Solutions to Exercise 4D Foundation questions

1a

Let $\frac{2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$\frac{2}{(x-1)(x+1)} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$

$2 = A(x+1) + B(x-1)$

When $x = 1$,

$2 = 2A$

$A = 1$

When $x = -1$,

$2 = -2B$

$B = -1$

Thus,

$\frac{2}{(x-1)(x+1)} = \frac{1}{x-1} - \frac{1}{x+1}$

1b

Let $\frac{1}{(x-4)(x-1)} = \frac{A}{x-4} + \frac{B}{x-1}$

$\frac{1}{(x-4)(x-1)} = \frac{A(x-1)+B(x-4)}{(x-4)(x-1)}$

$1 = A(x-1) + B(x-4)$

When $x = 4$,

$1 = 3A$

$A = \frac{1}{3}$

When $x = 1$,

$1 = -3B$

$B = -\frac{1}{3}$

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Thus,

$$\frac{1}{(x-4)(x-1)} = \frac{1}{3(x-4)} - \frac{1}{3(x-1)}$$

1c

$$\frac{4x}{x^2-9} = \frac{4x}{(x-3)(x+3)}$$

Let $\frac{4x}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$

$$\frac{4x}{(x-3)(x+3)} = \frac{A(x+3)+B(x-3)}{(x-3)(x+3)}$$

$4x = A(x+3) + B(x-3)$

When $x = 3$,

$12 = 6A$

$A = 2$

When $x = -3$,

$-12 = -6B$

$B = 2$

Thus,

$$\frac{4x}{x^2-9} = \frac{2}{x-3} + \frac{2}{x+3}$$

1d

$$\frac{x}{x^2-3x+2} = \frac{x}{(x-2)(x-1)}$$

Let $\frac{x}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$

$$\frac{x}{(x-2)(x-1)} = \frac{A(x-1)+B(x-2)}{(x-2)(x-1)}$$

$x = A(x-1) + B(x-2)$

When $x = 2$,

$A = 2$

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When $x = 1$,

$1 = -B$

$B = -1$

Thus,

$$\frac{x}{x^2 - 3x + 2} = \frac{2}{x - 2} - \frac{1}{x - 1}$$

1e

$$\frac{x - 1}{x^2 + x - 6} = \frac{x - 1}{(x - 2)(x + 3)}$$

Let $\frac{x - 1}{(x - 2)(x + 3)} = \frac{A}{x - 2} + \frac{B}{x + 3}$

$$\frac{x - 1}{(x - 2)(x + 3)} = \frac{A(x + 3) + B(x - 2)}{(x - 2)(x + 3)}$$

$x - 1 = A(x + 3) + B(x - 2)$

When $x = 2$,

$1 = 5A$

$A = \frac{1}{5}$

When $x = -3$,

$-4 = -5B$

$B = \frac{4}{5}$

Thus,

$$\frac{x - 1}{x^2 + x - 6} = \frac{1}{5(x - 2)} + \frac{4}{5(x + 3)}$$

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1f

Let $\frac{3x+1}{(x-1)(x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+3}$

$\frac{3x+1}{(x-1)(x^2+3)} = \frac{A(x^2+3)+(Bx+C)(x-1)}{(x-1)(x^2+3)}$

$3x+1 = A(x^2+3)+(Bx+C)(x-1)$

When $x = 1$,

$4 = 4A$

$A = 1$

Equating coefficients of x^2 yields:

$0 = A + B$

$0 = 1 + B$

$B = -1$

When $x = 0$,

$1 = 3A - C$

$1 = 3 - C$

$C = 2$

Thus,

$\frac{3x+1}{(x-1)(x^2+3)} = \frac{1}{x-1} + \frac{2-x}{x^2+3}$

2a

Let $\frac{2}{(x-4)(x-2)} = \frac{A}{x-4} + \frac{B}{x-2}$

$\frac{2}{(x-4)(x-2)} = \frac{A(x-2)+B(x-4)}{(x-4)(x-2)}$

$2 = A(x-2)+B(x-4)$

When $x = 4$,

$2 = 2A$

$A = 1$

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When $x = 2$,

$2 = -2B$

$B = -1$

Thus,

$$\int \frac{2}{(x-4)(x-2)}\,dx$$

$$= \int \left(\frac{1}{x-4} - \frac{1}{x-2}\right) dx$$

$$= \int \frac{1}{x-4}\,dx - \int \frac{1}{x-2}\,dx$$

$$= \ln|x-4| - \ln|x-2| + C$$

2b

$$\frac{4}{x^2+4x+3} = \frac{4}{(x+1)(x+3)}$$

Let $\frac{4}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$

$$\frac{4}{(x+1)(x+3)} = \frac{A(x+3)+B(x+1)}{(x+1)(x+3)}$$

$4 = A(x+3) + B(x+1)$

When $x = -1$,

$4 = 2A$

$A = 2$

When $x = -3$,

$4 = -2B$

$B = -2$

Thus,

$$\int \frac{4}{x^2+4x+3}\,dx$$

$$= \int \left(\frac{2}{x+1} - \frac{2}{x+3}\right) dx$$

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$$= 2\int \frac{1}{x+1}dx - 2\int \frac{1}{x+3}dx$$

$$= 2\ln|x+1| - 2\ln|x+3| + C$$

2c

Let $\frac{3x-2}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$

$$\frac{3x-2}{(x-1)(x-2)} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

$$3x-2 = A(x-2)+B(x-1)$$

When $x = 1$,

$$1 = -1A$$

$$A = -1$$

When $x = 2$,

$$4 = B$$

$$B = 4$$

Thus,

$$\int \frac{3x-2}{(x-1)(x-2)}dx$$

$$= \int \left(\frac{-1}{x-1} + \frac{4}{x-2}\right)dx$$

$$= -\int \frac{1}{x-1}dx + 4\int \frac{1}{x-2}dx$$

$$= -\ln|x-1| + 4\ln|x-2| + C$$

$$= 4\ln|x-2| - \ln|x-1| + C$$

2d

$$\frac{2x+10}{x^2+2x-3} = \frac{2x+10}{(x-1)(x+3)}$$

Let $\frac{2x+10}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

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$$\frac{2x+10}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$2x+10 = A(x+3)+B(x-1)$

