SYDNEY BOYS HIGH SCHOOL

2023

YEAR 10 STAGE 5.3
TERM 2
HALF YEARLY ASSESSMENT

Mathematics

General Instructions: **Time Allowed:** 90 minutes

- All questions may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using a black non-erasable pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: AMG & PSP

Name: _____

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Question	Marks
1	/16
2	/16
3	/16
4	/15
5	/13
Total	**/76**

(a) Simplify

(i) $8^{\frac{2}{3}} \times 5^0$ **1**

……

……

(ii) $\dfrac{12x^4y}{4x^2y^3}$. **1**

……

……

(iii) $8 - 2(x - 1)$. **1**

……

……

(b) Calculate $\sqrt{\dfrac{m}{\pi g}}$ correct to 5 significant figures, given $m = 1.45$ and $g = 9.8$. **1**

……

……

(c) How many of the following are true statements? Circle the correct number. **1**

I $a - a \times a = a$ II $a + a - a = a$

III $a \times a \div a = a$ IV $a \div a \times a = a$

A. 1 B. 2 C. 3 D. 4

(d) If a discount of 15% on the marked price of an article saves \$8.40, what is the marked price? **1**

……

……

……

……

(e) Markham invests $8500 for 3 years in a savings account. **1**
He gets 2.3% p.a. with interest compounding yearly.
How much will he have in his savings account at the end of 3 years?

……

……

(f) Find the slope of the line $x + cy + 3 = 0$, given that it passes through (3, –2). **2**

……

……

……

……

……

……

(g) What is the bearing from Derby to Hamilton? Circle the correct response. **1**



A. 030° T B. 060° T C. 150° T D. 210° T

(h) Solve the following quadratic equation, leave your answers in surd form. **2**

$$2x^2 = 3x + 7$$

……

……

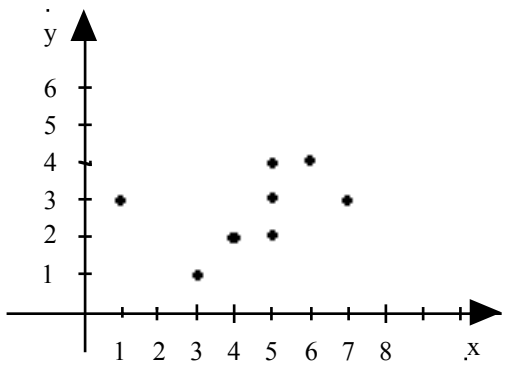
……

……

……

……

(i) If one of the points in the diagram is chosen at random, what is the probability that it lies on the line $x + y - 6 = 0$? **2**



……

……

……

……

……

……

(j) Twenty-four scores are recorded in a stem-and-leaf plot. **1**

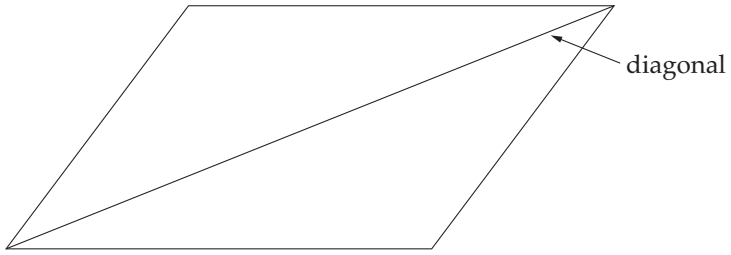
7	1 2 3
6	4 7 8 9
5	2 2 2 6 6 7 8
4	2 6 8 8 9 9
3	3 3 7 9

A score of 63 is added.

Which of the following will increase? Circle the correct response.

A. Mean B. Median C. Mode D. Range

(k) This is a parallelogram with one diagonal drawn. **1**



Consider the following statements:

I The diagonals are equal

II There are no axes of symmetry

III The opposite angles of the parallelogram are equal

IV The diagonals meet at right angles

How many of the above statements are true? Circle the correct response.

A. 0 B. 1 C. 2. D. 3

End of Question 1

Extra Working Page for Question 1 only

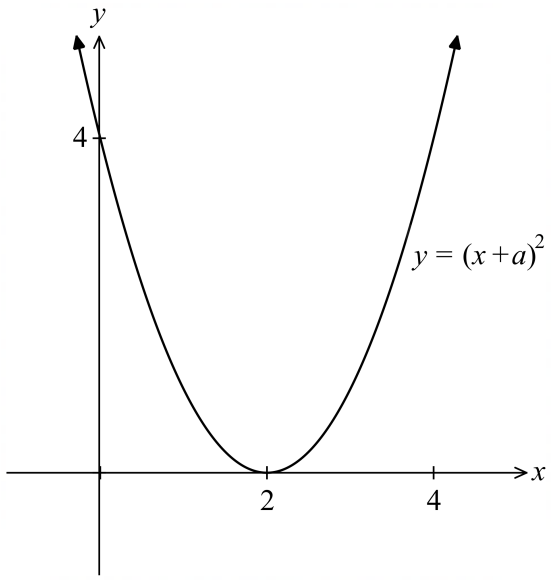
Question 2

Name: _____

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	**/16**

(a) What is the value of a? Circle the correct response. **1**



A. 2 B. –2 C. 4 D. 0

(b) In the diagram, ABC is a right-angled triangle and DEF is a semicircular arc. **3**

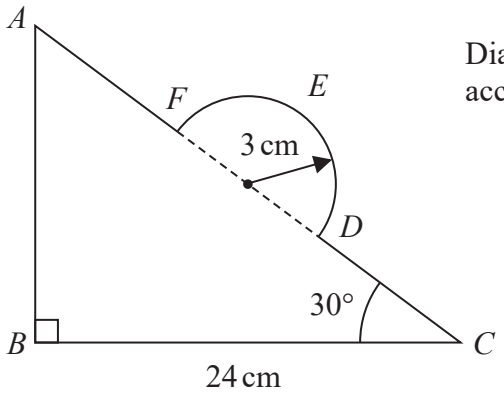


Diagram **NOT** accurately drawn

In triangle ΔABC, $BC = 24$ cm, $\angle ABC = 90°$, and $\angle BCA = 30°$.
The points D and F lie on AC so that DF is the diameter of the semicircular arc DEF. The radius of the semicircular arc is 3 cm.
Calculate the exact length of $AFEDC$.

……

……

……

……

……

……

……

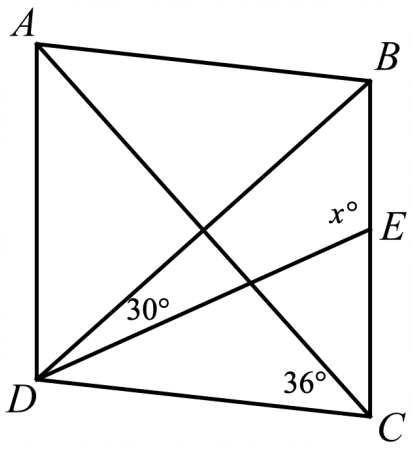
……

……

……

(c) $ABCD$ is a rhombus. **1**

$\angle BDE = 30°$, $\angle ACD = 36°$, and $\angle BED = x°$.



NOT DRAWN
TO SCALE

What is the value of x? Circle the correct response.

A. 90 B. 96 C. 102 D. 108

(d) Which of the following is the equation of a circle with centre (2, 5) and radius 3? **1**
Circle the correct response.

A. $(x-2)^2 + (y-5)^2 = 9$ B. $(x-2)^2 + (y-5)^2 = 3$

C. $(x+2)^2 + (y+5)^2 = 9$ D. $(x+2)^2 + (y+5)^2 = 3$

(e) Find the total surface area of a regular triangular pyramid (tetrahedron) of side 2 cm. **2**

Leave your answer in the form $a\sqrt{3}$, where a is an integer.

……

……

……

……

……

……

(f) Simplify $\dfrac{x^2-1}{(x-1)^2}$. **1**

……

……

……

……

(g) What are the centre and radius of the circle below? **2**

$$x^2 + y^2 + 6y + 5 = 0$$

……

……

……

……

……

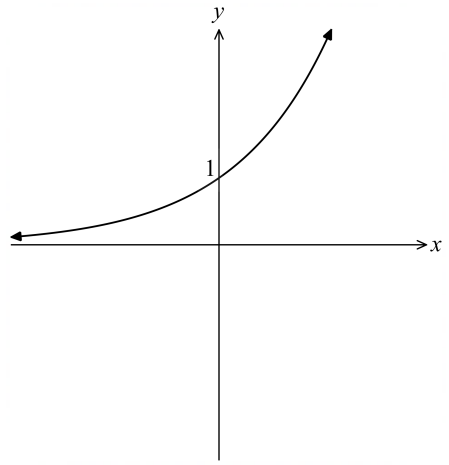
……

……

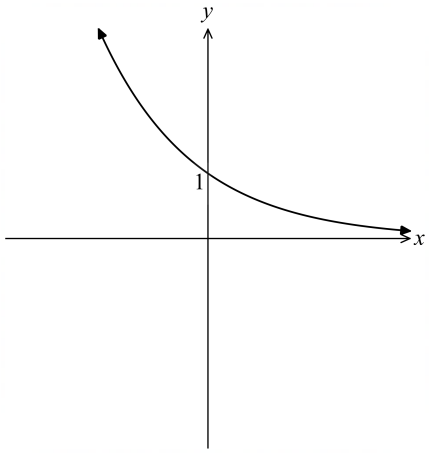
……

(h) Which graph could best represent $y = -2^{-x}$? Circle the correct response. **1**

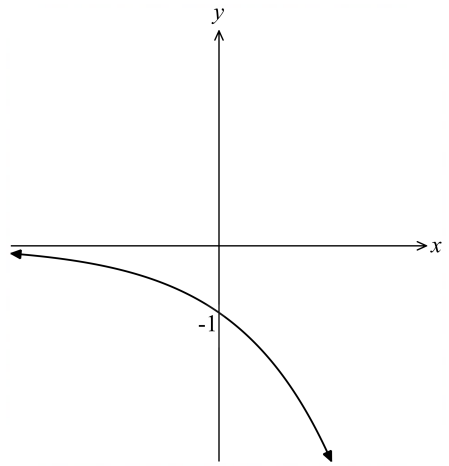
A.



