SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2009

Year 10
Half - Yearly Examination

Mathematics

General Instructions

- Working time - 90 minutes
- Write using black or blue pen
- Board Approved calculators may be used.
- **All necessary** working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.
- Attempt all questions.
- Clearly indicate your class by placing an X next to your class.
- All answers are to be given in simplified exact form unless otherwise stated.

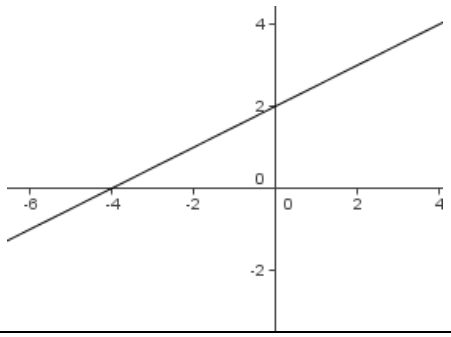
NAME _____

Examiner: *R Boros*

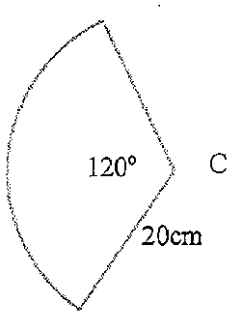
Class	Teacher	X
10A	Mr McQuillan	
10B	Ms Roessler	
10C	Ms Nesbitt	
10D	Mr Fuller	
10E	Mr Hespe	
10F	Mr Gainford	
10G	Ms Evans	

Question	Marks
1	/18
2	/15
3	/15
4	/13
5	/10
6	/9
7	/9
TOTAL	/89

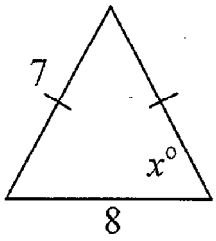
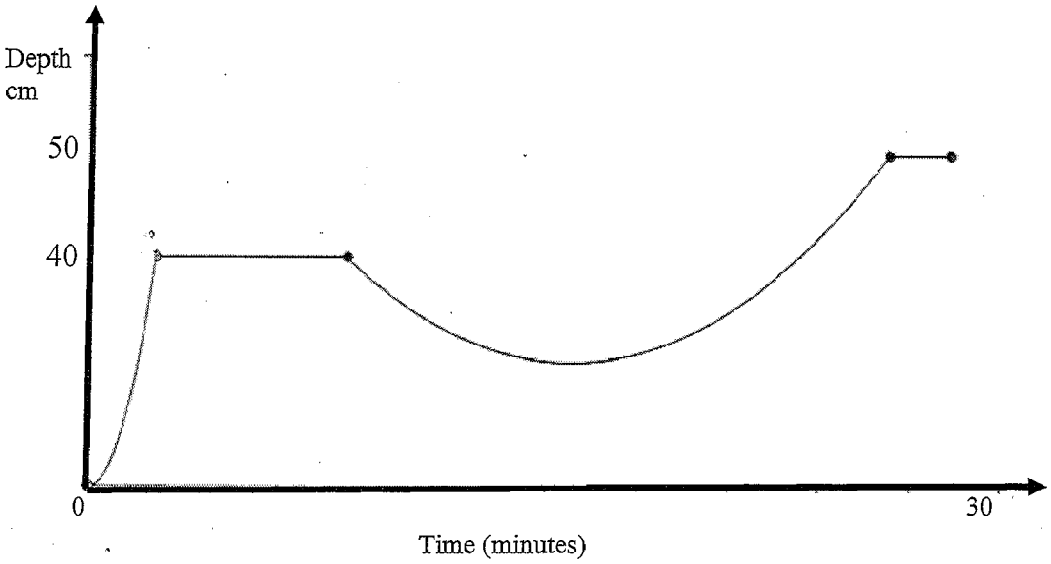
Question One (19 Marks)		**Answers**	**Marks**
a)	Find the simplest expression for: $-4a+7b-b+3a$		1
b)	$x \longrightarrow$ [A] $\longrightarrow$ [B] $\longrightarrow$ [Answer] This diagram represents two number machines. Machine A adds 7 to any number entering it. Machine B squares any number entering it. Write down the final expanded answer when x enters the machines.		2
c)	Fully factorise: $20xy^2-15xy$		1
d)	Which expression cannot be factorised: **A.** x^2+16 **B.** x^2-7 **C.** x^2-2x+1 **D.** $(x+2)^2-4$		1
e)	Fully factorise: $75a^2-27$		2
f)	Given that $\sqrt{8}=2\sqrt{2}$, fully factorise $\sqrt{8}xy-\sqrt{2}y$.		2
g)	Express $3x\sqrt{y}$ as an entire surd.		1

h)	Solve $9-3x>-6$ and graph the solution on a number line.		1
i)	Express $\dfrac{2}{2\sqrt{2}+5}$ with a rational denominator.		2
j)	What is the gradient of this line:  		2
k)	Which one of the following lines has zero gradient: **A.** $y=x$ **B.** $y=10$ **C.** $x=-3$ **D.** $x+y=0$		1
l)	Solve this pair of equations simultaneously: $3x+2y=-2$ $x+2y=2$		2
	End of Question One		

Question Two (15 Marks)		**Answers**	**Marks**
a)	Find the value of $\sin 44^o 30'$ correct to 4 decimal places.		1
b)	If $\cos\theta = 0.8000$, find the acute angled θ correct to the nearest minute.		1
c)	Solve $x^2 - 7x = 0$		2
d)	By showing all working and using factorisation only, solve: $4x^2 + 19x - 5 = 0$		3
e)	To complete the square for $x^2 - x$, find the third term.		2
f)	In her job as a supermarket supervisor Chris is paid a set wage of \$387.50 for a 36 hour week, plus time and half for overtime. In one week she worked 44 hours, find her wage.		2