When $x = 1$,

$12 = 4A$

$A = 3$

When $x = -3$,

$4 = -4B$

$B = -1$

Thus,

$$\int \frac{2x+10}{x^2+2x-3}\,dx$$

$$= \int \left(\frac{3}{x-1} - \frac{1}{x+3}\right)dx$$

$$= 3\int \frac{1}{x-1}\,dx - \int \frac{1}{x+3}\,dx$$

$$= 3\ln|x-1| - \ln|x+3| + C$$

2e

Let $\frac{4x+5}{(2x+3)(x+1)} = \frac{A}{2x+3} + \frac{B}{x+1}$

$$\frac{4x+5}{(2x+3)(x+1)} = \frac{A(x+1)+B(2x+3)}{(2x+3)(x+1)}$$

$4x+5 = A(x+1)+B(2x+3)$

When $x = -\frac{3}{2}$,

$-1 = -\frac{A}{2}$

$A = 2$

When $x = -1$,

$B = 1$

Thus,

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$$\int \frac{4x+5}{(2x+3)(x+1)}\,dx$$

$$= \int \left(\frac{2}{2x+3} + \frac{1}{x+1}\right) dx$$

$$= \int \frac{2}{2x+3}\,dx + \int \frac{1}{x+1}\,dx$$

$$= \ln|2x+3| + \ln|x+1| + C$$

$$= \ln|x+1| + \ln|2x+3| + C$$

2f

$$\frac{10x}{2x^2 - x - 3} = \frac{10x}{(x+1)(2x-3)}$$

Let $\frac{10x}{(x+1)(2x-3)} = \frac{A}{x+1} + \frac{B}{2x-3}$

$$\frac{10x}{(x+1)(2x-3)} = \frac{A(2x-3) + B(x+1)}{(x+1)(2x-3)}$$

$10x = A(2x-3) + B(x+1)$

When $x = -1$,

$-10 = -5A$

$A = 2$

When $x = \frac{3}{2}$,

$15 = \frac{5B}{2}$

$B = 6$

Thus,

$$\int \frac{10x}{2x^2 - x - 3}\,dx$$

$$= \int \left(\frac{2}{x+1} + \frac{6}{2x-3}\right) dx$$

$$= 2\int \frac{1}{x+1}\,dx + 3\int \frac{2}{2x-3}\,dx$$

$$= 2\ln|x+1| + 3\ln|2x-3| + C$$

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3a

$$\frac{1}{x^2-4}=\frac{1}{(x-2)(x+2)}$$

Let $\frac{1}{(x-2)(x+2)}=\frac{A}{x-2}+\frac{B}{x+2}$

$$\frac{1}{(x-2)(x+2)}=\frac{A(x+2)+B(x-2)}{(x-2)(x+2)}$$

$$1=A(x+2)+B(x-2)$$

When $x=2$,

$$1=4A$$

$$A=\frac{1}{4}$$

When $x=-2$,

$$1=-4B$$

$$B=-\frac{1}{4}$$

Thus,

$$\int_4^6 \frac{1}{x^2-4}\,dx$$

$$=\int_4^6 \left(\frac{1}{4(x-2)}-\frac{1}{4(x+2)}\right)dx$$

$$=\frac{1}{4}[\ln|x-2|-\ln|x+2|]_4^6$$

$$=\frac{1}{4}\left[\ln\left|\frac{x-2}{x+2}\right|\right]_4^6$$

$$=\frac{1}{4}\left[\ln\frac{4}{8}-\ln\frac{2}{6}\right]$$

$$=\frac{1}{4}\ln\left(\frac{1}{2}\div\frac{1}{3}\right)$$

$$=\frac{1}{4}\ln\frac{3}{2}$$

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3b

$$\frac{3}{x^2+x-2}=\frac{3}{(x-1)(x+2)}$$

Let $\frac{3}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$

$$\frac{3}{(x-1)(x+2)}=\frac{A(x+2)+B(x-1)}{(x-1)(x+2)}$$

$3=A(x+2)+B(x-1)$

When $x=1$,

$3=3A$

$A=1$

When $x=-2$,

$3=-3B$

$B=-1$

Thus,

$$\int_2^4 \frac{3}{x^2+x-2}\,dx$$

$$=\int_2^4 \left(\frac{1}{x-1}-\frac{1}{x+2}\right)dx$$

$$=[\ln|x-1|-\ln|x+2|]_2^4$$

$$=\left[\ln\left|\frac{x-1}{x+2}\right|\right]_2^4$$

$$=\ln\left(\frac{3}{6}\right)-\ln\left(\frac{1}{4}\right)$$

$$=\ln\left(\frac{1}{2}\div\frac{1}{4}\right)$$

$=\ln 2$

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3c

$$\frac{11}{2x^2+5x-12}=\frac{11}{(2x-3)(x+4)}$$

Let $\frac{11}{(2x-3)(x+4)}=\frac{A}{2x-3}+\frac{B}{x+4}$

$$\frac{11}{(2x-3)(x+4)}=\frac{A(x+4)+B(2x-3)}{(2x-3)(x+4)}$$

$$11=A(x+4)+B(2x-3)$$

When $x=\frac{3}{2}$,

$$11=\frac{11A}{2}$$

$$A=2$$

When $x=-4$,

$$11=-11B$$

$$B=-1$$

Thus,

$$\int_2^5 \frac{11}{2x^2+5x-12}\,dx$$

$$=\int_2^5 \left(\frac{2}{2x-3}-\frac{1}{x+4}\right)dx$$

$$=[\ln|2x-3|-\ln|x+4|]_2^5$$

$$=\left[\ln\left|\frac{2x-3}{x+4}\right|\right]_2^5$$

$$=\ln\left(\frac{7}{9}\right)-\ln\left(\frac{1}{6}\right)$$

$$=\ln\left(\frac{7}{9}\div\frac{1}{6}\right)$$

$$=\ln\frac{14}{3}$$

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3d

$$\frac{1}{3x^2-4x+1}=\frac{1}{(x-1)(3x-1)}$$

Let $\frac{1}{(x-1)(3x-1)}=\frac{A}{x-1}+\frac{B}{3x-1}$

$$\frac{1}{(x-1)(3x-1)}=\frac{A(3x-1)+B(x-1)}{(x-1)(3x-1)}$$

$$1=A(3x-1)+B(x-1)$$

When $x=1$,

$$1=2A$$

$$A=\frac{1}{2}$$

When $x=\frac{1}{3}$,

$$1=-\frac{2B}{3}$$

$$B=-\frac{3}{2}$$

Thus,

$$\int_{-1}^{0}\frac{1}{3x^2-4x+1}dx$$

$$=\int_{-1}^{0}\left(\frac{1}{2(x-1)}-\frac{3}{2(3x-1)}\right)dx$$

