NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL



2018 Half-Yearly Examination

FORM IV

MATHEMATICS

Wednesday 30th May 2018

General Instructions

- Writing time — 2 hours.
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 108 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

4A: WJM	4B: RDP	4C: LYL	4D: BDD
4E: RR	4F: LRP	4G: LL	4H: SDP
4I: DWH	4J: REJ		

Checklist

- Writing paper — approx. 18 sheets
- Candidature — 191 boys

Examiner

SDP

QUESTION ONE (12 marks) Start a new page. **Marks**

(a) Simplify $\dfrac{\sqrt{8}}{\sqrt{2}}$. **1**

(b) Expand $-7(9-x)$. **1**

(c) What is the gradient of the line $y-5x+6=0$? **1**

(d) Expand and simplify $(x-4)(x+5)$. **1**

(e) Solve:

(i) $x(x-5)=0$ **1**

(ii) $x^2=49$ **1**

(f) Rationalise the denominator of $\dfrac{4}{\sqrt{3}}$. **1**

(g) Solve $3x+8>23$. **1**

(h) What is the equation of the vertical line through $(8,2)$? **1**

(i) A cube has side length 3 cm.

(i) What is its volume? **1**

(ii) What is its surface area? **1**

(j) Expand and simplify $\sqrt{5}\left(\sqrt{5}-2\right)$. **1**

Examination continues next page ...

QUESTION TWO (12 marks) Start a new page. **Marks**

(a) Find the exact volume of a sphere of radius 4 cm. **1**

(b) Expand and simplify the expression $\frac{2}{3}(x-9)+\frac{1}{3}x$. **1**

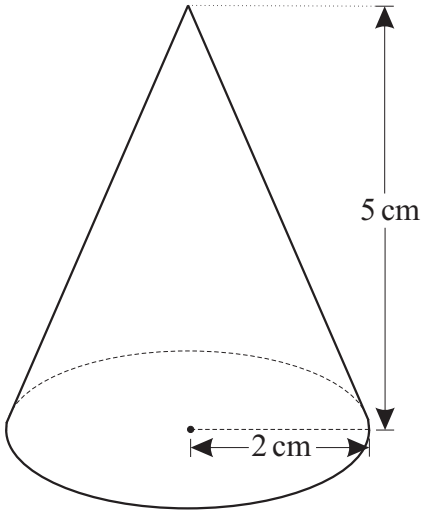
(c) Consider points $A(1,3)$ and $B(5,5)$ in the Cartesian plane.

(i) Find the midpoint of AB. **1**

(ii) Find the length of the interval AB. Leave your answer as a simplified surd. **1**

(d) Factorise $4a(b-2)-3(b-2)$. **1**

(e) **1**



The diagram above shows a cone with base radius 2 cm and perpendicular height 5 cm. Find its exact volume.

(f) Consider the line given by the equation $6x-5y-30=0$.

(i) Find the x- and y-intercepts of the line. **2**

(ii) Hence sketch the line, clearly labelling the intercepts with the axes. **2**

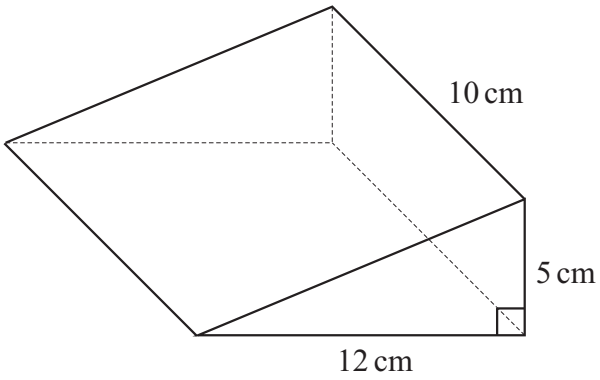
(g) Solve $x^2+6x+4=0$ by completing the square. Leave your answers in exact form. **2**

Examination continues overleaf ...

QUESTION THREE (12 marks) Start a new page. **Marks**

(a) Solve $11(x-4) \leq 16-4x$. **1**

(b)



The diagram above shows a solid right triangular prism.

(i) Find the volume of the prism. **1**

(ii) Find the surface area of the prism. **2**

(c) Expand and simplify $\left(\sqrt{2}+5\sqrt{3}\right)\left(\sqrt{2}-3\sqrt{3}\right)$. **2**

(d) Consider the parabola given by the equation $y = x^2 - 2x - 15$.

(i) State its concavity. **1**

(ii) Find the y-intercept. **1**

(iii) Find any x-intercepts. **1**

(iv) Find the coordinates of the vertex. **1**

(v) Hence sketch the parabola showing the information in parts (i) to (iv) clearly. **2**

Examination continues next page ...

QUESTION FOUR (12 marks) Start a new page. **Marks**

(a) Solve each of the following quadratic equations by factorising:

(i) $x^2 + x - 30 = 0$ **1**

(ii) $x^2 - 1 = 0$ **1**

(iii) $2x^2 - 7x + 6 = 0$ **2**

(b) Use the quadratic formula to solve $3x^2 - 2x - 7 = 0$. Give your answers correct to two decimal places. **2**

(c) A parabola has equation $y = x^2 + 2x - 1$.

(i) Find the equation of the axis of symmetry. **1**

(ii) Find the coordinates of the vertex. **1**

(d) Rationalise the denominator of $\dfrac{5}{3 - \sqrt{2}}$. **2**

(e) Consider the parabola $y = 2x^2 + 3x + 2$.

