Answers to Exercises

Chapter One

Exercise 1A (Page 8)

1(a) -1 **(b)** 1 **(c)** $-i$ **(d)** i
(e) i **(f)** -1 **(g)** 1 **(h)** 0

2(a) $-2i$ **(b)** $3-i$ **(c)** $1+i$ **(d)** $5+3i$ **(e)** $-3-2i$

3(a) $12-2i$ **(b)** $-6+2i$ **(c)** $1+5i$ **(d)** $7-11i$

4(a) $-5+4i$ **(b)** $5+5i$ **(c)** $14+5i$ **(d)** $-26+82i$
(e) $24+10i$ **(f)** $-5-12i$ **(g)** $2+11i$ **(h)** -4
(i) $28-96i$

5(a) 5 **(b)** 17 **(c)** 29 **(d)** 65

6(a) $-i$ **(b)** $1-2i$ **(c)** $3+2i$ **(d)** $1-2i$ **(e)** $-1+3i$
(f) $-\frac{1}{5}+\frac{3}{5}i$

7(a) $-2-i$ **(b)** $4-3i$ **(c)** $3+7i$ **(d)** 3 **(e)** $-3+4i$

8(a) $6+2i$ **(b)** 18 **(c)** $19-22i$ **(d)** $8-i$ **(e)** $1+2i$

9(a) $22+19i$ **(b)** $6+15i$ **(c)** $4-2i$ **(d)** $2-3i$
(e) 6

10(a) $x=3$ and $y=-2$ **(b)** $x=2$ and $y=-1$
(c) $x=6$ and $y=2$ **(d)** $x=\frac{14}{5}$ and $y=\frac{3}{5}$
(e) $x=\frac{35}{2}$ and $y=-\frac{39}{2}$

11(a) $\frac{9}{10}-\frac{13}{10}i$ **(b)** 1 **(c)** $-\frac{8}{29}$ **(d)** $-4-\frac{5}{2}i$

16(a) $\frac{x-iy}{x^2+y^2}$ **(b)** $\frac{x^2-y^2-2ixy}{(x^2+y^2)^2}$ **(c)** $\frac{x^2+y^2-1+2iy}{(x+1)^2+y^2}$

Exercise 1B (Page 16)

1(a) $z=\pm 3i$ **(b)** $z=2\pm 4i$ **(c)** $z=-1\pm 2i$
(d) $z=3\pm i$ **(e)** $z=\frac{1}{2}\pm\frac{1}{4}i$ **(f)** $z=-\frac{3}{2}\pm 2i$

2(a) $(z-6i)(z+6i)$ **(b)** $(z-2\sqrt{2}\,i)(z+2\sqrt{2}\,i)$
(c) $(z-1-3i)(z-1+3i)$ **(d)** $(z+2-i)(z+2+i)$
(e) $(z-3+\sqrt{5}\,i)(z-3-\sqrt{5}\,i)$
(f) $(z+\frac{1}{2}-\frac{\sqrt{3}}{2}\,i)(z+\frac{1}{2}+\frac{\sqrt{3}}{2}\,i)$

3(a) $z^2+2=0$ **(b)** $z^2-2z+2=0$
(c) $z^2+2z+5=0$ **(d)** $z^2-4z+7=0$

4(a) $\pm(1+i)$ **(b)** $\pm(2+i)$ **(c)** $\pm(-1+3i)$
(d) $\pm(6+i)$ **(e)** $\pm(2+3i)$ **(f)** $\pm(5-i)$ **(g)** $\pm(1-4i)$
(h) $\pm(5-4i)$

5(a) $\pm(1-2i)$ **(b)** $z=2-i$ or $1+i$

6(a) $\pm(1+3i)$ **(b)** $z=4+i$ or $3-2i$

7(a) $z=1-i$ or i **(b)** $z=-3+2i$ or $-2i$ **(c)** $z=4+i$ or $2-i$ **(d)** $z=-2+i$ or $\frac{1}{2}(3-i)$ **(e)** $z=-5+i$ or $3-2i$ **(f)** $z=3+i$ or $-1-3i$

8(a) $w=-1$ **(b)** $a=-6$ and $b=13$
(c) $k=8-i$ and the other root is $2+3i$.