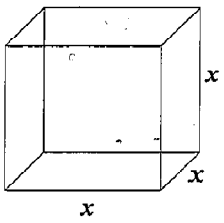
g)	 This is a piece of ham and pineapple pizza, left for Darren to eat at a party. ***C*** is the centre of this circular pizza. Find the area of pizza eaten by everyone else correct to the nearest cm^2.		2
h)	Dave decides to invest \$15000 at a compound interest rate of 9%p.a., calculated annually. Over 4 years, correct to the nearest dollar, what will his investment grow to?		2
	End of Question Two		

Question Three (15 Marks)		Answers	Marks
a)	**CHOCOLATE** The letters of the word 'chocolate' were written on cards, which were then shuffled. A student then selected one card at random. Find *P*(that the card is a vowel).		1
b)	In a set of 30 students; 20 studied Art; 15 studied Drama; 4 studied neither Art nor Drama. If a student was selected at random, find *P*(that the student studied both Art and Drama).		3
c)	Solve, using the quadratic formula, leaving your answer in simplest surd form: $x^2 + 8x + 10 = 0$.		3
d)	A bank client decides to invest \$20,000 in a scheme in which compound interest was offered at 14%p.a., calculated quarterly, for a period of 10 years. Find the amount of interest, to the nearest dollar, that would be earned.		3

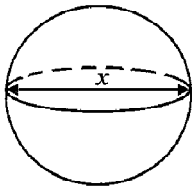
e)	Find x correct to the nearest minute.  		2
f)	The line $y=1$ crosses the parabola $y=x^2-3$ at 2 points called P and Q. Find the x values of the points P and Q.		2
g)	This graph represents the depth of water in a bath over time.  An explanation for this graph could be: **A.** Someone in the bath started splashing a lot to make bigger and bigger waves. **B.** Someone got out and was replaced by someone bigger. **C.** Someone got out and was replaced by someone smaller. **D.** Water was let out, but the bath was then filled to overflowing.		1
	End of Question Three		

Question Four (13 Marks)	Answers	Marks
a)	Circle which of the following has the greatest volume? (all measurements in cm)	2

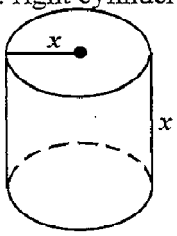
A: cube



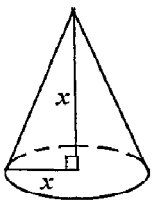
B: sphere



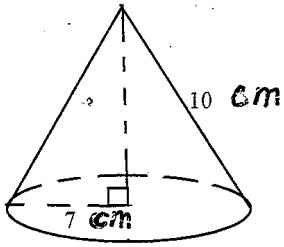
C: right cylinder



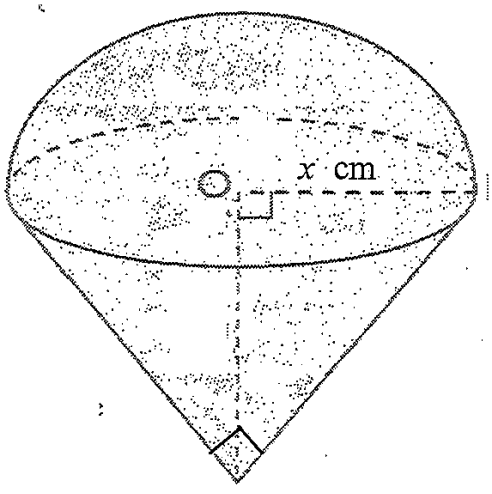
D: right cone



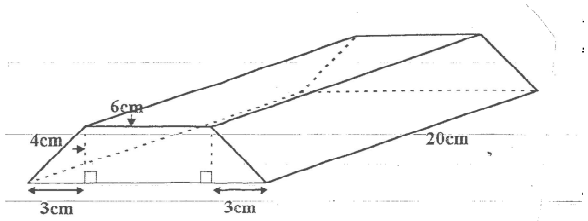
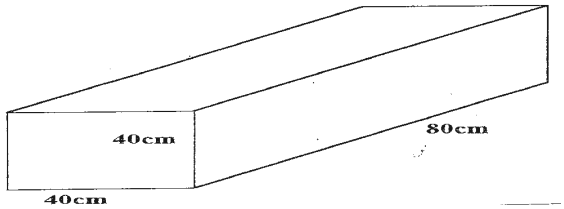
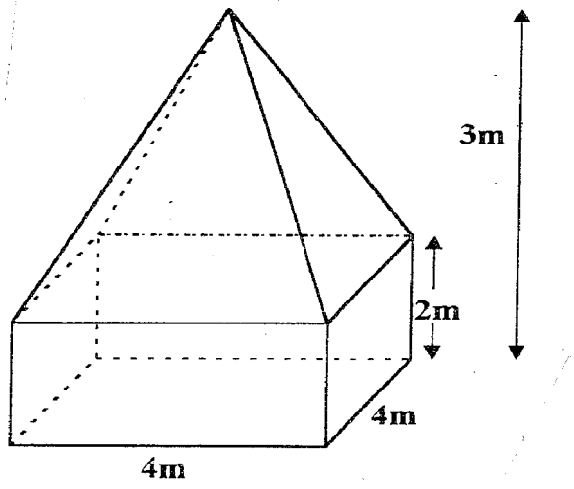
b) In cm^2 what is the <u>total</u> surface area of this cone (answer to the nearest whole number). 2

