SYDNEY BOYS HIGH SCHOOL

2021

YEAR 10 STAGE 5.3
TERM 2
HALF YEARLY ASSESSMENT

Mathematics

General Instructions: **Time Allowed:** 90 minutes

- All questions may be attempted.
- Marks may be deducted for careless or badly arranged work.
- Show ALL relevant mathematical reasoning and/or calculations.
- Write using black pen.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of the booklet.
  Show the number and part of the answer being rewritten.
- Leave your answers in the simplest exact form, unless otherwise stated.
- NESA approved calculators may be used.
- Clearly indicate your class by placing a cross **X** next to your class.

Examiner: AYW

Name: _____

Class	Teacher	
10Ma**A**	B Kilmore	
10Ma**B**	T Broockmann	
10Ma**C**	Z Zeng	
10Ma**P**	S Gurjar	
10Ma**L**	R Wang	
10Ma**U**	AM Gainford/D Taylor	
10Ma**S**	J James	

Question	Marks
1	/14
2	/15
3	/13
4	/13
5	/14
6	/11
Total	**/80**

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SYDNEY BOYS HIGH SCHOOL

Name: ……...……

10Ma .....

Question 1

(/14 Marks)

(a) Find the value of 0.085×0.62 correct to two significant figures. **1**

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(b) Fully factorise $36a^3b^2 - 24a^2b^3$ **1**

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(c) Lawrie's pay sheet for last week is shown below.

Thursday: 11 hours at normal rates
Friday: 5 hours at double time
Saturday: 8 hours at time-and-a-half

Which day does Lawrie earn the greatest amount? **1**

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(d) What is the gradient of the line, $4x - 2y - 3 = 0$? **1**

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(e) Simplify $4m^0 - (2m)^0$. **1**

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(f) In the formula $p = \frac{m^2}{n^2}$, p has a value of 4, m has a value of 10 and $n < 0$.
What is the value of n? **1**

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(g) Factorise the following:

(i) $9x^2 - 4$ **1**

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(ii) $x^3 - y^3$ **1**

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(h) Two cards are drawn at random in succession from a standard deck of playing cards, with replacement. What is the probability of getting two black cards? **1**

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(i) Find the GST (10%) contained in the retail price of $165. **1**

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(j) Find the equation of the line that has a y-intercept of –2 and is perpendicular to $y = \frac{1}{4}x + 5$. **1**

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(k) Find the number of sides in a regular polygon that has an exterior angle of 18°. **1**

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(l) If $p \propto q$ then $p = kq$; find k if $q = 7.9$ and $p = 37.92$. **1**

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(m) Give one additional property that will make a rectangle a square. **1**

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End of Question 1

Extra Working Page for Question 1 only

SYDNEY BOYS HIGH SCHOOL

Name: ……...……

10Ma .....

Question 2

(/15 Marks)

(a) Write the coordinates of the point of intersection of the lines: **2**

$$y = 4x - 1 \quad \text{and} \quad y = 9 - x$$

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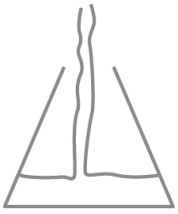
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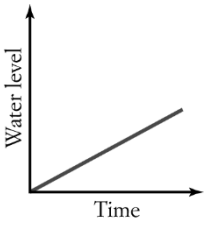
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(b) Water fills the bottle below at a constant rate and the height level of the water is graphed.

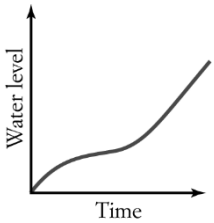


Which graph below shows this? (Circle the correct answer below). **1**

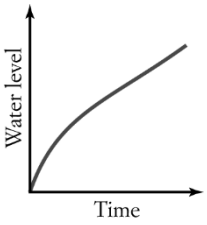
(A)



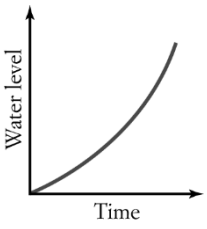
(B)



(C)

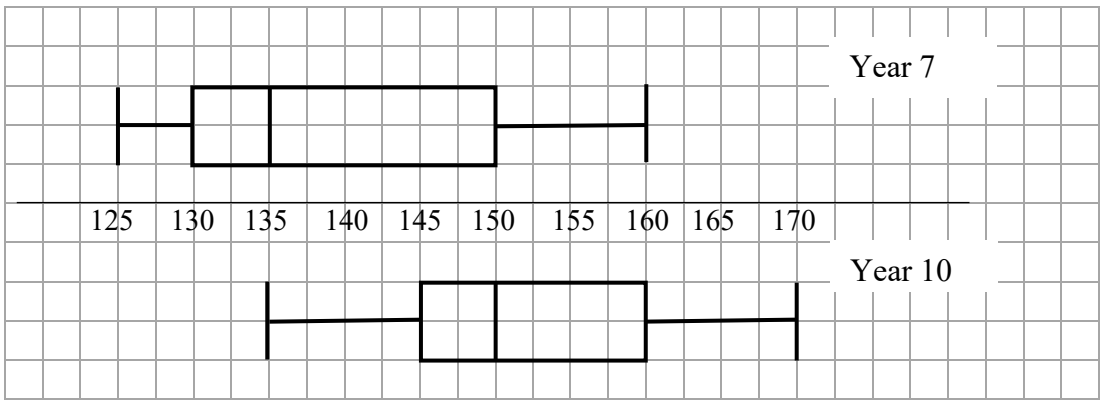


(D)



(c) The following graph shows the comparison of students' heights (cm) from Year 7 and

Year 10 at a local high school.



(i) What is the difference in the median heights of students in each year? **1**

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(ii) What is the difference in the interquartile ranges between Year 7 and Year 10? **2**

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(iii) What can be concluded about the result from question (ii) about the heights of Year 10 compared to Year 7? **1**

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(d) \$600 interest was earned on a \$1500 investment over 5 years. **2**
What was the annual flat rate of interest paid on the investment?

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(e) Find the coordinates of the vertex of $y = 2x^2 - 4x + 1$. **2**

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(f) Connie borrowed \$100 000 at a monthly interest rate (r) of 1% to purchase an investment property. Her monthly repayments (R) are \$1 100.