9 $z=\pm(2+i)$

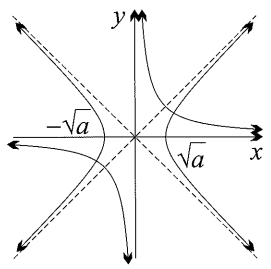
10(a) $\cos\theta+i\sin\theta$ or $\cos\theta-i\sin\theta$

11(a) $z=-1$ or $\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$ **(b)** $z=i$ or $\pm\frac{\sqrt{3}}{2}-\frac{1}{2}i$

12(a) $z=\omega$ satisfies the equation. **(c)** They are complex conjugates.

13(a) $\overline{\alpha}$

14(a)(i)



15(a) $\pm\frac{1}{\sqrt{2}}(1-i)$ **(b)** $\pm\sqrt{2}(1+2i)$ **(c)** $\pm(\sqrt{3}+i)$
(d) $\pm\sqrt{2}(3-2i)$
(e) $\pm\left(\sqrt{\sqrt{5}+1}-i\sqrt{\sqrt{5}-1}\right)$

16(a) $-2-i\pm\left(\sqrt{\sqrt{2}+1}+i\sqrt{\sqrt{2}-1}\right)$
(b) $1+i\pm\left(\sqrt{\sqrt{5}-1}-i\sqrt{\sqrt{5}+1}\right)$
(c) $-1+i\sqrt{3}\pm\left(\sqrt{2}-i\sqrt{6}\right)$
(d) $\frac{1}{2}\left(-1+i\pm\left(\sqrt{\sqrt{13}+2}-i\sqrt{\sqrt{13}-2}\right)\right)$

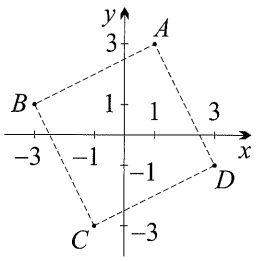
19 The term $b/|b|$ is the sign of b.
It is 1 when $b>0$, and -1 when $b<0$.

Exercise **1C** (Page 21)

1(a) $(2, 0)$ **(b)** $(0, 1)$ **(c)** $(-3, 5)$ **(d)** $(2, -2)$ **(e)** $(-5, -5)$ **(f)** $(-1, 2)$

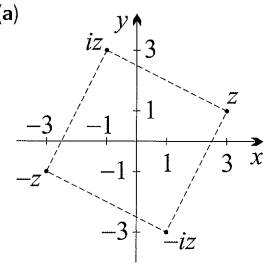
2(a) $-3 + 0i = -3$ **(b)** $0 + 3i = 3i$ **(c)** $7 - 5i$ **(d)** $a + bi$

3(a)

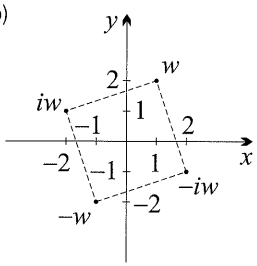


(b) A square. **(c)** An anticlockwise rotation of $90°$ about the origin.

4(a)

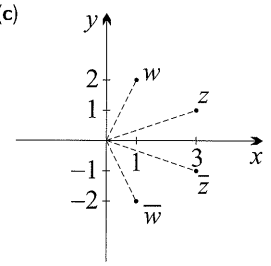


(b)

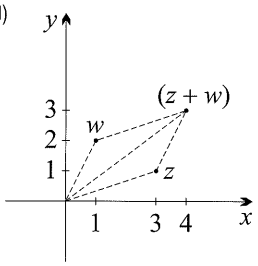


In (a) and (b) the points form a square.

(c)



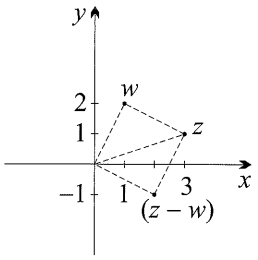
(d)



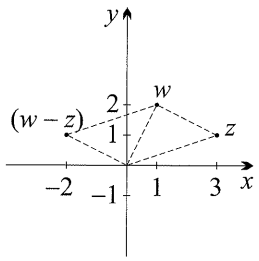
Conjugate pairs are reflections in the real axis.

With O the points form a parallelogram.

(e)

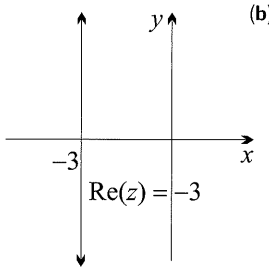


(f)

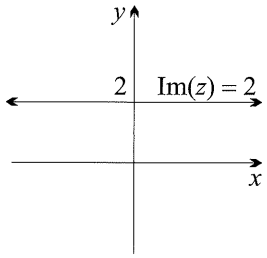


Again, in (e) and (f) the points are the vertices of a parallelogram.

