SYDNEY BOYS HIGH SCHOOL

2022

YEAR 10 STAGE 5.3
TERM 2
HALF YEARLY ASSESSMENT

Mathematics

General Instructions: **Time Allowed:** 90 minutes

- All questions may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using black pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: JB

Name: _____

Class	Teacher	
10Ma**A**	Mr. A Wang	
10Ma**B**	Ms. A Ward	
10Ma**C**	Ms. J Millar & Mr. E Choy	
10Ma**P**	Mr. R Wang	
10Ma**L**	Mr. A Fuller	
10Ma**U**	Mr. J James	
10Ma**S**	Mr. M Gainford	

Question	Marks
1	/16
2	/16
3	/16
4	/14
5	/10
Total	**/72**

(a) What is the value of $5.6^{-\frac{2}{3}}$ correct to two decimal places? **1**

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(b) Calculate $\sqrt{\frac{3}{500\pi}}$ and correct to 4 significant figures. **1**

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(c) Find the simple interest earned on $\$8200$ at 4% p.a. for 3 years. **1**

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(d) Find a monic quadratic equation that has two roots equal to 5 and -7. **1**

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(e) P is inversely proportional to the square of s. When $P = 50$, $s = 3$. **2**
Calculate the value of P when $s = 12$.

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(f) Solve the following quadratic equation, leave your answers in surd form. **2**

$$4x^2 - 3x - 2 = 0$$

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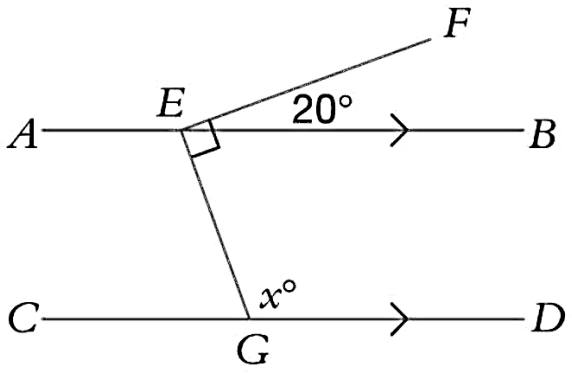
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(g) Find the value of x. Give reasons for your answers. **2**



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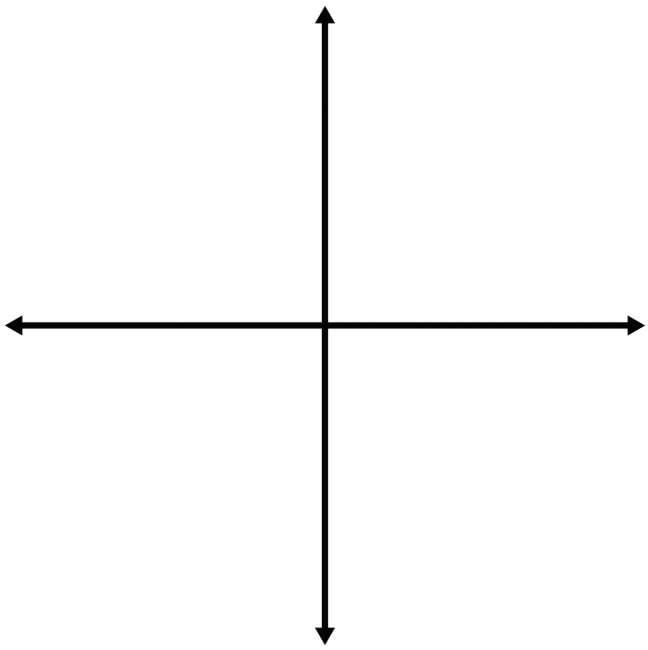
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(h) Sketch the following parabola showing the vertex and any intercepts. **2**

$$y = -\left(x + \frac{1}{2}\right)^2 - 1$$



(i) The average temperatures (Fahrenheit) of Lake Michigan recorded from January to December is given below.

Stem	Leaf			
3	7	7	9	9
4	1	3	8	
5	4	7		
6	6	6		
7	0			

(i) What is the median temperature recorded in Lake Michigan? **1**

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(ii) How many months have an average temperature of less than 60°F in Lake Michigan? **1**

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(iii) Calculate the interquartile range. **2**

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End of Question 1

Extra Working Page for Question 1 only

Question 2

Name: _____

Class	Teacher	
10Ma**A**	Mr. A Wang	
10Ma**B**	Ms. A Ward	
10Ma**C**	Ms. J Millar & Mr. E Choy	
10Ma**P**	Mr. R Wang	
10Ma**L**	Mr. A Fuller	
10Ma**U**	Mr. J James	
10Ma**S**	Mr. M Gainford	

Mark	**/16**

(a) Find the amount of compound interest earned if $\$8000$ is invested at 12% p.a for 2 years compounded quarterly. **2**

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(b) Find the points of intersection of the equations: **2**

$$x^2 + y^2 = 4 \text{ and } y = x$$

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(c) What are the centre and radius of the circle below? **2**

$$x^2 + y^2 - 4x - 6y - 36 = 0$$

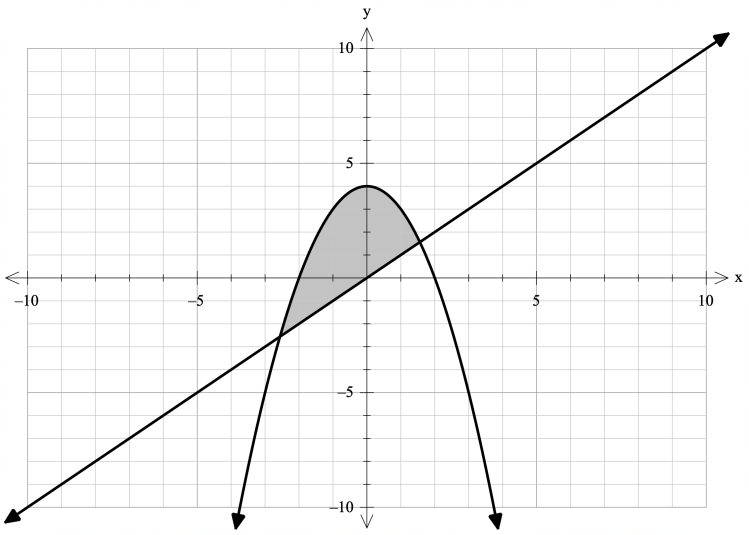
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(d) The shaded region in the diagram is enclosed by $y = 4 - x^2$ and $y = x$. **1**



Which of the following is the set of inequations that satisfy the shaded region?