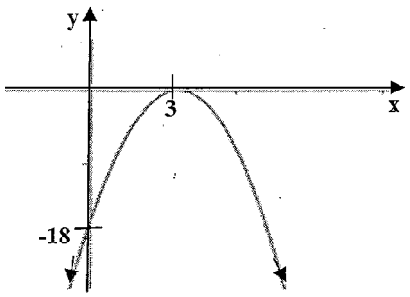
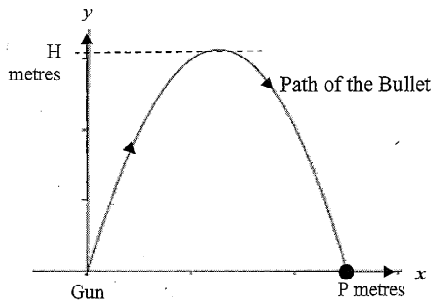


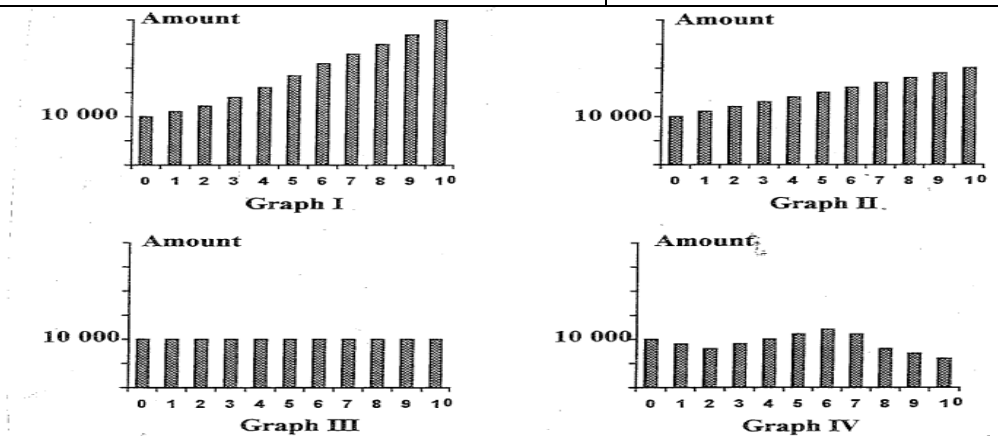
c) This figure shows a solid formed by a hemisphere and a right circular cone. Find the volume of this solid in terms of π and x 3

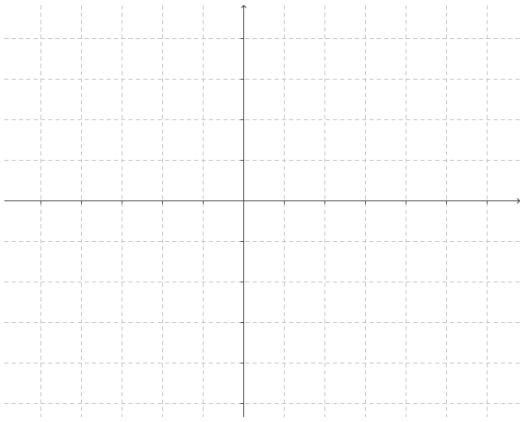
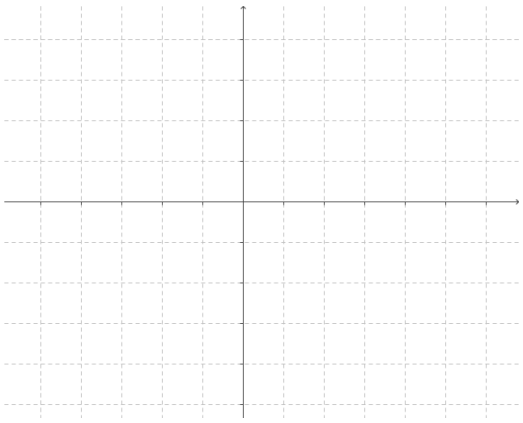
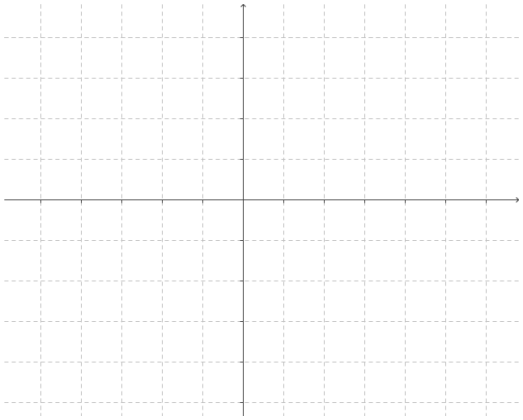
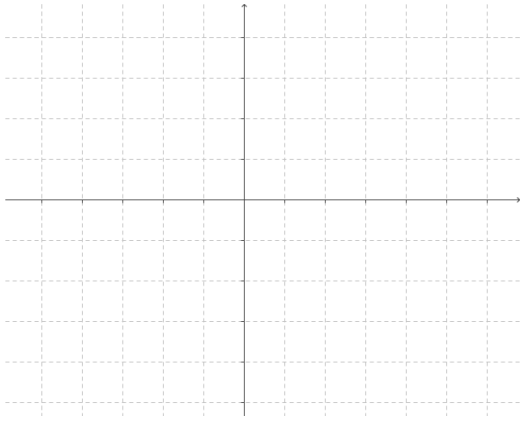


d)	If a coin is tossed 7 times find the probability of getting tails 7 times in a row.		2
e)	Marbles numbered 0 to 44, inclusive, were placed in a barrel for a draw. i.) If one marble was taken out and found to be an odd number, find the probability that the next was even, if the first was replaced. ii.) If 5 marbles were removed without replacing them, and each was an odd number, find the probability that the next marble to be removed is the '0'. iii.) If 4 marbles were removed, one by one and not replaced, find the probability that the numbers were '0', then '1', then '2', then '3'.		 1 1 2
	End of Question Four		