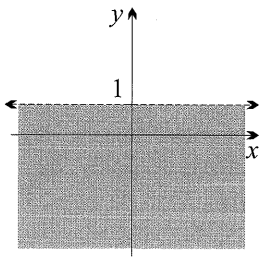
5(a)



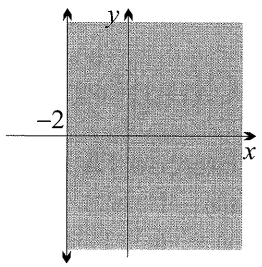
(b)



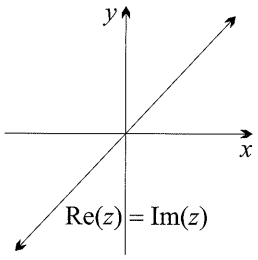
(c)



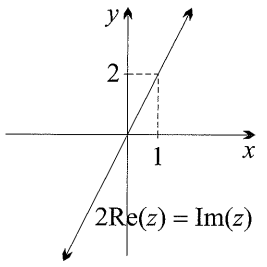
(d)



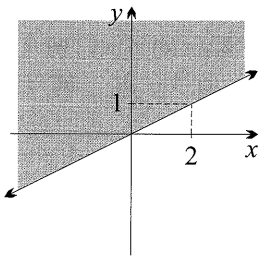
(e)



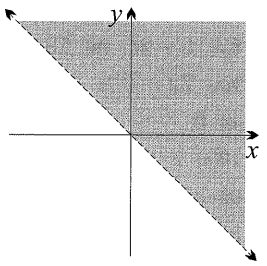
(f)



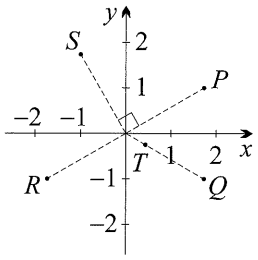
(g)



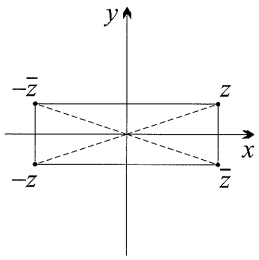
(h)



6



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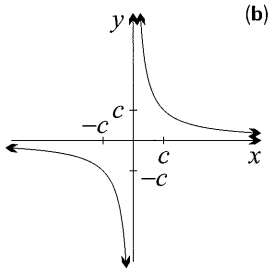


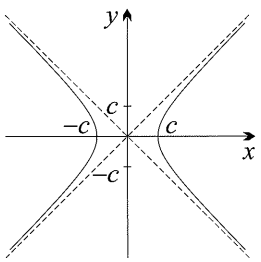
10(c) right-isosceles

11 It is the circle centre $(0, -1)$ with radius 1, omitting the origin.

12 It is the circle centre $(3, 0)$ with radius 3, omitting the origin.

14 It is a parabola with focus the origin and directrix $x = 1$.

15(a)


(b)


Exercise **1D** (Page 28)

1(a) 3 **(b)** 5 **(c)** $\sqrt{2}$ **(d)** 2 **(e)** 5 **(f)** 17

2(a) π **(b)** $\frac{\pi}{2}$ **(c)** $-\frac{\pi}{4}$ **(d)** $\frac{\pi}{3}$ **(e)** $\frac{3\pi}{4}$ **(f)** $-\frac{5\pi}{6}$

3(a) $2\operatorname{cis}\frac{\pi}{2}$ **(b)** $4\operatorname{cis}\pi$ **(c)** $\sqrt{2}\operatorname{cis}\frac{\pi}{4}$ **(d)** $2\operatorname{cis}\left(-\frac{\pi}{6}\right)$ **(e)** $2\operatorname{cis}\frac{2\pi}{3}$ **(f)** $\operatorname{cis}\left(-\frac{3\pi}{4}\right)$

4(a) $5\operatorname{cis}(0{\cdot}93)$ **(b)** $13\operatorname{cis}(-0{\cdot}39)$ **(c)** $\sqrt{5}\operatorname{cis}(2{\cdot}68)$ **(d)** $\sqrt{10}\operatorname{cis}(-1{\cdot}89)$

5(a) 3 **(b)** $-5i$ **(c)** $2\sqrt{2}+2\sqrt{2}\,i$ **(d)** $3\sqrt{3}-3i$ **(e)** $-\sqrt{2}+\sqrt{2}\,i$ **(f)** $-1-\sqrt{3}\,i$

