NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL



2016 Half-Yearly Examination

FORM IV

MATHEMATICS

Tuesday 24th May 2016

General Instructions

- Writing time — 2 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 108 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and Master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name, class and Master on this question paper and hand it in with your answers.

4A: SG	4B: BR	4C: REJ	4D: SDP
4E: DWH	4F: BDD	4G: GMC	4H: WJM
4I: LRP/LYL	4J: NL		

Checklist

- Writing paper required.
- Candidature — 191 boys

Examiner

GMC

QUESTION ONE (12 marks) Start a new page.

(a) Simplify $\sqrt{50}$.

(b) Simplify $2\sqrt{3} \times \sqrt{7}$.

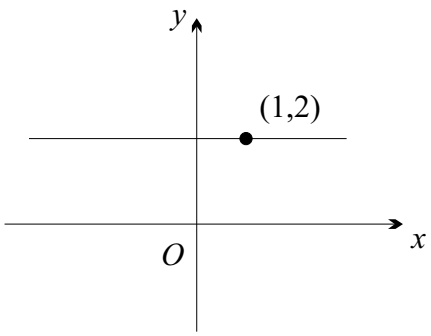
(c) Factorise fully each of the following expressions:

(i) $x^2 + 8x + 15$

(ii) $2x - 6x^2$

(d) Expand and simplify $(x + 3)^2$.

(e)



Write down the equation of the horizontal line passing through $(1, 2)$ as shown in the diagram above.

(f) Solve:

(i) $3x = -12$

(ii) $(x - 2)(x + 4) = 0$

(g) Express $\sqrt{a}$ in index form.

(h) Express 2^{-3} as a simplified fraction.

(i) Write down the y-intercept of the line $y = 5x - 3$.

Examination continues next page ...

QUESTION TWO (12 marks) Start a new page.

(a) Solve $2x + 3 > 1$.

(b) Factorise:

(i) $x^2 - 4y^2$

(ii) $ab - 2a + 3b - 6$

(c) Express $\dfrac{3}{\sqrt{7}}$ with a rational denominator.

(d) Consider the line given by the equation $2x + 3y + 12 = 0$.

(i) Find the x-intercept and y-intercept of the line.

(ii) Hence sketch the line.

(e) Consider the formula $v = u + at$.

(i) Make a the subject of the formula.

(ii) Find a if $v = 12{\cdot}6$, $u = 1{\cdot}9$ and $t = 7{\cdot}3$. Give your answer correct to one decimal place.

QUESTION THREE (12 marks) Start a new page.

(a) Write 67 943 in scientific notation correct to three significant figures.

(b) Solve the following simultaneous equations:

$$\begin{aligned} 8x + 3y &= 18 \\ y &= 4 - 2x \end{aligned}$$

(c) Factorise $2x^2 + 11x - 6$.

(d) Solve $\dfrac{x}{2} + \dfrac{x}{3} = 1$.

(e) Simplify $\sqrt{18} - 3\sqrt{8}$.

(f) Find the gradient of the line $3x + 4y - 2 = 0$.

Examination continues overleaf ...

QUESTION FOUR (12 marks) Start a new page.

(a) Use the quadratic formula to solve $4x^2 - 12x + 3 = 0$. Express your answer in simplest form.

(b) Expand and simplify:

(i) $\sqrt{5}\left(2\sqrt{5}+3\right)$

(ii) $\left(2\sqrt{3}+\sqrt{7}\right)\left(\sqrt{3}-2\sqrt{7}\right)$

(c) Consider the parabola given by the equation $y = x^2 + 4x - 5$.

(i) Find the y-intercept of the parabola.

(ii) Find any x-intercepts the parabola may have.

(iii) Find the equation of the axis of symmetry of the parabola.

(iv) Find the coordinates of the vertex of the parabola.

(v) Hence sketch the parabola showing the information in parts (i) to (iv) clearly.

The Examination continues over the page.

QUESTION FIVE (12 marks) Start a new page.