Question Five (10 Marks)		Answers	Marks
a)	 Find the volume of the prism shown.		3
b)	 A vase of volume 20,000cm^2 was carefully packed into this cardboard box, with the spaces around it filled with shredded paper to protect it from breakage. The parcel was then sealed and posted. i.) Find the volume occupied by the shredded paper. ii.) Find the area of the cardboard used to make the box.		2 2
c)	A tent, of total height 3m, is constructed with a sewn-in canvas floor. Find the volume of air enclosed by the tent (answer to the nearest unit). 		3
	End of Question Five		

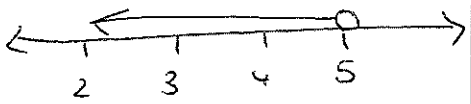
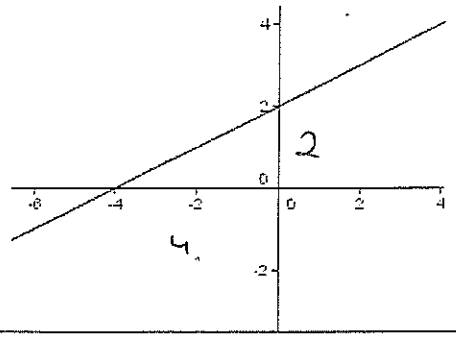
Question Six (9 Marks)		Answers	Marks
a)	Find the equation of this graph.  		2
b)	A bullet fires out of a big gun so that its height y metres is related to its distance along the ground from the gun x metres by the rule: $y = -2(x-20)^2 + 800$  i) The bullet lands at P. Find the distance that P is from the gun.		2
	ii) Hence, find the maximum height H metres, of the bullet.		2
	iii) A rocket races after the bullet, the instant the bullet leaves the gun. Its path has the rule $y = 60x$. Find how far along the horizontal the bullet and the rocket meet and how far above the ground this happens.		3
	End of Question Six		

Question 7 (9 Marks)	Answers	Marks
a)	 These graphs represent the amount 4 different customers have in their account at a certain bank over a period of 10 years. Each customer has chosen a different investment scheme. The amount has been recorded on 31st December each year. All four customers initially invested \$10,000 and interest is calculated yearly.	
	i.) One bank client decides to receive an annuity – that is, the interest is posted out to him to use for living expenses, but the capital in untouched. Find the graph which represents the amount in his bank account over time. ii.) Another client has her money invested in a compound interest scheme. Find the graph which represents the amount in his bank account over time. iii.) A third client has sometimes had to take advantage of the bank's offer to have 20% at call, due to various unexpected personal expenses. Find the graph which represents the amount in his bank account over time.	1 1 1
b)	Sketch the following on separate number planes showing at least two points on each. i.) $y = \frac{5}{x}$ ii.) $2x^2 + 2y^2 = 50$ (Use next page) iii.) $y = 5^{-x}$ iv.) $y = -x^2 + 6x - 10$	1 2 1 2

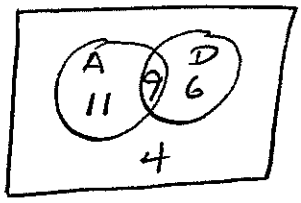
	Question 7 Answers	
(i)		
(ii)		
(iii)		
(iv)		
	End of Question 7	
	End Of Examination	

	Additional Space for Answers	
Question Number	Put "**see back**" at original question.	

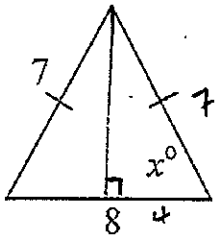
Question One (19 Marks)		Answers	Marks
a)	Find the simplest expression for: $-4a+7b-b+3a$	$6b-a$	1
b)	$x \longrightarrow$ [A] $\longrightarrow$ [B] $\longrightarrow$ [Answer] This diagram represents two number machines. Machine A adds 7 to any number entering it. Machine B squares any number entering it. Write down the final expanded answer when x enters the machines.	$(x+7)^2$ $= x^2+14x+49$	2
c)	Fully factorise: $20xy^2-15xy$	$5xy(4y-3)$	1
d)	Which expression cannot be factorised: **A.** x^2+16 **B.** x^2-7 **C.** x^2-2x+1 **D.** $(x+2)^2-4$	A	1
e)	Fully factorise: $75a^2-27$	$3(25a^2-9)$ $3(5a+3)(5a-3)$	2
f)	Given that $\sqrt{8}=2\sqrt{2}$, fully factorise $\sqrt{8}xy-\sqrt{2}y$.	$2\sqrt{2}xy-\sqrt{2}y$ $=\sqrt{2}y(2x-1)$	2
g)	Express $3x\sqrt{y}$ as an entire surd.	$\sqrt{(3x)^2}\times\sqrt{y}$ $=\sqrt{9x^2y}$	1

h)	Solve $9-3x>-6$ and graph the solution on a number line. $-3x > -15$	$x < 5$  	1
i)	Express $\frac{2}{2\sqrt{2}+5}$ with a rational denominator.	$\frac{2}{2\sqrt{2}+5} \times \frac{2\sqrt{2}-5}{2\sqrt{2}-5}$ $= \frac{4\sqrt{2}-10}{8-25}$ $= \frac{4\sqrt{2}-10}{17}$	2
j)	What is the gradient of this line:  	$\frac{\text{rise}}{\text{run}} = \frac{2}{4}$ $= 1/2$	2
k)	Which one of the following lines has zero gradient: **A.** $y=x$ **B.** $y=10$ **C.** $x=-3$ **D.** $x+y=0$	B	1
l)	Solve this pair of equations simultaneously: $3x+2y=-2$ (1) $x+2y=2$ (2) (1) − (2) $2x=-4$ $\boxed{x=-2}$	sub into (2) $-2+2y=2$ $2y=4$ $\boxed{y=2}$	2
	End of Question One		