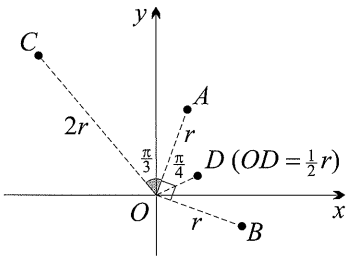
6(a) $\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$ **(b)** $\sqrt{2}\operatorname{cis}\frac{\pi}{4}$ **(c)** $\sqrt{2}\operatorname{cis}\frac{3\pi}{4}$ **(d)** $\sqrt{2}\operatorname{cis}\frac{\pi}{4}$ **(e)** $2\operatorname{cis}\left(-\frac{\pi}{2}\right)$ **(f)** $\frac{1}{\sqrt{2}}\operatorname{cis}\left(-\frac{\pi}{4}\right)$

7(a) $10\operatorname{cis}\frac{\pi}{3}$ **(b)** $9\operatorname{cis}3\theta$ **(c)** $2\operatorname{cis}\frac{\pi}{3}$ **(d)** $\frac{3}{2}\operatorname{cis}\alpha$ **(e)** $16\operatorname{cis}\frac{2\pi}{5}$ **(f)** $8\operatorname{cis}\frac{6\pi}{7}$

8(a) $2\sqrt{2}$ **(b)** $3\sqrt{2}$ **(c)** 6 **(d)** $4\sqrt{3}$ **(e)** 5 **(f)** $\sqrt{5}$

9(a) $\frac{\pi}{4}$ **(b)** $-\frac{3\pi}{4}$ **(c)** $-\frac{\pi}{3}$ **(d)** $\frac{\pi}{6}$ **(e)** 0·93 (53°) **(f)** −2·03 (−117°)

10(a) i **(b)** -1 **(c)** $\frac{1}{2}(1+i\sqrt{3})$ **(d)** $\frac{1}{\sqrt{2}}(-1+i)$ **(e)** $\frac{1}{2}(-\sqrt{3}+i)$ **(f)** $-i$ **(g)** $\frac{1}{\sqrt{2}}(1-i)$ **(h)** $-\frac{1}{2}(1+i\sqrt{3})$

11


13(a) $z_1 = 2\operatorname{cis}\frac{\pi}{6}$ and $z_2 = 4\operatorname{cis}\frac{\pi}{4}$ **(b)** $z_1z_2 = 8\operatorname{cis}\frac{5\pi}{12}$ and $\frac{z_2}{z_1} = 2\operatorname{cis}\frac{\pi}{12}$

14 $z_1 = 2\operatorname{cis}\frac{5\pi}{6}$, $z_2 = \sqrt{2}\operatorname{cis}(-\frac{3\pi}{4})$,
$z_1z_2 = 2\sqrt{2}\operatorname{cis}\frac{\pi}{12}$ and $\frac{z_2}{z_1} = \frac{\sqrt{2}}{2}\operatorname{cis}\frac{5\pi}{12}$

15(a) $\frac{1}{2}\Big((\sqrt{3}+1)+i(\sqrt{3}-1)\Big)$ **(b)** $\sqrt{2}\operatorname{cis}\frac{\pi}{12}$ **(c)** $\frac{1}{2\sqrt{2}}(\sqrt{3}+1)$

16(a) $\sqrt{2}$ **(b)** $\frac{\pi}{4}$ **(c)** $1+i$

24 $z+w = 2\cos\left(\frac{\theta-\phi}{2}\right)\operatorname{cis}\left(\frac{\theta+\phi}{2}\right)$

25(a) When $\operatorname{Im}(z) = 0$.

Exercise **1E** (Page 34)

1(a) $7+4i$ **(b)** $-3+2i$ **(c)** $3-2i$

2(a) $-3+4i$ **(b)** $1+7i$ **(c)** $-4-3i$ **(d)** $-7+i$

3 $-3+6i$

4(a) B represents $1+3i$, C represents $-1+2i$ **(b)** $-\sqrt{2}+2\sqrt{2}\,i$

5(a) $4+3i$ **(b)** $-3+4i$ **(c)** $2+7i$

6(a) $-5+12i$ **(b)** $-3-4i$

8 E represents w_2-w_1, F represents $i(w_2-w_1)$, C represents $w_2+i(w_2-w_1)$ and D represents $w_1+i(w_2-w_1)$.

9(a) Vectors BA and BC represent z_1-z_2 and z_3-z_2 respectively, and BA is the anticlockwise rotation of BC through 90° about B. So $z_1-z_2 = i(z_3-z_2)$. Squaring both sides gives the result. **(b)** $z_1-z_2+z_3$