Month (n)	Principal (P)	Interest (I)	$P+I$	$P+I-R$
1	100 000.00	1 000.00	101 000.00	99 900.00
2	99 900.00	999.00	100 899.00	99 799.00
3	99 799.00	997.99	100 796.99	99 696.99
4	99 696.99	996.97	100 693.96	99 593.96

(i) What is the amount owing after the fifth payment? **2**

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(ii) The loan is repaid after 241 instalments. Calculate the total amount of interest paid on this loan. **2**

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End of Question 2

Extra Working Page for Question 2 only

Name:

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Question 3

(/13 Marks)

(a) Solve $x^2 = -6x$. **1**

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(b) Solve $x^2 + 6x = 40$ by the method of completing the squares. **2**

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(c) Solve $2x^2 + 8x + 5 = 0$ using the quadratic formula. **2**

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(d) Find all the values of x that are solutions to $x^6 - 9x^3 + 8 = 0$. **2**

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(e) A survey was conducted asking a group of people whether they smoked or not.
The results are shown in the table below.

	Male	Female	Total
Smoker	225	190	415
Non-smoker	395	580	975
Total	620	770	1390

One of the respondents was chosen at random. Find the probability that they were:

(i) Male. **1**

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(ii) Male, or Non-smoker. **1**

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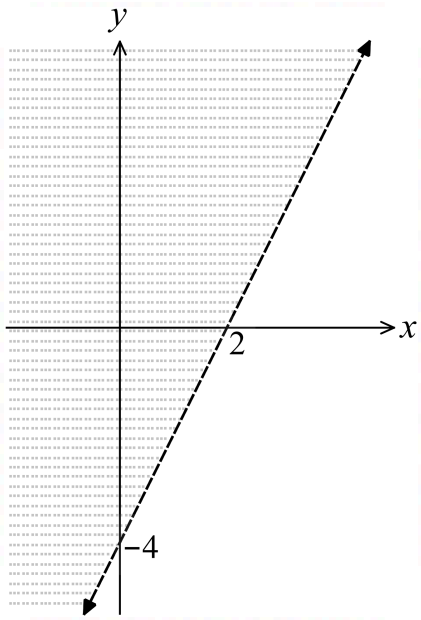
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(iii) Male, given he was a Non-smoker. **1**

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(f)



The shaded region above is best described by which inequality. **1**
(Circle the correct answer below).

(A) $2x - y - 4 < 0$ (B) $2x - y - 4 > 0$

(C) $2x - y - 4 \leq 0$ (D) $2x - y - 4 \geq 0$

(g) What is the equation of a circle with centre (1, –2) and radius 3 units? **1**

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(h) A sprinter runs 100 m in 10 seconds. What is her speed in km/h? **1**

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End of Question 3

Extra Working Page for Question 3 only

Name:

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Question 4 (/13 Marks)

(a) The table below is used by the SBHS Bank to calculate home loan repayments.

Monthly Repayments on a $1000 loan						
Rate p.a.	10 years	12 years	15 years	17 years	20 years	25 years
8.25%	$12.27	$10.96	$9.70	$9.13	$8.52	$7.88
8.5%	$12.40	$11.10	$9.85	$9.28	$8.68	$8.05
8.75%	$12.53	$11.24	$9.99	$9.43	$8.84	$8.22
9%	$12.67	$11.38	$10.14	$9.59	$9.00	$8.39
9.25%	$12.80	$11.52	$10.29	$9.74	$9.16	$8.56
9.5%	$12.94	$11.66	$10.44	$9.90	$9.32	$8.74
9.75%	$13.08	$11.80	$10.59	$10.05	$9.49	$8.91
10%	$13.22	$11.95	$10.75	$10.21	$9.65	$9.09
12%	$14.35	$13.15	$12.00	$10.55	$11.01	$10.35

(i) A bank will not allow a customer to borrow money if the repayments are more than 30% of the person's gross wage. Show with calculations that if Anthony earns $7400 per month, he will be able to borrow $250 000 at 8.5% for 20 years. **2**

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(ii) Calculate the amount of interest that Anthony will pay over the entirety of this loan. **2**

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(iii) Calculate the equivalent flat rate of interest as a percentage, correct to 1 decimal place. **2**

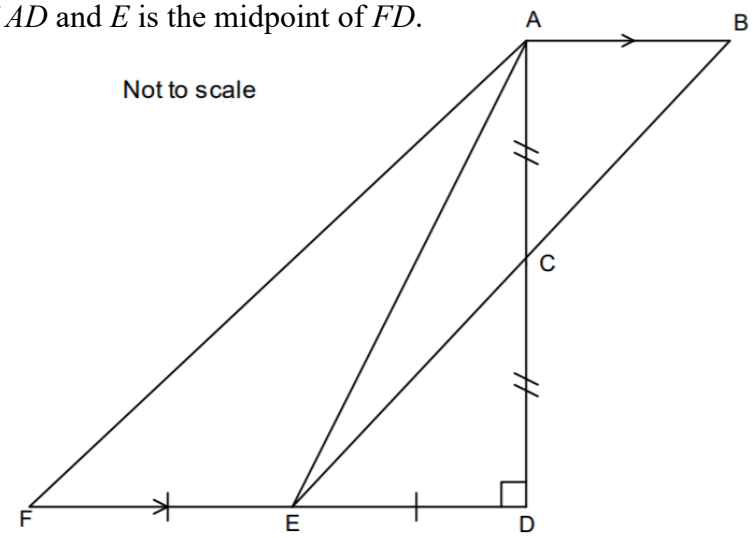
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(b) In the diagram below $AB \parallel FD$. ADF is a right-angled triangle with C the midpoint of AD and E is the midpoint of FD.