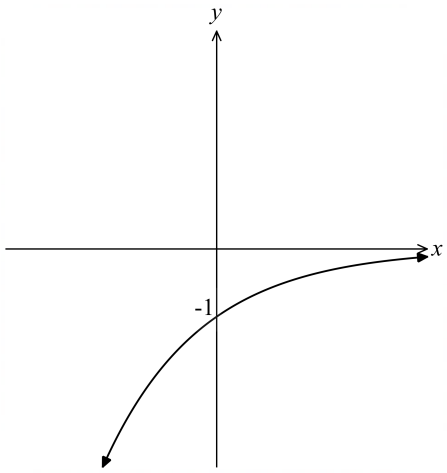
B.



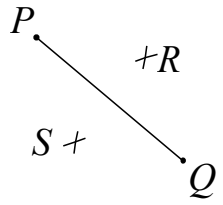
C.



D.



(i) Susan draws arcs of equal radius with centres P and Q. **1**
These arcs intersect at R and S as shown. She then makes the following statements.



I. The line RS must be perpendicular to PQ.

II. The line RS must bisect PQ.

In which of the above statements is Susan correct? Circle the correct response.

A. I only B. II only

C. Both I and II D. Neither I nor II

(j) A couple take out a home loan for \$275 000 for 25 years at 12% compound interest p.a., compounding monthly. They pay a regular monthly instalment of \$$Q$. **3**

The formula below is used to calculate Q, the monthly instalment, where P is the principal of the loan, R is $\left(1+\frac{r}{100}\right)$ where r% is the monthly interest rate, and n is the number of months of the loan.

$$Q=\frac{PR^n(R-1)}{R^n-1}$$

Find the total amount of interest that the couple will pay throughout their loan.

……

……

……

……

……

……

……

……

……

……

……

……

……

End of Question 2

Extra Working Page for Question 2 only

Question 3

Name: _____

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	**/16**

(a) Brendan buys a second-hand computer for $1600. The computer depreciates at 60% p.a. How much will it be worth after three years? **2**

……

……

……

……

……

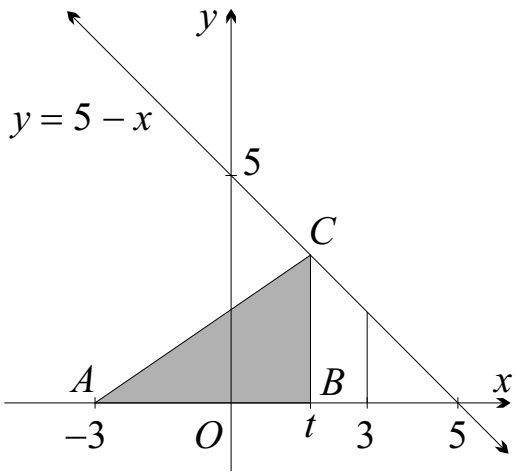
……

……

……

……

(b)



The graph above shows Δ *ABC* with one vertex at $A(-3, 0)$.
The second vertex $B(t, 0)$ lies between the origin and $x = 3$, so that $0 \leq t \leq 3$.
The third vertex *C* lies on the line $y = 5 - x$ and is directly above *B*.
Let *R* be the area of Δ *ABC*.

(i) Show that $2R = 15 + 2t - t^2$. **1**

……

……

……

……

……

……

(ii) Hence find the maximum area of Δ *ABC*. **2**

……

……

……

……

……

……

……

……

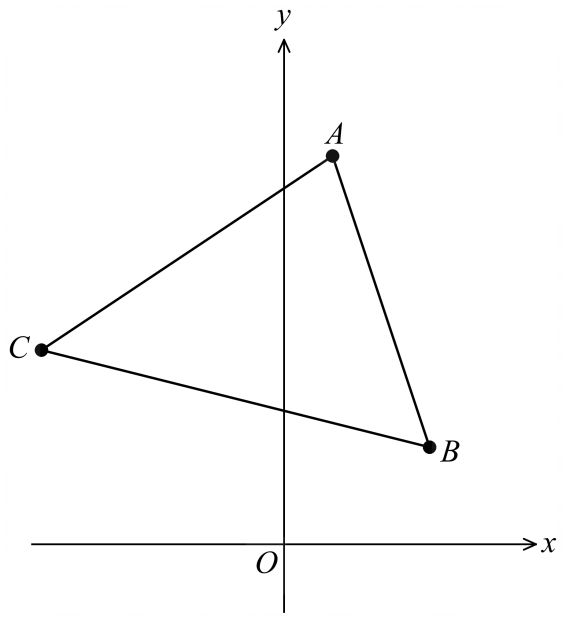
……

……

……

……

(c) In ΔABC, A is (1, 8), B is (3, 2), and C is (–5, 4).



(i) Find P, such that $APBC$ is a parallelogram. **2**

……

……

……

……

……

……

(ii) Find the equation of OB. **1**

……

……

……

……

(iii) Are O, B, and P collinear? Justify. **2**

……

……

……

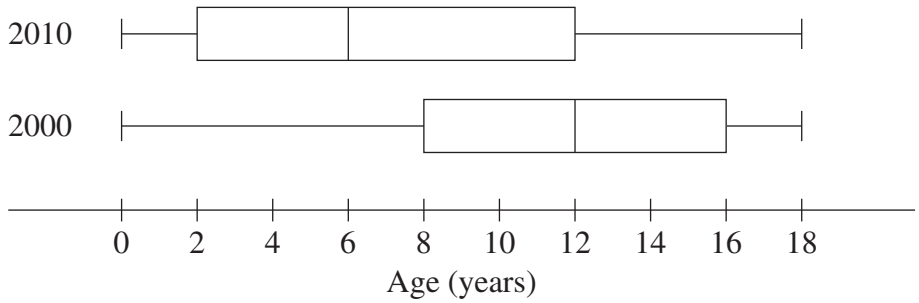
……

……

……

(d) The graphs show the distribution of the ages of children in Numbertown in 2000 and 2010. **2**

Distribution of the ages of children in Numbertown



Identify TWO changes in the distribution of ages between 2000 and 2010.
In your answer, refer to measures of location or spread or the shape of the distributions.

……

……

……

……

……

……

……

(e) (i) Factorise $x^2 + x - 6$. **1**

……

……

(ii) Hence solve $x^2 + x - 6 < 0$. **2**

……

……

……

……

……

……

(f) If $\left(ax^6\right)^{\frac{1}{n}} = 7x^3$, write down the exact value of a, where a is a positive number. **1**

……

……

End of Question 3

Extra Working Page for Question 3 only

Question 4

Name: _____

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	**/15**

(a) The time T (in days) taken to complete a particular harvest varies inversely as the number of harvesters H used for the harvest. **2**

If 10 harvesters take 12 days to complete the task, how long would it take 15 harvesters?

……

……

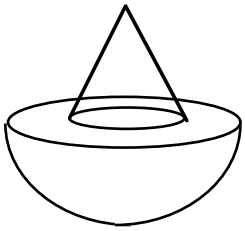
……

……

……

……

(b) The figure shows a child's glass top in the form of a hemisphere surmounted by a cone. **3**
The height of the cone and the radius of the hemisphere are both 14 cm.
The radius of the base of the cone is 8 cm.



Find the surface area of the solid, correct to two decimal places.

……

……

……

……

……

……

(c) Show that $\dfrac{6x^3}{9x^2-144} \div \dfrac{2x^4}{3(x-4)}$ can be written in the form $\dfrac{1}{x(x+r)}$, **2**

where r is an integer.

……

……

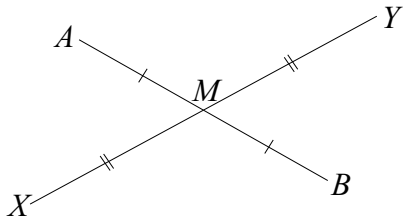
……

……

……

……

(d)



The diagram above shows two intervals AB and XY that bisect each other at M.