10(a) $2\omega i$ **(b)** $\frac{1}{2}\omega(1+2i)$

11 -2 and $1-\sqrt{3}\,i$

12(a) $w = -4+3i$ or $4-3i$ **(b)** $w = -1+7i$ or $7+i$ **(c)** $w = \frac{1}{2}(7+i)$ or $\frac{1}{2}(-1+7i)$

13 $-2+2i$, $12i$, 4

18(a) $z_1 = 2\operatorname{cis}\frac{\pi}{2}$, $z_2 = 2\operatorname{cis}\frac{\pi}{3}$ **(c)(i)** $\frac{5\pi}{12}$ **(ii)** $\frac{11\pi}{12}$

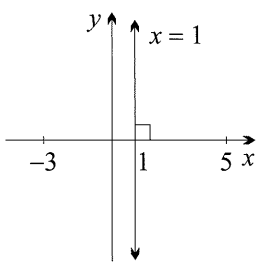
19(c) The sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

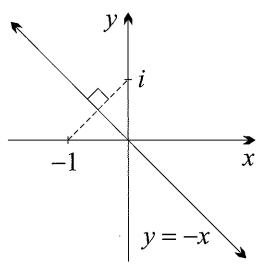
20(c) parallelogram **(d)** $\arg\frac{w}{z} = \frac{\pi}{2}$, so $\frac{w}{z}$ is purely imaginary.

23 Use the converse of the opposite angles of a cyclic quadrilateral.

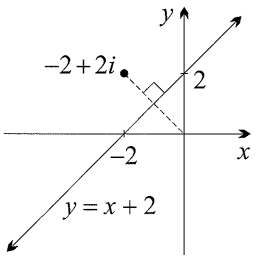
24 Take the argument of the fraction in the hint. The result is $\arg\left(\frac{z_3}{z_2}\right) - \arg\left(\frac{z_3-z_1}{z_2-z_1}\right)$. These are the angles at 0 and z_1 which, by the angles in the same segment theorem, are equal. Finally, use the result of Question 24.

Exercise **1F** (Page 42)

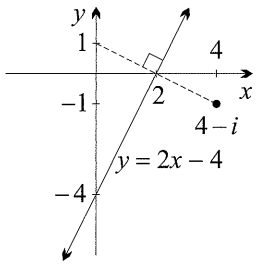
1(a)


(b)


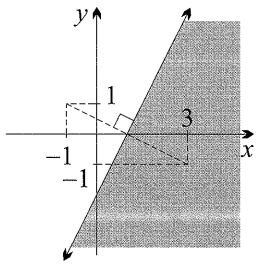
(c)



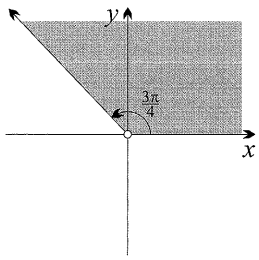
(d)



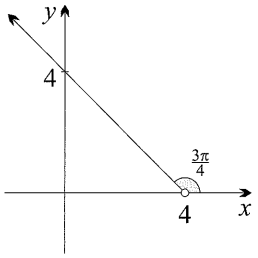
(c)



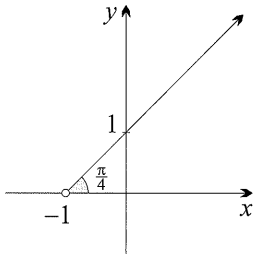
(d)



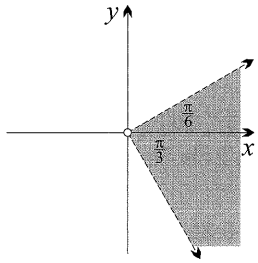
2(a)



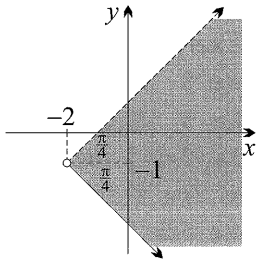
(b)



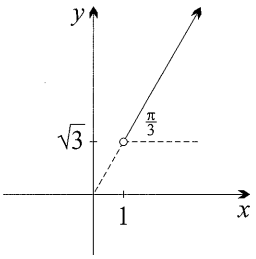
(e)



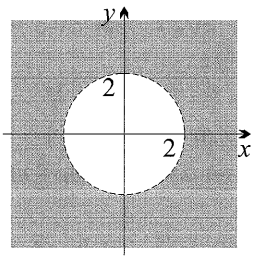
(f)