(i) Explain why $\angle CED = \angle ABC$. **1**

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(ii) Show that $\Delta CDE \equiv \Delta CAB$. **3**

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(iii) Prove that $ABEF$ is a parallelogram. **2**

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(c) For the curve $y = \dfrac{2}{x-5} - 6$, write down the equation of the vertical asymptote. **1**

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End of Question 4

Extra Working Page for Question 4 only

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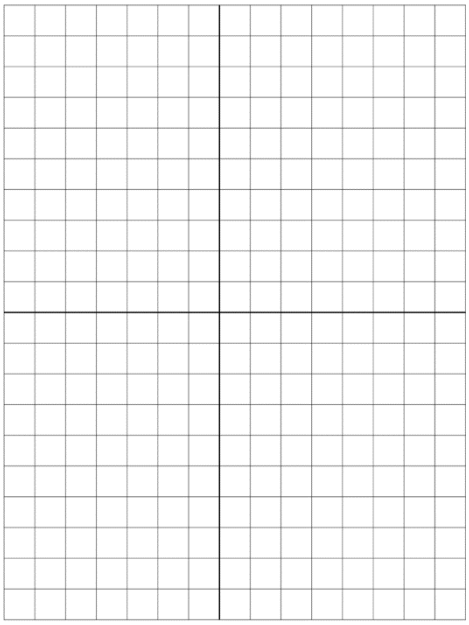
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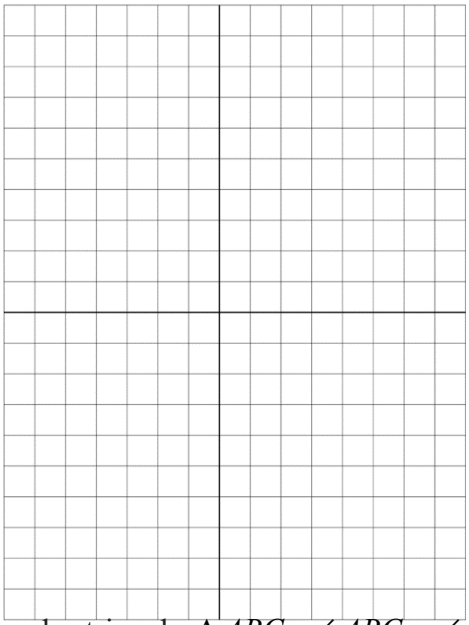
Question 5 (/14 Marks)

(a) Sketch the following graphs on separate number planes showing at least two points on each.

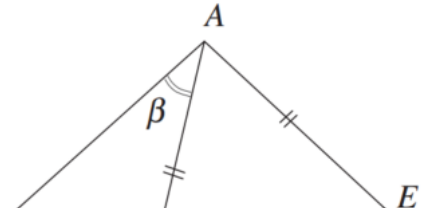
(i) $y = -3^{-x}$ **2**



(ii) $y = 2x^3 - 1$ **2**



(b) In the isosceles triangle ΔABC, $\angle ABC = \angle ACB = \alpha$. The points D and E lie on BC and AC, so that $AD = AE$, as shown in the diagram below. Let $\angle BAD = \beta$.



(i) Explain why $\angle ADC = \alpha + \beta$. **1**

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(ii) Find $\angle DAC$ in terms of α and β. **1**

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(iii) Hence, or otherwise, find $\angle EDC$ in terms of β. **2**

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(c) The triangle ABC has side lengths a, b, and c as shown in the diagram below.
The point D lies on AB, and CD is perpendicular to AB.



(i) Show that $a\sin B = b\sin A$. **1**

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(ii) Show that $c = a\cos B + b\cos A$ **2**

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(iii) If $c^2 = 4ab\cos A\cos B$, show that $a = b$. **3**

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End of Question 5

Extra Working Page for Question 5 only

SYDNEY BOYS HIGH SCHOOL

Name: ……...……

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Question 6

(/11 Marks)

(a) If 8 workers can plough a 6 hectare field in 5 hours, how long will it take to plough a 9 hectare field with 5 workers, working at the same rate? **2**

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(b) Find all the real values of x that are solutions to $x^4 - 3x^3 + x^2 - 3x = 0$. **2**

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(c) The prime factorization of 45 is $3^2 \times 5^1$. In general, the prime factorization of an integer $n \geq 2$ is of the form $p_1^{a_1} \times p_2^{a_2} \times p_3^{a_3} \times ... \times p_k^{a_k}$ where $p_1, p_2, p_3, ..., p_k$ are different prime numbers and $a_1, a_2, a_3, ..., a_k$ are positive integers.
Given an input of an integer $n \geq 2$, the Barbeau Process outputs the number equal to

$$n\left(\frac{a_1}{p_1} + \frac{a_2}{p_2} + \frac{a_3}{p_3} + ... + \frac{a_k}{p_k}\right).$$

For example, given an input of $45 = 3^2 \times 5$, the Barbeau Process outputs
$45\left(\frac{2}{3} + \frac{1}{5}\right) = 30 + 9 = 39.$

(i) Given an input of 280, what number does the Barbeau Process outputs? **2**

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(ii) Find the pair of prime numbers (p, q) such that the Barbeau Process with input $p^2 \times q$ outputs 135. **2**

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(d) A point is equidistant from the coordinate axes if the vertical distance from the **3**

point to the x-axis is equal to the horizontal distance from the point to the y-axis. The point of intersection of the vertical line $x = a$ (where a is a real number) with the line with equation $3x + 8y = 24$ is equidistant from the coordinate axes. What is the sum of all possible values of a?

End of Question 6

Extra Working Page for Question 6 only

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2021

YEAR 10
HALF YEARLY
ASSESSMENT TASK

Stage 5.3 Mathematics

Sample Solutions

NOTE: **Some of you may be disappointed with your mark.**
This process of checking your mark is NOT the opportunity to improve your marks.
Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown 'working' does not justify that your solution is worth any marks.

Question 1

(a) Find the value of 0.085×0.62 correct to two significant figures. **1**

The calculator gives 0.0527 which is 0.053 to 2 significant figures (or 5.3×10^{-2})

NOTE: This was not done as well as it should have been. Many students considered the zeroes in front of the 5 as significant OR they did it to 2 decimal places rather than what the question asks for.

(b) Fully factorise $36a^3b^2 - 24a^2b^3$ **1**

$12a^2b^2(3a - 2b)$

NOTE: The only factorisation here was to find all the common factors in the 2 terms. This was also done poorly by too many students.

(c) Lawrie's pay sheet for last week is shown below.
Thursday: 11 hours at normal rates
Friday: 5 hours at double time
Saturday: 8 hours at time-and-a-half

Which day does Lawrie earn the greatest amount? **1**

After converting all to normal rates, Saturday is the day that earns the greatest amount.

NOTE: The answer is Saturday, any students who only putting working that led towards that answer got some marks but not full marks.

(d) What is the gradient of the line, $4x - 2y - 3 = 0$? **1**

For a line in general form $ax + by + c = 0$, the gradient is $-\frac{a}{b}$.

Therefore, the gradient is 2.

NOTE: Pupils needed to state the gradient. If this was not done, then they may have gotten some marks for working that led to the correct result.