(a) Solve:

(i) $3^x = \dfrac{1}{9}$

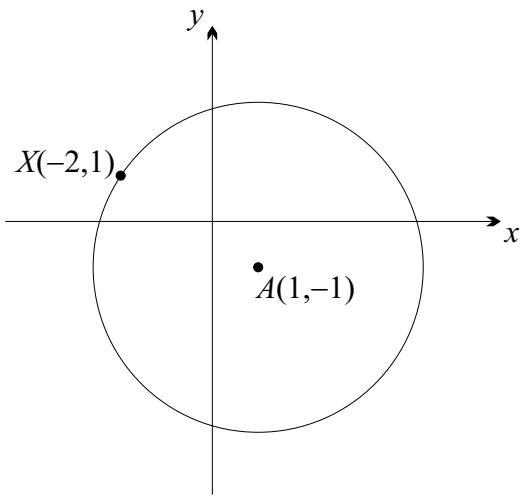
(ii) $\log_{10} x = 3$

(b) Simplify:

(i) $(2a + 3b)^2 - (2a - 3b)^2$

(ii) $\dfrac{1}{x-3} - \dfrac{1}{x+3}$

(c)



A circle with centre $A(1, -1)$ passes through the point $X(-2, 1)$ as shown in the diagram above. Find the length of the radius of the circle.

(d) Anne and Oliver eat an entire box of chocolates. Anne eats a third of the chocolates and Oliver eats 12 more than Anne. Let x represent the number of chocolates in the box.

(i) Write an equation describing this situation.

(ii) Hence find the number of chocolates eaten by Anne.

Examination continues overleaf ...

QUESTION SIX (12 marks) Start a new page.

(a) Solve $2(3-x) \leq 3(1-2x)$.

(b) Consider the interval joining the points $X(2,8)$ and $Y(-3,5)$.

(i) Find the midpoint of XY.

(ii) Find the gradient of XY.

(iii) Hence find the equation of the perpendicular bisector of XY.

(c) The area of a triangle is given by the formula $A = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

(i) Make h the subject of the equation.

(ii) Find h if $A = 3\sqrt{5}$ cm^2 and $b = 2\sqrt{7} - \sqrt{2}$ cm. Express your answer in simplest exact form with a rational denominator.

(d) Simplify $\dfrac{x^2 - xy}{y^2 - xy}$.

The Examination continues over the page.

Examination continues next page ...

QUESTION SEVEN (12 marks) Start a new page.

(a) (i) Write $\log_a 27 = 6$ in index form.

(ii) Hence evaluate $\log_a 3$.

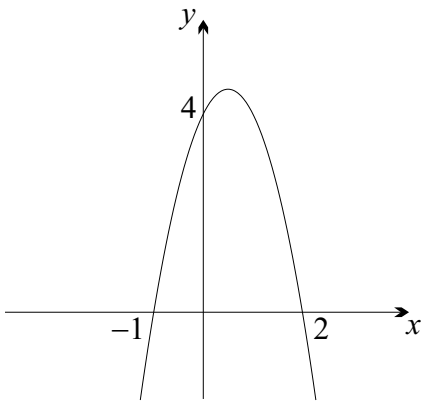
(b) Solve $2 - \dfrac{3x}{x+2} + \dfrac{2x+1}{x-4} = 0$.

(c) Reide jogs six hours a week to keep fit and has two routes to choose from: Route A and Route B. In one week, he jogs Route A twelve times and Route B eight times. The following week, he jogs Route A nine times and Route B twelve times.

(i) If x is the time it takes Reide to jog Route A and y is the time it takes Reide to jog Route B, write a pair of simultaneous equations describing this situation.

(ii) Hence determine how long it takes Reide to jog each route.

(d)



Find the equation of the parabola shown in the diagram above.

QUESTION EIGHT (12 marks) Start a new page.

(a) Solve:

(i) $\log_x 32 = \dfrac{5}{3}$

(ii) $4^{x-3} = \sqrt{8}$

(b) A parabola has equation $y = x^2 + kx + 3$ for some real number k.

(i) Is the parabola concave up or concave down?

(ii) Find the coordinates of the vertex of the parabola in terms of k.

(iii) Hence, or otherwise, find the values of k for which the parabola has exactly one x-intercept.

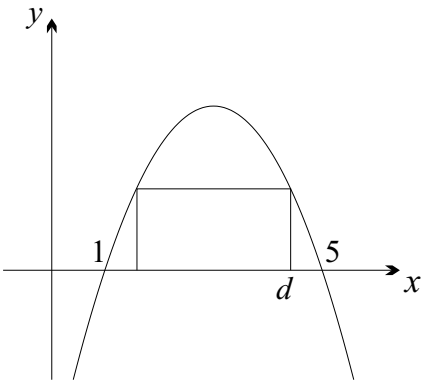
(c) Find the values of h such that the points $A(h, 2)$, $B(1, -3)$ and $C(10, h)$ are collinear.

Examination continues overleaf ...

QUESTION NINE (12 marks) Start a new page.