(i) Prove that the triangles AMX and BMY are congruent. **2**

……

……

……

……

……

……

……

……

……

……

……

……

(ii) Hence prove that AX and BY are parallel. **2**

……

……

……

……

……

……

……

……

……

……

……

……

(e) (i) Show that $\frac{2x-3}{x-3} = 2 + \frac{3}{x-3}$ **1**

……

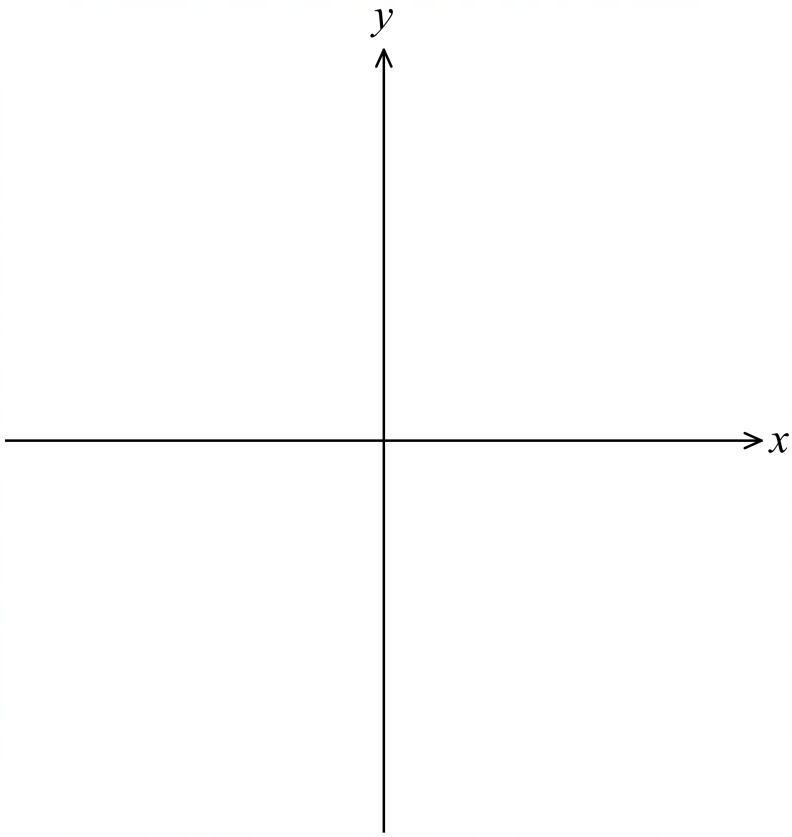
……

……

……

……

(ii) Hence or otherwise sketch $y = \frac{2x-3}{x-3}$, showing clearly any asymptotes and intercepts. **3**



End of Question 4

Extra Working Page for Question 4 only

Question 5

Name: _____

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	**/13**

(a) **2**

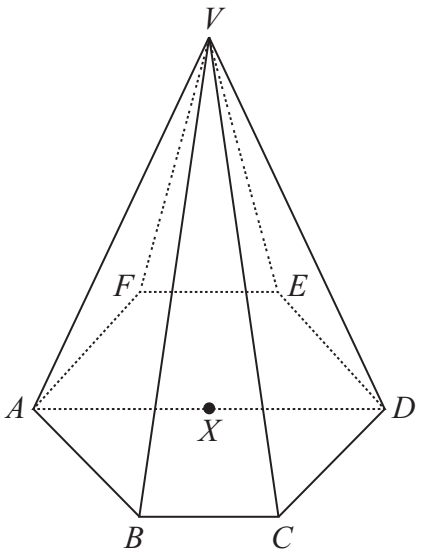


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a right pyramid with vertex V and base $ABCDEF$ which is a regular hexagon. The diagonal AD of the base is 10 cm and X is the mid-point of AD. The height VX of the pyramid is 12 cm.

Find the volume of the pyramid to two significant figures.

……

……

……

……

……

……

……

……

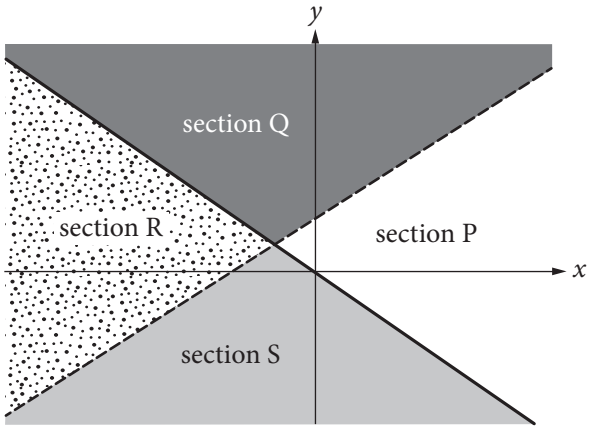
……

……

(b) A black jar contains 3 green marbles and P pink marbles, where $P > 1$. **3**
Chauncey takes two marbles out simultaneously.
If the probability that they are both the same colour is a half, find all possible values of P.

(c) Circle the correct response. **1**

$$\begin{cases} y \leq -x \\ 2y > 3x + 2 \end{cases}$$



A system of inequalities and a graph are shown above.
Which section(s) of the graph could represent all of the solutions to the system?

A. *R* B. *Q* & *S* C. *P* & *Q* D. *Q*, *R*, & *S*

(d) Simplify the following, leaving your answer in index form. **2**

$$(2+3)(2^2+3^2)(2^4+3^4)(2^8+3^8)(2^{16}+3^{16})(2^{32}+3^{32})(2^{64}+3^{64})$$

……

……

……

……

……

……

……

……

……

……

……

(e) (i) By factorising, or otherwise, show that **2**

$(p^2 - q^2 - r^2)^2 - 4q^2r^2 = (p - q - r)(p - q + r)(p + q - r)(p + q + r)$.

Part (e) continues on page 29

(e) (continued)

(ii) Hence solve $\left(p^2 - q^2 - r^2\right)^2 - 4q^2r^2 = 45$ for positive integers p, q, and r. **3**

End of paper

Extra Working Page for Question 5 only

SYDNEY BOYS HIGH SCHOOL

2023

YEAR 10 STAGE 5
TERM 2
HALF YEARLY

Sample Solutions

NOTE: Some of you may be disappointed with your mark.
This process of checking your mark is NOT the opportunity to improve your marks. Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done. Writing something down does not justify that your working can be linked to a mark.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

SYDNEY BOYS HIGH SCHOOL

2023

YEAR 10 STAGE 5.3
TERM 2
HALF YEARLY ASSESSMENT

Mathematics

General Instructions: **Time Allowed:** 90 minutes

- All questions may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using a black non-erasable pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: AMG & PSP

Name: SOLUTIONS

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong & J-Tan	X
10Ma**L**	D Taylor	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Question	Marks
1	16 /16
2	/16
3	/16
4	/15
5	/13
Total	**/76**

(a) Simplify

(i) $8^{\frac{2}{3}} \times 5^0$ use calc. **1**

$\sqrt[3]{8^2} \times 1 = 4$

(ii) $\frac{12x^4y^3}{4x^2y^3}$. **1**

$= \frac{3x^2}{y^2}$ or $3x^2y^{-2}$

(iii) $8 - 2(x-1)$. **1**

$= 8 - 2x + 2$

$= 10 - 2x$ or $2(5-x)$

(b) Calculate $\sqrt{\frac{m}{\pi g}}$ correct to 5 significant figures, given $m = 1.45$ and $g = 9.8$. **1**

$\sim 0.21701835\ldots$

~ 0.21702 (5SF: ½ mark to show rounding)

(c) How many of the following are true statements? Circle the correct number. **1**

I $a - a \times a = a$ ✗ II $a + a - a = a$ ✓

III $a \times a \div a = a$ ✓ IV $a \div a \times a = a$ ✓

A. 1 B. 2 C. 3 D. 4

(d) If a discount of 15% on the marked price of an article saves \$8.40, what is the marked price? **1**

$15\% = 8.40$ (first phrase translation)

$100\% = \frac{8.4}{15} \times 100$

$= \$56$

(e) Markham invests $8500 for 3 years in a savings account. 1
He gets 2.3% p.a. with interest compounding yearly.
How much will he have in his savings account at the end of 3 years?

$A = 8500(1+2{\cdot}3\%)^3$

$\sim 9100{\cdot}09$ or $9100{\cdot}10$

since this is money, rounding must be 2 D.P.

(f) Find the slope of the line $x + cy + 3 = 0$, given that it passes through $(3, -2)$. 2

If it goes through $(3,-2)$, then it can sub. into the equation and be true, ie.

$3 + c(-2) + 3 = 0$

$-2c = -6$

$c = 3$

$\therefore x + 3y + 3 = 0$

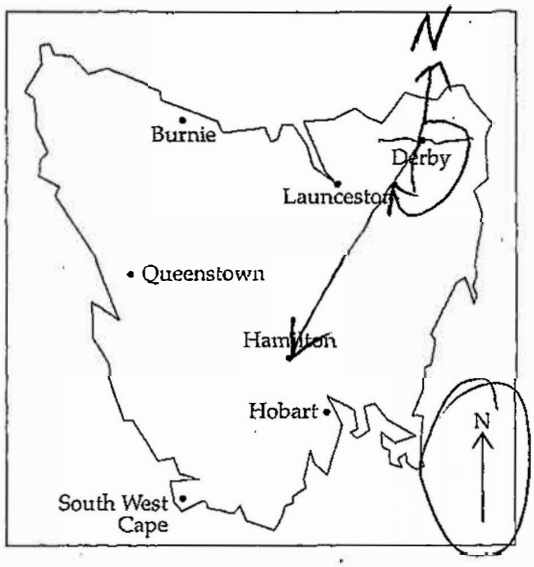
$3y = -x - 3$

$y = -\frac{1}{3}x - 1$

$\therefore m = -\frac{1}{3}$

Pls see extra working page for alt. soln. (no c needed)

(g) What is the bearing from Derby to Hamilton? Circle the correct response. 1



A. 030° T B. 060° T C. 150° T (D.) 210° T

no other answer possible

(h) Solve the following quadratic equation, leave your answers in surd form. 2

$2x^2 = 3x + 7$

$2x^2 - 3x - 7 = 0$

$a = 2, b = -3, c = -7$

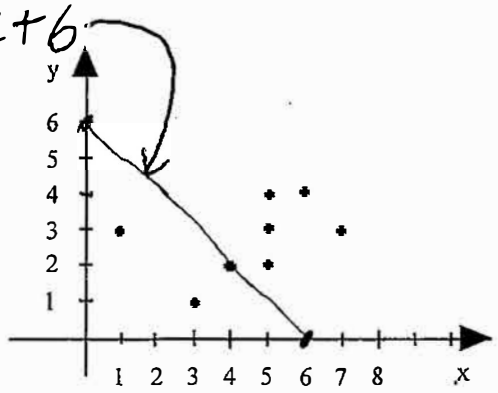
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$