(e) Simplify $4m^0 - (2m)^0$. **1**

Anything to the power of zero is 1. This expression simplifies to $4 \times 1 - 1 = 3$.

NOTE: One pupil noted that *m* cannot be equal to zero, but then of course, this would not simplify – and a simplification implies the expression can be defined. This again was not done well by too many pupils.

(f) In the formula $p = \frac{m^2}{n^2}$, p has a value of 4, m has a value of 10 and $n < 0$. **1**

What is the value of n?

Make n the subject: $n^2 = \frac{m^2}{p}$ and it follows that $n = \pm\frac{m}{\sqrt{p}}$ but we need a negative value for n.

Hence, we just take the negative of this expression.

$n = -\frac{10}{2}$, which is equal to –5.

NOTE: Once again, too many pupils got this wrong. A number failed to substitute the correct values into the pronumerals and others dropped the square in one of the pronumerals. Many did not correctly identify the solution as the negative of the final expression.

(g) Factorise the following:

(i) $9x^2 - 4$ **1**

$(3x)^2 - 2^2 = (3x-2)(3x+2)$

NOTE: A number of pupils failed to realise that the first term is the square of $3x$. A number of pupils just put $(3x-2)^2$ as the solution.

(ii) $x^3 - y^3$ **1**

$(x-y)(x^2+xy+y^2)$

NOTE: This was not remembered well. This is not easy to derive and pupils really needed to remember the factorisation.

(h) Two cards are drawn at random in succession from a standard deck of playing cards, with replacement. What is the probability of getting two black cards? **1**

These are independent events and thus $P(\text{Black then Black}) = P(\text{Black}) \times P(\text{Black})$

$P(\text{Black}) = \frac{26}{52} = \frac{1}{2}$, Therefore $P(\text{Black then Black}) = \frac{1}{4}$

NOTE: Many pupils worked this out 'without replacement' and could not receive full marks. Many pupils did not realise that half the deck is made up of black cards.

(i) Find the GST (10%) contained in the retail price of \$165. **1**

GST is 10% of the Selling price and the retail price is Selling Price + 10% of Selling Price.

There are 11 lots of 10% here so all that is necessary to find the GST is to divide by 1

$\$165 \div 11 = \$15$

NOTE: Many students' answer was 10% of the retail price. Which was not what the question wanted. Some students gave their answer as \$150 – which is not the GST amount.

(j) Find the equation of the line that has a *y*-intercept of –2 and is perpendicular to **1**
$y = \frac{1}{4}x + 5$.

The line above has a gradient of $\frac{1}{4}$. Two lines a perpendicular if the gradients obey the

expression $m_1 \times m_2 = -1$, where m_1 and m_2 are gradients of two different lines.

Therefore, the gradient for the line perpendicular to the one above is –4. Using the gradient

intercept form of a line with the fact the *y*–intercept must be –2:

$y = -4x - 2$ is the equation we are looking for.

NOTE: Many pupils did not find the gradient of the line perpendicular to the one in the question. Many pupils tried to find solutions rather than the equation of a line.

(k) Find the number of sides in a regular polygon that has an exterior angle of 18°. **1**

The exterior angle α of a *n*-sided polygon can be given by $\alpha = \frac{360°}{n}$

Making *n* the subject gives: $n = \frac{360°}{\alpha}$, and since $\alpha = 18°$, *n* must be 20.

NOTE: Many pupils used the formula for the sum of angles inside a polygon – this was the incorrect method. Since, the question is about external angles, pupils needed to use the fact that the sum of external angles on any polygon is $360°$.

(l) If $p \propto q$ then $p = kq$; find *k* if $q = 7.9$ and $p = 37.92$. **1**

Substitute the values for *p* and *q* into $p = kq$

$37.92 = k \times 7.9$, then $k = 4.8$

NOTE: This should have been simple but some pupils wrote down the numbers incorrectly and/or input the numbers incorrectly into a calculator.

(m) Give one additional property that will make a rectangle a square. **1**

Diagonals meet at right angles/All sides are equal

NOTE: Either of these are correct. Pupils should have only put one. Some pupils put 2 answers and were then taken as a full answer. If one or both were incorrect, full marks were no longer available. Please remember to only put the most correct answer you can think of. If pupils put 2 answers and expect the marker to choose which is correct, they should really get zero.

2021 YR10 HYE Q2 Solutions and Comments

General feedback:

It was disturbing to see an overall poor performance from the cohort in Question 2, with some students producing some truly lamentable results.

This performance is indicative of students not giving the necessary attention to the Review of Year 9 chapter in order to succeed.

Students would be wise to reflect on this experience in the leadup to the Yearly Exam.

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Beginning of Question 2 solutions and comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

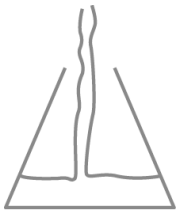
A. Write the coordinates of the point of intersection of the lines: 2

$$y = 4x - 1 \text{ and } y = 9 - x$$

Solution	Comment(s)
Since $y = 4x - 1$ and $y = 9 - x$: $4x - 1 = 9 - x$ $5x = 10$ $x = 2$ Substituting $x = 2$: $y = 9 - 2$ $= 7$ Hence, the point of intersection $P = (2, 7)$.	Common error(s): • Forgetting to solve for y after finding x.

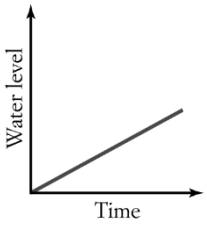
2021 YR10 HYE Q2 Solutions and Comments

B. Water fills the bottle below at a constant rate and the height level of the water is graphed. 1

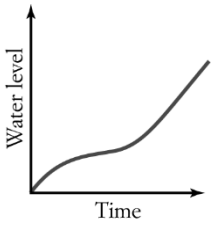


Which graph below shows this? (Circle the correct answer below)

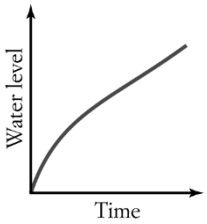
(A)



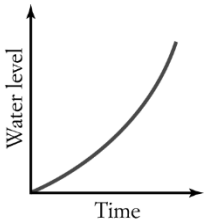
(B)



(C)



(D)