(a) Solve $8x^3 - \dfrac{27}{x^3} + 19 = 0$.

(b) A rectangle is inscribed between the x-axis and the positive part of the parabola $y = -x^2 + 6x - 5$ as shown in the diagram below.



(i) If the lower right hand coordinate of the rectangle is $(d, 0)$, show that the perimeter P of the rectangle is given by $P = -2d^2 + 16d - 22$.

(ii) Hence find the maximum perimeter of the rectangle.

(c) An astronaut goes on a long space voyage at an average speed of v km/h. When she returns home, she discovers that she and her twin sister are no longer the same age. While she has aged t years, her twin sister has aged T years according the formula:

$$T = \frac{t}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$ where c is the speed of light in km/h.

(i) Given that $v < c$, explain with reference to the formula why $T > t$.

(ii) Express $\dfrac{v}{c}$ in terms of t and T.

(iii) If the astronaut aged six years on the trip and her sister aged ten years, what fraction of the speed of light was she traveling?

END OF EXAMINATION

Form IV Half Yearly 2016.

Question 1

a) $\sqrt{50} = 5\sqrt{2}$ ✓

b) $2\sqrt{3} \times \sqrt{7} = 2\sqrt{21}$ ✓

c) i) $x^2 + 8x + 15 = (x+3)(x+5)$ ✓

ii) $2x - 6x^2 = 2x(1-3x)$ ✓

d) $(x+3)^2 = x^2 + 6x + 9$ ✓

e) $y = 2$ ✓

f) i) $3x = -12$

$x = -4$ ✓

ii) $(x-2)(x+4) = 0$

$x = 2, \quad x = -4$ ✓✓

g) $\sqrt{a} = a^{1/2}$ ✓

h) $2^{-3} = \frac{1}{8}$ ✓

i) -3 ✓

Question 2

a) $2x + 3 > 1$

$2x > -2$ ✓

$x > -1$ ✓

b) i) $x^2 - 4y^2 = (x - 2y)(x + 2y)$ ✓

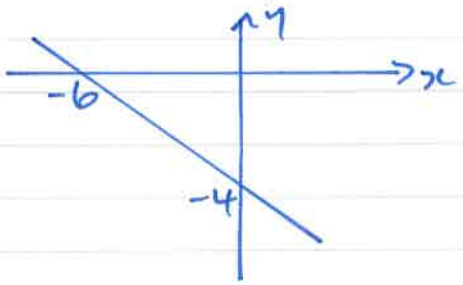
ii) $ab - 2a + 3b - 6 = a(b-2) + 3(b-2)$ ✓

$= (b-2)(a+3)$ ✓

c) $\frac{3}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$. ✓

d) i) x-intercept : -6; y-intercept : -4 ✓✓

ii)



✓

e) i) $v = u + at$

$a = \frac{v - u}{t}$ ✓✓

ii) $a = \frac{12.6 - 1.9}{7.3} = 1.5$ (1 d.p) ✓

Question 3

a) 6.79×10^4 ✓

b) $8x + 3y = 18$ — (1)

$y = 4 - 2x$ — (2)

sub (2) into (1)

$8x + 3(4 - 2x) = 18$ ✓

$2x = 6$

$x = 3$ ✓

$y = -2$ ✓

so $x = 3, y = -2$

c) $2x^2 + 11x - 6 = (2x - 1)(x + 6)$ ✓✓

d) $\frac{x}{2} + \frac{x}{3} = 1$

$3x + 2x = 6$ ✓

$5x = 6$

$x = 6/5$ ✓ or $x = 1\frac{1}{5}$

e) $\sqrt{18} - 3\sqrt{8} = 3\sqrt{2} - 6\sqrt{2}$ ✓

$= -3\sqrt{2}$ ✓

f) $3x + 4y - 2 = 0$

$4y = -3x + 2$

$y = -3/4\,x + 1/2$ ✓

so gradient is $-3/4$ ✓

Question 4

a) $4x^2 - 12x + 3 = 0$

$x = \dfrac{12 \pm \sqrt{144 - 4 \times 4 \times 3}}{8}$ ✓

$= \dfrac{12 \pm \sqrt{96}}{8}$ ✓

$= \dfrac{3 \pm \sqrt{6}}{2}$ ✓

b) i) $\sqrt{5}(2\sqrt{5} + 3) = 2 \times 5 + 3\sqrt{5}$

$= 10 + 3\sqrt{5}$ ✓

ii) $(2\sqrt{3} + \sqrt{7})(\sqrt{3} - 2\sqrt{7}) = 6 - 4\sqrt{21} + \sqrt{21} - 14$ ✓