Question Two (15 Marks)		Answers	Marks
a)	Find the value of $\sin 44°30'$ correct to 4 decimal places.	0.7009	1
b)	If $\cos\theta = 0.8000$, find the acute angled θ correct to the nearest minute.	$36°52'$	1
c)	Solve $x^2 - 7x = 0$ $x(x-7)=0$	$x = 0$ or 7	2
d)	By showing all working and using factorisation only, solve: $4x^2 + 19x - 5 = 0$ $4x \quad -1$ $x \quad 5$ $(4x-1)(x+5)=0$	$x = \frac{1}{4}$ or -5	3
e)	To complete the square for $x^2 - x$, find the third term. $x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$	$\frac{1}{4}$	2
f)	In her job as a supermarket supervisor Chris is paid a set wage of \$387.50 for a 36 hour week, plus time and half for overtime. In one week she worked 44 hours, find her wage. Overtime $= 44 - 36$ $= 8$ hrs Wage $= \$\left(387.50 + 8 \times 1.5 \times \frac{387.50}{36}\right)$	\$516.67	2

Question Three (15 Marks)		Answers	Marks
a)	**CHOCOLATE** The letters of the word 'chocolate' were written on cards, which were then shuffled. A student then selected one card at random. Find P(that the card is a vowel).	$P(\text{vowel}) = \frac{4}{9}$ ✓	1
b)	In a set of 30 students; 20 studied Art; 15 studied Drama; 4 studied neither Art nor Drama. If a student was selected at random, find P(that the student studied both Art and Drama). $30 - 4 = 26$ $26 - 20 - 15 = -9$ ✓	 $P(\text{both}) = \frac{9}{30}$ ✓ $= \frac{3}{10}$ ✓	3
c)	Solve, using the quadratic formula, leaving your answer in simplest surd form: $x^2 + 8x + 10 = 0$.	$x = \frac{-8 \pm \sqrt{64-40}}{2}$ ✓ $= \frac{-8 \pm \sqrt{24}}{2}$ ✓ $= \frac{-8 \pm 2\sqrt{6}}{2}$ $= -4 \pm \sqrt{6}$ ✓	3
d)	A bank client decides to invest \$20,000 in a scheme in which compound interest was offered at 14%p.a., calculated quarterly, for a period of 10 years. Find the amount of interest, to the nearest dollar, that would be earned.	$\text{Amt.} = \$20\,000\left(1 + \frac{14}{400}\right)^{40}$ ✓ $\doteq \$20\,000 \times 3.96926$ $\doteq \$79\,185.19$ ✓ $\therefore \text{Interest} = \$79185 - \$20\,000$ $= \$59185$ ✓	3

e) Find x correct to the nearest minute. — 2



$\cos x^\circ = 4/7$ ✓

$x \doteqdot 55.150095^{\circ}42$

$\doteqdot 55^\circ 9'$ ✓

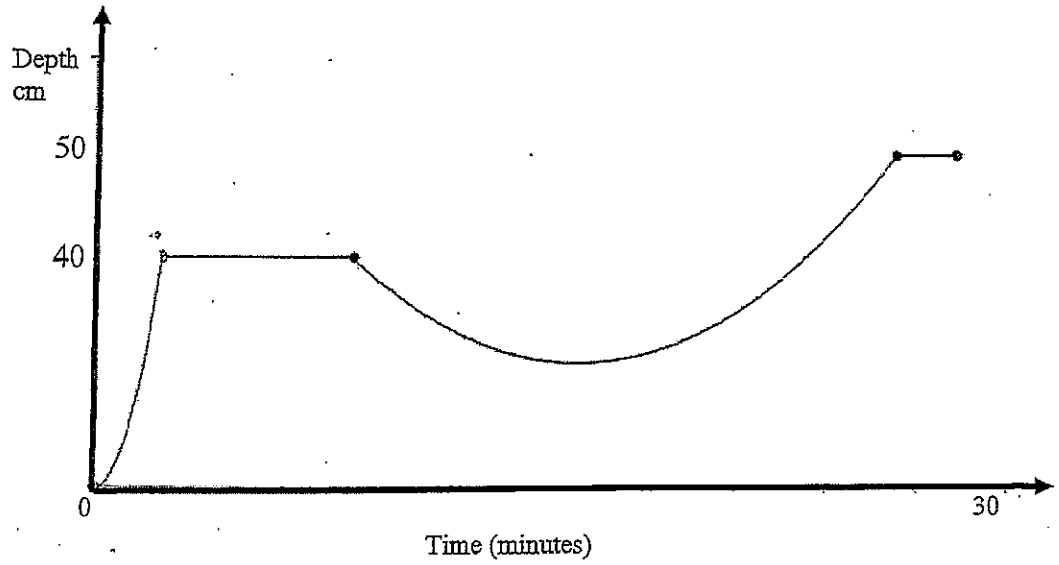
f) The line $y=1$ crosses the parabola $y=x^2-3$ at 2 points called P and Q. Find the x values of the points P and Q. — 2