$= \frac{3 \pm \sqrt{65}}{4}$ exact

(i) If one of the points in the diagram is chosen at random, what is the probability that it lies on the line $x + y - 6 = 0$? **2**

$y = -x + 6$



There are 8 dots → denom.
only 1 dot on line → $P = \frac{1}{8}$

(j) Twenty-four scores are recorded in a stem-and-leaf plot. **1**

7	1 2 3
6	3 4 7 8 9
5	2 2 2 6 6 7 8
4	2 6 8 8 9 9
3	3 3 7 9

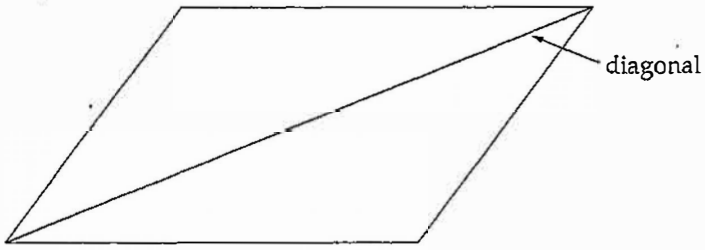
A score of 63 is added.

Which of the following will increase? Circle the correct response.

A. Mean B. Median C. Mode D. Range

Intuitively, the answer must be A because the old mean is 50 something, & 63 MUST bump the mean up.
The median gets pushed from between 52 onto a 52, ∴ no change
Mode is not affected by the 63 score.
Range = H − L no change b/c 63 neither H nor L.

(k) This is a parallelogram with one diagonal drawn. **1**



Consider the following statements:

I The diagonals are equal ✗

II There are no axes of symmetry ✓

III The opposite angles of the parallelogram are equal ✓

IV The diagonals meet at right angles ✗

How many of the above statements are true? Circle the correct response.

A. 0 B. 1 C. 2. D. 3

There were some flukes.

End of Question 1

Extra Working Page for Question 1 only

5) how to avoid calculating c.

$x + cy + 3 = 0$ & $(3, -2)$

what does $y =$ to cancel c?

$y = 0$

$\therefore x + 3 = 0$

$\therefore (-3, 0)$ is an intercept

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - -2}{-3 - 3}$$

$$= \frac{2}{-6}$$

$$= -\frac{1}{3}$$

Question 2

Name: Solutions

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	/16

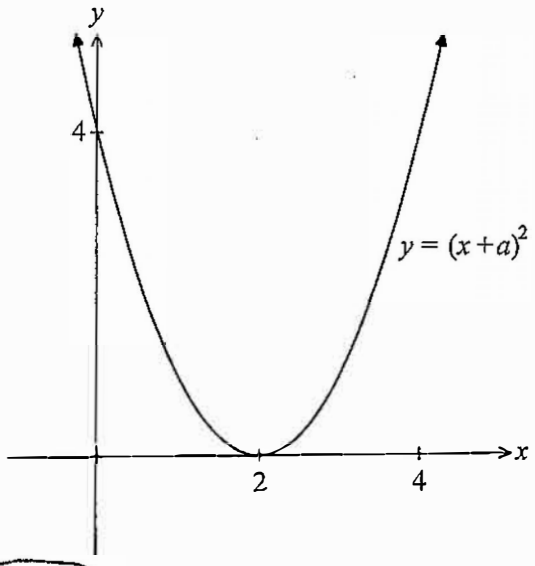
(a) What is the value of a? Circle the correct response. **1**

Point $(2,0)$ is on the curve $y=(x+a)^2$

So $0=(2+a)^2$

$\therefore 2+a=0$

$a=-2$



A. 2 B. −2 C. 4 D. 0

(b) In the diagram, ABC is a right-angled triangle and DEF is a semicircular arc. **3**

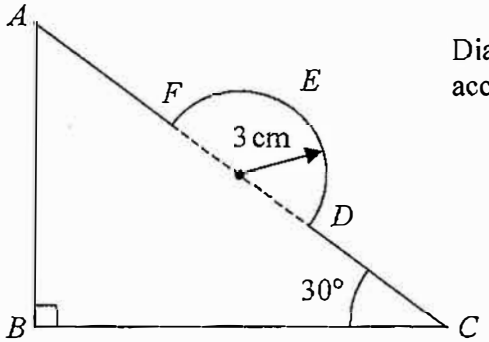


Diagram **NOT** accurately drawn

In triangle ΔABC, $BC = 24$ cm, $\angle ABC = 90°$, and $\angle BCA = 30°$.
The points D and F lie on AC so that DF is the diameter of the semicircular arc DEF. The radius of the semicircular arc is 3 cm.
Calculate the exact length of $AFEDC$.

$\cos 30^\circ = \frac{24}{AC}$

ie. $AC = \frac{24}{\cos 30^\circ}$

$= 24 \times \frac{2}{\sqrt{3}}$

$= 16\sqrt{3}$

So $AFEDC = 16\sqrt{3} + 3\pi - 6$.

You have to give the EXACT value as requested.

ie. no rounding off allowed!

Many students found AB.

This is just a waste of time.

In trig, given an angle + side, just find the required side by using the appropriate trig ratio. In this question you need to find the exact value of the trig function. You cannot leave the trig function.

BTW did I mention, you have to give the EXACT answer?

(c) $ABCD$ is a rhombus. **1**

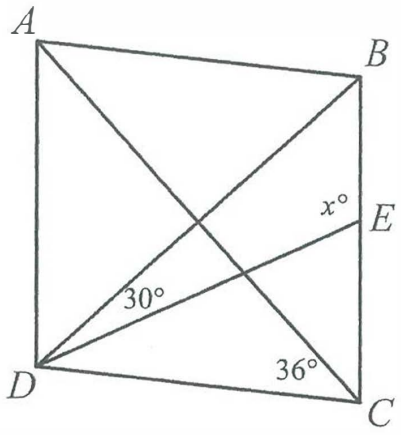
$\angle BDE = 30°$, $\angle ACD = 36°$, and $\angle BED = x°$.

$\angle DCB = 2 \times 36^\circ$ (rhombus diagonal bisects)
$= 72^\circ$

$\angle CBA = 180^\circ - \angle DCB$ (Supp in rhombus)
$= 108^\circ$

$\therefore \angle CBA = \frac{108^\circ}{2} = 54^\circ$

In $\triangle DBE$, $x = 180 - 30 - 54$
$= 96$



NOT DRAWN TO SCALE

What is the value of x? Circle the correct response.

A. 90 B. 96 C. 102 D. 108

(d) Which of the following is the equation of a circle with centre (2, 5) and radius 3? Circle the correct response. **1**

A. $(x-2)^2 + (y-5)^2 = 9 = 3^2$ B. $(x-2)^2 + (y-5)^2 = 3$

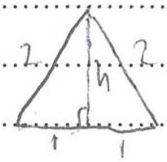
C. $(x+2)^2 + (y+5)^2 = 9$ D. $(x+2)^2 + (y+5)^2 = 3$

Done well

(e) Find the total surface area of a regular triangular pyramid (tetrahedron) of side 2 cm. **2**

Leave your answer in the form $a\sqrt{3}$, where a is an integer.

Face:



$2^2 = h^2 + 1^2$
$h = \sqrt{4-1}$
$= \sqrt{3}$

$A_F = \frac{1}{2} \times 2 \times \sqrt{3}$
$= \sqrt{3}$

$\therefore SA = 4 \times \sqrt{3}$ (4 faces)
$= 4\sqrt{3}$ cm^2

Remember a tetrahedron has 4 faces.

The question gives a hint the answer must have $\sqrt{3}$ in it.

(f) Simplify $\frac{x^2 - 1}{(x-1)^2}$. **1**

$= \frac{(x-1)(x+1)}{(x-1)(x-1)}$

$= \frac{x+1}{x-1}$

Be able to quickly identify the difference of two squares.

In fractions: factorise then cancel.

Write x clearly.

(g) What are the centre and radius of the circle below? **2**

$$x^2 + y^2 + 6y + 5 = 0$$

$x^2 + y^2 + 6y + 9 = 9 - 5$

$x^2 + (y+3)^2 = 4 = 2^2$

$\therefore$ centre is $(0, -3)$

and radius is 2.

remember to write the paretheses in $(0, -3)$.

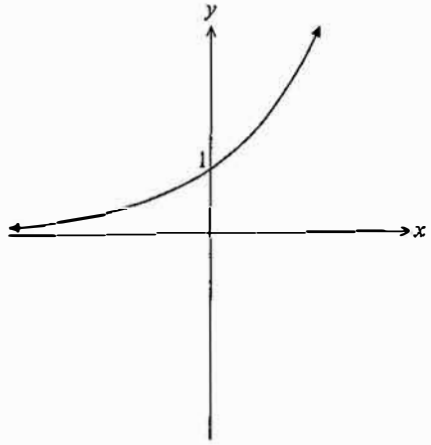
You find the 9 by halving 6 then squaring.

Remember to put it on both sides.

