 3x - 18} = \frac{2(x^2 - 9)}{(x+6)(x-3)}$ ✓ For any one factorisation

$= \frac{2(x-3)(x+3)}{(x+6)(x-3)}$

$= \frac{2x+6}{x+6}$ ✓ Accept factored.

d) i) $y = x^2 + 3x + 6$

$\Delta = 9 - 4 \times 1 \times 6$

$= -15$ ✓ requires both calculation and explanation.

As $b^2 - 4ac$ is <u>negative</u>, there are no real solutions.

ii) Min occurs when $x = \frac{-b}{2a}$

$x = \frac{-3}{2}$ ✓

$y = \left(\frac{-3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) + 6$

$= \frac{15}{4}$ ✓

Question Five (cont.)

e) $V = \pi \times 6^2 \times 20$

$= 720\pi \text{ cm}^3$ ✓

Let $\frac{4}{3}\pi r^3 = 720\pi$

$r^3 = 540$

$r \doteq 8.14 \text{ cm}$ ✓

Question Six

a) $a^3 + a^2b - ab^2 - b^3 = a^2(a+b) - b^2(a+b)$ ✓

$= (a^2 - b^2)(a+b)$

$= (a-b)(a+b)^2$ ✓

b) $m = \frac{t}{1+t}$

$m + mt = t$ ✓

$mt - t = -m$

$(m-1)t = -m$

$t = \frac{-m}{m-1}$

$t = \frac{m}{1-m}$

✓ Accept either

c) $y = x^2 - 5x - 6$

i) $(0, -6)$ ✓

ii) Let $x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = -1$ or $x = 6$

So x-ints at $(-1, 0)$ and $(6, 0)$ ✓

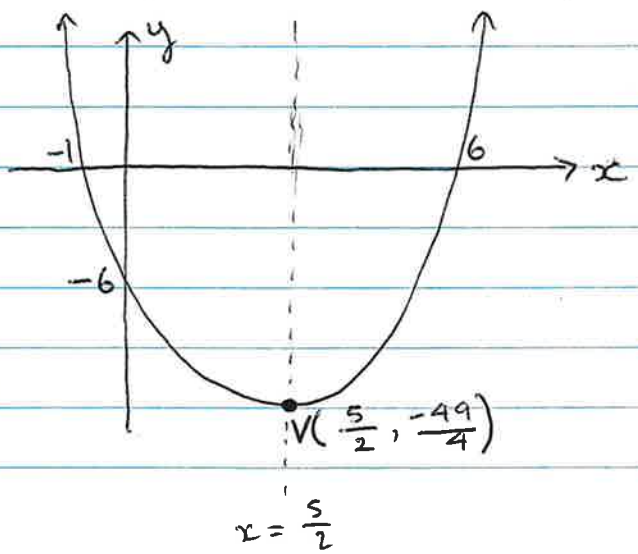
iii) $x = \frac{5}{2}$ ✓ is the equation of the axis of symmetry.

when $x = \frac{5}{2}$

$y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) - 6$

$= -\frac{49}{4}$ ✓ So $V\left(\frac{5}{2}, -\frac{49}{4}\right)$

iv)



✓ Correct shape/position

✓ All information shown

(Axis of symmetry not required)

Question Six (cont.)

d) Let there be x $20^{\text{¢}}$ coins & y $50^{\text{¢}}$ coins.