(A) $y \geq 4 - x^2, y \leq x$

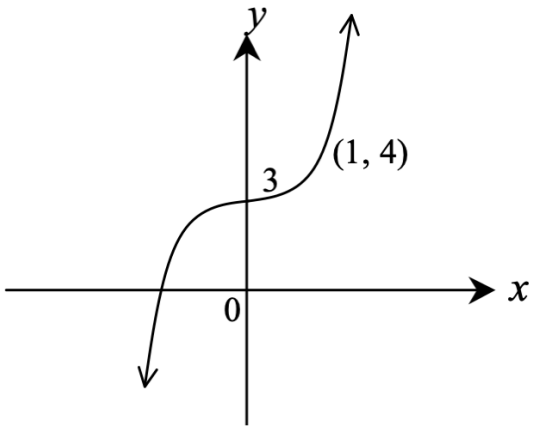
(B) $y \leq 4 - x^2, y \leq x$

(C) $y \leq 4 - x^2, y \leq x$

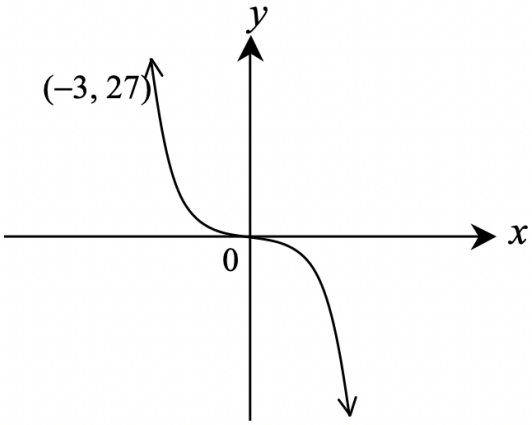
(D) $y \leq 4 - x^2, y \geq x$

(e) Match each equation to its graph. **4**

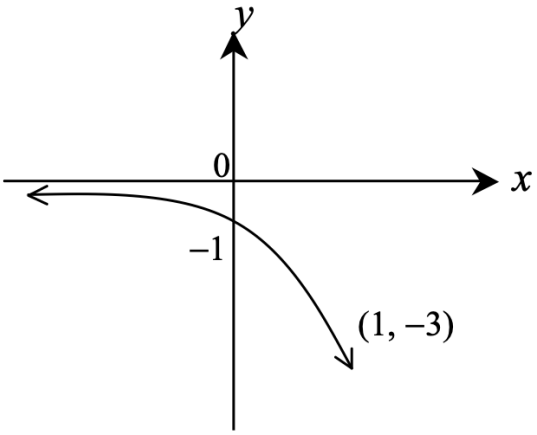
(A)



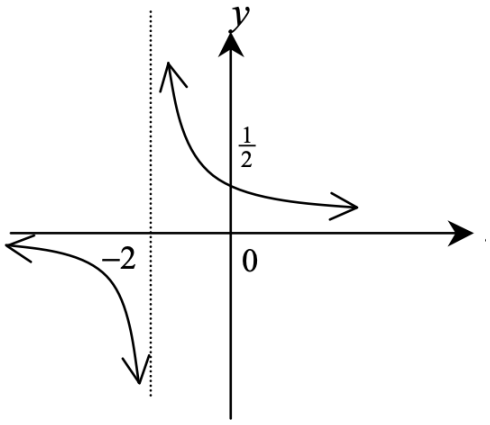
(B)



(C)



(D)



$y = -3^x$	
$y = \dfrac{1}{x+2}$	
$y = -x^3$	
$y = x^3 + 3$	

(f) An insurance company calculates the depreciation of a car using the formula:

$$A = P(0.86)^n$$

where P is the initial value and n is the number of years.

(i) What rate of depreciation is this? **1**

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(ii) If a car's value in April 2012 was $\$29200$, what will be its value in April 2022? Answer to the nearest dollar. **1**

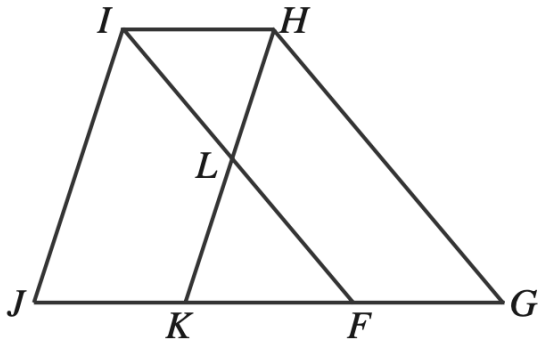
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(g) *HIJK* and *FGHI* are both parallelograms. *L* is the midpoint of *HK*. **3**
Prove that $GJ = 3 \times HI$.



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End of Question 2

Extra Working Page for Question 2 only

Question 3

Name: _____

Class	Teacher	
10Ma**A**	Mr. A Wang	
10Ma**B**	Ms. A Ward	
10Ma**C**	Ms. J Millar & Mr. E Choy	
10Ma**P**	Mr. R Wang	
10Ma**L**	Mr. A Fuller	
10Ma**U**	Mr. J James	
10Ma**S**	Mr. M Gainford	

Mark	**/16**

(a) Find p so that the line $3x - 4y - 1 + p(x + 2y - 3) = 0$ has a gradient of 3. **2**

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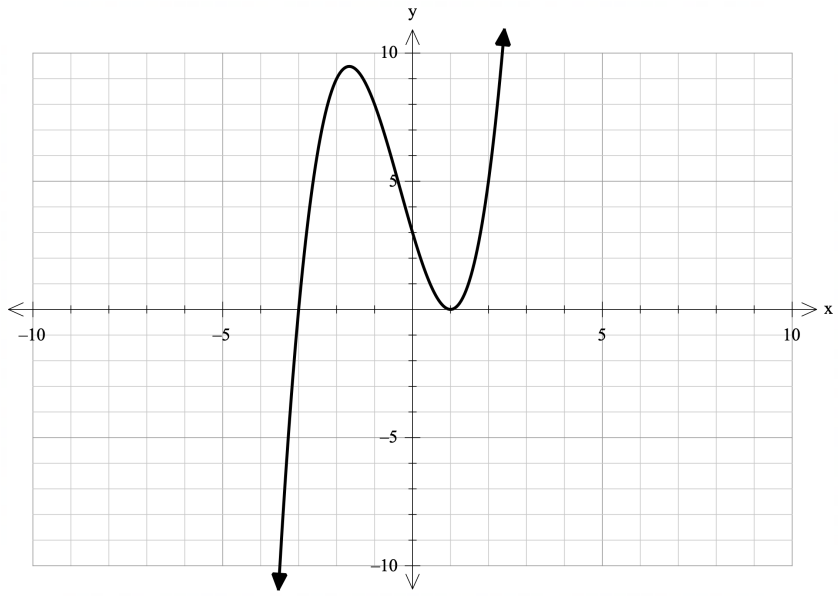
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(b) Write a possible equation of the curve shown. **1**



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(c) The results of a survey showing numbers of students playing sports.

Tennis	Netball	Football	Swimming	Golf	Cricket
5	10	20	45	15	5

(i) Complete the following table. **2**

Sport	**Frequency**	**Cumulative frequency**	**Cumulative percentage**
Swimming			
Football			
Golf			
Netball			
Cricket			
Tennis			

(ii) Draw a Pareto chart to represent the data above. **2**

(d) Suppose I discovered that my cat had a taste for the adorable little geckoes that live in the bushes and vines in my yard, back when I lived in Arizona.

In one month, suppose he deposited the following on my carpet: six grey geckoes, twelve geckoes that had dropped their tails in an effort to escape capture, and fifteen geckoes that he chewed on a little. Only one of the geckoes was grey, chewed on, and tailless; two were grey and tailless but not chewed on; two were grey and chewed on but not tailless.

There was a total of 24 geckoes left on my carpet that month. All of which were victims in some way to my cat.

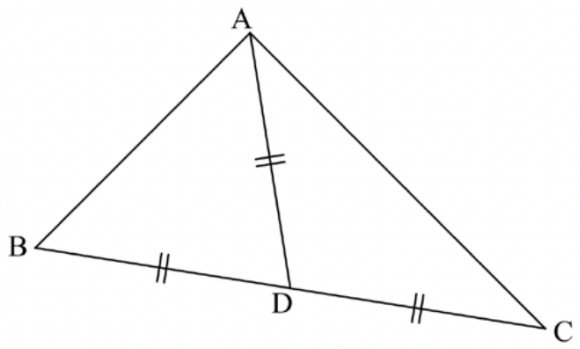
(i) Draw a Venn diagram to illustrate all the information given. **3**

(ii) What is the probability that if I chose one gecko at random, it would be chewed on given it was tailless? **1**

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(e) In the triangle ABC, $AD = BD = CD$.