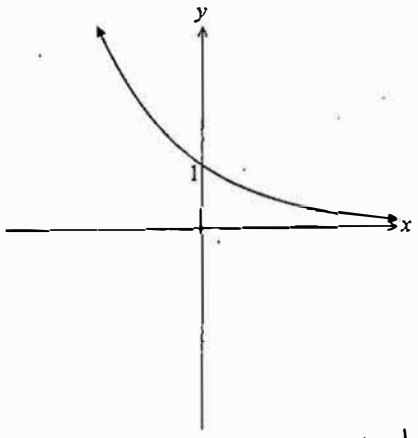
The x^2 is $(x+0)^2$.

(h) Which graph could best represent $y = -2^{-x}$? Circle the correct response. **1**

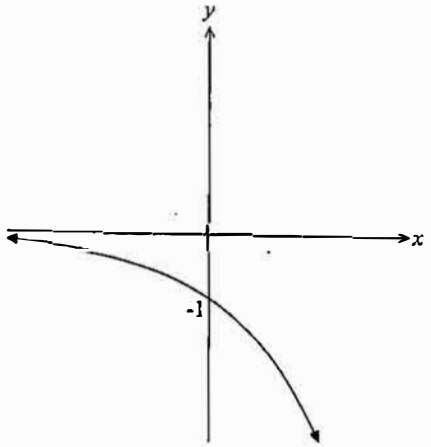
A.



B.

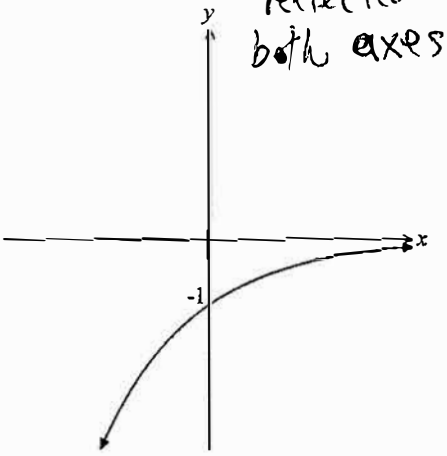


C.

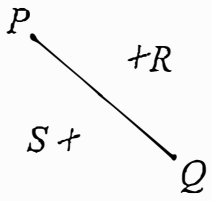


(D.)

reflected on both axes.



(i) Susan draws arcs of equal radius with centres P and Q. **1**
These arcs intersect at R and S as shown. She then makes the following statements.



Let r be the arc radii.
Then $r = PR = RQ = QS = SP$.
So PRSQ is a rhombus.
∴ The diagonals RS + PQ bisect each other at right angles.

I. The line RS must be perpendicular to PQ.

II. The line RS must bisect PQ.

In which of the above statements is Susan correct? Circle the correct response.

A. I only B. II only

C. Both I and II D. Neither I nor II

(j) A couple take out a home loan for \$275 000 for 25 years at 12% compound interest p.a., compounding monthly. They pay a regular monthly instalment of \$$Q$. **3**

The formula below is used to calculate Q, the monthly instalment, where P is the principal of the loan, R is $\left(1+\frac{r}{100}\right)$ where r% is the monthly interest rate, and n is the number of months of the loan.

$$Q=\frac{PR^n(R-1)}{R^n-1}$$

Find the total amount of interest that the couple will pay throughout their loan.

$R = 1 + \frac{r}{100}$, $r = \frac{12\%}{12 \text{ months}}$

$= 1.01$

$Q = \frac{275\,000(1.01)^{25\times12}(0.01)}{(1.01)^{25\times12}-1}$

$= \frac{2750(1.01)^{300}}{(1.01)^{300}-1}$

$= \$2896.366\ldots$ (keep in calculator)

Interest $= 300Q - 275\,000$

$= \$593\,909.92$

To avoid rounding errors keep the numbers in the calculator.

Remember r is monthly but the given rate is annual, so divide by 12.

End of Question 2

Year 10 HY Question 3 (16 marks) SOLUTIONS

(a) Brendan buys a second-hand computer for \$1600. The computer depreciates at 60% p.a. How much will it be worth after three years? 2

$A = P(1-r)^n$

$= \$1600(1-0.6)^3$

$= \$102.40$

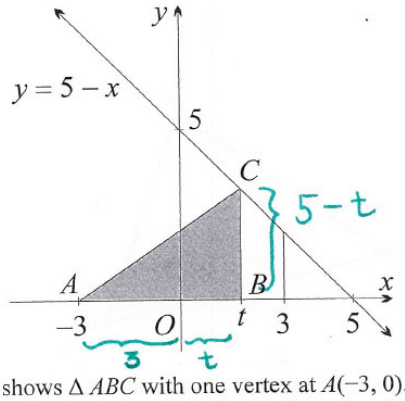
Marker's comment:

Very well done. Most students can apply the depreciation formula.

All monetary values should start with the '\$' sign and answer in two decimal places.
No marks were deducted in this instance, this time.

Criteria	Marks
Correct solution	2
Correct substitution into the depreciation formula	1

(b)



The graph above shows ΔABC with one vertex at $A(-3, 0)$.
The second vertex $B(t, 0)$ lies between the origin and $x = 3$, so that $0 \le t \le 3$.
The third vertex C lies on the line $y = 5 - x$ and is directly above B.
Let R be the area of ΔABC.

(i) Show that $2R = 15 + 2t - t^2$. 1

$R = \frac{1}{2} \times (t+3) \times (5-t)$

$2R = (3+t)(5-t)$

$\therefore 2R = 15 + 2t - t^2$ RTP

Marker's comment:

Students must show R is the area of the triangle. They must show why we have $2R$ on the right-hand side.
Students need to be able to identify the length and perpendicular height of the triangle.

Criteria	Marks
Show the correct solution using the area of the triangle	1
Correctly identify the length of BC	0.5

(ii) Hence find the maximum area of ΔABC. 2

$R = -t^2/2 + t + 15/2$

$a < 0 \therefore$ Concave down

max TP at $t = -b/2a$

ie $t = \frac{-1}{2(-1/2)} = \frac{-1}{-1} = 1$

max R at $t = 1$

$R = -\frac{1}{2}(1)^2 + (1) + \frac{15}{2}$

$= -\frac{1}{2} + 1 + \frac{15}{2}$

$\therefore$ max area of $\Delta ABC = 8$ units^2

Marker's comment:

Many students had a lot of issues with this question.

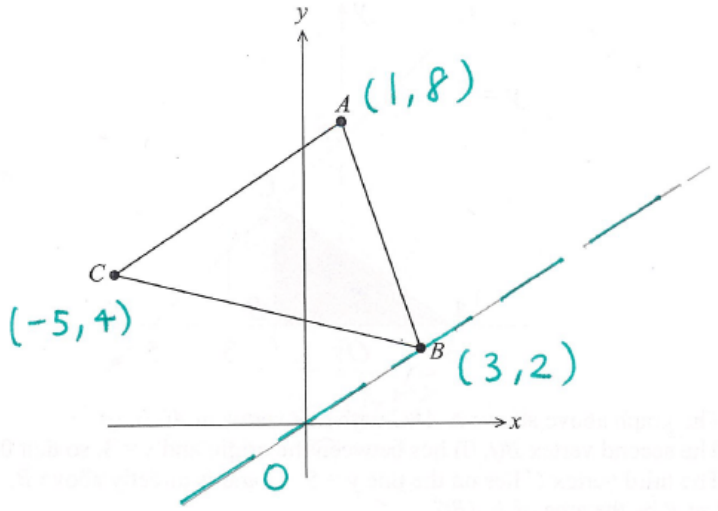
Most students needed to see the link between the value of t and the axis of symmetry.

Most candidates got the correct solution from trial and error.

This only worked because the value of t is an integer.

Criteria	**Marks**
Correct solution	2
Correctly substitute $t = 1$ into the formula of $2R$ with working out	1.5
Correctly substitute any values $0 \le t \le 3$ into the formula of R	1

(c) In ΔABC, A is $(1, 8)$, B is $(3, 2)$, and C is $(-5, 4)$.



(i) Find P, such that $APBC$ is a parallelogram. 2

Let $P(x, y)$

$AP \parallel BC$ $\quad \frac{y-8}{x-1} = \frac{2-4}{3--5} = \frac{-2}{8}$

$x - 1 = 8 \quad \therefore x = 9$

$y - 8 = -2 \quad \therefore y = 6$

$\therefore P = (9, 6)$

Marker's comment:

Most students struggled with this question. Students need to understand the order of this parallelogram. It is from A to P, then B, and then C; hence point P is in the first quadrant.

This question addresses the gradients, and it is the simplest method.

Criteria	Marks
Correct solution	2
Correct working but only one correct coordinate in $P = (9, 6)$	1.5
Identify $P = (-3, -2)$ with working ignoring the order of $APBC$	1
Identify $P = (-3, -2)$ with no working out	0.5

Part (c) continues on the next page

(c) (continued)

(ii) Find the equation of OB. 1

$m_{OB} = \frac{2}{3}$

$y = mx + c$ through the origin

$\therefore y = \frac{2}{3}x$

or $2x - 3y = 0$

Marker's comment:

Mostly well done.