$x^2 - 3 = 1$ ✓

$x^2 = 4$

$x = \pm 2$ ✓

g) This graph represents the depth of water in a bath over time. — 1



An explanation for this graph could be:

A. Someone in the bath started splashing a lot to make bigger and bigger waves.

B. Someone got out and was replaced by someone bigger.

C. Someone got out and was replaced by someone smaller.

D. Water was let out, but the bath was then filled to overflowing.

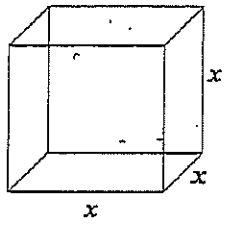
either of these. ✓

End of Question Three

Question Four (13 Marks)	Answers	Marks

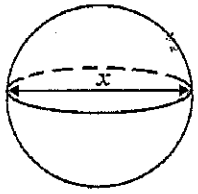
a) Circle which of the following has the greatest volume? (all measurements in cm) — 2

A: cube



$V_1 = x^3$

B: sphere

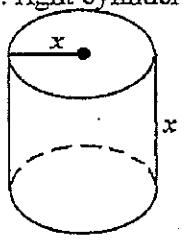


$V_2 = \frac{4}{3}\pi\left(\frac{x}{2}\right)^3$

$= \frac{4}{3}\pi\frac{x^3}{8}$

$= \frac{\pi}{6}x^3$

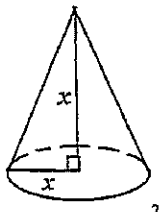
C: right cylinder



$V_3 = \pi(x)^2 x$

$= \pi x^3$

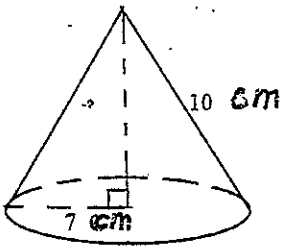
D: right cone



$V_4 = \frac{1}{3}\pi(x)^2 x$

$= \frac{\pi}{3}x^3$

b) In cm^2 what is the <u>total</u> surface area of this cone (answer to the nearest whole number). — 2



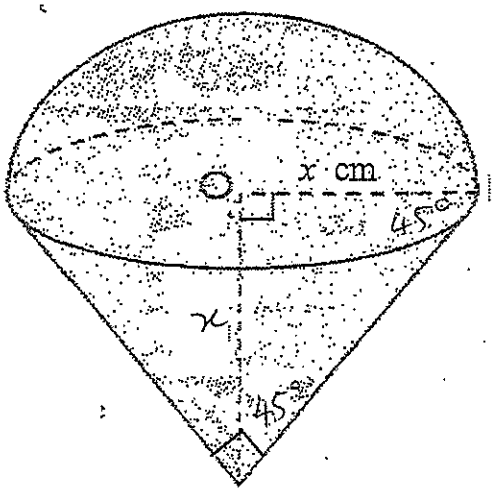
$SA = \pi r^2 + \pi r s$

$= \pi(7)^2 + \pi(7)(10)$

$= 119\pi$

$\approx 374 \text{ cm}^2$

c) This figure shows a solid formed by a hemisphere and a right circular cone. Find the volume of this solid in terms of π and x — 3

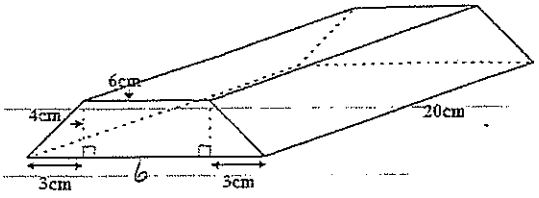
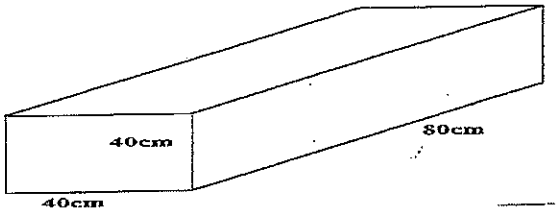
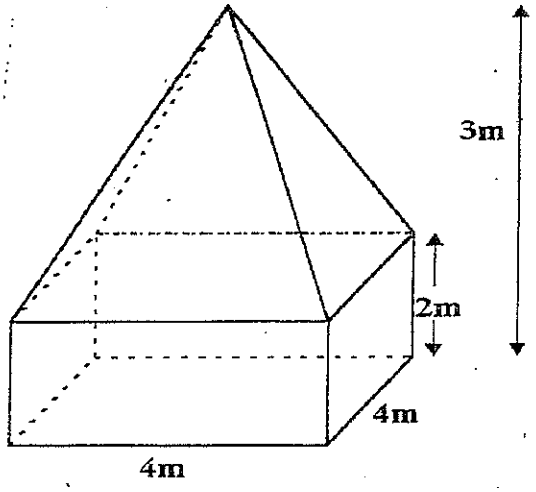


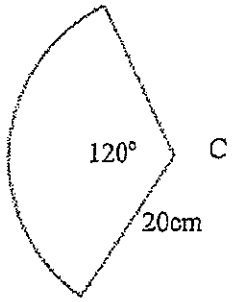
$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \frac{1}{3}\pi r^2 h$