(i) Prove $\angle BAC = 90°$. **2**

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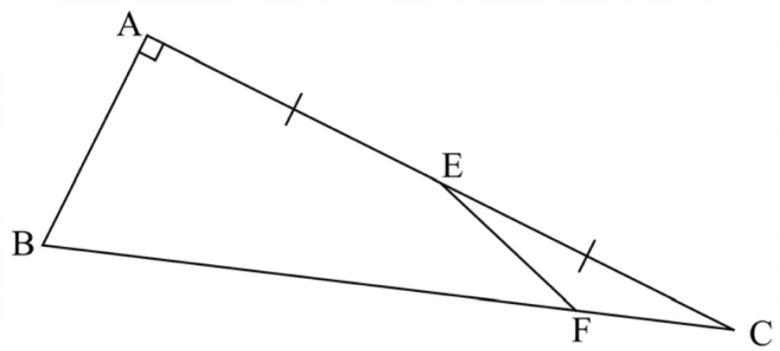
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(ii) In ΔABC, $\angle BAC = 90°$, $AE = CE$, $EF = 8$ and $\frac{BC}{FC} = 4$. **3**

By constructing an extra line in ΔABC and by using part (i), or otherwise, determine the length of BC.



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End of Question 3

Extra Working Page for Question 3 only

Question 4

Name: _____

Class	Teacher	
10Ma**A**	Mr. A Wang	
10Ma**B**	Ms. A Ward	
10Ma**C**	Ms. J Millar & Mr. E Choy	
10Ma**P**	Mr. R Wang	
10Ma**L**	Mr. A Fuller	
10Ma**U**	Mr. J James	
10Ma**S**	Mr. M Gainford	

Mark	**/14**

(a) A home loan of $\$300\,000$ is charged interest at the rate of 9% p.a.
The interest is calculated monthly and repayments of $\$2520$ are made at the end of each month so that the loan is paid off after 25 years.

(i) Calculate the amount still owing at the end of 3 months (after the first 3 repayments). **3**

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(ii) Determine the total amount of interest paid and the equivalent flat (simple) interest rate. **2**

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(b) If the measure of exterior angle of a regular polygon is $\frac{1}{5}$ times its interior angle. **3**
How many sides does the polygon have?

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(c) If $a+b \propto a-b$, prove that $a^2+b^2 \propto ab$. **3**

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(d) A black jar contains 3 green marbles and P pink marbles, where $P > 1$. **3**
Jay takes two marbles out simultaneously.
If the probability that they are both the same colour is a half, find all possible values of P.

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End of Question 4

Extra Working Page for Question 4 only

Question 5

Name: _____

Class	Teacher	
10Ma**A**	Mr. A Wang	
10Ma**B**	Ms. A Ward	
10Ma**C**	Ms. J Millar & Mr. E Choy	
10Ma**P**	Mr. R Wang	
10Ma**L**	Mr. A Fuller	
10Ma**U**	Mr. J James	
10Ma**S**	Mr. M Gainford	

Mark	**/10**

(a) By letting $u = x - \frac{2}{x}$, find the integer solutions of x in the following equation. **3**

$$\left(x^2 + \frac{4}{x^2}\right) - 4\left(x - \frac{2}{x}\right) - 9 = 0$$

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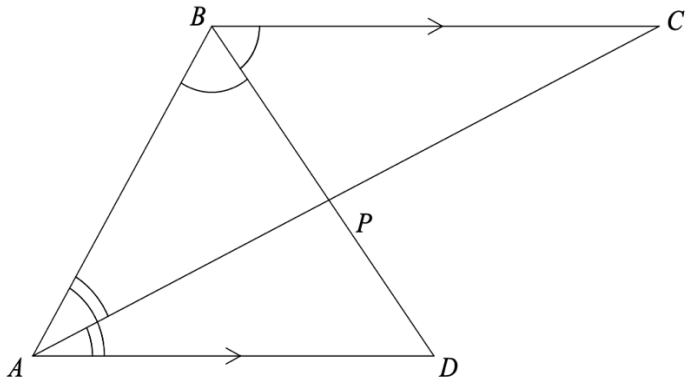
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(b) In the diagram, AD is parallel to BC, AC bisects $\angle BAD$ and BD bisects $\angle ABC$.

The lines AC and BD intersect at P.



(i) Prove that $\angle BAC = \angle BCA$. **1**

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(ii) Prove that $ABCD$ is a rhombus. **3**

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(c) By considering the factorisation of $x^2 - 2(a-1)x + a + 1 = 0$, find a such that the solutions of the quadratic equation x_1 and x_2 satisfy: **3**

$$\frac{x_1}{x_2} + \frac{x_2}{x_1} = 4.$$

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End of Question 5

Extra Working Page for Question 5 only

SYDNEY BOYS HIGH SCHOOL

2022

YEAR 10 STAGE 5.3
TERM 2
HALF YEARLY

Sample Solutions

NOTE: Some of you may be disappointed with your mark.
This process of checking your mark is NOT the opportunity to improve your marks. Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.

Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

2022 HYE Q1 Solutions and Comments

A. What is the value of $5.6^{\frac{-2}{3}}$ correct to two decimal places? 1

Solution(s)	Comment(s)
$5.6^{-\frac{2}{3}} = 0.32\ (2\ d.p.)$	No half marks. Students generally did well on this question.

B. Calculate $\sqrt{\frac{3}{500\pi}}$ and correct to 4 significant figures. 1

Solution(s)	Comment(s)
$\sqrt{\frac{3}{500\pi}} = 0.04370\ (4\ d.p.)$	Common error(s): • Dropping the zero following the seven. • Treating the first two zeroes as significant figures.

C. Find the simple interest earned on \$8 200 at 4% p.a. for 3 years. 1

Solution(s)	Comment(s)
$I = PRN$ $= 8200 \times 0.04 \times 3$ $= \$984$	No half marks. Students generally did well on this question.

D. Find a monic quadratic equation that has two roots equal to 5 and -7. 1

Solution(s)	Comment(s)
$(x-5)(x+7) = 0$	Students should note that: • The question asked for an equation, so $(x-5)(x+7)$ as an answer is insufficient. • The question states that the equation has roots of 5 and -7, so the correct equation is $(x-5)(x+7) = 0$, not $(x-5)(x+7) = y$. Common error(s): • $(x+5)(x-7) = 0$

2022 HYE Q1 Solutions and Comments

E. P is inversely proportional to the square of s. 2

When $P = 50, s = 3$.

Calculate the value of P when $s = 12$.

Solution(s)	Comment(s)
$$P \propto \frac{1}{s^2}$$ Hence, for some arbitrary constant k: $$P = \frac{k}{s^2}$$ $$k = Ps^2$$ Since k is a constant for all pairs of P and s: $$P_1 s_1^2 = P_2 s_2^2$$ $$P_2 = P_1 \frac{s_1^2}{s_2^2}$$ $$= 50 \times \frac{3^2}{12^2}$$ $$= \frac{25}{8}$$	Students should note that it isn't necessary to evaluate k. Common error(s): • Forgetting that P is proportional to the square of s, not just s.

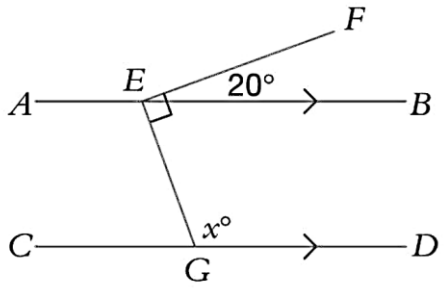
F. Solve the following quadratic equation, leaving your answers in surd form. 2

$$4x^2 - 3x - 2 = 0$$

Solution(s)	Comment(s)
By the quadratic formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ $$= \frac{3 \pm \sqrt{(-3)^2 - (4 \times 4 \times -2)}}{2 \times 4}$$ $$= \frac{3 \pm \sqrt{41}}{8}$$	Students generally did well on this question.

