Sydney Girls High School

MATHEMATICS

Year 10

HALF-YEARLY EXAMINATION

2011

Time Allowed: 60 minutes + 3 minutes reading time

Topics: Quadratic Equations, Probability, Consumer Arithmetic, Number Plane Graphs, Surface Area & Volumes

Total: 60 marks

Instructions:

- There are FIVE (5) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your name and your teacher's name clearly at the top of each question and clearly number each question.

Student's Name:_____ **Teacher's Name:**_____

Question 1 (12 marks)

a. Factorise and hence solve the following quadratic equations:

i) $2p^2 - 6p = 0$ 2

ii) $m^2 - 2m - 8 = 0$ 3

b. Sketch the following, on separate number planes, showing all relevant features:

i) $y = 3^{-x}$ 2

ii) $xy = -4$ 2

c. A sample of 100 matchboxes was taken and the number of matches in each box counted and recorded. The results are given in the table:

No. of matches	No. of matchboxes
47	5
48	13
49	26
50	45
51	10
52	1

Find the probability that a box selected at random from this sample contains:

i) exactly 50 matches? 1

ii) more than 50 matches? 1

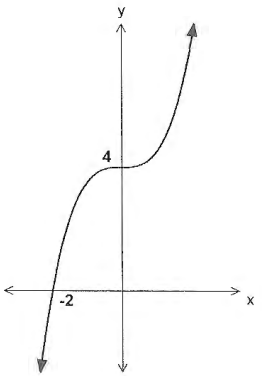
iii) 48 or fewer matches? 1

Question 2 (12 marks)

a. Find the simple interest earned if one million dollars is invested for 5 months at 0.75%p.a. 2

b. Use the quadratic formula to solve the equation $3x^2 = 6x - 2$. 3
Leave your answer in surd form.

c. Determine the cubic equation of the following graph: 2



d. The parabola $y = 2x^2 + kx - 7$ has an axis of symmetry with equation $x = 3$. 2
Find the value of k.

e. Find the compound interest earned on an investment of \$3000 at 3.8% p.a. over 2 years if the interest is calculated annually. 3

Question 3 (12 marks)

a. A 6-sided die has the numbers 0, 1, 2, 3, 4 and 5 on its faces.

In a game, two of these dice are rolled and the total of the faces is taken.

i) List the possible outcomes of the game. 2

ii) What is the chance of obtaining a total of 7? 1

iii) What total has the greatest probability? 1

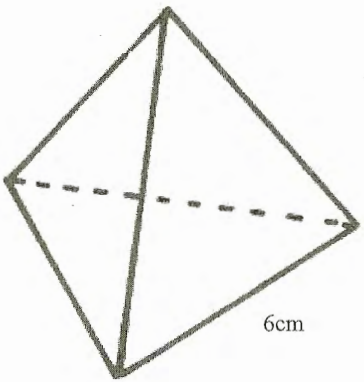
iv) What is the probability of getting a total of at least 3? 1

b. A computer bought for \$2500 was sold 5 years later for \$750. 3

Find its annual depreciation rate, correct to one decimal place.

c. i) Show that the height of an equilateral triangle with side length 6cm is $3\sqrt{3}$ cm. 2

ii) Hence, find the exact surface area of this tetrahedron. 2



Question 4 (12 marks)

a. Mary borrows $25000 to buy a car. She repays the loan in fortnightly instalments of $256 over 5 years. Calculate:

i) the total amount repaid. 1

ii) the amount of interest charged on the loan. 1

iii) the flat rate of interest (p.a.). 2

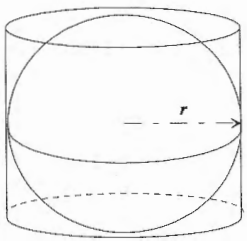
b. The members of a certain family have a tendency to inherit "ergophobia". When a child is born the chance of it being an ergophobiac or non-ergophobiac is equally likely.

i) Draw a tree diagram to show the outcomes of 3 children born. 2

ii) What is the probability that the family will have two ergophobiacs and one non-ergophobiac in any order? 1

iii) What is the probability of at least one ergophobiac being born? 2

c. A sphere with radius r fits tightly into a cylinder as shown.



Show that the ratio of the surface area of the sphere to the surface area of the cylinder is 2:3. 3

Question 5 (12 marks)

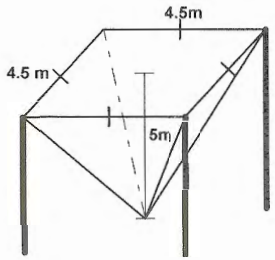
a. For the parabola with equation $y = -x^2 + 4x - 5$:

i) find any x or y intercepts. 2

ii) find the vertex. 2

iii) sketch its graph. 1

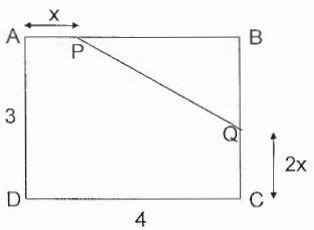
b. A grain hopper is in the shape of an inverted square pyramid as shown.
The perpendicular height of the pyramid is 5 metres.



i) Find the volume of the square pyramid. 2

ii) If there are 750kg of wheat per m^3, find the mass of grain in the hopper when it is filled to three-quarters of its capacity. 2
Give your answer in tonnes, correct to two decimal places.