$= -8 - 3\sqrt{21}$ ✓

c) $y = x^2 + 4x - 5$

i) y-intercept : -5 ✓ or $y = -5$ or $(0, -5)$

ii) $y = (x + 5)(x - 1)$

so x-intercepts : $-5, 1$ ✓ or $(-5, 0), (1, 0)$

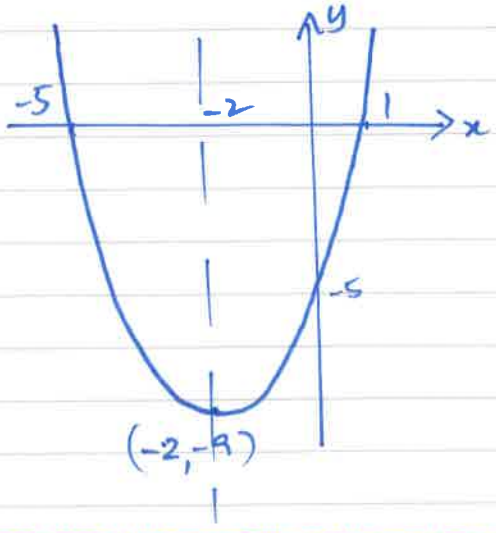
or $x = -5, x = 1$

iii) $x = \dfrac{-4}{2 \times 1}$

so axis of symmetry is $x = -2$ ✓

iv) when $x = -2$, $y = (-2)^2 + 4(-2) - 5 = -9$ ✓

v)



✓✓

Question 5.

a) i) $3^x = \frac{1}{9}$

$x = -2$ ✓

ii) $\log_{10} x = 3$

$x = 1000$ ✓

b) i) $(2a+3b)^2 - (2a-3b)^2$

$= 4a^2 + 12ab + 9b^2 - 4a^2 + 12ab - 9b^2$ ✓

$= 24ab$ ✓

ii) $\frac{1}{x-3} - \frac{1}{x+3} = \frac{x+3-(x-3)}{(x-3)(x+3)}$ ✓

$= \frac{6}{(x-3)(x+3)}$ ✓ [$\frac{6}{x^2-9}$ OK ALSO]

c) $AX = \sqrt{(-2-1)^2 + (1+1)^2}$ ✓

$= \sqrt{9+4}$ ✓

$= \sqrt{13}$ ✓

So radius is $\sqrt{13}$ units

d) i) $\frac{x}{3} + \frac{x}{3} + 12 = x$ ✓ or equivalent

ii) $\frac{2x}{3} + 12 = x$

$2x + 36 = 3x$

$x = 36$ ✓

So Anne eats $\frac{36}{3} = 12$ chocolates

✓

Question 6

a) $2(3-x) \leq 3(1-2x)$

$6-2x \leq 3-6x$ ✓

$3 \leq -4x$

$x \leq -3/4$ ✓

b) $X(2,8)$, $Y(-3,5)$

i) $MP = \left(\frac{2-3}{2}, \frac{8+5}{2}\right) = \left(-\frac{1}{2}, \frac{13}{2}\right)$ ✓

ii) $m_{XY} = \frac{5-8}{-3-2} = \frac{3}{5}$ ✓

iii) grad. of perp. bisector : $-5/3$

$y - \frac{13}{2} = -\frac{5}{3}\left(x + \frac{1}{2}\right)$ ✓

$y - \frac{13}{2} = -\frac{5x}{3} - \frac{5}{6}$

$y = -\frac{5x}{3} + \frac{17}{3}$ ✓ or $5x + 3y - 17 = 0$

c) i) $h = \frac{2A}{b}$ ✓

ii) $h = \frac{2 \times 3\sqrt{5}}{2\sqrt{7} - \sqrt{2}} \times \frac{2\sqrt{7} + \sqrt{2}}{2\sqrt{7} + \sqrt{2}}$ ✓