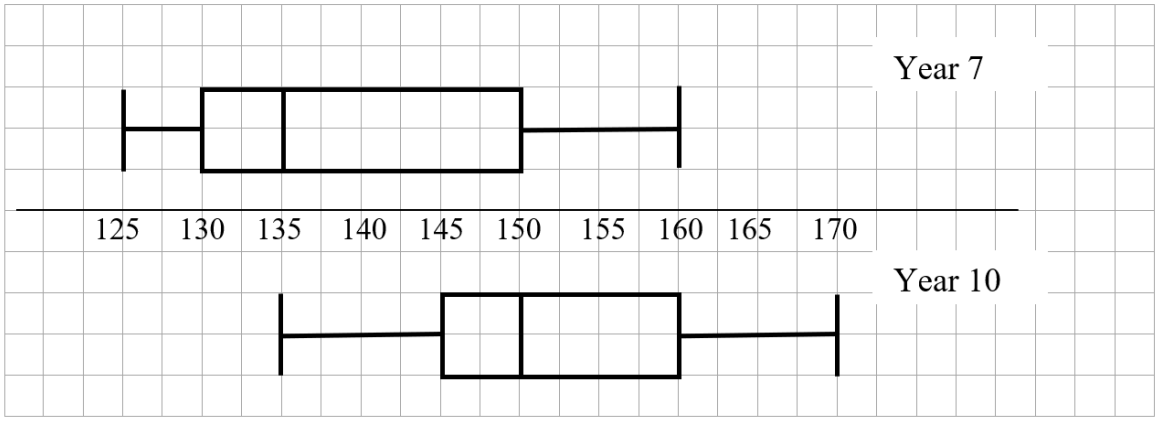


Solution	Comment(s)
(D), as indicated above.	No half marks. Common error(s): • Choosing (A), possibly due to forgetting that the question is asking for the rate at which the height is increasing, not the volume.

2021 YR10 HYE Q2 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

C. The following graph shows the comparison of students' heights (cm) from Year 7 to Year 10 at a local high school.



I. What is the difference in the median heights of students in each year? 1

Solution	Comment(s)
Median of YR7: 135 Median of YR10: 150 $\therefore 150 - 135 = 15\ cm$	No half marks. Students generally performed well on this question.

II. What is the difference in the interquartile ranges between Year 7 and Year 10? 2

Solution	Comment(s)
IQR of YR7: $150 - 130 = 20$ IQR of YR10: $160 - 145 = 15$ $\therefore 20 - 15 = 5\ cm$	Common error(s): • Stating the quartiles instead of calculating the IQRs.

III. What can be concluded about the result from Question II about the heights of Year 10 compared to Year 7? 1

Solution	Comment(s)
The spread of heights in YR7 is larger than the spread of heights in YR10.	No half marks. Students **must** describe the spread of the data sets using the IQR, as indicated in the question, to be awarded the mark. Sample answers that would **not** be awarded the mark: • "YR10 students are generally taller than YR7 students." • "YR10 students have a higher median height than YR7 students." • "The heights of YR7 students are more positively skewed."

2021 YR10 HYE Q2 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

D. \$600 interest was earned on a \$1 500 investment over 5 years. 2
What was the annual flat rate of interest paid on the investment?

Solution	Comment(s)
$I = PRN$ $\therefore R = \frac{I}{PN}$ $= \frac{600}{1500 \times 5}$ $= 8\% \, p.a.$	Common error(s): • Using the compound interest formula. • Setting $I = 600 + 1\,500$ instead of $I = 600$. • Writing the answer as 0.08%.

E. Find the coordinates of the vertex of $y = 2x^2 - 4x + 1$. 2

Solution	Comment(s)
Method 1 Completing the square: $y = 2x^2 - 4x + 2 - 2 + 1$ $= 2(x^2 - 2x + 1) - 1$ $= 2(x - 1)^2 - 1$ Hence, the vertex $V = (1, -1)$. **Method 2** Axis of symmetry is located at: $x = -\frac{b}{2a}$ $= \frac{4}{2 \times 2}$ $= 1$ Substituting $x = 1$: $y = 2(1)^2 - 4(1) + 1$ $= -1$ Hence, the vertex $V = (1, -1)$.	Students should **not** be attempting to find the intercepts to answer this question, as: • The intercepts are irrational, so rounding becomes a problem. • The intercepts are irrational, so it's easier to make silly mistakes. • It's slower, thus wasting time. Common error(s): • Forgetting to solve for y after finding x when using Method 2. • Incorrectly remembering the formula for the axis of symmetry as $x = \frac{2a}{-b}$ or variations. • Factorising to $y = 2x(x + 2) + 1$ to incorrectly read the vertex as $V = (-2, 1)$.

2021 YR10 HYE Q2 Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

F. Connie borrowed \$100 000 at a monthly interest rate (r) of 1% to purchase an investment property. Her monthly repayments (R) are \$1 100.

Month (n)	Principal (P)	Interest (I)	$P+I$	$P+I-R$
1	100 000.00	1 000.00	101 000.00	99 900.00
2	99 900.00	999.00	100 899.00	99 799.00
3	99 799.00	997.99	100 796.99	99 696.99
4	99 696.99	996.97	100 693.96	99 593.96

I. What is the amount owing after the fifth payment? 2

Solution				Comment(s)
Using the table, at the end of the fifth month:				Students risked being penalised by not rounding to 2 decimal places.
n	P	I	$P+I-R$	
5	99 593.96	995.94	99 489.9	
Hence, the amount owing after the fifth payment is \$99 489.90.				

II. The loan is repaid after 241 instalments. 2
Calculate the total amount of interest paid on this loan.

Solution	Comment(s)
Total amount of money paid: $1\,100 \times 241 = 265\,100$ Since \$100 000 was borrowed: $265\,100 - 100\,000 = 165\,100$ Hence, the total amount of interest paid is \$165 100.	Common error(s): • Using the compound interest formula.

End of Question 2 solutions and comments

SYDNEY BOYS HIGH SCHOOL

Name: .....

10Ma .....