$$=\frac{1}{2}\int_{-1}^{0}\left(\frac{1}{x-1}-\frac{3}{3x-1}\right)dx$$

$$=\frac{1}{2}[\ln|x-1|-\ln|3x-1|]_{-1}^{0}$$

$$=\frac{1}{2}\left[\ln\left|\frac{x-1}{3x-1}\right|\right]_{-1}^{0}$$

$$=\frac{1}{2}\left(\ln\left(\frac{-1}{-1}\right)-\ln\left(\frac{-2}{-4}\right)\right)$$

$$=\frac{1}{2}\left(\ln 1-\ln\frac{1}{2}\right)$$

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$$= \frac{1}{2}\ln\left(1 \div \frac{1}{2}\right)$$

$$= \frac{1}{2}\ln 2$$

4a

Let $\dfrac{x^2 - 2x + 5}{(x-2)(x^2+1)} = \dfrac{A}{x-2} + \dfrac{Bx + C}{x^2+1}$

$$\frac{x^2 - 2x + 5}{(x-2)(x^2+1)} = \frac{A(x^2+1) + (Bx+C)(x-2)}{(x-2)(x^2+1)}$$

$x^2 - 2x + 5 = A(x^2+1) + (Bx+C)(x-2)$

When $x = 2$,

$5 = 5A$

$A = 1$

Equating coefficients of x^2 yields:

$1 = A + B$

$1 = 1 + B$ (since $A = 1$)

$B = 0$

When $x = 0$,

$5 = A - 2C$

$5 = 1 - 2C$

$C = -2$

Thus,

$$\int \frac{x^2 - 2x + 5}{(x-2)(x^2+1)}\,dx$$

$$= \int \left(\frac{1}{x-2} - \frac{2}{x^2+1}\right)dx$$

$$= \int \frac{1}{x-2}\,dx - 2\int \frac{1}{x^2+1}\,dx$$

$$= \ln|x-2| - 2\tan^{-1} x + C$$

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4b

Let $\frac{6-x}{(2x+1)(x^2+3)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+3}$

$\frac{6-x}{(2x+1)(x^2+3)} = \frac{A(x^2+3)+(Bx+C)(2x+1)}{(2x+1)(x^2+3)}$

$6-x = A(x^2+3)+(Bx+C)(2x+1)$

When $x = -\frac{1}{2}$,

$\frac{13}{2} = \frac{13A}{4}$

$A = 2$

Equating coefficients of x^2 yields:

$0 = A + 2B$

$0 = 2 + 2B$ (since $A = 2$)

$B = -1$

When $x = 0$,

$6 = 3A + C$

$6 = 6 + C$

$C = 0$

Thus,

$\int \frac{6-x}{(2x+1)(x^2+3)}\,dx$

$= \int \left(\frac{2}{2x+1} - \frac{x}{x^2+3}\right) dx$

$= \int \frac{2}{2x+1}\,dx - \frac{1}{2}\int \frac{2x}{x^2+3}\,dx$

$= \ln|2x+1| - \frac{1}{2}\ln(x^2+3) + C$ (since x^2+3 is positive)

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4c

$$\frac{x^2+x+3}{x^3+x}=\frac{x^2+x+3}{x(x^2+1)}$$

Let $\frac{x^2+x+3}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$

$$\frac{x^2+x+3}{x(x^2+1)}=\frac{A(x^2+1)+(Bx+C)(x)}{x(x^2+1)}$$

$$x^2+x+3=A(x^2+1)+(Bx+C)(x)$$

When $x=0$,

$$A=3$$

Equating coefficients of x^2 yields:

$$1=A+B$$

$1=3+B$ (since $A=3$)

$$B=-2$$

Equating coefficients of x yields:

$$C=1$$

Thus,

$$\int\frac{x^2+x+3}{x^3+x}dx$$

$$=\int\left(\frac{3}{x}+\frac{-2x+1}{x^2+1}\right)dx$$

$$=\int\frac{3}{x}dx-\int\frac{2x}{x^2+1}dx+\int\frac{1}{x^2+1}dx$$

$=3\ln|x|-\ln(x^2+1)+\tan^{-1}x+C$ (since x^2+1 is positive)

$$=\tan^{-1}x+3\ln|x|-\ln(x^2+1)+C$$

5a

Let $\frac{1+2x-4x^2}{(x+1)(4x^2+1)}=\frac{A}{x+1}+\frac{Bx+C}{4x^2+1}$

$$\frac{1+2x-4x^2}{(x+1)(4x^2+1)}=\frac{A(4x^2+1)+(Bx+C)(x+1)}{(x+1)(4x^2+1)}$$

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$1 + 2x - 4x^2 = A(4x^2 + 1) + (Bx + C)(x + 1)$

When $x = -1$,

$-5 = 5A$

$A = -1$

Equating coefficients of x^2 yields:

$-4 = 4A + B$

$-4 = -4 + B$ (since $A = -1$)

$B = 0$

When $x = 0$,

$1 = A + C$

$1 = -1 + C$

$C = 2$

Thus,

$$\int_0^{\frac{1}{2}} \frac{1 + 2x - 4x^2}{(x + 1)(4x^2 + 1)}\,dx$$

$$= \int_0^{\frac{1}{2}} \left(\frac{-1}{x + 1} + \frac{2}{4x^2 + 1}\right) dx$$

$$= \int_0^{\frac{1}{2}} \left(\frac{2}{4x^2 + 1} - \frac{1}{x + 1}\right) dx$$

$$= \int_0^{\frac{1}{2}} \frac{2}{(2x)^2 + 1}\,dx - \int_0^{\frac{1}{2}} \frac{1}{x + 1}\,dx$$

$$= [\tan^{-1} 2x]_0^{\frac{1}{2}} - [\ln|x + 1|]_0^{\frac{1}{2}}$$

$$= (\tan^{-1} 1 - \tan^{-1} 0) - \left(\ln\frac{3}{2} - \ln 1\right)$$

$$= \frac{\pi}{4} - 0 - \ln\frac{3}{2} - 0$$

$$= \frac{\pi}{4} - \ln\frac{3}{2}$$

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5b

Let $\frac{7-x}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$

$\frac{7-x}{(x+3)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x+3)}{(x+3)(x^2+1)}$