c. ABCD is a rectangle, CD = 4 and AD = 3. If $AP = \frac{1}{2}QC = x$:



i) find an expression for the area of pentagon APQCD. 1

ii) hence, find its maximum area. 2

-------END OF TEST------

Question 1

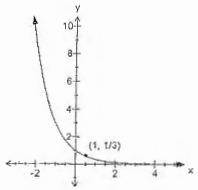
a. i) $2p(p-3)=0$

$p=0$ or $p=3$

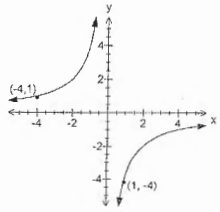
ii) $(m-4)(m+2)=0$

$m=4$ or $m=-2$

b. i)



ii)



c. i) $\frac{45}{100}=\frac{9}{20}$

ii) $\frac{11}{100}$

iii) $\frac{18}{100}=\frac{9}{50}$

Question 2

a. $I=\$1000000\times 0.75\%\times\frac{5}{12}$

$=\$3125$

b. $3x^2-6x+2=0$

$x=\frac{6\pm\sqrt{6^2-4(3)(2)}}{2(3)}$

$=\frac{6\pm\sqrt{12}}{6}$

$=\frac{3\pm\sqrt{3}}{3}$

c. $y=ax^3+d$

$y=ax^3+4$

When $y=0, x=-2$:

$0=a(-2)^3+4$

$-4=-8a$

$a=\frac{1}{2}$

$\therefore y=\frac{1}{2}x^3+4$

d. $x=\frac{-b}{2a}$

$3=\frac{-k}{4}$

$\therefore k=-12$

e. $CI=\$3000\times(1+3.8\%)^2-\$3000$

$=\$3000(1.038)^2-\$3000$

$=\$3232.332-\$3000$

$=\$232.33$

Question 3

a. i)

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

ii) $P(7)=\frac{4}{36}=\frac{1}{9}$

iii) 5

iv) $P(\geq 3)=\frac{30}{36}=\frac{5}{6}$

b. $\$750=\$2500(1-r)^5$

$0.3=(1-r)^5$

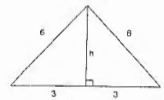
$0.3^{\frac{1}{5}}=1-r$

$r=1-0.3^{\frac{1}{5}}$

$r=0.2139969144$

$=21.4\%$

c. i)



$6^2=h^2+3^2$

$h^2=27$

$h=3\sqrt{3}$ cm

ii) $SA=4\times\frac{1}{2}\times 6\times 3\sqrt{3}$

$=36\sqrt{3}$ cm^2

Question 4

a. i) Total paid $=\$250\times 26\times 5$

$=\$33280$

ii) Interest $=\$33280-\$25000$

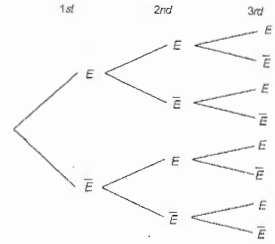
$=\$8280$

iii) Flat interest rate p.a

$=\frac{8280}{25000}\div 5\times 100$

$\doteq 6.6\%$ p.a.

b. i)



ii) $P(EE\bar{E})=\frac{3}{8}$

iii) $P(\text{at least one } E)=1-P(\bar{E}\bar{E}\bar{E})$

$=1-\frac{1}{8}$

$=\frac{7}{8}$

c.

Surface Area of Sphere $=4\pi r^2$

Surface Area of Cylinder

$=2\pi r^2+2\pi rh \quad (h=2r)$

$=2\pi r^2+4\pi r^2$

$=6\pi r^2$

$\therefore$ SA of sphere : SA of cylinder

$4\pi r^2 : 6\pi r^2$

$2:3$

Question 5

a) i) y-intercept: $x=0,\ y=-5\ \therefore (0,-5)$

x-intercept: $y=0,$

$$0=-x^2+4x-5$$

$$x=\frac{-4\pm\sqrt{16-4(-1)(-5)}}{2(-1)}$$

$$=\frac{-4\pm\sqrt{16-20}}{-2}$$

$$=\frac{-4\pm\sqrt{-4}}{-2}$$

$\therefore$ No solution

$\therefore$ No x-intercept

ii) Axis of Symmetry: $x=\frac{-b}{2a}=\frac{-4}{2(-1)}=2$

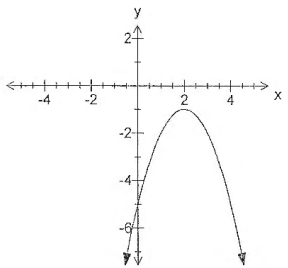
$$y=-(2)^2+4(2)-5$$

$$=-4+8-5$$

$$=-1$$

$\therefore$ Vertex$=(2,-1)$

iii)



b) i)

$$V=\frac{1}{3}Ah$$

$$=\frac{1}{3}\times(4.5)^2\times5$$

$$=33.75\text{m}^3$$

ii)

$$\text{Mass}=33.75\times\frac{3}{4}\times750\text{kg}$$

$$=18984.375\text{kg}$$

$$=18.98\text{ tonnes}$$

c) i)

Area of Pentagon APQCD

= Area of rectangle – Area of triangle PQB

$$=3\times4-\frac{1}{2}\times(3-2x)(4-x)$$

$$=12-\frac{1}{2}[-12-3x-8x+2x^2]$$

$$=12-6+\frac{11}{2}x-x^2$$

$$=-x^2+\frac{11}{2}x+6$$

ii) Maximum area occurs when:

$$x=\frac{-b}{2a}=\frac{-\frac{11}{2}}{2(-1)}=\frac{11}{4}$$

$\therefore$ Maximum Area

$$=-\left(\frac{11}{4}\right)^2+\left(\frac{11}{2}\right)\left(\frac{11}{4}\right)+6$$

$=13.56\text{ units}^2$