$$x + y = 80 \quad (1)$$
$$0.2x + 0.5y = 31.90 \quad (2)$$

} ✓ For both equations

$2 \times (2)$: $0.4x + y = 63.80 \quad (3)$

$(1) - (3)$: $0.6x = 16.20$

$x = 27$

There are 27 $20^{\text{¢}}$ coins and 53 $50^{\text{¢}}$ coins. ✓

Question Seven

a) Slant length $= \sqrt{5^2 + 12^2}$

$= 13\text{cm}$

Surface Area $= \pi r^2 + 2\pi rh + \pi r\ell$ ✓

$= 25\pi + 120\pi + 65\pi$

$= 210\pi \text{ cm}^2$ ✓

b) $\frac{4}{x\sqrt{3}} - 1 = \sqrt{3}$

$\frac{4}{x\sqrt{3}} = 1 + \sqrt{3}$

$\frac{4}{\sqrt{3}(1+\sqrt{3})} = x$

$x = \frac{4}{3+\sqrt{3}}$ ✓ $\times \frac{3-\sqrt{3}}{3-\sqrt{3}}$

$= \frac{12 - 4\sqrt{3}}{9-3}$

$= \frac{12-4\sqrt{3}}{6}$

$= \frac{6-2\sqrt{3}}{3}$ ✓

c) $\frac{4-x^2}{x^2+x-6} \div \frac{x^2-x-12}{x^2+6x+9} = \frac{(2-x)(2+x)}{(x+3)(x-2)} \times \frac{(x+3)^2}{(x-4)(x+3)}$ ✓ At least two factorisations

$= \frac{(2-x)(2+x)}{(x-2)(x-4)}$ ✓ cancelling $\frac{2-x}{x-2} = -1$

$= \frac{-(2+x)}{x-4}$

$= \frac{x+2}{4-x}$ ✓ Accept $\frac{-x-2}{x-4}$

Question Seven (cont.)

d) $x^2 - (2\sqrt{t+3})x = 3$

$x^2 - (2\sqrt{t+3})x + t+3 = t+6$ ✓

$(x - \sqrt{t+3})^2 = t+6$

$x - \sqrt{t+3} = \pm\sqrt{t+6}$

$x = \sqrt{t+3} \pm \sqrt{t+6}$ ✓

e) i) $AC = \sqrt{(-1-6)^2 + (0-1)^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$BD = \sqrt{(0-5)^2 + (3-(-2))^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$= AC$ ✓ Both shown

ii) Midpoint of AC: $M_1\left(\frac{-1+6}{2}, \frac{0+1}{2}\right)$

$= \left(\frac{5}{2}, \frac{1}{2}\right)$ ✓ At least one midpoint found

Midpoint of BD: $M_2\left(\frac{0+5}{2}, \frac{3+(-2)}{2}\right)$

$= \left(\frac{5}{2}, \frac{1}{2}\right)$

Hence $M_1 = M_2$ so the diagonals intersect at their midpoints, and hence bisect each other. ✓ correct justification

Question Eight

a)i) l: $2x + 3y - 5 = 0$

$\Rightarrow \quad y = \frac{-2}{3}x + \frac{5}{3}$

so l has gradient $\frac{-2}{3}$

$m_{\perp} = \frac{3}{2}$, ✓

so the new line through $P(3,2)$ is

$y - 2 = \frac{3}{2}(x-3)$

$2y - 4 = 3x - 9$

$\Rightarrow \quad 3x - 2y - 5 = 0$ ✓

ii) Solve: $2x + 3y - 5 = 0$ (1)

$3x - 2y - 5 = 0$ (2)

Note this means $2x + 3y - 5 = 3x - 2y - 5$

$\Rightarrow \quad 5y = x$ (3)

sub (3) into (1):

$10y + 3y - 5 = 0$

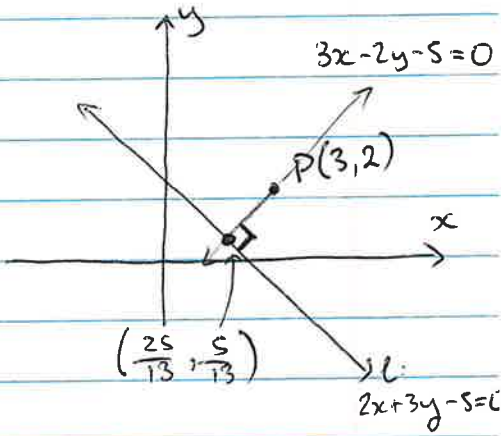
$13y = 5$

$y = \frac{5}{13}$ ✓

and so $x = \frac{25}{13}$ by (3).