(i) Calculate the value of $b^2 - 4ac$. **1**

(ii) Hence determine the number of x-intercepts of this parabola. **1**

Examination continues overleaf ...

QUESTION FIVE (12 marks) Start a new page. **Marks**

(a) Solve the following simultaneous equations: **2**

$$x = 3y - 7$$
$$5x - 3y = 25$$

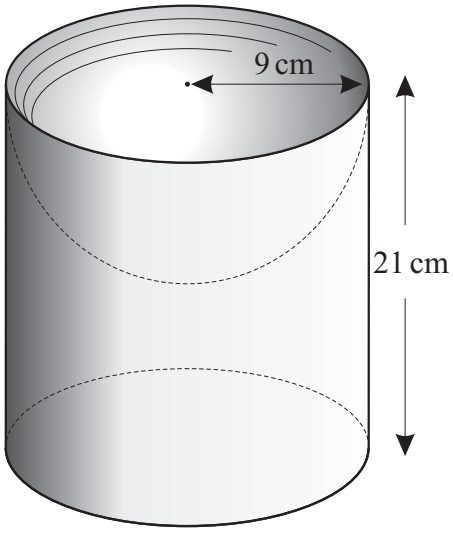
(b) A rectangular prism has volume of $132\,\text{cm}^3$ and surface area of $178\,\text{cm}^2$. All the side lengths of the prism are enlarged by a factor of 3.

(i) What is the volume of the larger prism? **1**

(ii) What is the surface area of the larger prism? **1**

(c) Rearrange and solve $x^2 - 10 = 5x$ by completing the square. Simplify fully and leave your answer as a single fraction. **3**

(d)



The diagram above shows an egg cup, which is a solid cylinder with a hemisphere removed from the top. Find:

(i) the exact volume. **2**

(ii) the exact surface area. **3**

Examination continues next page ...

QUESTION SIX (12 marks) Start a new page. **Marks**

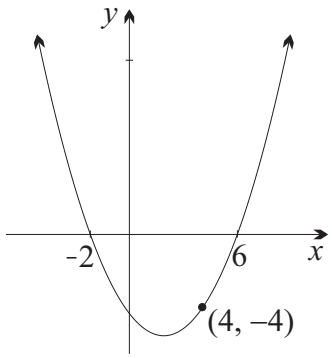
(a) Solve the following simultaneous equations: **2**

$$\begin{aligned} x + 2y &= 1 \\ 2x + \ y &= 5 \end{aligned}$$

(b) Find the equation of the line which passes through the points $A(1,3)$ and $B(4,-3)$. Leave your answer in gradient–intercept form. **2**

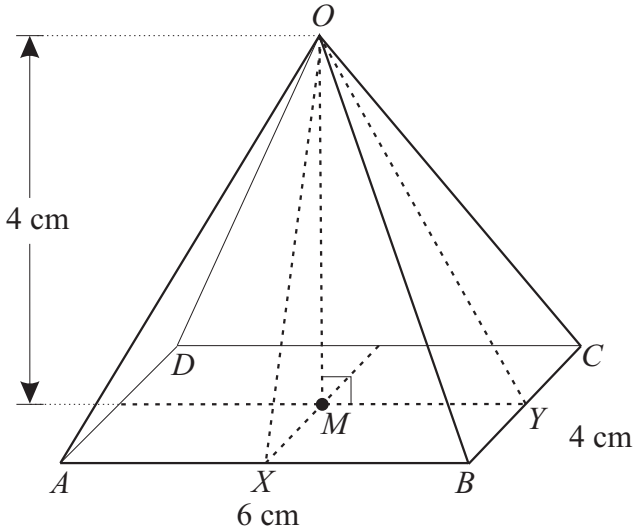
(c) Simplify $\dfrac{5x+5}{x^2-x-6} \div \dfrac{5x+15}{x^2-9}$. **2**

(d) **2**



Determine the equation of the parabola graphed above. Give your answer in expanded form.

(e)



$OM = 4\,\text{cm}$
$AB = 6\,\text{cm}$
$BC = 4\,\text{cm}$

The diagram above shows a rectangular pyramid with height OM, with $OX \perp AB$ and $OY \perp BC$.

(i) Find OX and OY, leaving your answers in exact form. **2**

(ii) Hence find the total surface area of the pyramid. Give your answer correct to two decimal places. **2**

Examination continues overleaf ...

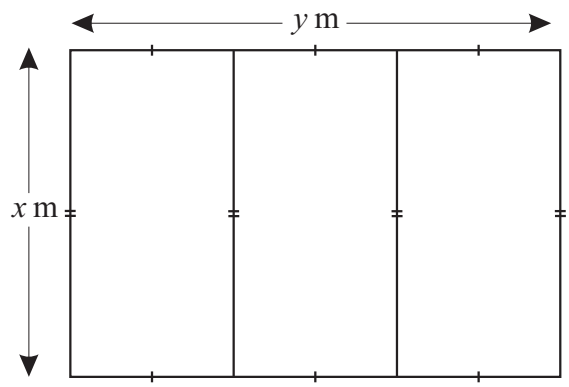
QUESTION SEVEN (12 marks) Start a new page. **Marks**

(a) Find the minimum value of the expression $2x^2 + 3x - 1$. **2**

(b) If Brian's age is multiplied by 5, it exceeds Nigel's age by 21 years. One-quarter of Nigel's age is less than Brian's age by 4 years. **3**

Let Brian's age be b years and Nigel's age be n years. By forming simultaneous equations, find their ages.

