Solutions to Exercise 5G Foundation questions

1a $c = (6, -9)$

$\underset{\sim}{c} = \begin{bmatrix} 6 \\ -9 \end{bmatrix}$

$r = 2\sqrt{7}$

For a circle centred on (a, b)

$(x-a)^2 + (y-b)^2 = r^2$

So

$(x-6)^2 + (y+9)^2 = \left(2\sqrt{7}\right)^2$

$(x-6)^2 + (y+9)^2 = 28$

1b $c = (6, -9)$

$\underset{\sim}{c} = \begin{bmatrix} 6 \\ -9 \end{bmatrix}$

$r = 2\sqrt{7}$

$|\underset{\sim}{r} - \underset{\sim}{c}| = r$

so

$\left|\underset{\sim}{r} - \begin{bmatrix} 6 \\ -9 \end{bmatrix}\right| = 2\sqrt{7}$

1c For $|\underset{\sim}{a}| = |\underset{\sim}{b}| = r^2$, where $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$\underset{\sim}{r} - \underset{\sim}{c} = \underset{\sim}{a}\cos\theta + \underset{\sim}{b}\sin\theta$

$\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 6 \\ -9 \end{bmatrix} = \begin{bmatrix} 2\sqrt{7} \\ 0 \end{bmatrix}\cos\theta + \begin{bmatrix} 0 \\ 2\sqrt{7} \end{bmatrix}\sin\theta$

So the two parametric equations are:

$x = 6 + 2\sqrt{7}\cos\theta$

$y = 2\sqrt{7}\sin\theta - 9$

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Chapter 5 worked solutions – Vectors

2a $c = (-2, 7, -4)$

$\underset{\sim}{c} = \begin{bmatrix} -2 \\ 7 \\ -4 \end{bmatrix}$

$r = 9$

$(x+2)^2 + (y-7)^2 + (z+4)^2 = 81$

2b $|\underset{\sim}{r} - \underset{\sim}{c}| = r$

$\left|\underset{\sim}{r} - \begin{bmatrix} -2 \\ 7 \\ -4 \end{bmatrix}\right| = 9$

3a $\left|\underset{\sim}{r} - \begin{bmatrix} -5 \\ -10 \end{bmatrix}\right| = 3\sqrt{5}$

$(x+5)^2 + (y+10)^2 = \left(3\sqrt{5}\right)^2$

$(x+5)^2 + (y+10)^2 = 45$

3b $\left|\underset{\sim}{r} - \begin{bmatrix} 3 \\ -1 \\ 8 \end{bmatrix}\right| = 11$

$(x-3)^2 + (y+1)^2 + (z-8)^2 = 11^2$

$(x-3)^2 + (y+1)^2 + (z-8)^2 = 121$

4 $x = 5 + 2\sqrt{2}\cos\theta$

$y = 2\sqrt{2}\sin\theta - 3$

For $|\underset{\sim}{a}| = |\underset{\sim}{b}| = r^2$, where $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$\underset{\sim}{r} - \underset{\sim}{c} = \underset{\sim}{a}\cos\theta + \underset{\sim}{b}\sin\theta$

$x - 5 = 2\sqrt{2}\cos\theta$

$y - (-3) = 2\sqrt{2}\sin\theta$

So the vector equation will be:

$\left|\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 5 \\ -3 \end{bmatrix}\right| = 2\sqrt{2}$

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The Cartesian equation is:

$(x-5)^2+(y+3)^2=\left(2\sqrt{2}\right)^2$

$(x-5)^2+(y+3)^2=8$

5a $x^2+y^2-6x+8y=0$

$(x^2-6x)+(y^2+8y)=0$

Completing the square gives

$(x^2-6x+9)-9+(y^2+8y+16)-16=0$

$(x-3)^2-9+(y+4)^2-16=0$

$(x-3)^2+(y+4)^2=25$

So

$r=5$

$\underset{\sim}{c}=\begin{bmatrix}3\\-4\end{bmatrix}$

$|\underset{\sim}{r}-\underset{\sim}{c}|=r$

So

$\left|\underset{\sim}{r}-\begin{bmatrix}3\\-4\end{bmatrix}\right|=5$

5b $x^2+y^2+z^2+x-2y-5z=0$

$(x^2+x)+(y^2-2y)+(z^2-5z)=0$

Completing the square gives

$\left(x^2+x+\frac{1}{4}\right)-\frac{1}{4}+(y^2-2y+1)-1+\left(z^2-5z+\frac{25}{4}\right)-\frac{25}{4}=0$