$\left(\frac{25}{13}, \frac{5}{13}\right)$ ✓

iii) $\left(\frac{25}{13}, \frac{5}{13}\right)$ is the closest point to P.



$$d = \sqrt{\left(3 - \frac{25}{13}\right)^2 + \left(2 - \frac{5}{13}\right)^2}$$

$$= \sqrt{\left(\frac{14}{13}\right)^2 + \left(\frac{21}{13}\right)^2}$$

(Noting that $\left(\frac{7}{13}\right)^2$ is a common factor)

$$= \frac{7}{13}\sqrt{4+9}$$

$$= \frac{7\sqrt{13}}{13} \text{ units}$$ ✓

Question Eight (cont.)

b) $4x - \frac{4}{x-1} = 4 + 7\sqrt{2}$

i) $4(u+1) - \frac{4}{u+1-1} = 4 + 7\sqrt{2}$

$4u + 4 - \frac{4}{u} = 4 + 7\sqrt{2}$

$4u - \frac{4}{u} = 7\sqrt{2}$ ✓

$4u^2 - 4 = 7\sqrt{2}\,u$

$4u^2 - 7\sqrt{2}\,u - 4 = 0$

ii) $u = \frac{7\sqrt{2} \pm \sqrt{98 + 64}}{8}$

$= \frac{7\sqrt{2} \pm \sqrt{162}}{8}$ ✓

$= \frac{7\sqrt{2} \pm 9\sqrt{2}}{8}$

$u = 2\sqrt{2}$ or $u = \frac{-\sqrt{2}}{4}$

$x = u + 1$

so $x = 2\sqrt{2} + 1$ or $x = 1 - \frac{\sqrt{2}}{4}$ ✓

c) i) $a + b + (a + 1) + (b + 5) = 28$ ✓

$2a + 2b = 22$

$2b = 22 - 2a$

$b = 11 - a$

ii) Area $= (a+1)(b+5)$

$= (a+1)(11-a+5)$

$= (a+1)(16-a)$ } Accept either ✓

$= 16 + 15a - a^2$ }

Question Eight (cont.)

c) iii) Area $= -a^2 + 15a + 16$

This is a concave down parabola so maximum occurs at vertex.

So $a = \frac{-15}{-2}$

$= 7.5$

but a is whole, so $a = 7$ or $a = 8$ (both equal by symmetry).

If $a = 7$

Area $= (7+1)(16-7)$

$= 8 \times 9$

$= 72m^2$ ✓

iv) The dimensions will be 8m x 9m ✓ (or 9m x 8m)

Question Nine

a) $(a-b)^3 = (a-b)(a-b)(a-b)$

$= (a^2-2ab+b^2)(a-b)$

$= a^3-2a^2b+ab^2-a^2b+2ab^2-b^3$

$= a^3-3a^2b+3ab^2-b^3$ ✓

b) $\sqrt{1-x}-\sqrt{1+x}=1$

Squaring gives

$(1-x)-2\sqrt{(1-x)(1+x)}+(1+x)=1$

$2-2\sqrt{1-x^2}=1$ ✓

$1=2\sqrt{1-x^2}$

Squaring again gives

$1=4(1-x^2)$

$1=4-4x^2$

$\Rightarrow 4x^2=3$

$x^2=\frac{3}{4}$ ✓

$x=\pm\frac{\sqrt{3}}{2}$

But check

If $x=\frac{\sqrt{3}}{2}$

$\sqrt{1-\frac{\sqrt{3}}{2}}-\sqrt{1+\frac{\sqrt{3}}{2}}=-1$ (check on calculator)

If $x=\frac{-\sqrt{3}}{2}$

$\sqrt{1-\frac{-\sqrt{3}}{2}}-\sqrt{1+\frac{-\sqrt{3}}{2}}=1$ (check on calculator)

so $x=\frac{-\sqrt{3}}{2}$ is only solution. ✓

Question Nine (cont.)

c) i) At the intersection points $y = x^2 + 1$

and $y = kx + k$

Solving simultaneously gives

$$x^2 + 1 = kx + k$$

$$x^2 - kx + (1-k) = 0$$

If there is only one solution then $\Delta = 0$, so

$$k^2 - 4(1-k) = 0$$

$$\Rightarrow k^2 + 4k - 4 = 0$$

ii) $k = \frac{-4 \pm \sqrt{16+16}}{2}$

$= \frac{-4 \pm 4\sqrt{2}}{2}$

$= -2 \pm 2\sqrt{2}$

and $k > 0$ (as B is in the first quadrant)