(c)



Total area = 720 m^2

A rectangular piece of land of area 720 m^2 is to be enclosed by a wall, and then divided into three equal regions by partition walls, as shown above. The total length of all the walls is 196 m. Let the width of the land be y m and the length of the land be x m.

(i) Show that $y = 98 - 2x$. **1**

(ii) Find all possible answers for the dimensions of the land. **2**

(d) The volume and surface area of a steel drum are 8 m^3 and 12 m^2 respectively. **2**

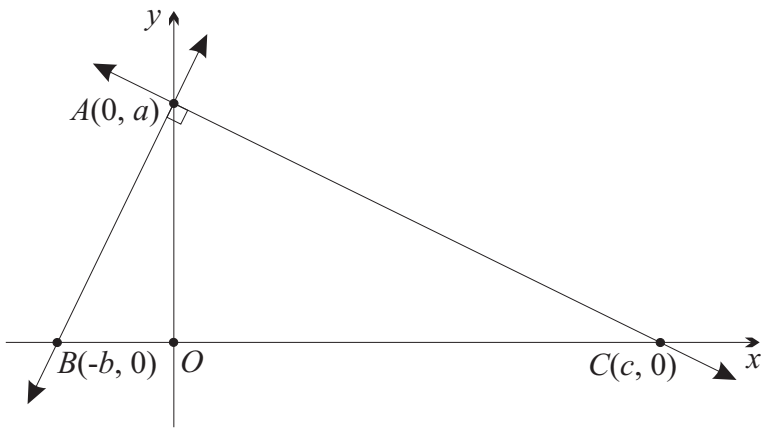
A scale model of the drum is made. If the model has volume of 343 cm^3, find its surface area in square centimetres.

(e) Factorise $(2x + 5y)^2 - 9(x - 2y)^2$. **2**

Examination continues next page ...

QUESTION EIGHT (12 marks) Start a new page. **Marks**

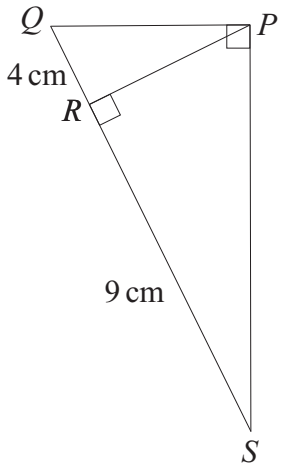
(a)



In the diagram above, the lines AC and AB are perpendicular.

(i) Show that the equation of line AC can be written $y = -\dfrac{b}{a}x + a$. **1**

(ii) Hence show that the lengths AO, BO and CO satisfy $AO^2 = BO \times CO$. **2**



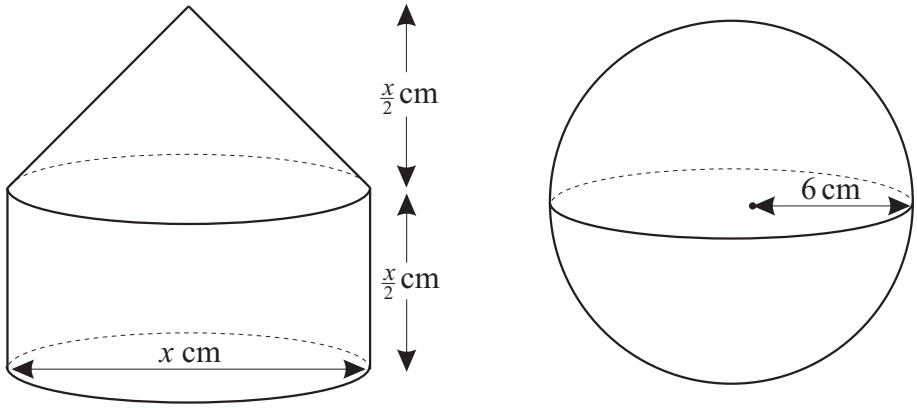
(iii) Find the area of triangle PQS above. **1**

(b) Sam runs a surfboard rental business. He currently charges \$12 per surfboard per day. On average, he rents out 36 surfboards a day. A business expert says that for every 50 cent decrease in rental price the business will gain two rentals a day. Sam wants to maximise his income. How much should he charge to rent a surfboard? Let x be the number of times Sam decreases the price by 50 cents. **3**

Examination continues overleaf ...

QUESTION EIGHT (Continued)

(c) **2**



The diagram above shows a cylinder with a cone on top, and a sphere. The cylinder has radius of $\frac{x}{2}$ cm and height of $\frac{x}{2}$ cm. The cone also has height of $\frac{x}{2}$ cm. The sphere has radius 6 cm. Both solids have the same volume.

Find the value of x.

(d) Solve $\left(x^2 - 4x\right)^2 + 8\left(x^2 - 4x\right) = 48$. **3**

Examination continues next page ...

QUESTION NINE (12 marks) Start a new page. **Marks**

(a) Two aeroplanes have the same size fuel tanks, but one will travel for 21 hours on a full tank and the other for 35 hours on a full tank. They set off at the same time with full tanks and both planes burn fuel at a constant rate. After how long will one have three times as much fuel left as the other? **3**

(b) Solve the following equations simultaneously to find x and y in terms of p and q: **3**

$$\frac{x}{p} + \frac{qy}{p^2} = 2q$$
$$qy - px = 0$$

(c) David had a piece of wire of length 200 cm and cut it into two pieces. He then formed the first piece into an equilateral triangle and the second piece into a square, such that the combined area enclosed by the separate pieces was a minimum. **3**

Find the exact perimeter of each of the shapes and find the combined area to two decimal places.