$\left(x+\frac{1}{2}\right)^2+(y-1)^2+\left(z-\frac{5}{2}\right)^2=\frac{30}{4}$

So

$\underset{\sim}{c}=\begin{bmatrix}-\frac{1}{2}\\1\\\frac{5}{2}\end{bmatrix}$

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$$r^2 = \frac{30}{4}$$

$$r = \frac{\sqrt{30}}{2}$$

$$|\underset{\sim}{r} - \underset{\sim}{c}| = r$$

So

$$\left| \underset{\sim}{r} - \begin{bmatrix} -\frac{1}{2} \\ 1 \\ \frac{5}{2} \end{bmatrix} \right| = \frac{\sqrt{30}}{2}$$

6 $P(8, -5, 2)$

$$\left| \underset{\sim}{r} - \begin{bmatrix} 5 \\ -3 \\ -4 \end{bmatrix} \right| = 7$$

$$\left| \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 5 \\ -3 \\ -4 \end{bmatrix} \right| = 7$$

For the point P:

$$\left| \begin{bmatrix} 8 \\ -5 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ -3 \\ -4 \end{bmatrix} \right|$$

$$= \left| \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} \right|$$

$$= \sqrt{3^2 + (-2)^2 + 6^2}$$

$$= \sqrt{9 + 4 + 36}$$

$$= \sqrt{49}$$

$$= 7$$

Thus point P is a point on the surface of the sphere.

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7 $A(-4, -5, 6)$

$$\left|\underset{\sim}{r} - \begin{bmatrix}-2\\4\\-1\end{bmatrix}\right| = 3\sqrt{15}$$

$$\left|\begin{bmatrix}x\\y\\z\end{bmatrix} - \begin{bmatrix}-2\\4\\-1\end{bmatrix}\right| = 3\sqrt{15}$$

For the point A:

$$\left|\begin{bmatrix}-4\\-5\\6\end{bmatrix} - \begin{bmatrix}-2\\4\\-1\end{bmatrix}\right|$$

$$= \left|\begin{bmatrix}-2\\-9\\7\end{bmatrix}\right|$$

$$= \sqrt{(-2)^2 + (-9)^2 + 7^2}$$

$$= \sqrt{4 + 81 + 49}$$

$$= \sqrt{134}$$

Since $\sqrt{134} < 3\sqrt{15}$, point A lies inside the circle.

8 $\left(\underset{\sim}{r} - \left(2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right)\right) \cdot \left(\underset{\sim}{r} - \left(2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right)\right) = 20$

$$\begin{bmatrix}x-2\\y-1\\z+1\end{bmatrix} \cdot \begin{bmatrix}x-2\\y-1\\z+1\end{bmatrix} = 20$$

$$(x-2)^2 + (y-1)^2 + (z+1)^2 = 20$$

$$r^2 = 20$$

$$r = \sqrt{20}$$
$$r = 2\sqrt{5}$$

So the centre of the circle is $(2, 1, -1)$

The radius of the circle is $2\sqrt{5}$.

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9a $\underset{\sim}{r}(t) = (2\cos t + 1)\underset{\sim}{\imath} + (2\sin t - 1)\underset{\sim}{\jmath}$

$x = 2\cos t + 1$

$y = 2\sin t - 1$

9b $x = 2\cos t + 1$

$\cos t = \dfrac{x-1}{2}$

$y = 2\sin t - 1$

$\sin t = \dfrac{y+1}{2}$

Since $\sin^2 t + \cos^2 t = 1$,

$\left(\dfrac{x-1}{2}\right)^2 + \left(\dfrac{y+1}{2}\right)^2 = 1$

$(x-1)^2 + (y+1)^2 = 4$

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Solutions to Exercise 5G Development questions

10a $\underset{\sim}{r}(t) = (t-2)\underset{\sim}{i} + (t^2-2)\underset{\sim}{j}$, for $t \geq 0$

$x = t - 2$

$t = x + 2$

$y = t^2 - 2$

$y = (x+2)^2 - 2$

10b Since $t \geq 0$ and $x = t - 2$,

$x \geq 0 - 2$

$x \geq -2$

So domain is $[-2, \infty)$.