$7-x = A(x^2+1)+(Bx+C)(x+3)$

When $x = -3$,

$10 = 10A$

$A = 1$

Equating coefficients of x^2 yields:

$0 = A + B$

$0 = 1 + B$ (since $A = 1$)

$B = -1$

When $x = 0$,

$7 = A + 3C$

$7 = 1 + 3C$

$C = 2$

Thus,

$\int_{-1}^{1} \frac{7-x}{(x+3)(x^2+1)}\,dx$

$= \int_{-1}^{1} \left(\frac{1}{x+3} + \frac{2-x}{x^2+1}\right)dx$

$= \int_{-1}^{1} \left(\frac{1}{x+3} + \frac{2}{x^2+1} - \frac{x}{x^2+1}\right)dx$

$= \int_{-1}^{1} \frac{1}{x+3}\,dx + 2\int_{-1}^{1} \frac{1}{x^2+1}\,dx - \frac{1}{2}\int_{-1}^{1} \frac{2x}{x^2+1}\,dx$

$= [\ln|x+3|]_{-1}^{1} + 2[\tan^{-1}x]_{-1}^{1} - \frac{1}{2}[\ln(x^2+1)]_{-1}^{1}$ (since x^2+1 is positive)

$= (\ln 4 - \ln 2) + 2(\tan^{-1}1 - \tan^{-1}(-1)) - \frac{1}{2}(\ln 2 - \ln 2)$

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$$= \ln 2 + 2\left(\frac{\pi}{4} - \left(\frac{\pi}{4}\right)\right) - 0$$

$$= \ln 2 + \pi$$

$$= \pi + \ln 2$$

5c

$$\frac{x^2 - 4}{x^3 + 2x} = \frac{x^2 - 4}{x(x^2 + 2)}$$

Let $\frac{x^2 - 4}{x(x^2 + 2)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 2}$

$$\frac{x^2 - 4}{x(x^2 + 2)} = \frac{A(x^2 + 2) + (Bx + C)(x)}{x(x^2 + 2)}$$

$$x^2 - 4 = A(x^2 + 2) + (Bx + C)(x)$$

When $x = 0$,

$$-4 = 2A$$

$$A = -2$$

Equating coefficients of x^2 yields:

$$1 = A + B$$

$$1 = -2 + B \qquad \text{(since } A = -2\text{)}$$

$$B = 3$$

Equating coefficients of x yields:

$$C = 0$$

Thus,

$$\frac{x^2 - 4}{x^3 + 2x} = \frac{3x}{x^2 + 2} - \frac{2}{x}$$

Hence,

$$\int_1^{\sqrt{2}} \frac{x^2 - 4}{x^3 + 2x} dx$$

$$= \int_1^{\sqrt{2}} \left(\frac{3x}{x^2 + 2} - \frac{2}{x}\right) dx$$

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$$= \frac{3}{2}\int_1^{\sqrt{2}} \frac{2x}{x^2+2}\,dx - 2\int_1^{\sqrt{2}} \frac{1}{x}\,dx$$

$$= \frac{3}{2}[\ln(x^2+2)]_1^{\sqrt{2}} - 2[\ln|x|]_1^{\sqrt{2}} \qquad (\text{since } x^2+2 \text{ is positive})$$

$$= \frac{3}{2}(\ln 4 - \ln 3) - 2\left(\ln\sqrt{2} - \ln 1\right)$$

$$= \frac{3}{2}\ln 4 - \frac{3}{2}\ln 3 - 2\ln\sqrt{2} + 0$$

$$= \frac{3}{2}\ln 4 - \frac{3}{2}\ln 3 - \ln\left(\sqrt{2}\right)^2$$

$$= 3\ln(4)^{\frac{1}{2}} - \frac{3}{2}\ln 3 - \ln 2$$

$$= 3\ln 2 - \ln 2 - \frac{3}{2}\ln 3$$

$$= 2\ln 2 - \frac{3}{2}\ln 3$$

$$= \ln 2^2 - \frac{3}{2}\ln 3$$

$$= \ln 4 - \frac{3}{2}\ln 3$$

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Solutions to Exercise 4D Development questions

6a

$$\int \frac{2x+3}{(x-1)(x-2)(2x-3)}dx$$

$$\frac{2x+3}{(x-1)(x-2)(2x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{2x-3}$$

Using cover-up rule:

$$A = \frac{2(1)+3}{(1-2)(2(1)-3)} = 5$$

$$B = \frac{2(2)+3}{(2-1)(2(2)-3)} = 7$$

$$C = \frac{2\left(\frac{3}{2}\right)+3}{\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)} = -24$$

Hence

$$\frac{2x+3}{(x-1)(x-2)(2x-3)} = \frac{5}{x-1} + \frac{7}{x-2} - \frac{24}{2x-3}$$

$$\int \frac{2x+3}{(x-1)(x-2)(2x-3)}dx$$

$$= \int \left(\frac{5}{x-1} + \frac{7}{x-2} - \frac{24}{2x-3}\right)dx$$

$$= 5\ln|x-1| + 7\ln|x-2| - 12\ln|2x-3| + C$$

6b

$$\int \frac{4x+12}{x^3-6x^2+8}dx$$

$$= \int \frac{4x+12}{x(x^2-6x+8)}dx$$

$$= \int \frac{4x+12}{x(x-4)(x-2)}dx$$

$$\frac{4x+12}{x(x-4)(x-2)} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x-2}$$

Using cover-up rule:

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$$A = \frac{4(0)+12}{(0-4)(0-2)} = \frac{3}{2}$$

$$B = \frac{4(4)+12}{(4)(4-2)} = \frac{7}{2}$$

$$C = \frac{4(2)+12}{(2)(2-4)} = -5$$

Hence

$$\frac{4x+12}{x(x-4)(x-2)} = \frac{\frac{3}{2}}{x} + \frac{\frac{7}{2}}{(x-4)} - \frac{5}{x-2}$$

$$\int \frac{4x+12}{x(x-4)(x-2)}\,dx$$

$$= \int \left(\frac{3}{2x} + \frac{7}{2(x-4)} - \frac{5}{x-2}\right) dx$$

$$= \frac{3}{2}\ln|x| + \frac{7}{2}\ln|x-4| - 5\ln|x-2| + C$$

7a

$$\int_2^7 \frac{3x+5}{(x-1)(x+2)(x+1)}\,dx$$

$$\frac{3x+5}{(x-1)(x+2)(x+1)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+1}$$

Using cover-up rule:

$$A = \frac{3(1)+5}{(1+2)(1+1)} = \frac{4}{3}$$

$$B = \frac{3(-2)+5}{(-2-1)(-2+1)} = -\frac{1}{3}$$

$$C = \frac{3(-1)+5}{(-1-1)(-1+2)} = -1$$

Hence

$$\frac{3x+5}{(x-1)(x+2)(x+1)} = \frac{\frac{4}{3}}{x-1} - \frac{\frac{1}{3}}{x+2} - \frac{1}{x+1}$$

Uncorrected proofs

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$$\int_2^7 \frac{3x+5}{(x-1)(x+2)(x+1)}\,dx$$

$$= \int_2^7 \left(\frac{\frac{4}{3}}{x-1} - \frac{\frac{1}{3}}{x+2} - \frac{1}{x+1}\right)dx$$

$$= \left[\frac{4}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| - \ln|x+1|\right]_2^7$$

$$= \frac{4}{3}\ln 6 - \frac{1}{3}\ln 9 - \ln 8 - \frac{4}{3}\ln 1 + \frac{1}{3}\ln 4 + \ln 3$$

$$= \frac{4}{3}\ln 3 + \frac{4}{3}\ln 2 - \frac{2}{3}\ln 3 - 3\ln 2 + \frac{2}{3}\ln 2 + \ln 3$$

$$= \frac{5}{3}\ln 3 - \ln 2$$

7b

$$\int_1^2 \frac{13x+6}{x^3-x^2-6x}\,dx$$

$$= \int_1^2 \frac{13x+6}{x(x^2-x-6)}\,dx$$

$$= \int_1^2 \frac{13x+6}{x(x-3)(x+2)}\,dx$$

$$\frac{13x+6}{x(x-3)(x+2)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+2}$$

Using cover-up rule:

$$A = \frac{13(0)+6}{(0-3)(0+2)} = -1$$

$$B = \frac{13(3)+6}{3(3+2)} = 3$$

$$C = \frac{13(-2)+6}{(-2)(-2-3)} = -2$$

Hence

$$\frac{13x+6}{x(x-3)(x+2)} = -\frac{1}{x} + \frac{3}{x-3} - \frac{2}{x+2}$$

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$$\int_1^2 \frac{13x+6}{x(x-3)(x+2)}dx$$

$$= \int_1^2 \left(-\frac{1}{x}+\frac{3}{x-3}-\frac{2}{x+2}\right)dx$$

$$= [-\ln|x| + 3\ln|x-3| - 2\ln|x+2|]_1^2$$

$$= -\ln 2 + 3\ln 1 - 2\ln 4 + \ln 1 - 3\ln 2 + 2\ln 3$$

$$= -4\ln 2 - 4\ln 2 + 2\ln 3$$

$$= -8\ln 2 + 2\ln 3$$

8a i

$$\frac{2x^2+1}{(x-1)(x+2)} = A + \frac{B}{x-1} + \frac{C}{x+2}$$

$$2x^2 + 1 = A(x-1)(x+2) + B(x+2) + C(x-1)$$

Let $x = -2$,

$$9 = -3C$$

$$C = -3$$

Let $x = 1$,

$$3 = 3B$$

$$B = 1$$

Let $x = 0$,

$$1 = -2A + 2 + 3$$

$$A = 2$$

Hence

$$\frac{2x^2+1}{(x-1)(x+2)} = 2 + \frac{1}{x-1} - \frac{3}{x+2}$$

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8a ii

$$\int \frac{2x^2+1}{(x-1)(x+2)}\,dx$$

$$= \int \left(2 + \frac{1}{x-1} - \frac{3}{x+2}\right) dx$$

$$= 2x + \ln|x-1| - 3\ln|x+2| + C$$

8b i

$$\int \frac{x^2-2x+3}{(x+1)(x-2)}\,dx$$

$$\frac{x^2-2x+3}{(x+1)(x-2)} = A + \frac{B}{x+1} + \frac{C}{x-2}$$

$$x^2-2x+3 = A(x+1)(x-2) + B(x-2) + C(x+1)$$

Let $x = 2$,

$$3 = 3C$$

$$C = 1$$

Let $x = -1$,

$$6 = -3B$$

$$B = -2$$

Let $x = 0$,

$$3 = -2A + 4 + 1$$

$$A = 1$$

Hence

$$\frac{x^2-2x+3}{(x+1)(x-2)} = 1 - \frac{2}{x+1} + \frac{1}{x-2}$$

$$\int \frac{x^2-2x+3}{(x+1)(x-2)}\,dx$$

$$= \int \left(1 - \frac{2}{x+1} + \frac{1}{x-2}\right) dx$$

$$= x - 2\ln|x+1| + \ln|x-2| + C$$

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8b ii

$$\int \frac{3x^2 - 66}{(x+4)(x-5)}\,dx$$

$$\frac{3x^2 - 66}{(x+4)(x-5)} = A + \frac{B}{x+4} + \frac{C}{x-5}$$

$$3x^2 - 66 = A(x+4)(x-5) + B(x-5) + C(x+4)$$

Let $x = 5$,

$$9 = 9C$$

$$C = 1$$

Let $x = -4$,

$$-18 = -9B$$

$$B = 2$$

Let $x = 0$,

$$-66 = -20A - 10 + 4$$

$$A = 3$$

Hence

$$\frac{3x^2 - 66}{(x+4)(x-5)} = 3 + \frac{2}{x+4} + \frac{1}{x-5}$$

$$\int \frac{3x^2 - 66}{(x+4)(x-5)}\,dx$$

$$= \int \left(3 + \frac{2}{x+4} + \frac{1}{x-5}\right) dx$$

$$= 3x + 2\ln|x+4| + \ln|x-5| + C$$

9a i

$$\frac{x^3 - 3x^2 - 4}{(x+1)(x-3)} = Ax + B + \frac{C}{x+1} + \frac{D}{x-3}$$

$$x^3 - 3x^2 - 4 = (Ax+B)(x+1)(x-3) + C(x-3) + D(x+1)$$

Equating x^3 coefficients gives $A = 1$.