Question 3

(/13 Marks)

(a) Solve $x^2 = -6x$. **1**

$x^2+6x=0$

$x(x+6)=0$

$x=0,-6$

(b) Solve $x^2+6x=40$ by the method of completing the squares. **2**

$x^2+6x+9=40+9$

$(x+3)^2=49$

$x+3=\pm 7$

$x=-3\pm 7$

$=-10,4$

(c) Solve $2x^2+8x+5=0$ using the quadratic formula. **2**

$x=\dfrac{-8\pm\sqrt{8^2-4\times 2\times 5}}{2\times 2}$

$=\dfrac{-8\pm\sqrt{24}}{4}$

$=\dfrac{-8\pm 2\sqrt{6}}{4}$

$=\dfrac{-4\pm\sqrt{6}}{2}$

(d) Find all the values of x that are solutions to $x^6 - 9x^3 + 8 = 0$. **2**

(A) let $u = x^3$

$u^2 - 9u + 8 = 0$

$(u-1)(u-8) = 0$

$u = 1, 8$

$x^3 = 1, 8$

$x = 1, 2$

(B) $(x^3)^2 - 9x^3 + 8 = 0$

$(x^3 - 1)(x^3 - 8) = 0$

$x^3 = 1, 8$

$x = 1, 2$

(e) A survey was conducted asking a group of people whether they smoked or not.
The results are shown in the table below.

	Male	Female	Total
Smoker	225	190	415
Non-smoker	395	580	975
Total	620	770	1390

One of the respondents was chosen at random. Find the probability that they were:

(i) Male. **1**

$\frac{620}{1390} = \frac{62}{139}$

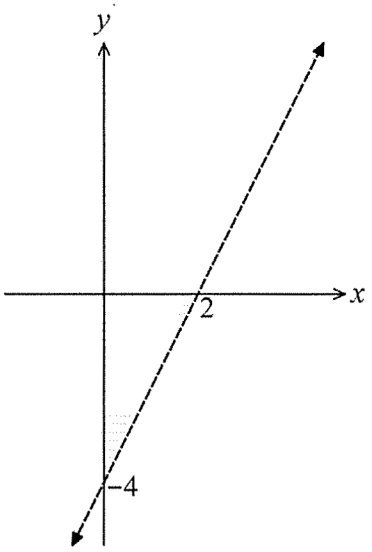
(ii) Male, or Non-smoker. **1**

$\frac{225+395+580}{1390} = \frac{1200}{1390} = \frac{120}{139}$

(iii) Male, given he was a Non-smoker. **1**

$\frac{395}{975} = \frac{79}{195}$

(f)



The shaded region above is best described by which inequality. **1**
(Circle the correct answer below).

(A) $2x - y - 4 < 0$ (B) $2x - y - 4 > 0$

(C) $2x - y - 4 \leq 0$ (D) $2x - y - 4 \geq 0$

(g) What is the equation of a circle with centre (1, –2) and radius 3 units? **1**

$(x-1)^2 + (y+2)^2 = 3^2$

(h) A sprinter runs 100 m in 10 seconds. What is her speed in km/h? **1**

100m/10 seconds = 360×100m/360×10 seconds
= 36,000m/3,600sec
= 36km/hr

End of Question 3

SYDNEY BOYS HIGH SCHOOL

Name: SOLNS

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Question 4

(/13 Marks)

(a) The table below is used by the SBHS Bank to calculate home loan repayments.

Monthly Repayments on a $1000 loan						
Rate p.a.	10 years	12 years	15 years	17 years	20 years	25 years
8.25%	$12.27	$10.96	$9.70	$9.13	$8.52	$7.88
8.5%	$12.40	$11.10	$9.85	$9.28	$8.68	$8.05
8.75%	$12.53	$11.24	$9.99	$9.43	$8.84	$8.22
9%	$12.67	$11.38	$10.14	$9.59	$9.00	$8.39
9.25%	$12.80	$11.52	$10.29	$9.74	$9.16	$8.56
9.5%	$12.94	$11.66	$10.44	$9.90	$9.32	$8.74
9.75%	$13.08	$11.80	$10.59	$10.05	$9.49	$8.91
10%	$13.22	$11.95	$10.75	$10.21	$9.65	$9.09
12%	$14.35	$13.15	$12.00	$10.55	$11.01	$10.35

(i) A bank will not allow a customer to borrow money if the repayments are more than 30% of the person's gross wage. Show with calculations that if Anthony earns $7400 per month, he will be able to borrow $250 000 at 8.5% for 20 years. **2**

$30\% \times \$7400 = \$2220$ ✓

Monthly repayments on $\$250000 = 250 \times \$8.68$ ✓
$= \$2170$ which is less than $\$2220$.

$\therefore$ can borrow $\$250000$

Most students did this well. Some found $2170 as a percentage of $7400 and that was also correct.

(ii) Calculate the amount of interest that Anthony will pay over the entirety of this loan. **2**

Total repayments = $2170 × 240 months
= $520800
∴ Interest = $520800 − $250000
= $270800 over 20 yrs

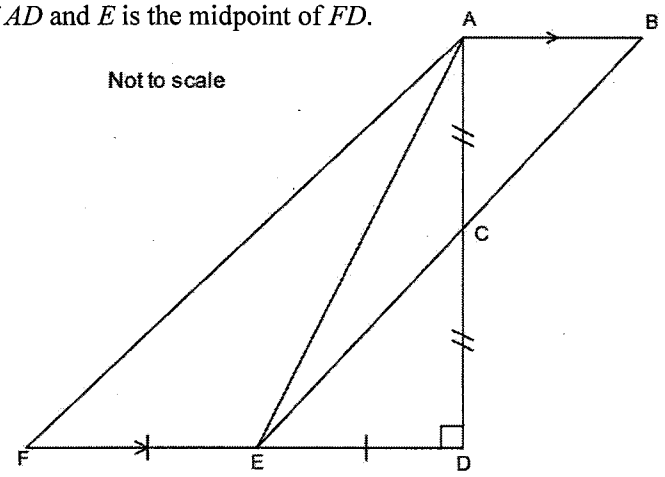
Many students found the repayments but did not subtract to find the interest. Some incorrectly used the compound interest formula forgetting that there were repayments.

(iii) Calculate the equivalent flat rate of interest as a percentage, correct to 1 decimal place. **2**

$I = Prt$
$270800 = 250000 \times r \times 20$
$r = 0.05416$
$= 5.4\%$

Some incorrectly used the compound interest formula- they did not read the question.

(b) In the diagram below $AB \parallel FD$. ADF is a right-angled triangle with C the midpoint of AD and E is the midpoint of FD.



(i) Explain why $\angle CED = \angle ABC$. **1**

They are alternate angles AB//FD

Done well.