(d) Find all solutions to the following equation: **3**

$$\left(x^2 - 5x + 5\right)^{x^2-11x+30} = 1.$$

END OF EXAMINATION

Form IV Half-Yearly 2018

Question One

a) $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{4}$

$= 2$ ✓

b) $-7(9-x) = -63+7x$

or $= 7x-63$ ✓

c) $y-5x+6=0$

$y = 5x-6$

gradient is 5 ✓

d) $(x-4)(x+5) = x^2-4x+5x-20$

$= x^2+x-20$ ✓

ei) $x(x-5)=0$

$x=0$ or $x-5=0$

$x=5$ ✓ both

ii) $x^2 = 49$

$x = \pm\sqrt{49}$

$x = 7$ or -7 ✓ both

f) $\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$ ✓

g) $3x+8 > 23$

$3x > 15$

$x > 5$ ✓

1h) $x = 8$ ✓

i i) $V =$ length × width × height

$= 3cm \times 3cm \times 3cm$

$= 27cm^3$ ✓ lose one mark throughout paper for incorrect units

ii) $SA =$ face area $\times 6$

$= 3cm \times 3cm \times 6$

$= 54cm^2$ ✓

i) $\sqrt{5}(\sqrt{5}-2) = 5 - 2\sqrt{5}$ ✓

Question Two

a) $V_{sphere} = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 4^3$

$= \frac{256}{3}\pi \ cm^3$ ✓ lose one mark throughout paper for not exact form

b) $\frac{2}{3}(x-9) + \frac{1}{3}x = \frac{2}{3}x - 6 + \frac{1}{3}x$

$= x - 6$ ✓

ci) $M_{AB} = \left(\frac{1+5}{2}, \frac{3+5}{2}\right)$

$= (3, 4)$ ✓

ii) Length $AB = \sqrt{(5-1)^2 + (5-3)^2}$

$= \sqrt{4^2 + 2^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$

$= 2\sqrt{5}$ ✓

2d) $4a(b-2) - 3(b-2) = (4a-3)(b-2)$ ✓

e) $V_{cone} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 2^2 \times 5$

$= \frac{20}{3}\pi \text{ cm}^3$ ✓

f i) $6x - 5y - 30 = 0$

set $x = 0$

$-5y - 30 = 0$

$-30 = 5y$

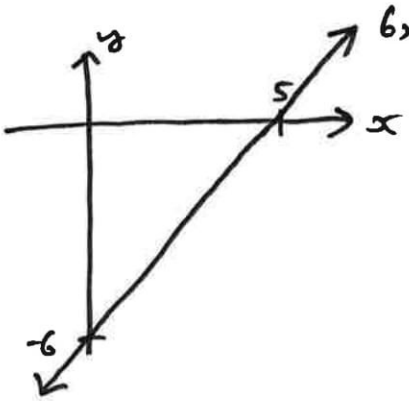
$y = -6$ y-intercept $(0, -6)$ ✓

set $y = 0$ $6x - 30 = 0$

$6x = 30$

$x = 5$ x-intercept $(5, 0)$ ✓

ii)



✓ labelled intercepts

✓ good sketch

g) $x^2 + 6x + 4 = 0$

$x^2 + 6x + 9 - 9 + 4 = 0$

$(x+3)^2 - 9 + 4 = 0$ ✓

$(x+3)^2 - 5 = 0$

$(x+3)^2 = 5$

$x + 3 = \sqrt{5}$ $\quad x + 3 = -\sqrt{5}$

$x = -3 + \sqrt{5}$ $\quad x = -3 - \sqrt{5}$ ✓

Question Three

a)

$$11(x-4) \leq 16-4x$$

$$11x-44 \leq 16-4x$$

$$15x-44 \leq 16$$

$$15x \leq 60$$

$$x \leq 4$$ ✓

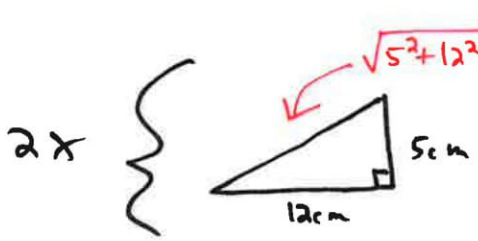
b i)

V_{prism} = cross-sectional area × depth

$$= \frac{1}{2} \times 5 \times 12 \times 10$$

$$= 300\,cm^3$$ ✓

ii)