Let $x = 3$,

$$-4 = 4D$$

Uncorrected proofs

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$D = -1$

Let $x = -1$,

$-8 = -4C$

$C = 2$

Let $x = 0$,

$-4 = -3B - 6 - 1$

$B = -1$

9a ii

$$\int_0^1 \frac{x^3 - 3x^2 - 4}{(x+1)(x-3)}\,dx$$

$$= \int_0^1 \left(x - 1 + \frac{2}{x+1} - \frac{1}{x-3}\right) dx$$

$$= \left[\frac{1}{2}x^2 - x + 2\ln|x+1| - \ln|x-3|\right]_0^1$$

$$= \frac{1}{2} - 1 + 2\ln 2 - \ln 2 + \ln 3$$

$$= \ln 2 + \ln 3 - \frac{1}{2}$$

9b

$$\int_2^4 \frac{x^3 + 4x^2 + x - 3}{(x+2)(x-1)}\,dx$$

$$\frac{x^3 + 4x^2 + x - 3}{(x+2)(x-1)} = Ax + B + \frac{C}{x+2} + \frac{D}{x-1}$$

$x^3 + 4x^2 + x - 3 = (Ax + B)(x+2)(x-1) + C(x-1) + D(x+2)$

Equating x^3 coefficients gives $A = 1$.

Let $x = 1$,

$3 = 3D$

$D = 1$

Let $x = -2$,

Uncorrected proofs

$3 = -3C$

$C = -1$

Let $x = 0$,

$-3 = -2B + 1 + 2$

$B = 3$

Hence

$$\int_2^4 \frac{x^3 + 4x^2 + x - 3}{(x+2)(x-1)}\,dx$$

$$= \int_2^4 \left(x + 3 - \frac{1}{x+2} + \frac{1}{x-1}\right)dx$$

$$= \left[\frac{1}{2}x^2 + 3x - \ln|x+2| + \ln|x-1|\right]_2^4$$

$$= 8 + 12 - \ln 6 + \ln 3 - 2 - 6 + \ln 4 - \ln 1$$

$$= 12 + \ln 2$$

10ai

$$\frac{3x^2 - 10}{x^2 - 4x + 4} = A + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{3x^2 - 10}{(x-2)^2} = A + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$3x^2 - 10 = A(x-2)^2 + B(x-2) + C$

Let $x = 2$,

$12 - 10 = C$

$C = 2$

$3x^2 - 10 = A(x^2 - 4x + 4) + Bx - 2B + 2$

$3x^2 - 10 = Ax^2 - 4Ax + 4A + Bx - 2B + 2$

$3x^2 - 10 = Ax^2 + (B - 4A)x + 4A - 2B + 2$

Equating x^2 coefficients give $A = 3$.

Equating x coefficients give:

$B - 12 = 0$

Uncorrected proofs

$B = 12$

10a ii

$$\int \frac{3x^2 - 10}{x^2 - 4x + 4}\,dx$$

$$= \int \left(3 + \frac{12}{x-2} + \frac{2}{(x-2)^2}\right) dx$$

$$= 3x + 12\ln|x-2| - \frac{2}{x-2} + C$$

10b i

$$\frac{3x+7}{(x-1)^2(x-2)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} + \frac{D}{(x-2)^2}$$

$3x + 7 = A(x-1)(x-2)^2 + B(x-2)^2 + C(x-1)^2(x-2) + D(x-1)^2$

Let $x = 1$,

$3 + 7 = B$

$B = 10$

Let $x = 2$,

$6 + 7 = D$

$D = 13$

$3x + 7 = A(x-1)(x-2)^2 + 10(x-2)^2 + C(x-1)^2(x-2) + 13(x-1)^2$

Equating x^3 coefficients gives $A + C = 0$

Let $x = 0$,

$7 = -4A + 40 - 2C + 13$

$-46 = -4A - 2C$

$23 = 2A + C$

$A = -C$ from previous calculation

$23 = -2C + C = -C$

$C = -23$

Uncorrected proofs

$A = 23$

10b ii

$$\int \frac{3x+7}{(x-1)^2(x-2)^2}\,dx$$

$$= \int \left(\frac{23}{x-1} + \frac{10}{(x-1)^2} - \frac{23}{x-2} + \frac{13}{(x-2)^2}\right) dx$$

$$= 23\ln|x-1| - \frac{10}{x-1} - 23\ln|x-2| - \frac{13}{x-2} + C$$

$$= 23\ln\left|\frac{x-1}{x-2}\right| - \frac{10}{x-1} - \frac{13}{x-2} + C$$

11a

$$\int_4^6 \frac{x^2-8}{x^3+4x}\,dx$$

$$= \int_4^6 \frac{x^2-8}{x(x^2+4)}\,dx$$

$$\frac{x^2-8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$x^2 - 8 = A(x^2+4) + (Bx+C)x$$