Criteria	Marks
Correct solution	1
Correctly shown the gradient of OB	0.5

(iii) Are O, B, and P collinear? Justify. 2

Sub $P(9,6)$ to $2x - 3y = 0$

$2(9) - 3(6) = 0$

$\therefore P$ is on OB

$\therefore O, B$ and P are collinear

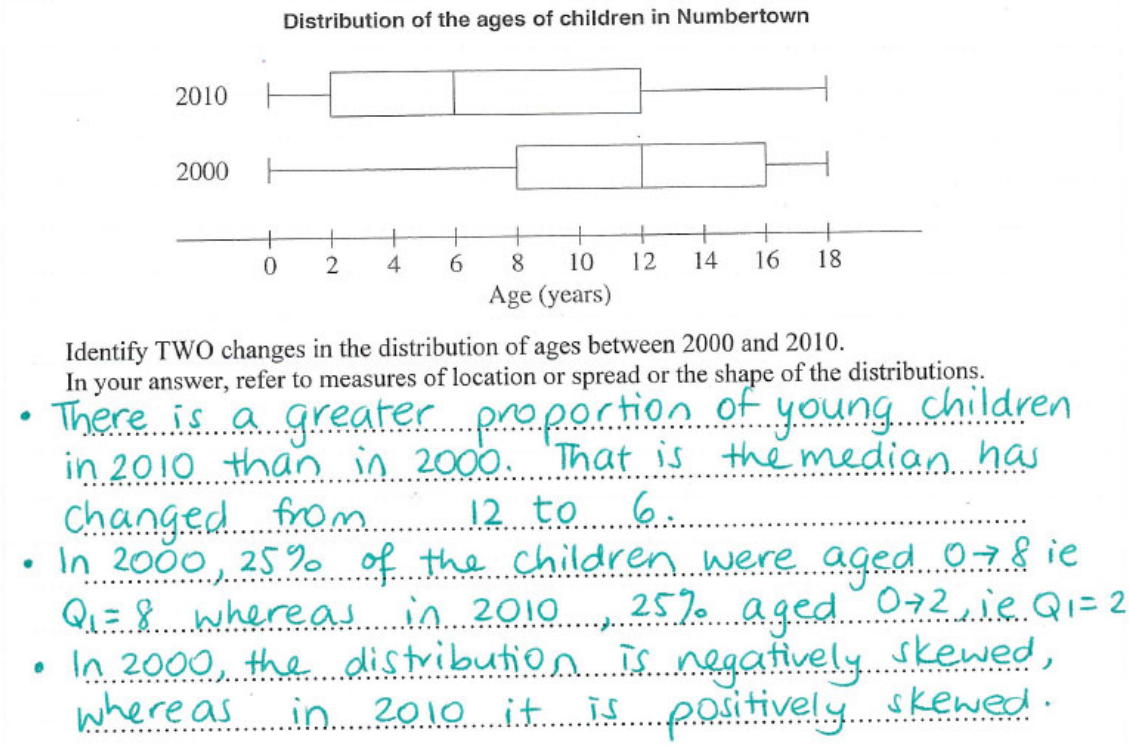
OR// $m_{OB} = m_{OP} = \frac{6}{9} = m_{BP} = \frac{4}{6} = \frac{2}{3}$.

Marker's comment:

Most students identified that the points are collinear but did not justify this mathematically.
They must show the point P is on the line of OB, or they have the same gradients.
Students must **SHOW** the gradients are equal or the point P is on the line OB, not just stating it.

Criteria	Marks
Correct solution with working out	2
State the point are collinear OR show that they are not collinear from consistent errors from previous sections.	1

(d) The graphs show the distribution of the ages of children in Numbertown in 2000 and 2010. **2**



Identify TWO changes in the distribution of ages between 2000 and 2010.
In your answer, refer to measures of location or spread or the shape of the distributions.

- There is a greater proportion of young children in 2010 than in 2000. That is the median has changed from 12 to 6.
- In 2000, 25% of the children were aged 0→8 ie $Q_1 = 8$ whereas in 2010, 25% aged 0→2, ie $Q_1 = 2$.
- In 2000, the distribution is negatively skewed, whereas in 2010 it is positively skewed.

Marker's comments:

Mostly well done.

Students need to CLEARLY identify TWO CHANGES, which is the compare the two different year groups.

Some students got confused with 'range' and 'interquartile range', they are not the same.

Criteria	Marks
Two correct comparisons that are age related.	2
One correct comparison that is age related.	1

(e) (i) Factorise $x^2 + x - 6$. 1

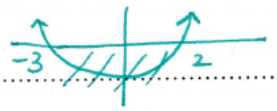
$(x+3)(x-2)$

Marker's comments:

Very well done. The question asked students to factorise, NOT to solve.

Criteria	Marks
Correct solution.	1
$(x + 3)$ OR $(x - 2)$	0.5

(ii) Hence solve $x^2 + x - 6 < 0$. 2



$\therefore -3 < x < 2$

or $x > -3$ and $x < 2$

Marker's comment:

Many students struggled solving this quadratic inequality.

The solutions are the section of the parabola that is below the y-axis because $y < 0$.

Student should NOT answer with $x < -3, 2$

Criteria	Marks
Correct solution	2
$x > -3$ OR $x < 2$	1

(f) If $(ax^6)^{\frac{1}{n}} = 7x^3$, write down the exact value of a, where a is a positive number. 1

$a^{1/n} \times x^{\frac{6}{n}} = 7x^3$ $\quad \frac{6}{n} = 3 \quad \therefore n = 2$

$a^{1/2} = 7$

$\therefore a = 49$

Marker's comment:

Most students could not equate the terms.

Criteria	Marks
Correct solution	1
Correctly identify $n = 2$	0.5

Question 4

Name: SOLUTIONS

Class	Teacher	
10Ma**A**	J James & J Millar	
10Ma**B**	A Ward	
10Ma**C**	R Wang	
10Ma**P**	R Wong	
10Ma**L**	D Taylor	
10Ma**U**	M Zheng	
10Ma**S**	A Fuller	

Mark	**/15**

2023 Y10 HY Question 4 Marking Notes

a) Very well done, nearly everyone got this correct.
Variation was the method of choice although some manipulated rates.
Layout showed a logical approach.

b) Poorly done, almost all students made an error at some stage.
Notation was poor many using *r* from both radii.
Generally poor layout, some with no formulae.
Some were not able to be deciphered and hence, some marks were not gained.
Some were just a jumble of numbers.
Remember, most marks are gained for communicating your solution, not just the answer.

c) More than half the students got full marks for this question.
Most recognised the method to use the difference of two squares.
Many cancelled factors so much that the solution was not readable.
If the method was unintelligible some marks were not gained.

d) (i) Very well done and laid out.

Many forgot to say which triangles they were working with.

Reasoning was generally well done.

(ii) A little less than half got full marks on this question.

Use of parallel lines to prove parallel lines was common.

Also, alternate angles need to be equal to prove parallel lines, not just alternate.

e) (i) Done well by more than half of the students.

Many students did not state what they started with and put down the first line of working as their start point.

This could have come from both the left or right hand side and could be read as proving LHS = LHS.

(ii) Done well by about half of the students.

Marks were awarded for asymptotes x2, Intercepts x2, shape and orientation.

(a) The time T (in days) taken to complete a particular harvest varies inversely as the number of harvesters H used for the harvest. **2**

If 10 harvesters take 12 days to complete the task, how long would it take 15 harvesters?

$T \propto \frac{1}{H}$

$T = \frac{K}{H}$

$12 = \frac{K}{10}$

$K = 120$

$T = \frac{120}{H}$

$= \frac{120}{15}$

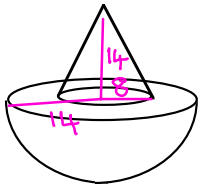
$= 8$ days

Very well done, nearly everyone got this correct.

Variation was the method of choice although some manipulated rates.

Layout showed a logical approach.

(b) The figure shows a child's glass top in the form of a hemisphere surmounted by a cone. **3**
The height of the cone and the radius of the hemisphere are both 14 cm.
The radius of the base of the cone is 8 cm.



$r = 14$

$R = 8$

Find the surface area of the solid, correct to two decimal places.

SA Hemisphere $= \frac{1}{2}(4\pi r^2) = 2\pi(14)^2 = 392\pi$

SA Cone $= \pi R l = \pi R(\sqrt{14^2 + 8^2}) = 8\pi\sqrt{260}$

Area of Annulus $= \pi r^2 - \pi R^2 = \pi(14)^2 - \pi(8)^2 = 132\pi$

S.A. $= 392\pi + 8\pi\sqrt{260} + 132\pi$

$= 652.9961\pi$

$= 2051.45$

Poorly done, almost all students made an error at some stage.

Notation was poor many using r from both radii.
Generally poor layout, some with no formulae.
Some were not able to be deciphered and hence, some marks were not gained.
Some were just a jumble of numbers.
Remember, most marks are gained for communicating your solution, not just the answer.

(c) Show that $\frac{6x^3}{9x^2-144} \div \frac{2x^4}{3(x-4)}$ can be written in the form $\frac{1}{x(x+r)}$, **2**

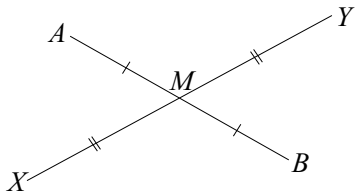
where r is an integer.

$= \frac{6x^3}{(3x+12)(3x-12)} \times \frac{3x-12}{2x^4}$

$= \frac{6x^3}{2x^4} \times \frac{3(x-4)}{9(x+4)(x-4)} = \frac{3}{x} \times \frac{1}{3(x+4)} = \frac{1}{x(x+4)}$

More than half the students got full marks for this question.
Most recognised the method to use the difference of two squares.
Many cancelled factors so much that the solution was not readable.
If the method was unintelligible some marks were not gained.

(d)



The diagram above shows two intervals AB and XY that bisect each other at M.