(ii) Show that $\Delta CDE \equiv \Delta CAB$. **3**

$\angle CED = \angle ABC$ (Alt Ls AB // FD)

$\angle BAC = \angle CDE = 90°$ (same reason) ✓

$AC = CD$ (given) ✓

$\therefore \Delta CDE \equiv \Delta CAB$ (AAS) ✓

This question was done very well.

(iii) Prove that *ABEF* is a parallelogram. **2**

$AB = FE$ (given)

$AB = ED$ (matching sides in cong Δs) ✓

One pair of opposite sides in ABEF are equal and parallel ✓

$\therefore$ ABEF is a parallelogram.

This was the minimum amount of evidence to prove it is a parallelogram. If students showed both but did not realise this was enough then they only got one mark. Some students did a lot more work in proving it was a parallelogram.

(c) For the curve $y = \frac{2}{x-5} - 6$, find the equation of the vertical asymptote. **1**

$x = 5$ ✓

About half the cohort incorrectly thought it was y=6.

End of Question 4

SYDNEY BOYS HIGH SCHOOL

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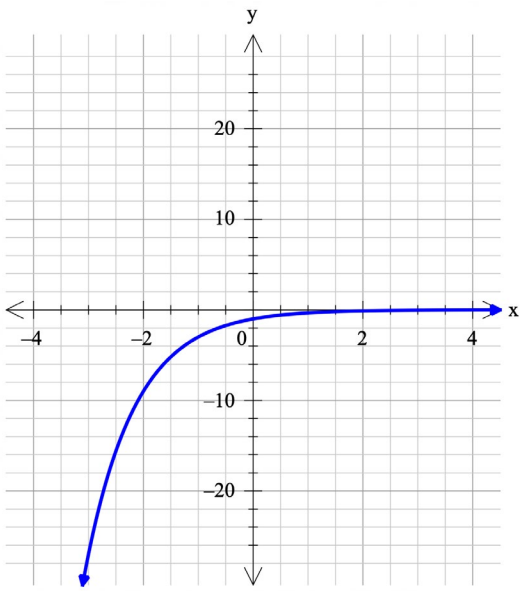
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Question 5

(/14 Marks)

(a) Sketch the following graphs on separate number planes showing at least two points on each.

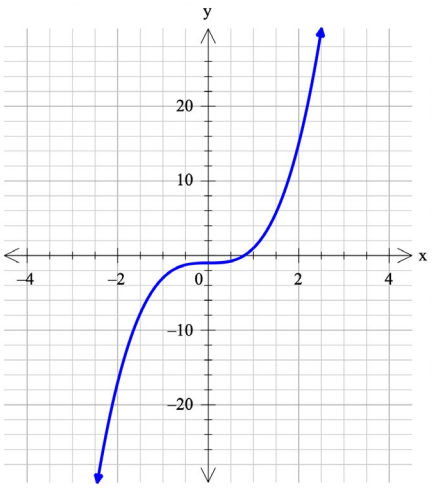
(i) $y = -3^{-x}$ **2**



x	–3	–2	–1	0	1	2	3
y	–27	–9	–3	–1	$-\frac{1}{3}$	$-\frac{1}{9}$	$-\frac{1}{27}$

[Generally well answered – some forgot to include two specific points.]

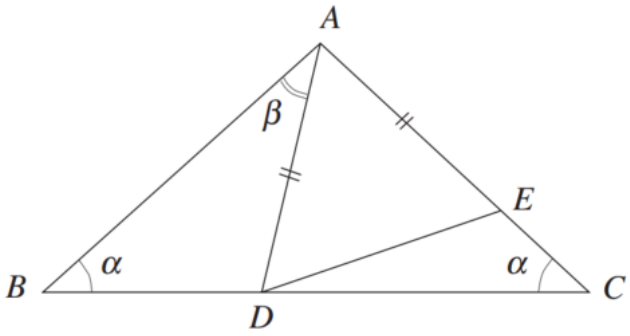
(ii) $y = 2x^3 - 1$ **2**



x	–3	–2	–1	0	1	2	3
y	–55	–17	–3	–1	1	15	53=

[Generally well answered – some forgot to include two specific points.]

(b) In the isosceles triangle ΔABC, $\angle ABC = \angle ACB = \alpha$. The points D and E lie on BC and AC, so that $AD = AE$, as shown in the diagram below. Let $\angle BAD = \beta$.



(i) Explain why $\angle ADC = \alpha + \beta$. **1**

$\angle ADC = \alpha + \beta$ (exterior angle of a triangle is equal to the sum of the remote interior angles)

[Well answered. Some indication of reasoning was expected.]

(ii) Find $\angle DAC$ in terms of α and β. **1**

$$2\alpha + \beta + \angle DAC = 180° \ (angle\ sum\ of\ \Delta ABC)$$

$$\therefore \angle DAC = 180° - 2\alpha - \beta$$

[Well answered, though some failed to get signs right with grouping symbols.]

(iii) Hence, or otherwise, find $\angle EDC$ in terms of β. **2**

$\angle EDC = \angle ADC - \angle ADE$

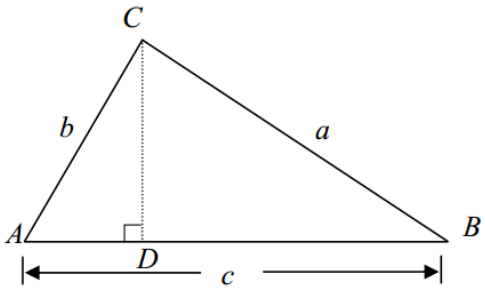
$= \alpha + \beta - \frac{1}{2}(180° - (180° - 2\alpha - \beta))$ (isosceles triangle)

$= \alpha + \beta - \frac{1}{2}(2\alpha + \beta)$

$= \frac{\beta}{2}$

[Very few managed to work this through to the conclusion.]

(c) The triangle ABC has side lengths a, b, and c as shown in the diagram below.
The point D lies on AB, and CD is perpendicular to AB.



(i) Show that $a\sin B = b\sin A$. **1**

In ΔACD, $\frac{CD}{b} = \sin A$ In ΔBCD, $\frac{CD}{a} = \sin B$

$\therefore CD = b\sin A$ $\therefore CD = a\sin B$

$\therefore a\sin B = b\sin A$

[Some used unsatisfactory proofs, and some wrongly assumed $\angle C$ was a right angle.

This error cost all 6 marks. Otherwise well answered.]