$= \frac{12\sqrt{35} + 6\sqrt{10}}{28 - 2}$ ✓

$= \frac{6\sqrt{35} + 3\sqrt{10}}{13}$ cm ✓ $\frac{3\sqrt{5}}{13}(2\sqrt{7} + \sqrt{2})$ OK

d) $\frac{x^2 - xy}{y^2 - xy} = \frac{x(x-y)}{y(y-x)}$ ✓

$= \frac{x(x-y)}{-y(x-y)}$

$= -\frac{x}{y}$ ✓

Question 7

a) i) $\log_a 27 = 6 \Rightarrow a^6 = 27$ ✓

ii) $(a^2)^3 = 3^3$
So $a^2 = 3$ ✓
So $\log_a 3 = 2$ ✓

b) $2 - \frac{3x}{x+2} + \frac{2x+1}{x-4} = 0$

$2(x+2)(x-4) - 3x(x-4) + (2x+1)(x+2) = 0$ ✓
$2x^2 - 4x - 16 - 3x^2 + 12x + 2x^2 + 5x + 2 = 0$
$x^2 + 13x - 14 = 0$ ✓
$(x+14)(x-1) = 0$
$x = -14, \quad x = 1$ ✓

c) Let x be time to run Route A
Let y be time to run Route B
$12x + 8y = 360$ — ①
$9x + 12y = 360$ — ② } ✓
① × 6 : $72x + 48y = 2160$ — ③
② × 4 : $36x + 48y = 1440$ — ④
③ − ④ : $36x = 720$
$x = 20$ ✓
$y = (360 - 9 \times 20) \div 12 = 15$ ✓
So he runs Route A in 20 min & Route B in 15 min

d) let $y = k(x+1)(x-2)$ ✓
when $x = 0, \quad y = 4$
So $4 = k(1)(-2)$
$k = -2$ ✓
So $y = -2(x^2 - x - 2)$
$= -2x^2 + 2x + 4$ } ✓

Question 8

a) i) $\log_x 32 = 5/3$

$x^{5/3} = 32$ ✓

$x = 32^{3/5} = 8$ ✓

ii) $4^{x-3} = \sqrt{8}$

$2^{2x-6} = 2^{3/2}$ ✓

$2x - 6 = 3/2$

$2x = 15/2$

$x = 15/4$ ✓

b) $y = x^2 + kx + 3$

i) the parabola is concave up ✓

ii) axis of symmetry : $x = -\frac{k}{2}$ ✓

when $x = -k/2$, $y = (-\frac{k}{2})^2 + k(-\frac{k}{2}) + 3$

$= 3 - \frac{k^2}{4}$

so vertex is $(-k/2, 3 - k^2/4)$ ✓

iii) Solve $3 - k^2/4 = 0$ ✓

$k^2/4 = 3$

$k^2 = 12$

$k = \pm\sqrt{12}$ ✓ (or $\pm 2\sqrt{3}$)

c) $\frac{-3-2}{1-h} = \frac{h+3}{10-1}$ ✓

$-5 \times 9 = (1-h)(h+3)$

$-45 = h + 3 - h^2 - 3h$

$h^2 + 2h - 48 = 0$ ✓

$(h+8)(h-6) = 0$

$h = -8, 6.$ ✓

Question 9.

a) $8x^3 - \frac{27}{x^3} + 19 = 0$

$8(x^3)^2 + 19(x^3) - 27 = 0$

$(8x^3 + 27)(x^3 - 1) = 0$ ✓

$x^3 = -27/8$, $x^3 = 1$ ✓

$x = -3/2$, $x = 1$ ✓

b) i) axis of symmetry : $x = 3$

So perimeter $= 4(d-3) + 2(-d^2 + 6d - 5)$ ✓

$= 4d - 12 - 2d^2 + 12d - 10$ ✓

$= -2d^2 + 16d - 22$ as required

ii) Let $P = -2d^2 + 16d - 22$

$= -2(d^2 - 8d + 11)$

max P occurs when $d = \frac{-16}{2(-2)} = 4$ ✓

when $d = 4$, $P = 10$

So max perimeter is 10 units. ✓

c) i) Since $0 < \frac{v^2}{c^2} < 1$, $0 < 1 - \frac{v^2}{c^2} < 1$

So $0 < \sqrt{1 - \frac{v^2}{c^2}} < 1$

So $T > t$ ✓ or similar.

ii) $T^2 = \frac{t^2}{1 - \left(\frac{v}{c}\right)^2}$

$1 - \left(\frac{v}{c}\right)^2 = \frac{t^2}{T^2}$ ✓ [$\left(\frac{v}{c}\right)^2$ in numerator]

$\frac{v}{c} = \sqrt{1 - t^2/T^2}$ ✓

iii) Let $t = 6$, $T = 10$

So $v/c = \sqrt{1 - 6^2/10^2}$ ✓

$= \sqrt{64/100}$

$= 8/10$

So she travelled at 80% ✓ of the speed of light