2022 HYE Q1 Solutions and Comments

G. Find the value of x. Give reasons for your answer. 2

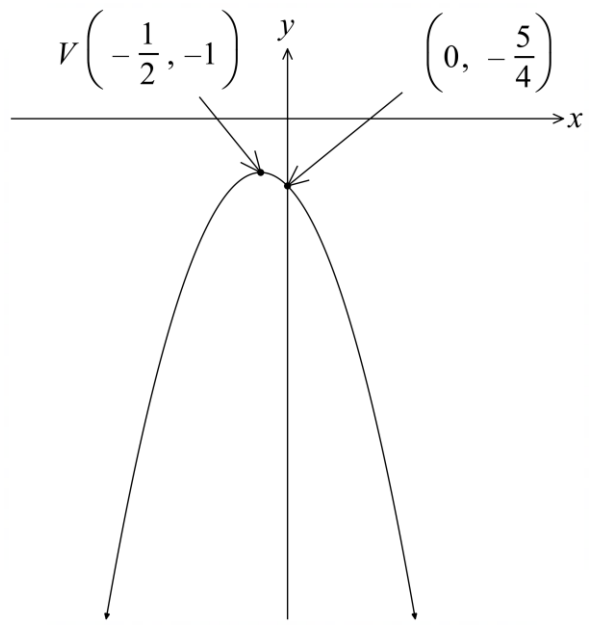


Solution(s)	Comment(s)
$\angle BEG + \angle BEF = 90°$ ($\angle FEG$ is a right angle) $\angle BEG + 20° = 90°$ $\angle BEG = 70°$ $\angle BEG + \angle DGE = 180°$ (Coint angles, $AB \parallel CD$) $70° + x° = 180°$ $x° = 110°$	It was incredibly disappointing to see most students failing to provide the correct geometrical reason to their proof. Common error(s): • Stating that $\angle BEG + \angle BEF = 90°$ because they're complementary instead of $\angle FEG$ being a right angle. • Forgetting that cointerior angles only add up to $180°$ on parallel lines.

2022 HYE Q1 Solutions and Comments

H. Sketch the following parabola, showing the vertex and any intercepts. 2

$$y = -\left(x + \frac{1}{2}\right)^2 - 1$$

Solution(s)	Comment(s)
The graph of $y = -\left(x + \frac{1}{2}\right)^2 - 1$ is below:  This can be obtained by either: • Recognising that $y = -\left(x + \frac{1}{2}\right)^2 - 1$ is in vertex form. • Recognising that $y = -\left(x + \frac{1}{2}\right)^2 - 1$ is a series of transformations from $y = x^2$.	Students who drew their graph in pencil will **not** have their marks changed. All marks were allocated to graph. **No** marks were allocated to the working out. Students should make use of the working out they did on the side to check that they've sketched the correct graph. As an example, some students graphed a concave up parabola with vertex $V\left(\frac{-1}{2}, -1\right)$ and y intercept $\left(0, \frac{-5}{4}\right)$, when recognising that $\frac{-5}{4} < -1$ would have informed these students that they've drawn an incorrect graph. Common error(s): • Misreading the vertex as $\left(\frac{1}{2}, 1\right)$ or variations instead of $\left(\frac{-1}{2}, 1\right)$ from the equation.

2022 HYE Q1 Solutions and Comments

I. The average temperatures (in Fahrenheit) of Lake Michigan from January to December is given below.

Stem	Leaf			
3	7	7	9	9
4	1	3	8	
5	4	7		
6	6	6		
7	0			

(I) What is the median temperature recorded in Lake Michigan? 1

Solution(s)	Comment(s)
There are 12 months in a year, so the median is midpoint between the 6th and 7th scores. Therefore: $T_M = \frac{43 + 48}{2}$ $= 45.5°\text{F}$	Students generally did well on this question.

(II) How many months have an average temperature less than $60°\text{F}$ in Lake Michigan? 1

Solution(s)	Comment(s)
From the stem and leaf plot, there are 9 scores with a value less than 60. Hence, there are 9 months when the average temperature is less than $60°\text{F}$.	Students generally did well on this question.

(III) Calculate the interquartile range. 2

Solution(s)	Comment(s)
$Q_1 = \frac{39 + 39}{2}$ $= 39$ $Q_3 = \frac{57 + 66}{2}$ $= 61.5$ $IQR = Q_3 - Q_1$ $= 22.5$	Common error(s): • Excluding the scores 43 and 48 when finding Q_1 and Q_3.

Question 2

Name: SOLUTIONS

Class	Teacher	
10Ma**A**	Mr. A Wang	
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10Ma**P**	Mr. R Wang	
10Ma**L**	Mr. A Fuller	
10Ma**U**	Mr. J James	
10Ma**S**	Mr. M Gainford	

Mark	**/16**

(a) Find the amount of compound interest earned if $8000 is invested at 12% p.a for 2 years compounded quarterly. **2**

3% per quarter

$A = P(1+r)^n$

$A = 8000(1.03)^8$

$= \$10134.16$

$I = A - P = 10134.16 - 8000 = \$2134.16$

(b) Find the points of intersection of the equations: **2**

$$x^2 + y^2 = 4 \text{ and } y = x$$

$x^2 + y^2 = 4$ ——— (1)

$y = x$ ——— (2)

sub (2) into (1)

$x^2 + (x)^2 = 4$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

sub into (2)

when $x = \sqrt{2}$

$y = \sqrt{2}$

when $x = -\sqrt{2}$

$y = -\sqrt{2}$

∴ points of intersection are $(\sqrt{2}, \sqrt{2})$ & $(-\sqrt{2}, -\sqrt{2})$

(c) What are the centre and radius of the circle below? **2**

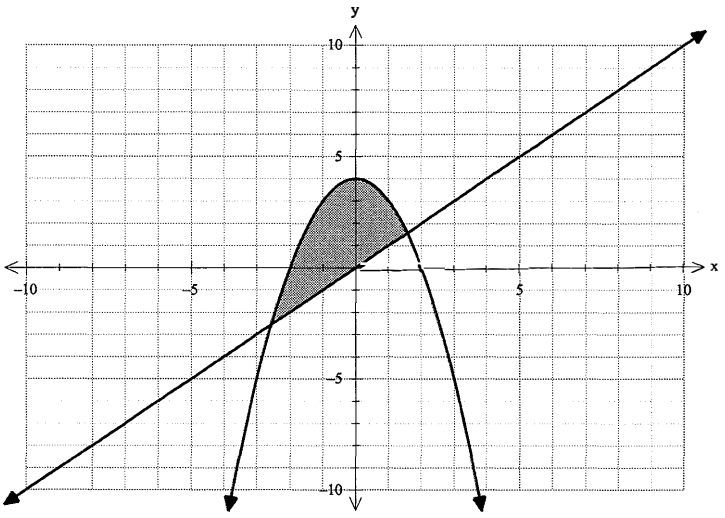
$$x^2 + y^2 - 4x - 6y - 36 = 0$$

$x^2 - 4x + 4 + y^2 - 6y + 9 = 36 + 4 + 9$

$(x-2)^2 + (y-3)^2 = 49$

centre $(2, 3)$, radius 7

(d) The shaded region in the diagram is enclosed by $y = 4 - x^2$ and $y = x$. **1**



Which of the following is the set of inequations that satisfy the shaded region?