10c Graph of $y = (x+2)^2 - 2$ for $x \geq -2$ shown below.

x-intercept: when $y = 0$,

$(x+2)^2 - 2 = 0$

$(x+2)^2 = 2$

$x + 2 = \pm\sqrt{2}$

$x = -2 \pm \sqrt{2}$

But $x \geq -2$, so x-intercept is $-2 + \sqrt{2}$.

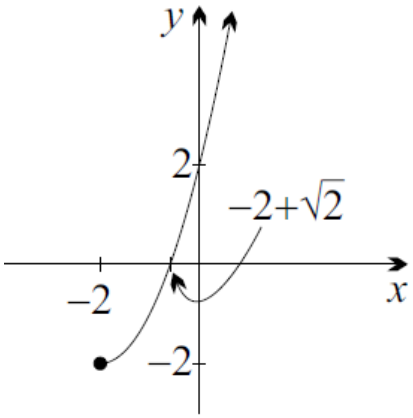
y-intercept: when $x = 0$,

$y = (0+2)^2 - 2 = 2$

So y-intercept is 2.

Uncorrected proofs

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11a $\underset{\sim}{a} = 3\underset{\sim}{i} - \underset{\sim}{j}$

For a circle with radius r, centred on the origin its vector equation will be:

$r = |\underset{\sim}{a}|$

$|\underset{\sim}{r}|^2 = 3^2 + (-1)^2$

$|\underset{\sim}{r}| = \sqrt{10}$

11b $\underset{\sim}{a} = 3\underset{\sim}{i} - \underset{\sim}{j}$

The tangent of the circle at point $á$ is perpendicular to the radius.

So $(\underset{\sim}{r} - \underset{\sim}{a}) \cdot \underset{\sim}{a} = 0$

$\left(\underset{\sim}{r} - \left(3\underset{\sim}{i} - \underset{\sim}{j}\right)\right) \cdot \left(3\underset{\sim}{i} - \underset{\sim}{j}\right) = 0$

11c $\underset{\sim}{a} = 3\underset{\sim}{i} - \underset{\sim}{j}$

$r = \sqrt{10}$

$(\underset{\sim}{r} - \underset{\sim}{a}) \cdot \underset{\sim}{a} = 0$

$\left(x\underset{\sim}{i} + y\underset{\sim}{j}\right) \cdot \left(3\underset{\sim}{i} - \underset{\sim}{j}\right) + 10 = 0$

$(3x - y) - 10 = 0$

So the Cartesian equation for the tangent is:

$y = 3x - 10$

Uncorrected proofs

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12a $\underset{\sim}{r} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

So the parametric equations are:

$x = 3\lambda + 1$

$y = 2\lambda - 1$

12b $\left| \underset{\sim}{r} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right| = \sqrt{13}$

$\underset{\sim}{r} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\left| \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right| = \sqrt{13}$

$\left| \lambda \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right| = \sqrt{13}$

$9\lambda^2 + 4\lambda^2 = 13$

$\lambda^2 = 1$

$\lambda = \pm 1$

Substituting $\lambda = \pm 1$ and solving for x and y:

$A = (3(1) + 1, 2(1) - 1)$

$A = (4, 1)$

$B = (3(-1) + 1, 2(-1) - 1)$

$B = (-2, -3)$

13 First find the Cartesian equations for these spheres.

$\left| \underset{\sim}{r} - \begin{bmatrix} 5 \\ -6 \\ 3 \end{bmatrix} \right| = 7$

$\left| \underset{\sim}{r} - \begin{bmatrix} -3 \\ 2 \\ 7 \end{bmatrix} \right| = 5$

$|\underset{\sim}{v} - \underset{\sim}{c}| = r$

So the Cartesian equations in terms of x will be:

$(x - 5)^2 = 7^2$

$(x + 3)^2 = 5^2$

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Equating these two equations to find the intersecting point on the x-axis

$(x-5)^2-49=(x+3)^2-25$

$x^2-10x+25-49=x^2+6x+9-25$

$16x=8$

$x=\frac{1}{2}$

As there is only a single value for the two spheres to intersect on the x-axis, the spheres touch each other at a single point.

14a The circle of intersection will be perpendicular to the axis the spheres are centred on as both spheres centre on the z-axis, one at $(0, 0, 0)$ and the other at $(0, 0, 5)$.

The sphere of intersection between the two spheres will be parallel to the xy-plane.