(i) Prove that the triangles AMX and BMY are congruent. **2**

In $\triangle$'s AMX and BMY
$\angle AMX = \angle BMY$ (vert. opp)
AM = BM (given)
MX = MY (given)
$\therefore \triangle AMX \equiv \triangle BMY$ (S.A.S)

(i) Very well done and laid out.
Many forgot to say which triangles they were working with.
Reasoning was generally well done.

(ii) Hence prove that AX and BY are parallel. **2**

Since $\triangle AMX \equiv \triangle BMY$ (proven above)
$\angle MAX = \angle MBY$ (corresponding angles of congruent $\triangle$'s)
$\therefore$ AX || YB Alternate $\angle$'s are equal

(ii) A little less than half got full marks on this question.
Use of parallel lines to prove parallel lines was common.
Also, alternate angles need to be equal to prove parallel lines, not just alternate.

(e) (i) Show that $\frac{2x-3}{x-3} = 2 + \frac{3}{x-3}$ **1**

RHS $= 2 + \frac{3}{x-3} = \frac{2(x-3)}{(x-3)} + \frac{3}{x-3} = \frac{2x-6+3}{x-3}$

$= \frac{2x-3}{x-3} =$ LHS

(i) Done well by more than half of the students.

Many students did not state what they started with and put down the first line of working as their start point.

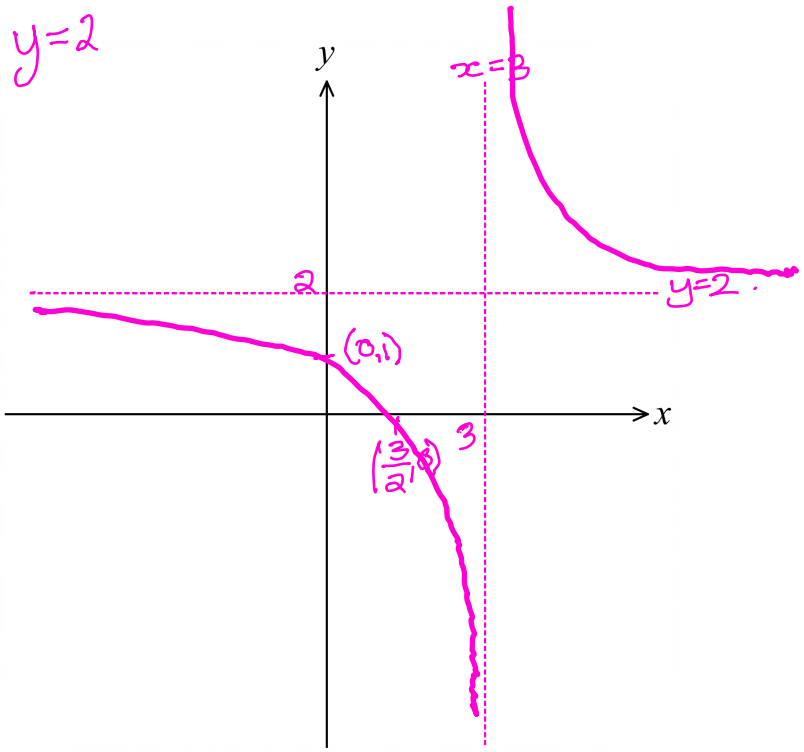
This could have come from both the left or right hand side and could be read as proving LHS = LHS.

(ii) Hence or otherwise sketch $y = \frac{2x-3}{x-3}$, showing clearly any asymptotes and intercepts. **3**

Asymptote $x=3$, $y=2$

$y=0$ $x=\frac{3}{2}$

$x=0$ $y=1$



(ii) Done well by about half of the students.

Marks were awarded for asymptotes, intercepts, shape and orientation.

Question 5 SOLUTIONS

(a) **2**

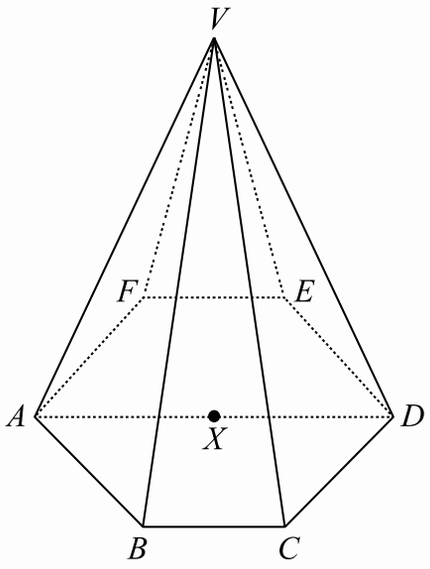


Diagram **NOT** accurately drawn

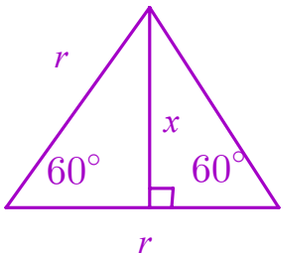
Figure 1

Figure 1 shows a right pyramid with vertex V and base $ABCDEF$ which is a regular hexagon. The diagonal AD of the base is 10 cm and X is the mid-point of AD. The height VX of the pyramid is 12 cm.

Find the volume of the pyramid to two significant figures.

Volume of "Regular" Pyramid is $\frac{h \times A}{3}$, where A is the area of the base.

The base is 6 isosceles triangles. Let each side length be r.



$$x = r \sin 60^\circ$$

Area of triangle is $\frac{1}{2} \times r \times r \sin 60^\circ$

$$A = 6 \times \frac{1}{2} r^2 \sin 60^\circ$$

$$r = 5,\ h = 12 \quad \text{both in cm.}$$

So $$A = 6 \times \frac{5^2}{2} \times \frac{\sqrt{3}}{2}$$

$$A = 75\frac{\sqrt{3}}{2}\ \text{cm}^2$$

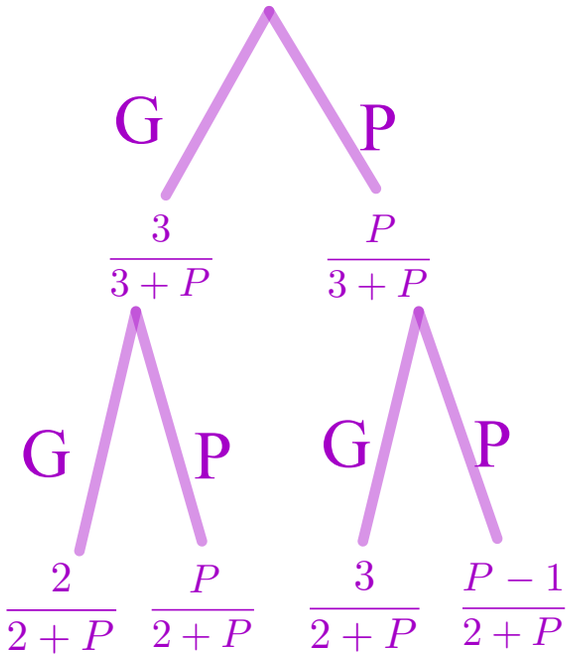
Then, $$V = \frac{12}{3} \times \frac{75\sqrt{3}}{2}$$

$$V = 150\sqrt{3}$$

$$V \approx 260\ \text{cm}^3$$

Comments: The fact that so many stage 5.3 pupils could not find the area of a regular hexagon is a bitter disappointment. It hurts to discuss the many and varied ways in which they failed to do this properly, so I leave it to You to ponder why this simple task eluded you (if you were challenged by this task). Too many pupils did not leave their answer as an approximation to two significant figures as was asked and did not receive full marks.

(b) A black jar contains 3 green marbles and P pink marbles, where $P > 1$. **3**
Chauncey takes two marbles out simultaneously.
If the probability that they are both the same colour is a half, find all possible values of P.



$$P(\text{same colour pairs}) = \overline{P(\text{same colour pairs})}$$

$$P(G,G) + P(P,P) = P(P,G) + P(G,P), \text{ where for } P(X,Y)$$

X is the top level of the probability tree and Y is the lower level.

$$\frac{3 \times 2}{(3+P)(2+P)} + \frac{P(P-1)}{(3+P)(2+P)} = \frac{2 \times 3P}{(3+P)(2+P)}$$

$$6 + P^2 - P = 6P$$

$$\Rightarrow P^2 - 7p + 6 = (P-6)(P-1)$$

1 is a solution, however $P > 1$, hence $P = 6$ is the only solution to the question.

Comments: Pupils ended up adding probabilities when they should have multiplied and vice versa. Some solved for only a single colour being paired or factorised quadratics that have no integer solutions, or worse still, no real solutions.

The problem was exacerbated by the difficulty of dealing with marbles picked simultaneously, which most found hard to manage. Look carefully at the solution for the A-team, it avoids the issue of choosing one marble, then another altogether - on next page.

Alternative Solution for the A-team

Let G_1, G_2 & G_3 be the 3 green balls.
Let each pink ball be numbered in the same way as the green balls.

Since $P > 1$, there are at least 2 pink balls.

This least case has these options:

$$(G_1, G_2), (G_1, G_3), (G_2, G_3), (G_1, P_1), (G_2, P_1), (G_3, P_1),$$
$$(G_1, P_2), (G_2, P_2), (G_3, P_2), (P_1, P_2)$$

Adding an *nth* pink ball adds *n* - 1 pink pairs & 3 more green/pink pairs.
There will always be only 3 green pairs. The green balls will have a 50% chance when *P* equals 1 only, since there are no pink pairs.
We need to compare the sum of same paired colours to the sum of pairs of different colours.

Sum of Green pairs + sum of pink pairs = sum of green pink pairs.

Sum of green pairs is 3.