(A) $y \geq 4 - x^2, y \leq x$

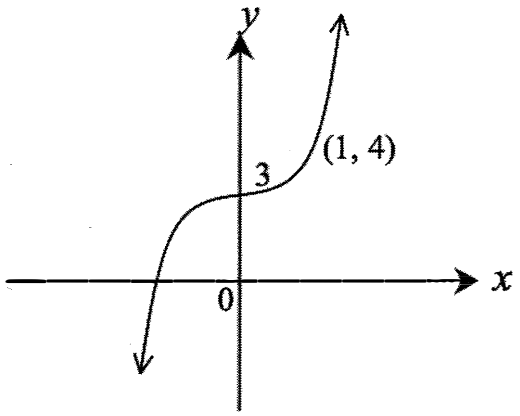
(B) $y \leq 4 - x^2, y \leq x$ } same.

(C) $y \leq 4 - x^2, y \leq x$ }

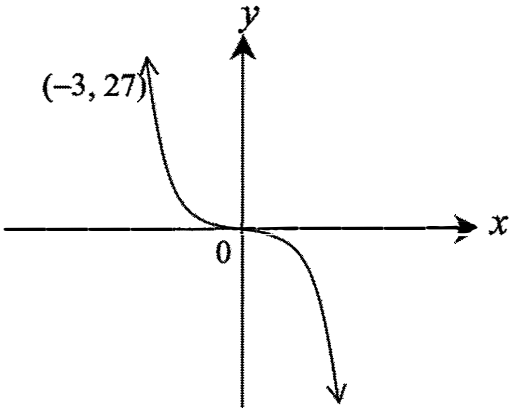
(D) $y \leq 4 - x^2, y \geq x$

(e) Match each equation to its graph. **4**

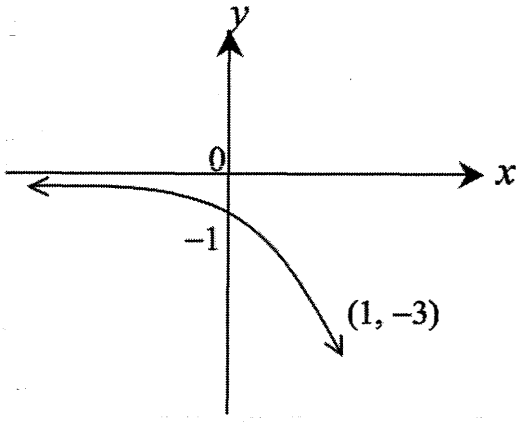
(A)



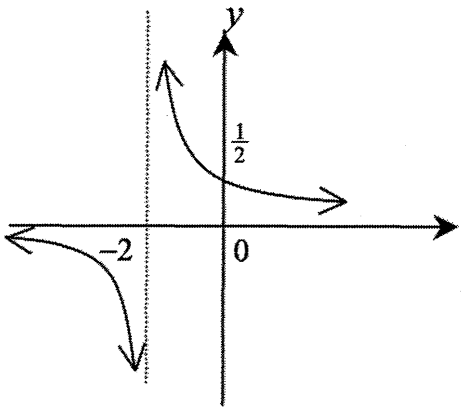
(B)



(C)



(D)



$y = -3^x$	C
$y = \frac{1}{x+2}$	D
$y = -x^3$	B
$y = x^3 + 3$	A

(f) An insurance company calculates the depreciation of a car using the formula:

$$A = P(0.86)^n$$

where P is the initial value and n is the number of years.

(i) What rate of depreciation is this? **1**

$A = P(1-r)^n$ ie 14% p.a

$\therefore 1 - r = 0.86$

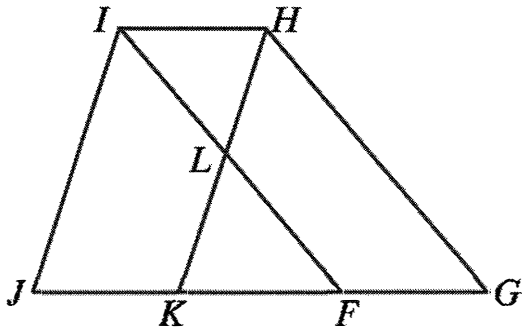
$r = 0.14$

(ii) If a car's value in April 2012 was \$29200, what will be its value in April 2022? Answer to the nearest dollar. **1**

$A = 29200(0.86)^{10}$

$= \$6462$ (to nearest dollar)

(g) *HIJK* and *FGHI* are both parallelograms. *L* is the midpoint of *HK*. **3**
Prove that $GJ = 3 \times HI$.



In $\triangle$s ILH & FLK

HL = KL (given L is the midpoint of HK)

$H\hat{I}L = K\hat{F}L$ (alternate angles, HI // GJ)

$H\hat{L}I = K\hat{L}F$ (vertically opposite angles)

$\triangle ILH \equiv \triangle FLK$ (AAS)

$\therefore HI = KF$ (corresponding sides, $\triangle ILH \equiv \triangle FLK$)

HI = JK (opposite sides, parallelogram HIJK)

HI = FG (opposite sides, parallelogram FGHI)

$HI + HI + HI = JK + KF + FG$

$3 \times HI = GJ$

End of Question 2

Extra Working Page for Question 2 only

COMMENTS:

Students need to read the question and pick up on any hints that are given in it.

a) Asks for the compound interest not the amount the money grows to.

b) Asks for the points of intersection suggesting there is more than one.

In terms of deductive geometry.

Most questions involve congruent triangles and so we should be looking to find such triangles.

Common mistakes:

- mistaking similarity for congruence
- assuming L is also the midpoint of IF

These are not necessarily problems. However, given that we are proving the result everything should be justified with reasons.

Question 3 (solutions)

(a) Find p so that the line $3x-4y-1+p(x+2y-3)=0$ has a gradient of 3. **2**

$$(3+p)x+(2p-4)y-(3p+1)=0$$

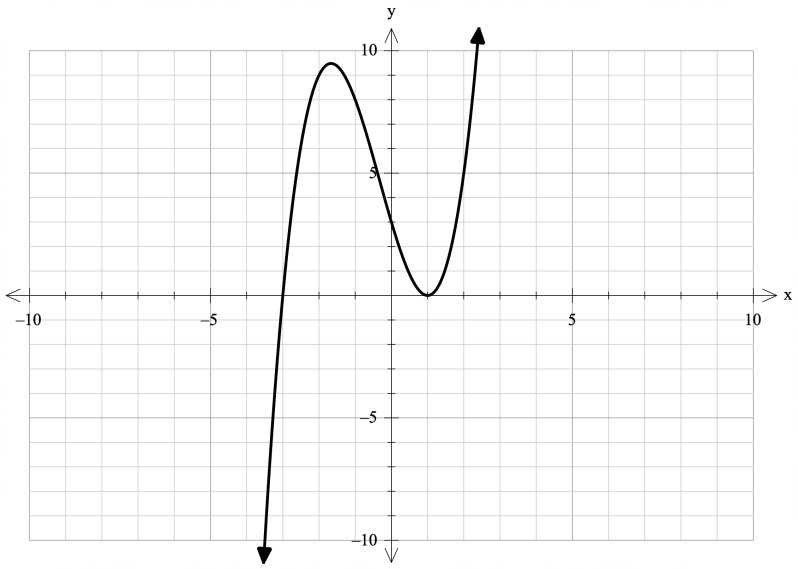
For an equation in general form (i.e. $ax+by+c=0$), $m=-\frac{a}{b}$, where a and b are the coefficients of x and y respectively and m is the gradient.

Hence:

$$3=-\frac{(3+p)}{(2p-4)}$$
$$3(2p-4)=-3-p$$
$$6p-12=-3-p$$
$$7p=9$$
$$p=\frac{9}{7}$$

NOTE: Pupils did not see that they needed to collect like terms with respect to the variables x and y.

(b) Write a possible equation of the curve shown. **1**



Looking at where the curve touches (or cuts) the x-axis, it is obvious that there is a single root at –3 and a double root at 1.