14b $|\underset{\sim}{r}|=3$

$$\left|\underset{\sim}{r}-\begin{bmatrix}0\\0\\5\end{bmatrix}\right|=4$$

$$\left|(z^2-5)\begin{bmatrix}0\\0\\5\end{bmatrix}\right|=4$$

$$\left\|\begin{bmatrix}x\\y\\z-5\end{bmatrix}\right\|=4$$

$x^2+y^2+z^2=9$

$y^2=9-x^2-z^2$ (1)

$x^2+y^2+(z-5)^2=16$ (2)

Substituting (1) into (2) gives us:

$x^2+(9-x^2-z^2)+(z-5)^2=16$

$9-z^2+(z-5)^2=16$

$9-z^2+z^2-10z+25=16$

$-18=-10z$

Uncorrected proofs

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$z = \frac{9}{5}$

Substituting this into equation (2) and solving for $x^2 + y^2$:

$x^2 + y^2 + \left(\frac{9}{5} - 5\right)^2 = 16$

$x^2 + y^2 + \frac{256}{25} = 16$

$x^2 + y^2 = \frac{144}{25}$

As $x^2 + y^2 = r^2$

$r = \frac{12}{5}$

The intersecting circle is centred on $(0, 0, z)$.

So, solving for the position of the intersecting circle on the z-axis using a point on the circle solve for z, say $x = \frac{12}{5}, y = 0$:

$x^2 + y^2 + z^2 = 9$

$\left(\frac{12}{5}\right)^2 + 0^2 + z^2 = 9$

$z^2 = 9 - \frac{144}{25}$

$z^2 = \frac{81}{25}$

$z = \frac{9}{5}$

So the intersecting circle is centred on

$C = \left(0, 0, \frac{9}{5}\right)$

Uncorrected proofs

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15 $(x-3)^2+(y+4)^2+(z+2)^2=81$

$$\left|\underset{\sim}{r}-\begin{bmatrix}3\\-4\\-2\end{bmatrix}\right|=81$$

$$\underset{\sim}{r}=\begin{bmatrix}-3\\16\\-9\end{bmatrix}+\lambda\begin{bmatrix}7\\-12\\3\end{bmatrix}$$

$$\left|\begin{bmatrix}-3\\16\\-9\end{bmatrix}+\lambda\begin{bmatrix}7\\-12\\3\end{bmatrix}-\begin{bmatrix}3\\-4\\-2\end{bmatrix}\right|=81$$

$$\left|\begin{bmatrix}-6+7\lambda\\20-12\lambda\\-7+3\lambda\end{bmatrix}\right|=81$$

$$(-6+7\lambda)^2+(20-12\lambda)^2+(-7+3\lambda)^2=81$$

$$36-84\lambda+49\lambda^2+200-480\lambda+144\lambda^2+49-42\lambda+9\lambda^2=81$$

$$202\lambda^2-606\lambda+404=0$$

$$\lambda^2-3\lambda+2=0$$

$$(\lambda-1)(\lambda-2)=0$$

So the line intersects with the sphere for the values of λ where

$$\lambda=1,2$$

Substituting these values back into the line equation $\underset{\sim}{r}$:

For $\lambda=1$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-3\\16\\-9\end{bmatrix}+\begin{bmatrix}7\\-12\\3\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\4\\-6\end{bmatrix}$$

For $\lambda=2$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-3\\16\\-9\end{bmatrix}+2\begin{bmatrix}7\\-12\\3\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}11\\-8\\-3\end{bmatrix}$$

Uncorrected proofs

The line intersects with the sphere at points:

$(4, 4, -6)$ and $(11, -8, -3)$

16a $\underset{\sim}{r} = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

$x = 3\lambda - 2$

$y = 4\lambda + 3$

$z = 5\lambda + 4$

16b Substituting the parametric equations into the equation for the plane:

$x = 3\lambda - 2$

$y = 4\lambda + 3$

$z = 5\lambda + 4$

$2x + 4y - z = 55$

$2(3\lambda - 2) + 4(4\lambda + 3) - (5\lambda + 4) = 55$

$6\lambda - 4 + 16\lambda + 12 - 5\lambda - 4 = 55$

$17\lambda + 4 = 55$

$\lambda = 3$

Substituting λ into $\underset{\sim}{r}$ will give us the point of intersection.