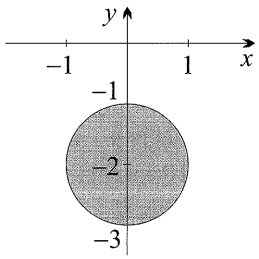
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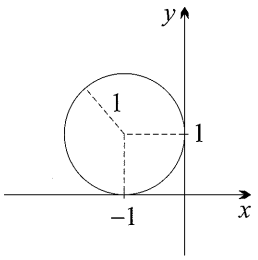
(g)



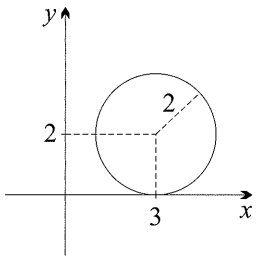
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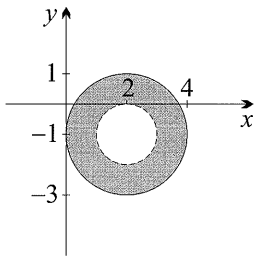
3(a)



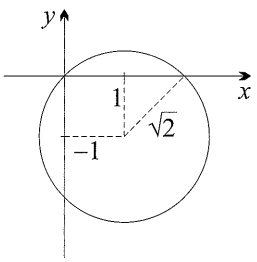
(b)



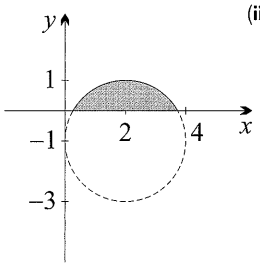
(i)



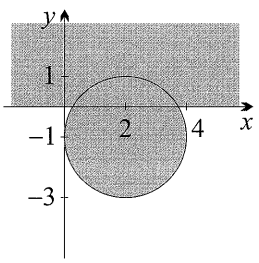
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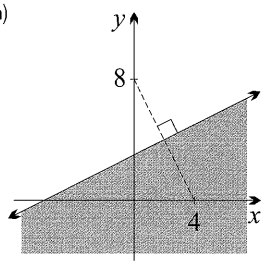
5(a)(i)



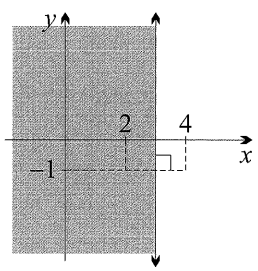
(ii)



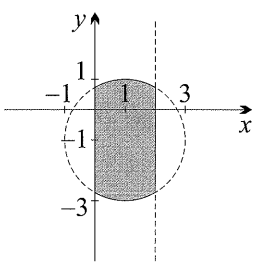
4(a)



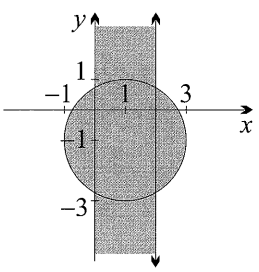
(b)



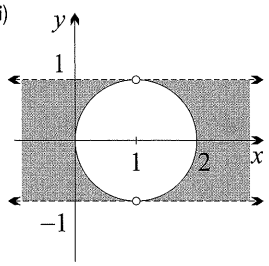
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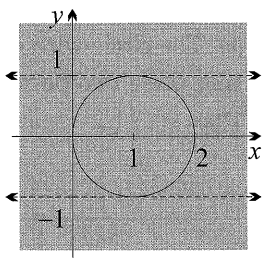
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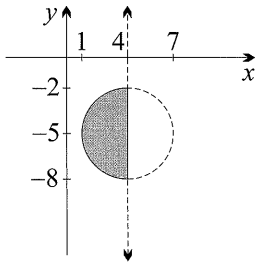
(c)(i)



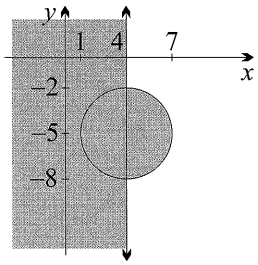
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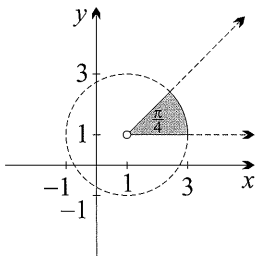
(d)(i)