Therefore, the equation looks like $y=a(x+3)(x-1)^2$

NOTE: For 1 mark, this is all that is necessary.
For those who went further than this, well done. The actual equation is when $a = 1$.
This is found by choosing any specific point on the curve and solving for a.
One could choose any of $(0,3)$, $(1,0)$, $(2,5)$ or $(-2,9)$, then substitute these values in for x and y appropriately and solve for a.

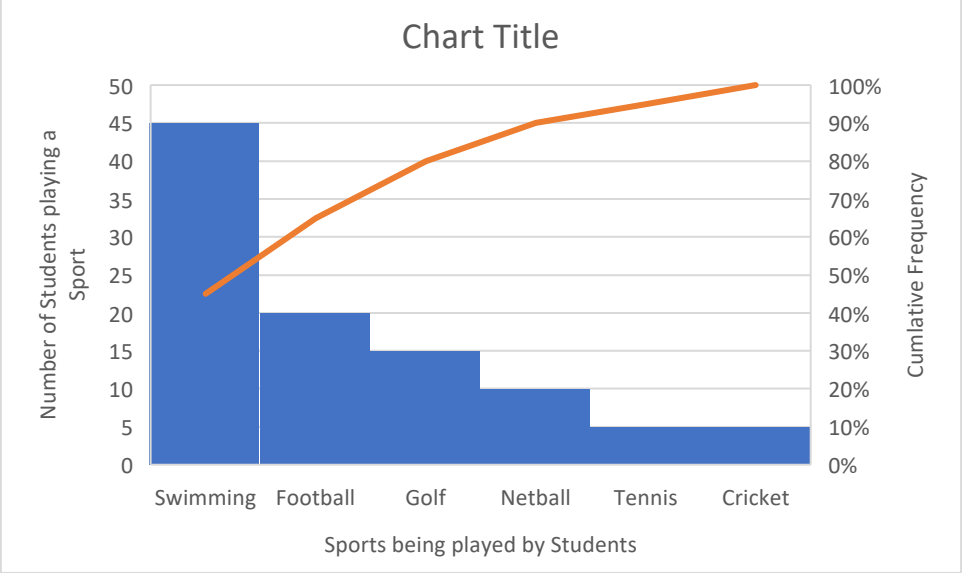
(c) The results of a survey showing numbers of students playing sports.

Tennis	Netball	Football	Swimming	Golf	Cricket
5	10	20	45	15	5

(i) Complete the following table. **2**

Sport	**Frequency**	**Cumulative Frequency**	**Cumulative Percentage**
Swimming	45	45	45%
Football	20	65	65%
Golf	15	80	80%
Netball	10	90	90%
Cricket	5	95	95%
Tennis	5	100	100%

(ii) Draw a Pareto chart to represent the data above. **2**



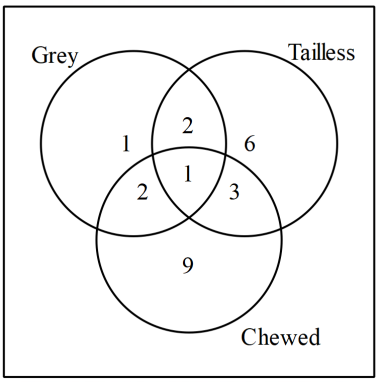
NOTE: No marks were awarded for the first column of the table, since all these values were given.

(d) Suppose I discovered that my cat had a taste for the adorable little geckoes that live in the bushes and vines in my yard, back when I lived in Arizona.

In one month, suppose he deposited the following on my carpet: six grey geckoes, twelve geckoes that had dropped their tails in an effort to escape capture, and fifteen geckoes that he chewed on a little. Only one of the geckoes was grey, chewed on, and tailless; two were grey and tailless but not chewed on; two were grey and chewed on but not tailless.

There was a total of 24 geckoes left on my carpet that month. All of which were victims in some way to my cat.

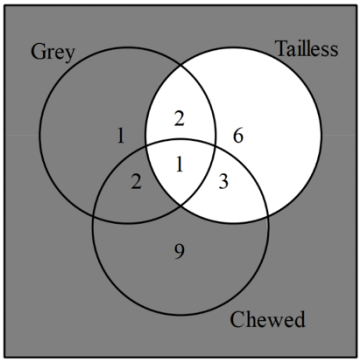
(i) Draw a Venn diagram to illustrate all the information given. **3**



(ii) What is the probability that if I chose one gecko at random, it would be chewed on given it was tailless? **1**

The answer to this question is dependent on *your* Venn Diagram. I marked it correct if pupils wrote this: $\frac{n(T \cap C)}{n(T)}$, where T is the set of Tailless geckos and C is the set of chewed geckos and the appropriate values were taken straight from the pupil's answer to part (i)

Reasoning: The restricted sample space for the probability is the tailless geckos.
The event space in this case is the number of chewed geckos in the restricted sample space.



For the correct Venn diagram, there are 12 tailless geckos and 4 of them have been chewed on.

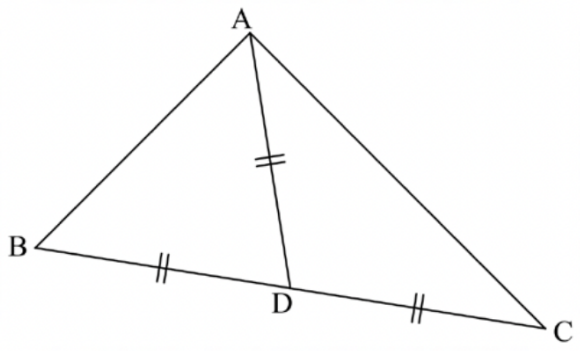
$$P(C \text{ given } T) = \frac{4}{12} = \frac{1}{3}$$

NOTE:

(i) Full marks could not be awarded if it was clear that the wrong sample space and event space were used to get the answer, given the Venn Diagram pupils gave as an answer for part (i)

(ii) Pupils managed to input the data for the areas mentioned into the Venn Diagram but generally were unable to work out all the other regions from the information.

(e) In the triangle ABC, $AD = BD = CD$.



(i) Prove $\angle BAC = 90°$. **2**

$\Delta\ ABD$ and $\Delta\ ACD$ are isosceles (given)

$\angle BAD = \angle ABD$ & $\angle CAD = \angle ACD$ (angles opposite equal sides in a triangle are equal)

Let $\angle BAD = \angle ABD = \alpha$ & $\angle CAD = \angle ACD = \beta$

$\angle BAC = \angle BAD + \angle CAD = \alpha + \beta$

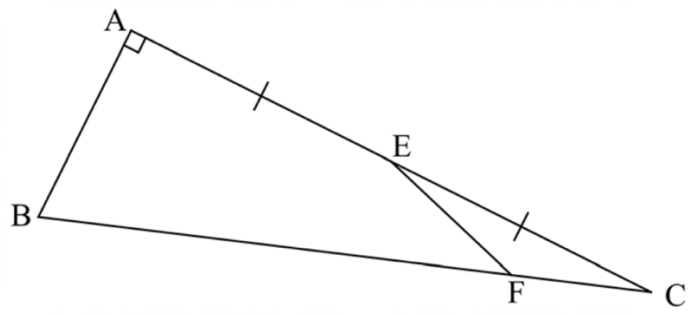
$180° = 2\alpha + 2\beta$ (angle sum of a triangle)

Hence $\alpha + \beta = 90°$ and $\angle BAC = 90°$.

NOTE: This was generally well done.
Some students used variable without stating what they were and just assumed the notes written they had written in the diagram would be enough to show what was meant in their proof.
A proof is succinct, logical and well defined.
If variables are used, they must be defined properly.