Sum of pink pairs is $(P-1)+(P-2)+\ldots+3+2+1=\frac{P^2-P}{2}$, for *P* odd & even. *

Sum of green/pink pairs is $3P$.

$$3+\frac{P^2-P}{2}=3P$$

$$\Rightarrow P^2-7p+6=(P-6)(P-1)$$

1 is a known solution, but *P* > 1, hence *P* = 6 is the only solution to the question.

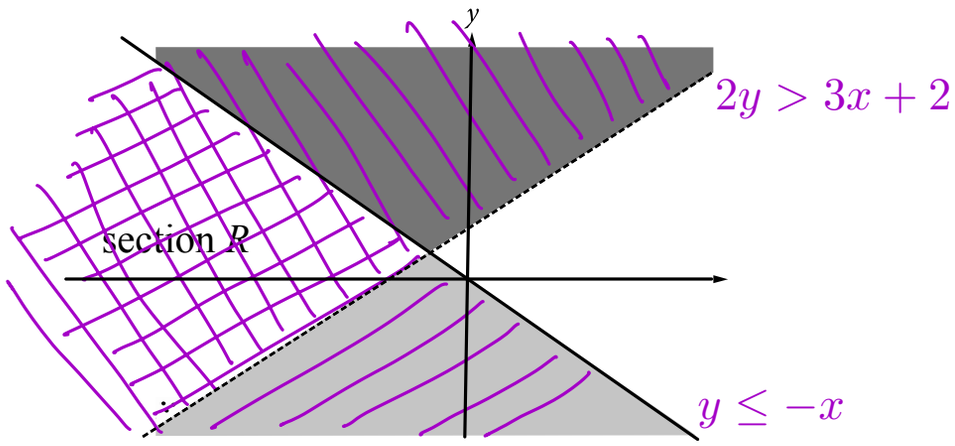
* For even *P*, there are an odd number of terms. The middle value is $\frac{P}{2}$ and there are $\frac{P-2}{2}$ lots of $(P-k)+k=P,\ where\ k=1,2,\ldots,\frac{P-2}{2}$.
For odd *P*, there are an even number of terms. There are $\frac{P-1}{2}$ lots of

$$(P-k)+k,\quad \text{where } k=1,2,\ldots\frac{P-1}{2}.$$

$$\frac{(P-2)}{2}P+\frac{P}{2}=\frac{(P-1)}{2}P$$

(c) Circle the correct response. **1**

$$\begin{cases} y \leq -x \\ 2y > 3x + 2 \end{cases}$$



A system of inequalities and a graph are shown above.
Which section(s) of the graph could represent all of the solutions to the system?

A. R B. Q & S C. P & Q D. Q, R, & S

(d) Simplify the following, leaving your answer in index form. **2**

$$(2+3)(2^2+3^2)(2^4+3^4)(2^8+3^8)(2^{16}+3^{16})(2^{32}+3^{32})(2^{64}+3^{64})$$

$$\begin{aligned} 2+3 &= (3+2)(3-2) \\ &= 3^2 - 2^2 \end{aligned}$$

$$(3^2-2^2)(3^2+2^2)(3^4+2^4)(3^8+2^8)(3^{16}+2^{16})(3^{32}+2^{32})(3^{64}+2^{64})$$

$$(3^4-2^4)(3^4+2^4)(3^8+2^8)(3^{16}+2^{16})(3^{32}+2^{32})(3^{64}+2^{64})$$

$$(3^8-2^8)(3^8+2^8)(3^{16}+2^{16})(3^{32}+2^{32})(3^{64}+2^{64})$$

$$(3^{16}-2^{16})(3^{16}+2^{16})(3^{32}+2^{32})(3^{64}+2^{64})$$

$$(3^{32}-2^{32})(3^{32}+2^{32})(3^{64}+2^{64})$$

$$(3^{64}-2^{64})(3^{64}+2^{64})$$

$$3^{128} - 2^{128}$$

Comments: It is not surprising to me that this was the question that pupils had the most difficulty with. It took me a while to see the solution and I actually doubt I would have done this in the given time for the exam. Only 2 students got full marks.

(e) (i) By factorising, or otherwise, show that **2**

$(p^2-q^2-r^2)^2-4q^2r^2=(p-q-r)(p-q+r)(p+q-r)(p+q+r)$.

$$\begin{aligned}\text{LHS} &= \left(p^2-\left(q^2+r^2\right)\right)-(2qr)^2\\ &= \left(p^2-\left(q^2+r^2\right)-2qr\right)\left(p^2-\left(q^2+r^2\right)+2qr\right)\\ &= \left(p^2-(q+r)^2\right)\left(p^2-(q-r)^2\right)\\ &= (p-q-r)(p+q+r)(p-q+r)(p+q-r)\\ &= \text{RHS}\end{aligned}$$

OR

$$\begin{aligned}\text{RHS} &= (p-(q+r))(p+(q+r))(p+(q-r))(p-(q-r))\\ &= \left(p^2-(q+r)^2\right)\left(p^2-(q-r)^2\right)\\ &= p^4+(q+r)^2(q-r)^2-p^2\left((q-r)^2+(q+r)^2\right)\\ &= p^4+\left(q^2-r^2\right)^2-p^2\left(2q^2+2r^2\right)\\ &= p^4+q^4+r^4-2q^2r^2-2p^2\left(q^2+r^2\right)\\ &= \left(p^2\right)^2-2p^2\left(q^2+r^2\right)+\left(q^2\right)^2+\left(r^2\right)^2-2q^2r^2\\ &= \left(p^2\right)^2-2p^2\left(q^2+r^2\right)+\left(\left(q^2\right)^2+2q^2r^2+\left(r^2\right)^2\right)-2\times 2r^2q^2\\ &= \left(p^2\right)^2-2p^2\left(q^2+r^2\right)+\left(q^2+r^2\right)^2-4q^2r^2\\ &= \left(p^2-\left(q^2+r^2\right)\right)^2-4q^2r^2\\ &= \left(p^2-q^2-r^2\right)^2-4q^2r^2\\ &= \text{LHS}\end{aligned}$$

<u>Comments</u>: Only one student did the lower solution (which was my first - I guess I like to think backwards). A number of students expanded both sides of the equation and showed these are the same. This is NOT elegant, but got full marks. One thing this question did highlight was whether pupils have reasonable algebra skills. If there is a cross on your paper, then You need to work on your algebra!!

Part (e) continues on page 29

(e) (continued)

(ii) Hence solve $\left(p^2-q^2-r^2\right)^2-4q^2r^2=45$ for positive integers p, q, and r. **3**

$$45 = 3^2 \times 5$$

This has only 3 factors, so it is necessary to use 1 as another factor. The largest factor will be the sum of all positive factors and the assumption here, is that the smallest factor will be the equation with two negatives.
The two other factors from (i) will likely be the non-unique factors or at the very least, one of them will be a unique factor.

Another assumption here, is that $45 = 1 \times 3 \times 3 \times 5$ is the factorisation related to (i).

Other possibilities are:
$$1 \times 1 \times 5 \times 9$$
$$1 \times 1 \times 1 \times 45$$
$$1 \times 1 \times 3 \times 15$$

However, none of these are useful since the smallest factor is not unique and will not lead to viable solutions, given the factorisation from (i). The matrix formed on the next page makes this obvious to one familiar with row reduction. **

$$p+q+r=5 \quad ①$$
$$p-q-r=1 \quad ②$$
$$p+q-r=3 \quad ③$$
$$① + ② \Rightarrow 2p=6 \Rightarrow p=3$$
$$\Rightarrow 3-q-r=1$$
$$q+r=2 \quad ④$$
$$③ \Rightarrow 3+q-r=3$$
$$q-r=0 \quad ⑤$$
$$④ + ⑤ \Rightarrow 2q=2 \Rightarrow q=1$$
$$\Rightarrow r=1$$

From (i), $p-q+r=3-1+1=3$ and is consistent.

$$p=3 \ \& \ q=r=1$$

End of paper

Comments: This was NOT the hardest question on the paper, but it did stump most. Pupils needed to relate positive factors of 45 to the factors in part (i). Then they had to make correct assumptions when equating the factors of 45 to the factors in (i). There was a bit of "guess & check" going on, which is expected but most of those pupils were quite capable of the mathematical reasoning necessary to recreate the working shown in the solutions.

OR

Recall Matrices from last year? $A\underline{x} = \underline{b}$. Here $\underline{x} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 3 \end{array}\right) \quad R1 \Rightarrow \frac{R1+R2}{2} \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 3 \end{array}\right)$$

$$R2 \Rightarrow \frac{R3-R2}{2} \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 3 \end{array}\right)$$

$$R3 \Rightarrow R1+R2-R3 \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

* *

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 1 & -1 & 3 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & 1 & -1 & 3 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & -1 & -2 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The last line (with all zeroes) means no 4th unknown is necessary. It is confirmation that the 4th equation satisfies the linear system with the three variables. Any other combinations of factors of 45 is an unsolvable linear system.

Note:

The right solution came about from making valid (or educated) assumptions about the value of the equations in factored form. But, we only need 3 out of the four equations to find the values of *p*, *q* and *r*. Any three equations could have been chosen to find the correct values for *p*, *q* and *r* as long as we had the right values for the equations.

Had we chosen the equations that equal 3 and 1, we would still get the same values for *p*, *q* and *r*.

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \end{array}\right) \quad \begin{array}{l} R1 \Rightarrow \dfrac{R1+R3}{2} \\ R3 \Rightarrow \dfrac{R2-R3}{2} \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & -1 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$R2 \Rightarrow R1+R3-R2 \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$p = 3 \ \ \& \ \ q = r = 1$$