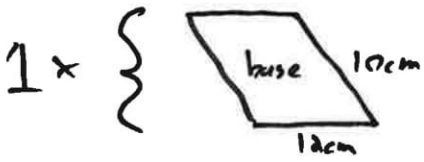


✓

$A = \frac{1}{2}bh$

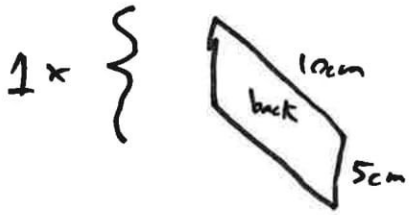
$A = \frac{1}{2} \times 12 \times 5$

$A = 30\,cm^2$



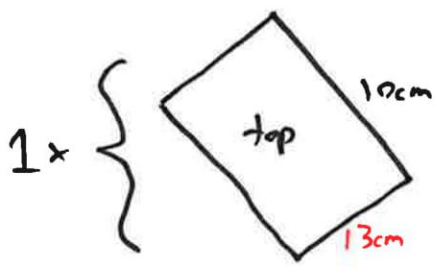
$A = 10 \times 12$

$A = 120\,cm^2$



$A = 10 \times 5$

$A = 50\,cm^2$



$A = 10 \times 13$

$A = 130\,cm^2$

Total SA $= 30 \times 2 + 120 + 50 + 130$

$= 360\,cm^2$ ✓

3c) $(\sqrt{2} + 5\sqrt{3})(\sqrt{2} - 3\sqrt{3})$

$= \sqrt{4} + 5\sqrt{6} - 3\sqrt{6} - 15\sqrt{9}$

$= 2 + 2\sqrt{6} - 45$

$= 2\sqrt{6} - 43$ ✓ surd ✓ rational part

di) $y = x^2 - 2x - 15$

concave up ✓ as the coefficient of x^2 is positive

ii) y-intercept when $x = 0$

$y = -15$ so y-intercept at $(0, -15)$ ✓

iii) x-intercepts when $y = 0$

$0 = x^2 - 2x - 15$

$0 = (x - 5)(x + 3)$

$x = 5$ or $x = -3$

so x-intercepts at $(5, 0)$ or $(-3, 0)$ ✓ both

iv) take average of x-intercepts

$x = \frac{5 + (-3)}{2}$

$x = 1$

OR

$x = -\frac{b}{2a}$

$x = -\frac{(-2)}{2 \times 1}$

$x = 1$

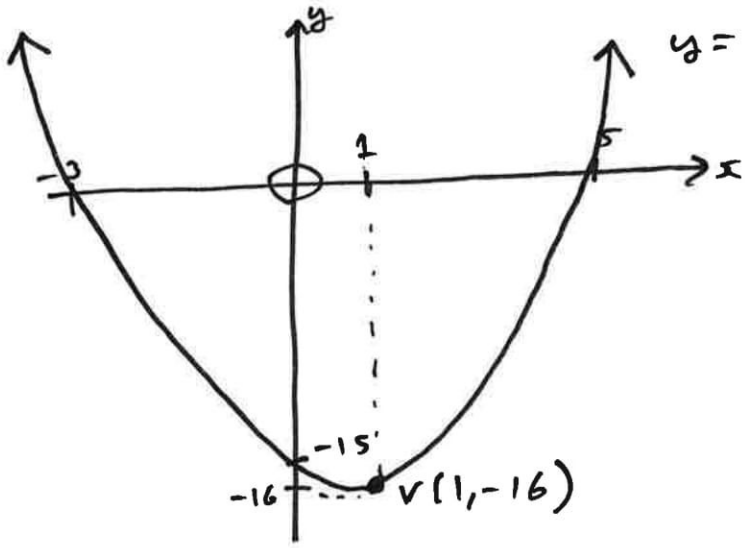
sub $x = 1$ into $y = x^2 - 2x - 15$

$y = 1^2 - 2 \times 1 - 15$

$y = -16$

Vertex $(1, -16)$ ✓

3dv)



✓ labelled intercepts, vertex

✓ good sketch

Question Four

ai) $x^2 + x - 30 = 0$

$(x+6)(x-5) = 0$

$x+6=0$ $\quad$ $x-5=0$

$x=-6$ $\quad$ $x=5$ ✓ both

ii) $x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x+1=0$ $\quad$ $x-1=0$

$x=-1$ $\quad$ $x=1$ ✓ both (must be done by factorising)

iii) $2x^2 - 7x + 6 = 0$

$2x^2 - 4x - 3x + 6 = 0$ ✓ correct progress

$2x(x-2) - 3(x-2) = 0$

$(2x-3)(x-2) = 0$

$2x-3=0$ $\quad$ $x-2=0$

$x = \frac{3}{2}$ or $x=2$ ✓ both

x to 12

+ to -7

-3, -4

4b) $3x^2 - 2x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times -7}}{2 \times 3}$$ ✓

$$x = \frac{2 \pm \sqrt{88}}{6}$$

$x = 1.90$ or $x = -1.23$ (2dp) ✓ both

ci) $y = x^2 + 2x - 1$

$$x = -\frac{b}{2a}$$

$$x = -\frac{2}{2 \times 1}$$

$x = -1$ ✓

ii) sub $x = -1$ into $y = x^2 + 2x - 1$

$y = (-1)^2 + 2(-1) - 1$

$y = -2$

Vertex $(-1, -2)$ ✓

d) $\frac{5}{3 - \sqrt{2}} \times \frac{3 + \sqrt{2}}{3 + \sqrt{2}}$ ✓ $= \frac{15 + 5\sqrt{2}}{9 + 3\sqrt{2} - 3\sqrt{2} - 2}$

$= \frac{15 + 5\sqrt{2}}{7}$ ✓

4ei) $y = 2x^2 + 3x + 2$

$b^2 - 4ac = 3^2 - 4 \times 2 \times 2$

$= -7$ ✓

ii) as $b^2 - 4ac < 0$ there are no x-intercepts ✓

Question Five

5a) $x = 3y - 7$ ①

$5x - 3y = 25$ ②

sub ① into ② $5(3y - 7) - 3y = 25$

$15y - 35 - 3y = 25$

$12y = 60$

$y = 5$ ✓

Sub $y = 5$ into ① $x = 3(5) - 7$

$x = 8$ ✓

bi) $V = 132 cm^3$ $SA = 178 cm^2$

Length scale factor = 3

Volume scale factor = 3^3

$V_{\text{larger prism}} = 132 \times 3^3$

$= 3564 cm^3$ ✓

ii) Length scale factor = 3

Area scale factor = 3^2

$SA_{\text{larger prism}} = 178 \times 3^2$

$= 1602 cm^2$ ✓

5c) $x^2 - 10 = 5x$

$x^2 - 5x - 10 = 0$

$x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 10 = 0$

$\left(x - \frac{5}{2}\right)^2 - \frac{65}{4} = 0$ ✓

$\left(x - \frac{5}{2}\right)^2 = \frac{65}{4}$

$x - \frac{5}{2} = \frac{\sqrt{65}}{2}$ $\qquad$ $x - \frac{5}{2} = -\frac{\sqrt{65}}{2}$ ✓ two correct solution