(e) (ii) In ΔABC, $\angle BAC = 90°$, $AE = CE$, $EF = 8$ and $\frac{BC}{FC} = 4$ **3**

By constructing an extra line in ΔABC and by using part (i), or otherwise, determine the length of BC.



Imagine that the diagram below is an exact copy of the diagram above.

Figure 1: If a line is drawn from A to the midpoint of BC then, using part (a), $AG = BG = GC$.

Figure 2: Next, draw a line from E to G.

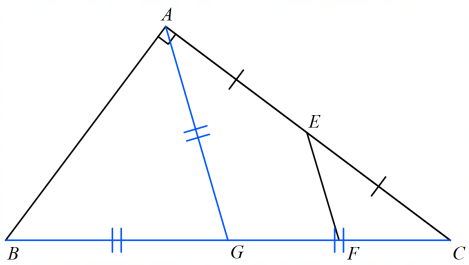


Figure 1

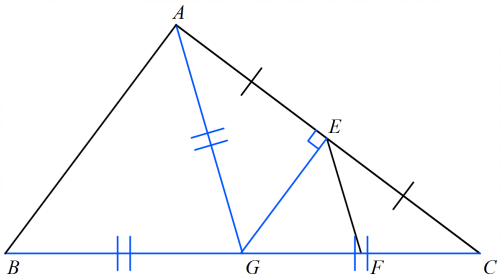


Figure 2

It is not hard to see that ΔAGE & ΔCGE are congruent (SSS).
By congruence, $\angle AEG = \angle CEG$.
Being adjacent supplementary, $\angle AEG = \angle CEG = 90°$.

From $\frac{BC}{FC} = 4$, F is the mid-point between GC.
From part (a), this implies that $EF = GF = FC = 8$.

Since $BC = 4 \times FC$, $BC = 32$.

NOTE: Many pupils guessed this answer without any working.
Any correct solutions that are practically bald answers (no substantial working) received no more than a single mark.
Not one pupil used congruent triangles correctly to show that $EF = FC$.
Many tried using similarity but were not successful.
Most who tried to prove the above, ended up using some false reasoning or simply assumed that $EF = FC$ with no proof.

(a) A home loan of $300 000 is charged interest at the rate of 9% p.a.
The interest is calculated monthly and repayments of $2520 are made at the end of each month so that the loan is paid off after 25 years.

(i) Calculate the amount still owing at the end of 3 months (after the first 3 repayments). **3**

$A_1 = 30000 \times \left(1 + \frac{0.09}{12}\right) - 2520 = 299730$

$A_2 = 299730 \times 1.0075 - 2520 = 299457.975$

$A_3 = 299457.975 \times 1.0075 - 2520 = 299183.9098$

$A_3 \doteqdot \$299183.91$

(ii) Determine the total amount of interest paid and the equivalent flat (simple) interest rate. **2**

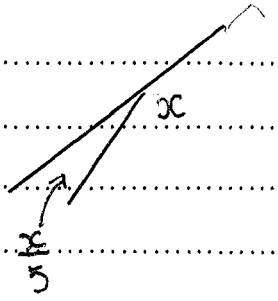
Total paid $= 2520 \times 25 \times 12 = 756000$

$I = 756000 - 300000 = \$456000$

$456000 = 300000 \times R \times 300$

$R = 0.50667\%$ per mth $= 6.08\%$ p.a.

(b) If the measure of exterior angle of a regular polygon is $\frac{1}{5}$ times its interior angle. **3**
How many sides does the polygon have?



Let x be the interior angle.
Let n be the number of sides

$x + \frac{x}{5} = 180$

$\frac{6x}{5} = 180$

$6x = 900$

$x = 150$

$\frac{x}{5} = 30$

$\frac{360}{n} = 30$ Ext. Angle

$n = 12$

(c) If $a+b \propto a-b$, prove that $a^2+b^2 \propto ab$. **3**

$a+b \propto a-b$

$a+b = k(a-b)$

$(a+b)^2 = k^2(a-b)^2$

$a^2+2ab+b^2 = k^2a^2+2k^2ab+k^2b^2$

$a^2-k^2a^2+b^2-k^2b^2 = -2ab-2k^2ab$

$a^2(1-k^2)+b^2(1-k^2) = -2ab(1+k^2)$

$a^2+b^2 = ab\dfrac{(-2(1+k^2)}{(1-k^2)}$ $\therefore a^2+b^2 \propto ab$

(d) A black jar contains 3 green marbles and P pink marbles, where $P > 1$. **3**
Jay takes two marbles out simultaneously.
If the probability that they are both the same colour is a half, find all possible values of P.

$PP + GG = \frac{1}{2}$

$\dfrac{P}{P+3} \times \dfrac{P-1}{P+2} + \dfrac{3}{P+3} \times \dfrac{2}{P+2} = \dfrac{1}{2}$

$\dfrac{P^2-P+6}{(P+3)(P+2)} = \dfrac{1}{2}$

$2P^2-2P+12 = P^2+5P+6$

$P^2-7P+6 = 0$

$(P-6)(P-1) = 0$

$P > 1$ so $P=1$ not a solution $\therefore P=6$

End of Question 4

2022 Year 10 Half Yearly Marking Notes – Question 4

(a) (i) This question was reasonably well done. Errors occurred when the question was answered as if it were compound interest. A value for each year could be given although some did it on the calculator and showed no working.

(ii) Many did not get this correct. Although the question was simple, there were problems interpreting the text. Some did not find both values. The error was carried forward (ECF) where the initial value was not calculated correctly

(b) This question was relatively poorly done. Most made it more difficult than necessary by ending up with a quadratic. A common mistake was to use the interior angle sum rather than the individual interior angle $\frac{(n-2)180}{n}$ .Others had the sum of the interior and exterior angle being 360^o not 180^o

(c) Extremely poorly done. Many did not know that $a+b \propto a-b$ implies that $a+b = k(a-b)$. Some tried to substitute for k.

(d) Relatively poorly done. Many did not understand that there needed to be a reduction in the sample space for drawing the second marble even though simultaneous this is so. A lot of students incorrectly factorised $p^2 - p + 6$ as $(p-3)(p+2)$

(a) By letting $u = x - \frac{2}{x}$, find the integer solutions of x in the following equation. 3

$$\left(x^2 + \frac{4}{x^2}\right) - 4\left(x - \frac{2}{x}\right) - 9 = 0$$

$$\left(x^2 + \frac{4}{x^2}\right) - 4\left(x - \frac{2}{x}\right) - 9 = 0 \quad \text{——— (1)}$$

Let $u = x - \frac{2}{x}$

$u^2 = \left(x^2 + \frac{4}{x^2} - 4\right)$

$\therefore$ Equation (1) is reduced to:

$$\left(x^2 + \frac{4}{x^2} - 4\right) - 4\left(x - \frac{2}{x}\right) - 5 = 0$$

ie $u^2 - 4u - 5 = 0$ ——— (2)

$(u-5)(u+1) = 0$

$\therefore$ $u = 5 \Rightarrow x - \frac{2}{x} = 5$

ie $x^2 - 5x - 2 = 0$ ——— (3)

$\therefore$ $x = \frac{5 \pm \sqrt{33}}{2}$

(obviously NOT integral soln.)