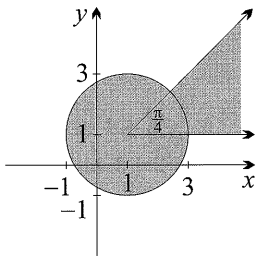
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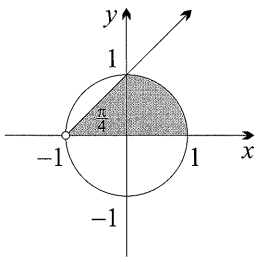
(e)(i)



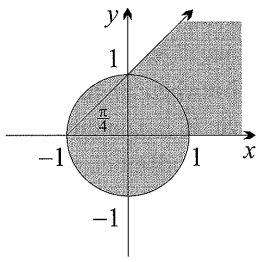
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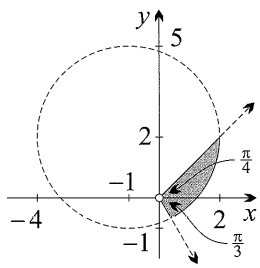
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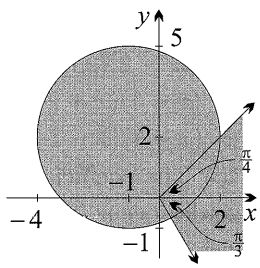
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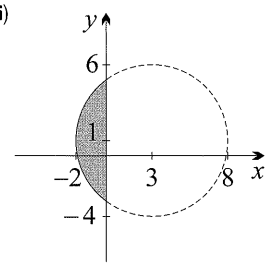
(g)(i)



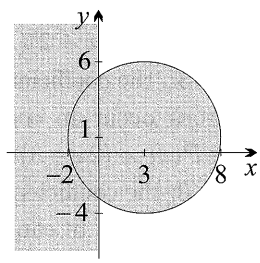
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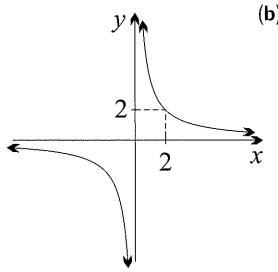
(h)(i)



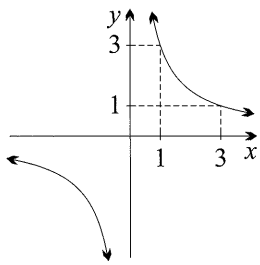
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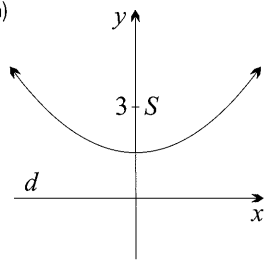
6(a)



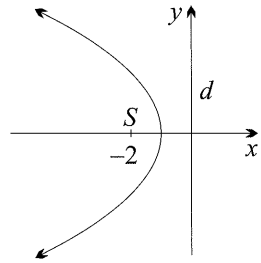
(b)



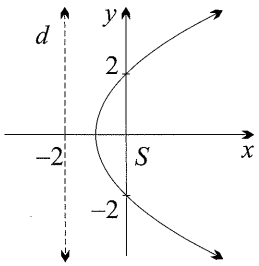
7(a)



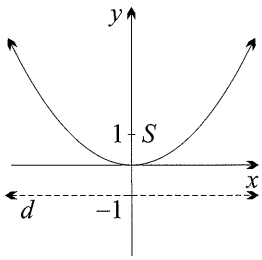
(b)



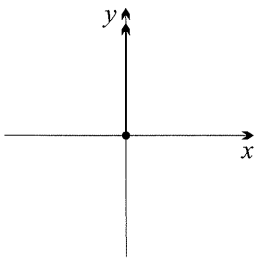
(c)



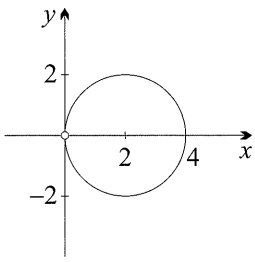
(d)



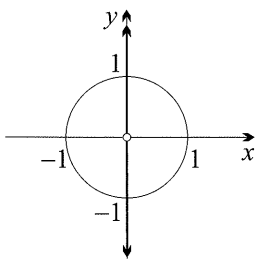
8(a)



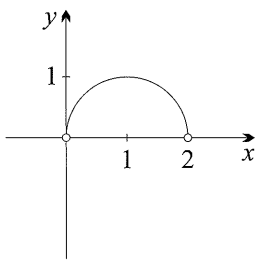
(b)