$x = \frac{5+\sqrt{65}}{2}$ $\qquad$ $x = \frac{5-\sqrt{65}}{2}$ ✓ fully simplified

di) $V_{cylinder} = \pi r^2 h$

$= \pi \times 9^2 \times 21$

$= 1701\pi \text{ cm}^3$

$V_{hemisphere} = \frac{1}{2} \times \frac{4}{3} \pi r^3$

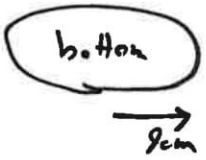
$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 9^3$ ✓ either

$= 486\pi \text{ cm}^3$

$V_{total} = 1701\pi - 486\pi$

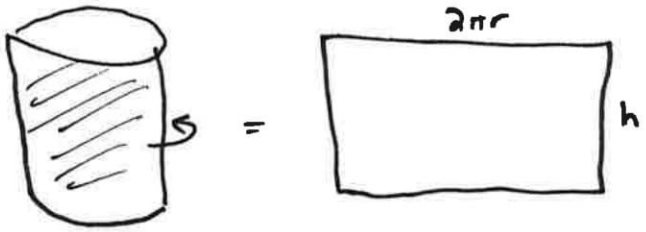
$= 1215\pi \text{ cm}^3$ ✓

5dii)



$A = \pi r^2$

$A = 81\pi \text{ cm}^2$



$A = 2\pi rh$

$A = 2 \times \pi \times 9 \times 21$

$A = 378\pi \text{ cm}^2$ ✓



$A = \frac{1}{2} \times 4\pi r^2$

$A = \frac{1}{2} \times 4 \times \pi \times 9^2$

$A = 162\pi \text{ cm}^2$ ✓

Total SA $= 81\pi + 378\pi + 162\pi$

$= 621\pi \text{ cm}^2$ ✓

Question Six

a) $x + 2y = 1$ ①

$2x + y = 5$ ②

① × 2 $2x + 4y = 2$ ③

③ − ② $3y = -3$

$y = -1$ ✓

Sub $y = 1$ into ① $x + 2 \times (-1) = 1$

$x = 3$ ✓

6b) A (1, 3) B (4, −3)

$$\text{gradient} = \frac{3-(-3)}{1-4}$$

$$= \frac{6}{-3}$$

$$= -2 \quad \checkmark$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x-1)$$

$$y - 3 = -2x + 2$$

$$y = -2x + 5 \quad \checkmark$$

c) $\frac{5x+5}{x^2-x-6} \div \frac{5x+15}{x^2-9} = \frac{5(x+1)}{(x-3)(x+2)} \times \frac{(x+3)(x-3)}{5(x+3)}$ ✓ flip & two correct factorising

$$= \frac{x+1}{x+2} \quad \checkmark$$

d) $y = a(x-\alpha)(x-\beta)$

$$y = a(x-(-2))(x-6)$$

$$y = a(x+2)(x-6) \quad \checkmark$$

sub $y = -4$, $x = 4$

$$-4 = a(4+2)(4-6)$$

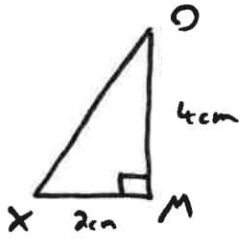
$$-4 = -12a$$

$$a = \frac{1}{3}$$

$$y = \frac{1}{3}(x+2)(x-6)$$

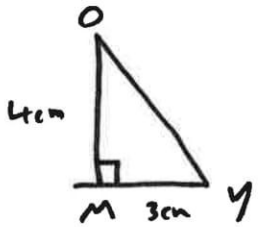
or $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ ✓

6 ei)



$OX = \sqrt{2^2+4^2}$

$= 2\sqrt{5}$ cm ($\sqrt{20}$ ok) ✓

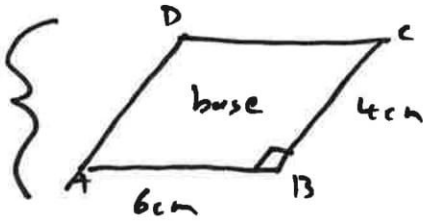


$OY = \sqrt{3^2+4^2}$

$= 5$ cm ✓

ii)