$$x^2 - 8 = Ax^2 + 4A + Bx^2 + Cx$$

$$x^2 - 8 = (A+B)x^2 + Cx + 4A$$

$$4A = -8$$

$$A = -2$$

$$-2 + B = 1$$

$$B = 3$$

$$C = 0$$

Hence

$$\int_4^6 \frac{x^2-8}{x(x^2+4)}\,dx$$

Uncorrected proofs

$$= \int_4^6 \left(\frac{-2}{x} + \frac{3x}{x^2+4}\right) dx$$

$$= \left[-2\ln|x| + \frac{3}{2}\ln|x^2+4|\right]_4^6$$

$$= -2\ln 6 + \frac{3}{2}\ln 40 + 2\ln 4 - \frac{3}{2}\ln 20$$

$$= 2\ln\frac{2}{3} + \frac{3}{2}\ln 2$$

$$= \frac{3}{2}\ln 2 - 2\ln\frac{3}{2}$$

11b

$$\int_0^2 \frac{1+4x}{(4-x)(x^2+1)} dx$$

$$\frac{1+4x}{(4-x)(x^2+1)} = \frac{A}{4-x} + \frac{Bx+C}{x^2+1}$$

$$1+4x = A(x^2+1) + (Bx+C)(4-x)$$

Let $x = 4$,

$$17 = 17A$$

$$A = 1$$

$$1+4x = x^2 + 1 - Bx^2 + 4Bx - Cx + 4C$$

$$1+4x = (1-B)x^2 + (4B-C)x + 4C + 1$$

$$\therefore 1 - B = 0$$

$$B = 1$$

$$4 - C = 4$$

$$C = 0$$

Hence

$$\int_0^2 \frac{1+4x}{(4-x)(x^2+1)} dx$$

$$= \int_0^2 \left(\frac{1}{4-x} + \frac{x}{x^2+1}\right) dx$$

Uncorrected proofs

$$= \left[-\ln|4-x| + \frac{1}{2}\ln|x^2+1|\right]_0^2$$

$$= -\ln 2 + \frac{1}{2}\ln 5 + \ln 4 - \frac{1}{2}\ln 1$$

$$= -\ln 2 + \frac{1}{2}\ln 5 + 2\ln 2$$

$$= \frac{1}{2}\ln 4 + \frac{1}{2}\ln 5$$

$$= \frac{1}{2}\ln 20$$

12a

$$\frac{x^2-1}{x^4+x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$\frac{x^2-1}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$x^2 - 1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$$

$$x^2 - 1 = Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$$

$$x^2 - 1 = (A+C)x^3 + (B+D)x^2 + Ax + B$$

Equating coefficients gives:

$$A + C = 0$$

$$B + D = 1$$

$$A = 0$$

$$B = -1$$

$$C = 0$$

$$D = 2$$

Uncorrected proofs

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12b

$$\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{x^2-1}{x^4+x^2}\,dx$$

$$= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \left(\frac{2}{x^2+1} - \frac{1}{x^2}\right) dx$$

$$= \left[2\tan^{-1}(x) + \frac{1}{x}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$$

$$= 2\tan^{-1}\left(\sqrt{3}\right) + \frac{1}{\sqrt{3}} - 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3}$$

$$= \frac{2\pi}{3} + \frac{1}{\sqrt{3}} - \frac{\pi}{3} - \sqrt{3}$$

$$= \frac{\pi}{3} + \frac{\sqrt{3}}{3} - \frac{3\sqrt{3}}{3}$$

$$= \frac{\pi}{3} - \frac{2\sqrt{3}}{3}$$

$$= \frac{1}{3}\left(\pi - 2\sqrt{3}\right)$$

13a

$$\int \frac{x^2+1}{x^2-1}\,dx$$

$$= \int \frac{x^2+1}{(x-1)(x+1)}\,dx$$

$$\frac{x^2+1}{(x-1)(x+1)} = A + \frac{B}{x-1} + \frac{C}{x+1}$$

$$x^2+1 = A(x-1)(x+1) + B(x+1) + C(x-1)$$

$$x^2+1 = Ax^2 - A + Bx + B + Cx - C$$

$$x^2+1 = Ax^2 + (B+C)x + B - A - C$$

Equating coefficients gives:

$$A = 1$$

$$B + C = 0$$

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$B - 1 - C = 1$

$B - C = 2$

$\therefore B = 1$

$\therefore C = -1$

Hence

$$\int \frac{x^2 + 1}{(x-1)(x+1)}\,dx$$

$$= \int \left(1 + \frac{1}{x-1} - \frac{1}{x+1}\right) dx$$

$$= x + \ln|x-1| - \ln|x+1| + C$$

13b

$$\int \frac{x^2 + 1}{x^2 - x}\,dx$$

$$= \int \frac{x^2 + 1}{x(x-1)}\,dx$$

$$\frac{x^2 + 1}{x(x-1)} = A + \frac{B}{x} + \frac{C}{x-1}$$

Using cover-up method:

$$B = \frac{0^2 + 1}{(0-1)} = -1$$

$$C = \frac{1^2 + 1}{1} = 2$$

$x^2 + 1 = Ax(x-1) - (x-1) + 2x$

Let $x = 2$,

$5 = 2A - 1 + 4$

$A = 1$

Hence

$$\int \frac{x^2 + 1}{x(x-1)}\,dx$$

$$= \int \left(1 - \frac{1}{x} + \frac{2}{x-1}\right) dx$$

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$= x - \ln|x| + 2\ln|x-1| + C$

13c

$$\int \frac{x^3+1}{x^3+x}\,dx$$

$$= \int \frac{x^3+1}{x(x^2+1)}\,dx$$

$$\frac{x^3+1}{x(x^2+1)} = A + \frac{B}{x} + \frac{Cx+D}{x^2+1}$$

$$x^3+1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x$$

$$x^3+1 = Ax^3 + Ax + Bx^2 + B + Cx^2 + Dx$$

$$x^3+1 = Ax^3 + (B+C)x^2 + (A+D)x + B$$

Equating coefficients:

$A = 1$

$B = 1$

$B + C = 0$

$C = -1$

$1 + D = 0$

$D = -1$

Hence

$$\int \frac{x^3+1}{x(x^2+1)}\,dx$$

$$= \int \left(1 + \frac{1}{x} + \frac{-x-1}{x^2+1}\right)dx$$

$$= \int \left(1 + \frac{1}{x} - \frac{x}{x^2+1} - \frac{1}{x^2+1}\right)dx$$

$$= x + \ln|x| - \frac{1}{2}\ln(x^2+1) - \tan^{-1}(x) + C$$

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13d

$$\int \frac{x^2}{x^2 - 5x + 6}\,dx$$

$$= \int \frac{x^2}{(x-2)(x-3)}\,dx$$

$$\frac{x^2}{(x-2)(x-3)} = A + \frac{B}{x-2} + \frac{C}{x-3}$$

Using cover-up method:

$$B = \frac{2^2}{2-3} = -4$$

$$C = \frac{3^2}{3-2} = 9$$

$$x^2 = A(x-2)(x-3) - 4(x-3) + 9(x-2)$$

Equating x^2 coefficients gives $A = 1$

Hence

$$\int \frac{x^2}{(x-2)(x-3)}\,dx$$

$$= \int \left(1 - \frac{4}{x-2} + \frac{9}{x-3}\right) dx$$

$$= x - 4\ln|x-2| + 9\ln|x-3| + C$$

13e

$$\int \frac{x^3 + 5}{x^2 + x}\,dx$$

$$= \int \frac{x^3 + 5}{x(x+1)}\,dx$$

$$\frac{x^3 + 5}{x(x+1)} = Ax + B + \frac{C}{x} + \frac{D}{x+1}$$

Using cover-up method:

$$C = \frac{0^3 + 5}{0+1} = 5$$

$$D = \frac{-1^3 + 5}{-1} = -4$$

Uncorrected proofs

$x^3 + 5 = (Ax + B)x(x + 1) + 5(x + 1) - 4x$

Equating x^3 coefficients gives $A = 1$

Let $x = 1$,

$6 = (1 + B)2 + 10 - 4$

$B = -1$

Hence

$$\int \frac{x^3 + 5}{x(x + 1)} dx$$

$$= \int \left(x - 1 + \frac{5}{x} - \frac{4}{x + 1}\right) dx$$

$$= \frac{1}{2}x^2 - x + 5\ln|x| - 4\ln|x + 1| + C$$

13f

$$\int \frac{x^4}{x^2 - 3x + 2} dx$$

$$= \int \frac{x^4}{(x - 2)(x - 1)} dx$$

$$\frac{x^4}{(x - 2)(x - 1)} = Ax^2 + Bx + C + \frac{D}{x - 2} + \frac{E}{x - 1}$$

Using cover-up method:

$$D = \frac{2^4}{2 - 1} = 16$$

$$E = \frac{1^4}{1 - 2} = -1$$

$x^4 = (Ax^2 + Bx + C)(x - 2)(x - 1) + 16(x - 1) - (x - 2)$

Equating x^4 coefficients gives $A = 1$

Equating x^3 coefficients gives $B - 3A = 0$

$B = 3$

Let $x = 0$,

$0 = 2C - 16 + 2$

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$C = 7$

Hence

$$\int \frac{x^4}{(x-2)(x-1)}\,dx$$

$$= \int \left(x^2 + 3x + 7 + \frac{16}{x-2} - \frac{1}{x-1}\right) dx$$

$$= \frac{1}{3}x^3 + \frac{3}{2}x^2 + 7x + 16\ln|x-2| - \ln|x-1| + C$$

Uncorrected proofs

Ext 2

Ex 4D

Q14. Let $\frac{5x-x^2}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

Then $5x - x^2 = A(x+1)^2 + B(x+1)(x-1) + C(x-1)$

When $x=1$ $\quad 4 = 4A$; $A=1$

When $x=-1$ $\quad -6 = -2C$; $C=3$

When $x=0$ $\quad 0 = A - B - C$

$B = A - C$

$= -2.$

so $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{5x-x^2}{(x+1)^2(x-1)}\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{x-1} - \frac{2}{x+1} + \frac{3}{(x+1)^2}\,dx$

$= \left[\log|x-1| - 2\log(x+1) - \frac{3}{x+1}\right]_{-\frac{1}{2}}^{\frac{1}{2}}$

$= \left(\log\frac{1}{2} - 2\log\frac{3}{2} - 2\right)$

$\quad - \left(\log\frac{3}{2} - 2\log\frac{1}{2} - 6\right)$

$= 4 + 3\log\frac{1}{2} - 3\log\frac{3}{2}$

$= 4 - 3\log 3.$

Ext 2

Ex 4D

(Q15 a) $C_k = \lim_{x \to a_k} P(x) \cdot \dfrac{(x-a_k)}{Q(x)}$

$= \lim_{x \to a_k} P(x) \cdot \dfrac{x - a_k}{Q(x) - Q(a_k)}$ (since $Q(a_k) = 0$)

$= P(a_k) \cdot \lim_{x \to a_k} \dfrac{1}{\left(\dfrac{Q(x) - Q(a_k)}{x - a_k}\right)}$

$= P(a_k) \cdot \dfrac{1}{Q'(a_k)}$

b) Q6b $P(x) = 4x + 12$

$Q(x) = x^3 - 6x^2 + 8x = x(x-4)(x-2)$

$Q'(x) = 3x^2 - 12x + 8$

so $C_0 = \dfrac{12}{8} = \dfrac{3}{2}$ $(x = 0)$

$C_2 = \dfrac{20}{-4} = -5$ $(x = 2)$

$C_4 = \dfrac{28}{8} = \dfrac{7}{2}$ $(x = 4)$

so $\displaystyle\int \frac{4x+12}{x^3 - 6x^2 + 8x}\, dx$

$= \displaystyle\int \frac{3/2}{x} - \frac{5}{x-2} + \frac{7/2}{x-4}\, dx$

$= \frac{3}{2} \log|x| - 5 \log|x-2| + \frac{7}{2} \log|x-4| + C$

Ex 1.2

Ex 4D

Q15 (b) (cont'd)

Q7 b

$P(x) = 13x + 6$

$Q(x) = x^3 - x^2 - 6x = x(x-3)(x+2)$

$Q'(x) = 3x^2 - 2x - 6$

so $C_0 = \frac{6}{-6} = -1$ $(x = 0)$

$C_3 = \frac{45}{15} = 3$ $(x = 3)$

$C_{-2} = \frac{-20}{10} = -2$ $(x = -2)$

so $\int_1^2 \frac{13x+6}{x^3 - x^2 - 6x}\, dx$

$= \int_1^2 \frac{3}{x-3} - \frac{1}{x} - \frac{2}{x+2}\, dx$

$= \left[3\log|x-3| - \log|x| - 2\log(x+2)\right]_1^2$

$= (3\log 1^{=0} - \log 2 - 2\log 4)$

$- (3\log 2 - \log 1^{=0} - 2\log 3)$

$= 2\log 3 - \log 2 - 4\log 2 - 3\log 2$

$= 2\log 3 - 8\log 2.$