so $k = -2 + 2\sqrt{2}$ ✓

Now $x^2 - kx + (1-k) = 0$ gives

$$x = \frac{k \pm \sqrt{\Delta}}{2}$$

so if $k = -2 + 2\sqrt{2}$ and $\Delta = 0$ then

$x = -1 + \sqrt{2}$

and $y = x^2 + 1$

so $y = (-1+\sqrt{2})^2 + 1$

$= 4 - 2\sqrt{2}$

$B(-1+\sqrt{2}, 4-2\sqrt{2})$ ✓

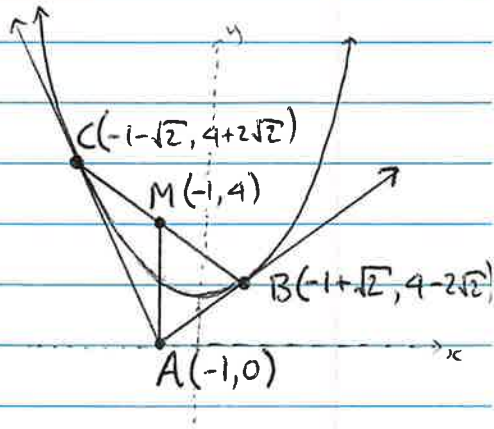
iii) By similar reasoning, C occurs when $k = -2 - 2\sqrt{2}$.

This gives $x = -1 - \sqrt{2}$

and $y = 4 + 2\sqrt{2}$

so $C(-1-\sqrt{2}, 4+2\sqrt{2})$ ✓

Question Nine (cont.)



c) iv) Note that the midpoint of BC, M(-1,4) is vertically above A(-1,0).

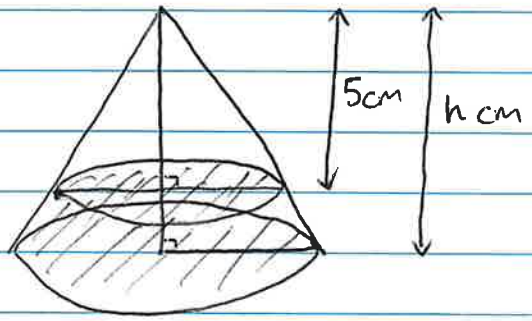
ΔAMB and ΔAMC have base AM and "height" of $\sqrt{2}$ in the horizontal direction.

So $\text{Area}_{\Delta AMB} = \text{Area}_{\Delta AMC} = \frac{1}{2} \times 4 \times \sqrt{2}$

$= 2\sqrt{2}$

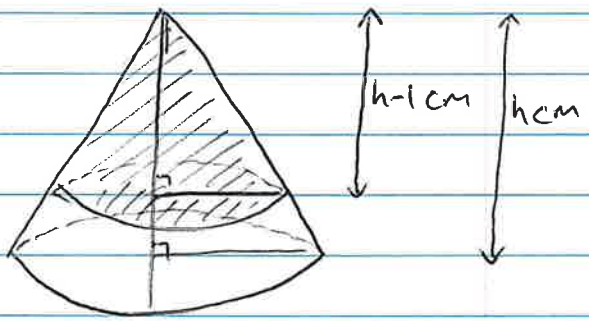
Thus $\text{Area}_{\Delta ABC} = 4\sqrt{2}$ units^2. ✓

d) Suppose the total volume of the cone is V cm^3, and the height is $h$cm.



Let the unfilled volume be V_1.

By similarity $V_1 = \left(\frac{5}{h}\right)^3 V$ } ✓either shown



Let the filled volume be V_2.

{ By similarity $V_2 = \left(\frac{h-1}{h}\right)^3 V$

Since the volume of the liquid is constant,

$$V - V_1 = V_2$$

$$V - \left(\frac{5}{h}\right)^3 V = \left(\frac{h-1}{h}\right)^3 V$$

$$1 - \frac{125}{h^3} = \frac{(h-1)^3}{h^3}$$ ✓ equation involving h only

$$\Rightarrow h^3 - 125 = h^3 - 3h^2 + 3h - 1$$

$$\Rightarrow 3h^2 - 3h - 124 = 0$$

$$h = \frac{3 \pm \sqrt{9 + 12 \times 124}}{6}$$

$$\doteq 6.95 \text{ cm}$$ ✓