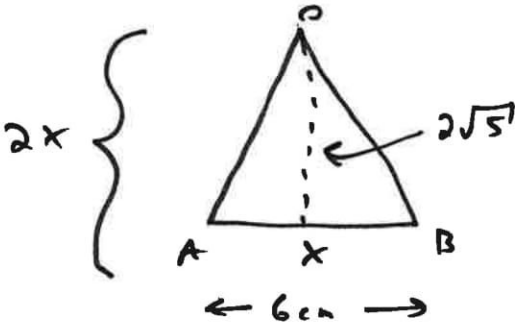
1×



Area $= 6 \times 4$

$= 24\text{cm}^2$

2×

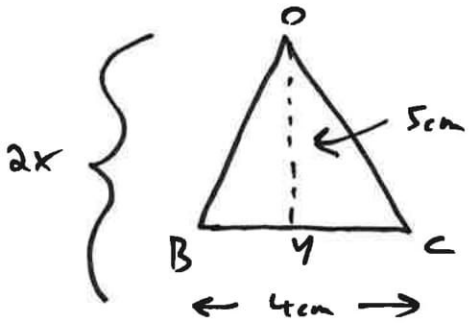


$A = \frac{1}{2} \times 6 \times 2\sqrt{5}$

$= 6\sqrt{5}\ \text{cm}^2$

✓either

2×



$A = \frac{1}{2} \times 4 \times 5$

$A = 10\text{cm}^2$

Total SA $= 24 + 2 \times 6\sqrt{5} + 2 \times 10$

$= 44 + 12\sqrt{5}$

$= 70.83\ \text{cm}^2$ (2dp) ✓

Question Seven

a) $2x^2 + 3x - 1$

axis of symmetry $= -\frac{b}{2a}$

$= -\frac{3}{2\times 2}$

$= -\frac{3}{4}$ ✓

Sub $x = -\frac{3}{4}$ into $2x^2 + 3x - 1 = -\frac{17}{8}$

minimum value is $-\frac{17}{8}$ ✓

b) $5b = n + 21$ ①

$\frac{1}{4}n + 4 = b$ ② ✓ correct equation

② $\times 4$ $n + 16 = 4b$

$4b = n + 16$ ③

① − ③ $b = 5$ ✓

sub $b = 5$ into ② $\frac{1}{4}n + 4 = 5$

$\frac{1}{4}n = 1$

$n = 4$ ✓

Brian is 5 years old
Nigel is 4 years old

ci) wall length: $196 = 4x + 2y$

$2y = 196 - 4x$

$y = 98 - 2x$ ✓ show that

7 c ii) Area: $xy = 720$

$x(98 - 2x) = 720$

$98x - 2x^2 = 720$

$2x^2 - 98x + 720 = 0$ ✓ form quadratic

$x^2 - 49x + 360 = 0$

$(x - 40)(x - 9) = 0$

$x = 40$ $\quad$ $x = 9$

$y = 18$ $\quad$ $y = 80$ ✓ both

Land measures 40m × 18m

or 9m × 80m

d) $V_{drum} = 8\ m^3$ $\quad$ $SA_{drum} = 12m^2$

$= 8,000,000\ cm^3$ $\quad$ $= 120,000\ cm^2$

$V_{model} = 343\ cm^3$

Volume scale factor $= \frac{8,000,000}{343}$ ✓

Length scale factor $= \sqrt[3]{\frac{8000000}{343}}$

$= \frac{200}{7}$

Area scale factor $= \left(\frac{200}{7}\right)^2$

$= \frac{40,000}{49}$

$SA_{model} = 120,000 \div \frac{40,000}{49}$

$= 147\ cm^2$ ✓

7e) $(2x+5y)^2 - 9(x-2y)^2$

difference of two squares

$= ((2x+5y)+3(x-2y))((2x+5y)-3(x-2y))$ ✓

$= (2x+5y+3x-6y)(2x+5y-3x+6y)$

$= (5x-y)(11y-x)$ ✓

Question Eight

ai) gradient $AB = \frac{a-0}{0-(-b)}$

$= \frac{a}{b}$

so gradiat $AC = -\frac{b}{a}$

$y - y_1 = m(x-x_1)$

$y - a = -\frac{b}{a}(x-0)$

$y = -\frac{b}{a}x + a$ ✓ show that

ii) x-intercept of line AC

$0 = -\frac{b}{a}x + a$

$a = \frac{b}{a}x$

$x = \frac{a^2}{b}$ so $c = \frac{a^2}{b}$ ✓

LHS $= AO^2 = a^2$

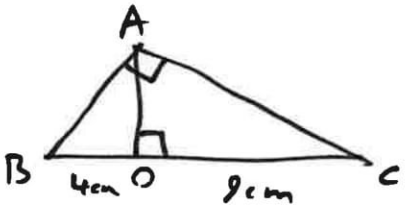
RHS $= BO \times CO = b \times c$

$= b \times \frac{a^2}{b}$

$= a^2$

LHS = RHS so correct ✓ (or other method)

8 aiii) rotate triple, looks same as above diagram



$\therefore$ $AO^2 = 4 \times 9$

$= 36$

$AO = 6$ (not -6 as length)

So area $= \frac{1}{2} \times (4+9) \times 6$

$= 39 cm^2$ ✓

b) If Sam reduces the price 'x' times each surfboard costs $\$(12 - 0.5x)$

He gains 2 rentals per 'x' decrease in price

so rents $(36 + 2x)$ surfboards.

So he makes $\$(36+2x)(12-0.5x)$ ✓

$= (18+x)(24-x)$

$= -x^2 + 6x + 432$

max at axis of symmetry $= -\frac{6}{2 \times -1}$

$= 3$ ✓

he should decrease the price 3 times.

so $\$(12 - 3 \times 0.5) = \$10.50$

Sam should change $\$10.50$ to rent a surfboard. ✓

8c) $V_{sphere} = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 6^3$

$V_{solid} = \pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \frac{4}{3}\pi r^2 h$