(ii) Show that $c = a\cos B + b\cos A$ **2**

In ΔACD, $\frac{AD}{b} = \cos A$ In ΔBCD, $\frac{DB}{a} = \cos B$

$\therefore AD = b\cos A$ $\therefore DB = a\cos B$

Now $c = AD + DB$ $\therefore$ c = $a\cos B + b\cos A$

[Other than reasons above, well answered.]

(iii) If $c^2 = 4ab\cos A\cos B$, show that $a = b$. **3**

$c^2 = (a\cos B + b\cos A)^2$

$\therefore c^2 = a^2\cos^2 B + 2ab\cos B\cos A + b^2\cos^2 A$ ——(1)

$c^2 = 4ab\cos B\cos A$ (given)——(2)

By subtraction (1) – (2)

$0 = a^2\cos^2 B - 2ab\cos B\cos A + b^2\cos^2 A$

$\therefore (a\cos B - b\cos A)^2 = 0$

$\therefore$ $a\cos B = b\cos A$ ——(3)

From above $a\sin B = b\sin A$ ——(4)

Now (4) ÷ (3) $\tan B = \tan A$, and so $B = A$ (angles in Δ)

$\therefore$ $a = b$ (isosceles ΔACB)

[Very few (<10 say) were able to give this complete answer.]

End of Question 5

Question 6

(a) If 8 workers can plough a 6 hectare field in 5 hours, how long will it take to plough a 9 hectare field with 5 workers, working at the same rate? 2

We start with the number of works (W) per Number of hectares (H) per number of hours (T).

W	H	T
8	6	5

Since number of workers and time are inverse relationships, we can switch them, while keeping the number of hours constant. This gives:

W	H	T
5	6	8

Similarly, since Number of hectares and time are linear relationships, we can divide each by 6, which gives:

W	H	T
5	1	$\frac{8}{6}$

We then multiply hectares and number of hours by 9, which gives

W	H	T
5	9	$\frac{8}{6}\times 9$

$\therefore \frac{8\times 9}{6} = 12$ hours

Alternatively:
1 worker completes 0.75 Ha in 5 hours.
There are 12 lots of 0.75 in 9, therefor one worker completes 9 Ha in $12 \times 5 = 60$ hours.
Five workers will work at 5 times this rate, and so will take 12 hours.

About half the cohort got this question right.
A common mistake was treating time and number of workers as a linear relationship.

(b) Find all the real values of x that are solutions to $x^4 - 3x^3 + x^2 - 3x = 0$. **2**

$$x^4 - 3x^3 + x^2 - 3x = 0$$
$$\left(x^2(x^2+1) - 3x(x^2+1)\right) = 0$$
$$(x^2 - 3x)(x^2+1) = 0$$
$$x(x-3)(x^2+1) = 0$$

$x = 0, 3$

Generally well done. A handful of students incorrectly gave -1 as solution. Some students factorized the expression then neglected to give 0 as a solution.
A few students incorrectly factorized the expression.

(c) The prime factorization of 45 is $3^2 \times 5^1$. In general, the prime factorization of an integer $n \geq 2$ is of the form $p_1^{a_1} \times p_2^{a_2} \times p_3^{a_3} \times ... \times p_k^{a_k}$ where $p_1, p_2, p_3, ..., p_k$ are different prime numbers and $a_1, a_2, a_3, ..., a_k$ are positive integers.
Given an input of an integer $n \geq 2$, the Barbeau Process outputs the number equal to

$$n\left(\frac{a_1}{p_1}+\frac{a_2}{p_2}+\frac{a_3}{p_3}+...+\frac{a_k}{p_k}\right).$$

For example, given an input of $45 = 3^2 \times 5$, the Barbeau Process outputs
$45\left(\frac{2}{3}+\frac{1}{5}\right) = 30 + 9 = 39.$

(i) Given an input of 280, what number does the Barbeau Process outputs? **2**

$2^3 \times 5^1 \times 7^1 = 280$
Ouput = $280(\frac{3}{2}+\frac{1}{5}+\frac{1}{7}) = 516$

This was quite well done by the cohort as a whole.

(ii) Find the pair of prime numbers (p, q) such that the Barbeau Process with input $p^2 \times q$ outputs 135. **2**

$p^2 q(\frac{2}{p}+\frac{1}{q}) = 135$
$2pq + p^2 = 135$
$p(2q + p) = 135 = 3^3 \times 5$
If $p = 3,\ 2q + 3 = 45 \rightarrow q = 21$ Which is not prime

Generally not well done.

Common errors were setting $p^2 q$ equal to 135, or attempting to solve $X\left(\frac{2}{p}+\frac{1}{q}\right) = 135$.

Students were expected to get the correct solution, without any flaws in their deductive reasoning. For example, some students incorrectly deduced that $p = 5$ is the only possible factorisation of $p(2q + p) = 3^5 \times 5$.

Alternate solution:
$2pq + p^2 = 135$
$p^2 + 2pq - 135 = 0$
$p = \frac{-2q \pm \sqrt{4q^2 + 540}}{2}$
The first prime q which makes $4q^2 + 540$ equal to a perfect square is 11.
As $p = \frac{-2 \times 11 \pm \sqrt{121 + 540}}{2} = 5$ is also prime, we conclude that $(p, q)=(5, 11)$.
A handful of students attempted to use the quadratic formulae to find (p, q).

(iii) Determine the three triples (a, b, c) of positive integers such that the Barbeau Process with input $2^a \times 3^b \times 5^c$ outputs $4 \times 2^a \times 3^b \times 5^c$. **3**

$$2^a \times 3^b \times 5^c \left(\frac{a}{2} + \frac{b}{3} + \frac{c}{5}\right) = 4 \times 2^a \times 3^b \times 5^c$$

$$\frac{a}{2} + \frac{b}{3} + \frac{c}{5} = 4$$

As 2,3 and 5 have no common divisor, $\frac{a}{2}, \frac{b}{3} \text{ and } \frac{c}{5}$ must be integers. There is only one way that we can write 4 as a sum of positive integers, and that is 4=2+1+1. The table below shows each of three possible solutions.

$\frac{a}{2}$	$\frac{b}{3}$	$\frac{c}{5}$	Solution
2	1	1	$a = 4, b = 3, c = 5$
1	2	1	$a = 2, b = 6, c = 5$
1	1	2	$a = 2, b = 3, c = 10$

Generally not done well.
Only one or two students got all three correct solutions.

End of Solutions