$= \frac{2\pi}{3}x^3 + \frac{\pi x^3}{3}$

$= \pi x^3 \text{ cm}^3$

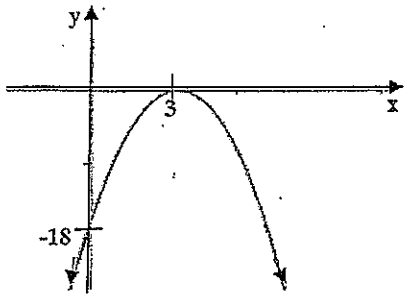
d)	If a coin is tossed 7 times find the probability of getting tails 7 times in a row.	$\frac{1}{2^7} = \frac{1}{128}$	2
e)	Marbles numbered 0 to 44, inclusive, were placed in a barrel for a draw.		
	i.) If one marble was taken out and found to be an odd number, find the probability that the next was even, if the first was replaced.	$\frac{22}{45}$	1
	ii.) If 5 marbles were removed without replacing them, and each was an odd number, find the probability that the next marble to be removed is the '0'.	$\frac{1}{40}$	1
	iii.) If 4 marbles were removed, one by one and not replaced, find the probability that the numbers were '0', then '1', then '2', then '3'.	$\frac{1}{45} \times \frac{1}{44} \times \frac{1}{43} \times \frac{1}{42}$ $= \frac{1}{3575880}$	2
	End of Question Four		

Question Five (10 Marks)		Answers	Marks
a)	 Find the volume of the prism shown.	$V = \frac{1}{2}(6+12)4 \times 20$ $= 720\ cm^3$	3
b)	 A vase of volume 20,000cm^2 was carefully packed into this cardboard box, with the spaces around it filled with shredded paper to protect it from breakage. The parcel was then sealed and posted. i.) Find the volume occupied by the shredded paper. ii.) Find the area of the cardboard used to make the box.	(i) $40 \times 40 \times 80 - 20000$ $= 108\,000\ cm^3$ (ii) $2(40 \times 40) + 4(40 \times 80)$ $= 3200 + 12800$ $= 16000\ cm^2$	2 2
c)	A tent, of total height 3m, is constructed with a sewn-in canvas floor. Find the volume of air enclosed by the tent (answer to the nearest unit). 	$4 \times 4 \times 2 + \frac{1}{3} \times 4 \times 4 \times 1$ $32 + 5\frac{1}{3}$ $37\ m^3$ (nearest m)	3
	End of Question Five		

g)	 This is a piece of ham and pineapple pizza, left for Darren to eat at a party. ***C*** is the centre of this circular pizza. Find the area of pizza eaten by everyone else correct to the nearest cm^2. Area Missing $= \frac{2}{3} \times \pi r^2 = \frac{2}{3} \times \pi (20)^2$	$\simeq 838\ cm^2$ [419 = 1]	2
h)	Dave decides to invest \$15000 at a compound interest rate of 9%p.a., calculated annually. Over 4 years, correct to the nearest dollar, what will his investment grow to? $A = \$15000\left(1+\frac{9}{100}\right)^4$	\$21173	2
	End of Question Two		

Question Six (9 Marks) | **Answers** | **Marks**

a) Find the equation of this graph.



$y = -a(x-3)^2$

$-18 = -a(0-3)^2$

$= -a \times 9$ (1)

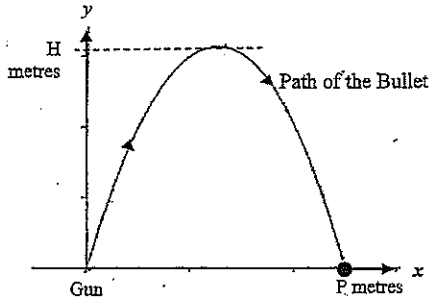
$a = 2$

$y = -2(x-3)^2$ (1)

$= -2x^2 + 12x - 18$ *

2

b) A bullet fires out of a big gun so that its height y metres is related to its distance along the ground from the gun x metres by the rule: $y = -2(x-20)^2 + 800$



i) The bullet lands at P. Find the distance that P is from the gun.

(i) $0 = -2(x-20)^2 + 800$ (1)

$= -2x^2 + 80x$

$= x^2 - 40x$

$= x(x-40)$ (1)

P is 40m from the gun

2

ii) Hence, find the maximum height H metres, of the bullet.

H is at turning pt (1)

$x = 20, y = 800$

$H = 800$ (1)

2

iii) A rocket races after the bullet, the instant the bullet leaves the gun. Its path has the rule $y = 60x$. Find how far along the horizontal the bullet and the rocket meet and how far above the ground this happens.

$y = 60x$

$y = -2x^2 + 80x$ (1)

$60x = -2x^2 + 80x$

$x^2 - 10x = 0$ (1)

$x(x-10) = 0$

Along 10

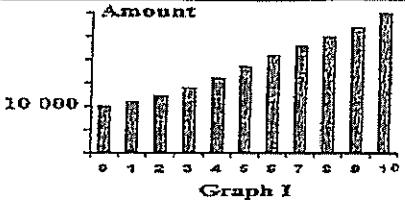
Above 600 (1)

3

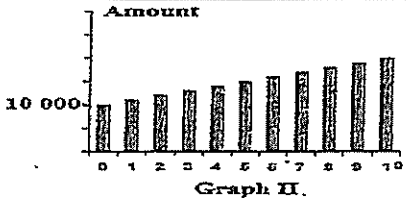
End of Question Six

Question 7 (9 Marks)	Answers	Marks

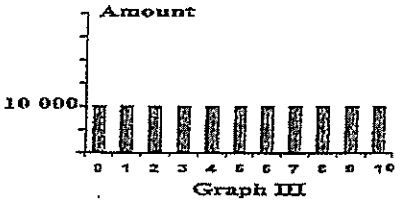
a)