$= \frac{4}{3}\pi \times (\frac{x}{2})^2 \times (\frac{x}{2})$

$= \frac{4}{3}\pi (\frac{x}{2})^3$ ✓

so $\frac{4}{3}\pi \times 6^3 = \frac{4}{3}\pi \times (\frac{x}{2})^3$

so $6 = \frac{x}{2}$

$x = 12$ ✓

d) $(x^2 - 4x)^2 + 8(x^2 - 4x) = 48$

set $u = x^2 - 4x$

$u^2 + 8u - 48 = 0$ ✓

$(u + 12)(u - 4) = 0$

$u + 12 = 0$ $\quad u - 4 = 0$

$x^2 - 4x + 12 = 0$ or $x^2 - 4x - 4 = 0$

$b^2 - 4ac = -32$

so no solutions ✓

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times -4}}{2 \times 1}$

$x = \frac{4 \pm \sqrt{32}}{2}$

$x = 2 + 2\sqrt{2}$ or $2 - 2\sqrt{2}$ ✓

Question Nine

9a) Shorter plane burns fuel at $\frac{1}{21}$ tanks per hour

After 't' hours it has $\left(1-\frac{t}{21}\right)$ of a tank left

Longer plane burns fuel at $\frac{1}{35}$ tanks per hour

After 't' hours it has $\left(1-\frac{t}{35}\right)$ of a tank left

✓ rates

Shorter plane must have less fuel so...

$$3\left(1-\frac{t}{21}\right) = 1-\frac{t}{35} \quad \checkmark$$

$$3-\frac{t}{7} = 1-\frac{t}{35}$$

$$2 = \frac{4}{35}t$$

$$t = 17\tfrac{1}{2} \text{ hours}$$

After $17\frac{1}{2}$ hours ✓

b) $\frac{x}{p}+\frac{qy}{p^2}=2q$ (1)

$qy - px = 0$ (2)

(2) $\Rightarrow$ $qy = px$ (3) ✓

sub (3) into (1)

$\frac{x}{p}+\frac{px}{p^2}=2q$

$\frac{2x}{p}=2q$

$x = pq$ ✓

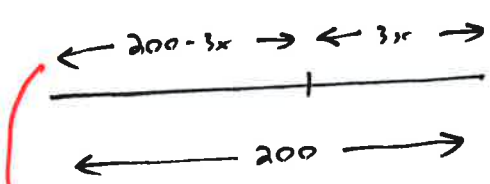
sub $x=pq$ into (3)

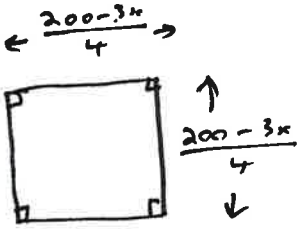
$qy = p^2q$

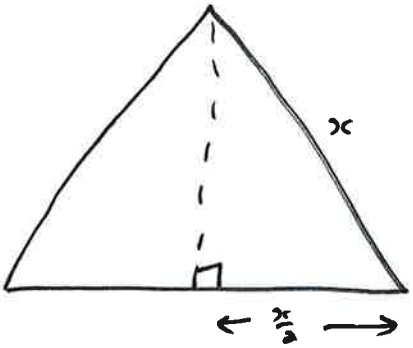
$y = p^2$ ✓

(or similar method)

9c)







$$\text{Area} = \left(\frac{200-3x}{4}\right)^2$$

$$= \frac{9x^2 - 1200x + 40000}{16}$$

$$h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2}$$

$$h = \frac{\sqrt{3}}{2}x$$

$$\text{Area} = \frac{1}{2} \times \frac{\sqrt{3}}{2}x \times x$$

$$= \frac{\sqrt{3}}{4}x^2$$

$$\text{Total Area} = \left(\frac{9+4\sqrt{3}}{16}\right)x^2 - 75x + 2500 \quad \checkmark$$

$$\text{Axis of symmetry} = -\frac{-75}{2\left(\frac{9+4\sqrt{3}}{16}\right)}$$

$$= 75 \times \frac{8}{9+4\sqrt{3}} \times \frac{9-4\sqrt{3}}{9-4\sqrt{3}}$$

$$= 75 \times \frac{72-32\sqrt{3}}{33}$$

$$= \frac{1800-800\sqrt{3}}{11}$$

(c continued ...) Min when $x = \frac{1800 - 800\sqrt{3}}{11}$

Perimeter $\Delta = \frac{5400 - 2400\sqrt{3}}{11}$ cm

Perimeter $\square = 200 - 3\left(\frac{1800 - 800\sqrt{3}}{11}\right)$

$= 200 - \frac{5400 - 2400\sqrt{3}}{11}$

$= \frac{2400\sqrt{3} - 3200}{11}$ cm

✓ both

Min Area : sub $x = \frac{1800 - 800\sqrt{3}}{11}$

into $\left(\frac{9 + 4\sqrt{3}}{16}\right)x^2 - 75x + 2500$

$= 1087.41$ cm^2 ✓

9d) $(x^2 - 5x + 5)^{x^2 - 11x + 30} = 1$

3 options : 1) $x^2 - 5x + 5 = 1$ (as $1^x = 1$ for any x)

2) $x^2 - 11x + 30 = 0$ (as $x^0 = 1$ for any x) not $x = 0$

3) $x^2 - 5x + 5 = -1$ and $x^2 - 11x + 30$ even (as $(-1)^x = 1$ for any even value of x)

1) $x^2 - 5x + 5 = 1$

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$ $x = 1$ or $x = 4$ ✓

2) $x^2 - 11x + 30 = 0$

$(x - 6)(x - 5) = 0$ $x = 5$ or $x = 6$ ✓

(checked and $x^2 - 5x + 5 \neq 0$ for $x = 5$ or $x = 6$)

3) $x^2 - 5x + 5 = -1$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$ $x = 2$ or $x = 3$ ✓

(checked and $x^2 - 11x + 30$ is even for $x = 2, 3$)

So $x = 1, 2, 3, 4, 5$ or 6