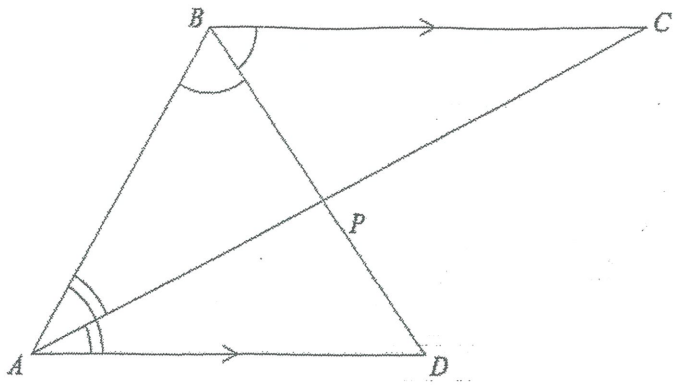
OR $u = -1 \Rightarrow x - \frac{2}{x} = -1$

ie $x^2 + x - 2 = 0$ ——— (4)

$(x+2)(x-1) = 0$

$\therefore$ $x = -2, 1$ are the integer solns.

(b) In the diagram, AD is parallel to BC, AC bisects $\angle BAD$ and BD bisects $\angle ABC$. The lines AC and BD intersect at P.



(i) Prove that $\angle BAC = \angle BCA$.

(ii) Prove that $ABCD$ is a rhombus.

(i) $\angle BAC = \angle CAD$
(AC bisects $\angle BAD$, given)
$\angle CAD = \angle BCA$ (alt. $\angle$'s $BC \parallel AD$)
$\therefore \angle BAC = \angle BCA$.

(ii) (General approach)
Show that $\triangle ABP \equiv \triangle CBP$.
AND $\triangle BCP \equiv \triangle DAP$.

THEN show one of the following:

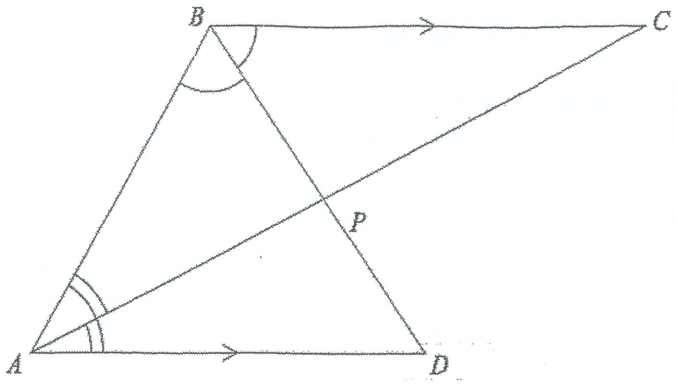
(a) * • Join CD → step (1)
• Show that quadrilateral ABCD has all sides equal.

OR (b) • Show that quadrilateral ABCD is a parallelogram with adjacent sides equal

OR • (c) Show that the diagonals of ABCD are perpendicular bisectors.

OR • (d) Show that diagonals of quadrilaterals ABCD bisects each vertex angles

(b) In the diagram, AD is parallel to BC, AC bisects $\angle BAD$ and BD bisects $\angle ABC$. The lines AC and BD intersect at P.



(i) Prove that $\angle BAC = \angle BCA$.

(ii) Prove that $ABCD$ is a rhombus.

In $\triangle ABP$, $\triangle CBP$

$\angle ABP = \angle CBP$
(BD bisects $\angle ABC$)
$\angle BAC = \angle BCA$
(proven in (i))
BP is common

$\therefore \triangle ABP \equiv \triangle CBP$
(AAS)

In $\triangle BCP$, DAP
$\angle CBD = \angle ADB$
(alternate $\angle$'s $BC \parallel AD$)
$\angle BPC = \angle DPA$
(vertically opp. $\angle$'s
$\angle P = AP$
(Matching sides
$\triangle ABP \equiv \triangle CBP$

$\therefore \triangle BCP \equiv \triangle DAP$
(AAS)

* Two sample methods are given below to prove $ABCD$ is a rhombus.

Method ① (Method in previous (c) table)

$\angle APB = \angle CPB$
(Matching $\angle$'s
$\triangle ABP \equiv \triangle CBP$)

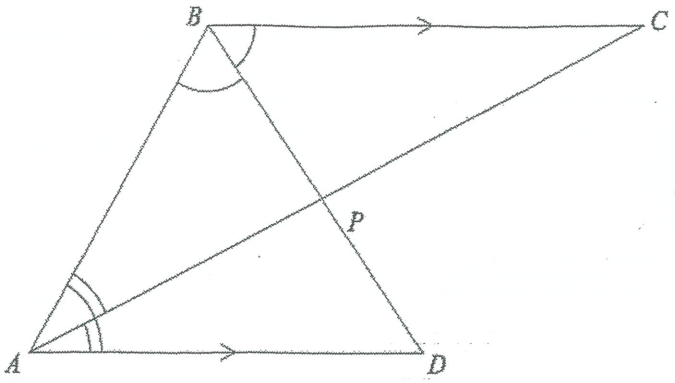
$\angle APB + \angle CPB = 180°$
(straight angle)
$\therefore \angle APB + \angle CPB = 180°$
$\therefore \angle APB = \angle CPB = 90°$

Similarly, $\angle APB = \angle APD$
$= 90°$
$\therefore AC \perp BD$

$BP = DP$
(Matching sides
$\triangle BCP \equiv \triangle DAP$)

$AP = CP$
(Matching sides,
$\triangle ABP \equiv \triangle CBP$
$\therefore AC$ bisects BD at $90°$
$\therefore ABCD$ is a rhombus
(diagonals bisect each other at $90°$)

(b) In the diagram, AD is parallel to BC, AC bisects $\angle BAD$ and BD bisects $\angle ABC$. The lines AC and BD intersect at P.



(i) Prove that $\angle BAC = \angle BCA$.

(ii) Prove that $ABCD$ is a rhombus.

Method (2)

Method (b) in the previous table.

- $BC = AD$

(corresponding sides $\triangle BCP \equiv \triangle DAP$)

- $BC \parallel AD$ (given)

$\therefore$ $ABCD$ is a parallelogram

(one pair of opposite sides equal and parallel.)

BUT - $BC = BA$

(corresponding, $\triangle ABP \equiv \triangle CBP$)

$\therefore$ $ABCD$ is a rhombus

(A parallelogram with a pair of adjacent sides equal.)

(c) By considering the factorisation of $x^2-2(a-1)x+a+1=0$, find a such that the solutions of the quadratic equation x_1 and x_2 satisfy: 3

$$\frac{x_1}{x_2}+\frac{x_2}{x_1}=4.$$

$$x^2-2(a-1)x+(a+1)=0$$

(1) —

$$\begin{cases} x_1+x_2=2(a-1) \\ x_1x_2=a+1 \end{cases}$$

Proof OTP

Now

$$\frac{x_1}{x_2}+\frac{x_2}{x_1}=4$$

$$\frac{x_1^2+x_2^2}{x_1x_2}=4$$

The numerator is equivalent to:

$$\frac{(x_1+x_2)^2-2x_1x_2}{x_1x_2}=4 \quad *$$

(2) —

Subst (1) into (2), we have:

$$\frac{4(a-1)^2-2(a+1)}{a+1}=4$$

$$4(a^2-2a+1)-2a-2=4a+4$$

$$4a^2-14a-2=0$$

(3) —

$$\therefore a=\frac{7\pm\sqrt{57}}{2}$$

Proof:

x_1 and x_2 are solutions (roots) of $x^2-2(a-1)x+a+1=0$.

i.e. $x_1, x_2 = \dfrac{2(a-1)\pm\sqrt{4(a-1)^2-4(a+1)}}{2}$

$x_1, x_2 = (a-1)+\sqrt{a^2-3a}, (a-1)-\sqrt{a^2-3a}$

$\therefore x_1+x_2 = 2(a-1)$

$$\begin{aligned}\therefore x_1x_2 &= \left[(a-1)+\sqrt{a^2-3a}\right]\times\left[(a-1)-\sqrt{a^2-3a}\right]\\ &= (a-1)^2-(a^2-3a)\\ &= a^2-2a+1-a^2+3a\\ &= a+1\end{aligned}$$