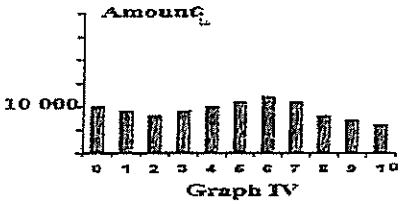
Graph I



Graph II



Graph III



Graph IV

These graphs represent the amount 4 different customers have in their account at a certain bank over a period of 10 years. Each customer has chosen a different investment scheme. The amount has been recorded on 31st December each year. All four customers initially invested \$10,000 and interest is calculated yearly.

i.)	One bank client decides to receive an annuity – that is, the interest is posted out to him to use for living expenses, but the capital in untouched. Find the graph which represents the amount in his bank account over time.	III	1
ii.)	Another client has her money invested in a compound interest scheme. Find the graph which represents the amount in his bank account over time.	I	1
iii.)	A third client has sometimes had to take advantage of the bank's offer to have 20% at call, due to various unexpected personal expenses. Find the graph which represents the amount in his bank account over time.	IV	1

b) Sketch the following on separate number planes showing at least two points on each.

i.) $y=\frac{5}{x}$ — 1

ii.) $2x^2+2y^2=50$ (Use next page) — 2

iii.) $y=5^{-x}$ — 1

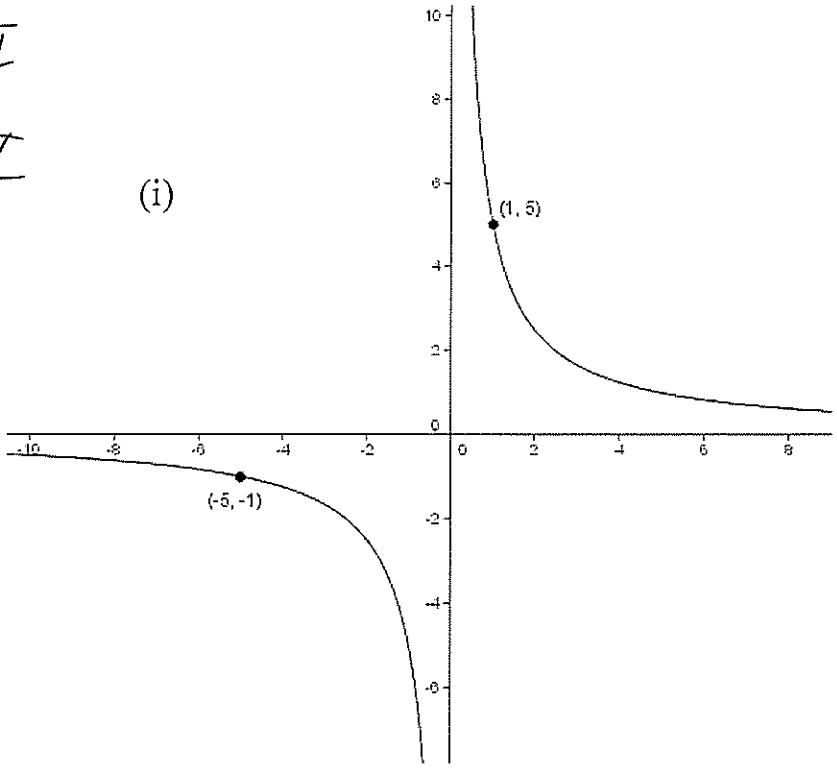
iv.) $y=-x^2+6x-10$ — 2

III

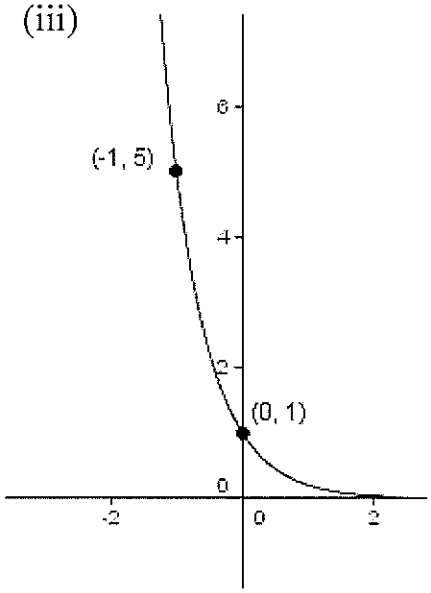
I

IV

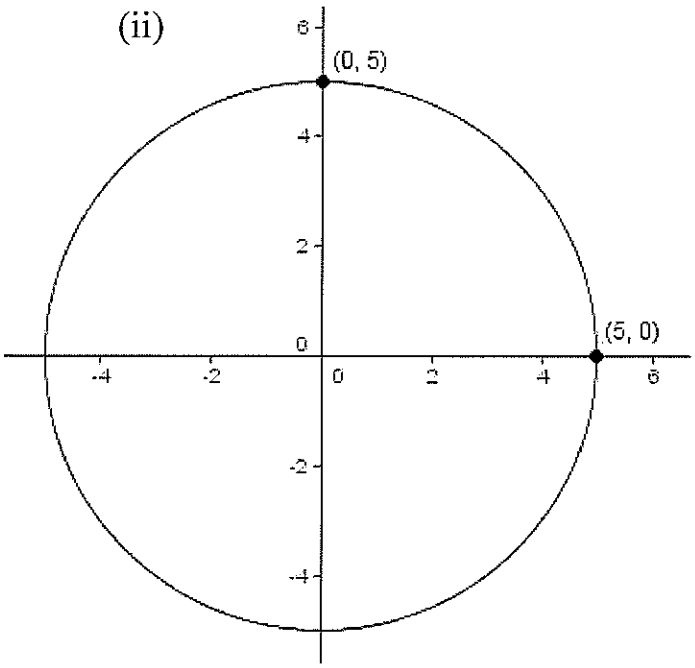
(i)



(iii)



(ii)



(iv)