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix} + 3 \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \\ 19 \end{bmatrix}$

So the point of intersection between the line and the plane is:

$(7, 15, 19)$

Uncorrected proofs

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17a $\underset{\sim}{r} = \begin{bmatrix} \frac{1}{2}(e^t + e^{-t}) \\ \frac{1}{2}(e^t + e^{-t}) \end{bmatrix}$

$x = \frac{1}{2}(e^t + e^{-t})$

$x^2 = \frac{1}{4}(e^t + e^{-t})^2$

$x^2 = \frac{1}{4}(e^{t+t} + e^{t-t} + e^{-t+t} + e^{-t-t})$

$x^2 = \frac{1}{4}(e^{2t} + 2 + e^{-t})$

$x^2 - \frac{1}{2} = \frac{1}{4}(e^{2t} + e^{-t})$

$y = \frac{1}{2}(e^t - e^{-t})$

$y^2 = \frac{1}{4}(e^t - e^{-t})^2$

$y^2 = \frac{1}{4}(e^{t+t} - e^{t-t} - e^{-t+t} + e^{-t-t})$

$y^2 = \frac{1}{4}(e^{2t} + e^{-2t} - 2)$

$y^2 + \frac{1}{2} = \frac{1}{4}(e^{2t} + e^{-2t})$

Equating the LHS for x and y gives:

$x^2 - \frac{1}{2} = y^2 + \frac{1}{2}$

$x^2 - y^2 = 1$

17b $\underset{\sim}{r} = \begin{bmatrix} 2\sin t \\ 2\sin t \tan t \end{bmatrix}$

$x = 2\sin t$

We know that, $\cos^2 t + \sin^2 t = 1$

So

$x^2 = 4\sin^2 t$

Uncorrected proofs

$$x^2 = 4 - 4\cos^2 t$$

$$\frac{4 - x^2}{4} = \cos^2 t$$

$$\tan t = \frac{\sin t}{\cos t}$$

$$y = 2\sin t \tan t$$

$$y = \frac{2\sin^2 t}{\cos t}$$

$$y^2 = 4\sin^4 t \times \frac{1}{\cos^2 t}$$

$$y^2 = \frac{1}{4}x^4 \times \frac{1}{\left(\frac{4 - x^2}{4}\right)}$$

$$y^2 = \frac{x^4}{(4 - x^2)}$$

$$y = \pm\frac{x^2}{\sqrt{(4 - x^2)}}$$

Uncorrected proofs

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5G

(18)(a) Possible direction vectors are

$$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{AC} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}.$$

So a possible equation for P is

$$\underset{\sim}{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}, \text{ where } \lambda, \mu \in \mathbb{R}.$$

(There is an infinite number of correct answers to (a). In each case the corresponding Cartesian equation is the same.)

(b) A set of parametric equations of P is

$$x = 1 + \lambda + 2\mu \quad (1)$$

$$y = -1 + 4\lambda + 5\mu \quad (2)$$

$$z = \lambda - 2\mu \quad (3)$$

(1) − (3): $x - z = 1 + 4\mu$

so $\mu = \frac{1}{4}(x - z - 1)$

Substituting into (3),

$$z = \lambda - \frac{1}{2}(x - z - 1)$$

$$\lambda = \frac{1}{2}(x - z - 1) + z$$

$$= \frac{1}{2}(x + z - 1).$$

Substituting into (2),

$$y = -1 + 2(x + z - 1) + \frac{5}{4}(x - z - 1)$$

$$4y = -4 + 8x + 8z - 8 + 5x - 5z - 5$$

$$13x - 4y + 3z = 17.$$

5G

(19) We want a parametric vector equation of the plane.

Introduce 2 parameters λ and μ by letting

$$y = \lambda \text{ and } z = \mu.$$

Then $ax + b\lambda + c\mu = d$

so $x = \frac{1}{a}(d - b\lambda - c\mu)$

so a parametric vector equation is

$$\underset{\sim}{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{a}(d - b\lambda - c\mu) \\ \lambda \\ \mu \end{pmatrix}$$

i.e. $$\underset{\sim}{r} = \begin{pmatrix} \frac{d}{a} \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -\frac{b}{a} \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -\frac{c}{a} \\ 0 \\ 1 \end{pmatrix}.$$

So two non-parallel direction vectors are

$$\begin{pmatrix} -\frac{b}{a} \\ 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} -\frac{c}{a} \\ 0 \\ 1 \end{pmatrix}.$$

Now, $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} -\frac{b}{a} \\ 1 \\ 0 \end{pmatrix} = -b + b$

$= 0$

and $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} -\frac{c}{a} \\ 0 \\ 1 \end{pmatrix} = -c + c$

$= 0$

so the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is perpendicular to the plane.

(20) The curves are graphed in the Answers to Exercises in the text book.