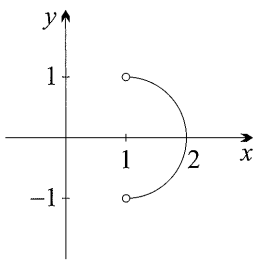
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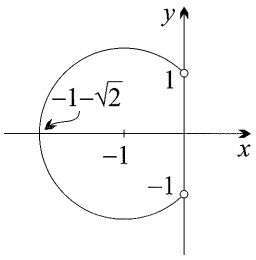


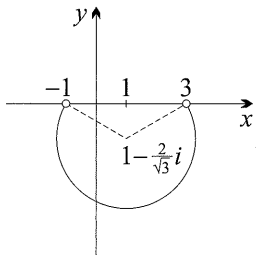
9(a)

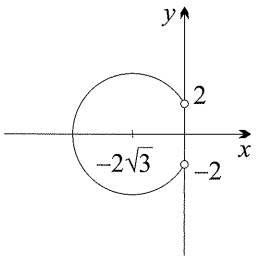


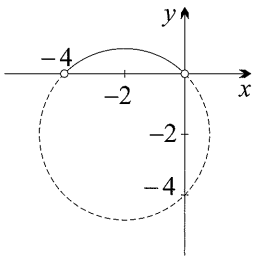
(b)



(c)


(d)


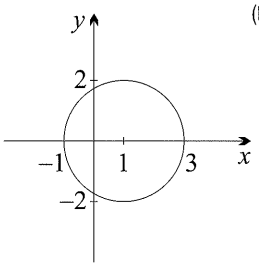
(e)


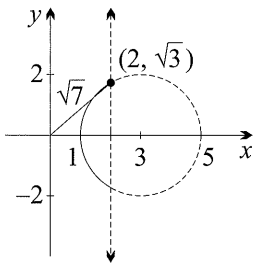
(f)


10(b) $\sqrt{3}\,\mathrm{cis}\,\frac{\pi}{3} = \frac{\sqrt{3}}{2}(1 + i\sqrt{3})$

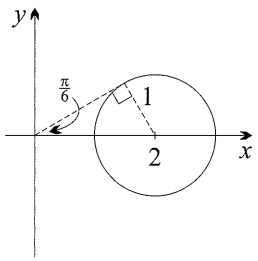
11(a) $\arg(z+3) = \frac{\pi}{3}$ **(b)** $|z| = \frac{3\sqrt{3}}{2}$, $\arg z = \frac{5\pi}{6}$

(c) $-\frac{9}{4} + \frac{3\sqrt{3}}{4}i$

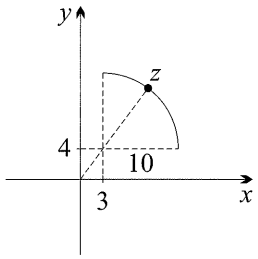
12(a)


(b)


3 and 1

13(a)(i)


(b) This is simply part (a) shifted left by 2.

14(a)


(b) 15 **(c)** $9 + 12i$

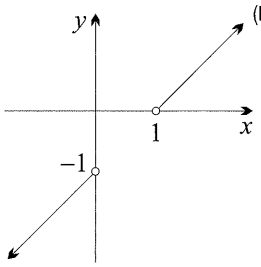
15(b)(i) $|z+2| = 2$, centre -2, radius 2

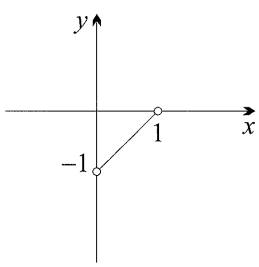
(ii) $|z-(1+i)| = 1$, centre $1+i$, radius 1

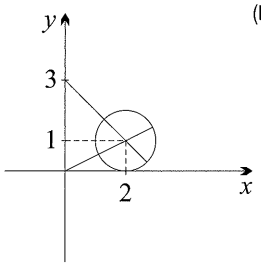
(iii) $|z-1| = 1$, centre 1, radius 1

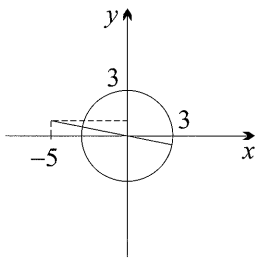
16(a) The line through 1 and i, omitting i.

(b) The circle with diameter joining 1 and i, omitting these two points.

17(a)


(b)


18(a)


(b)


(i) $\sqrt{5}+1$ and $\sqrt{5}-1$ $\sqrt{26}+3$ and $\sqrt{26}-3$

(ii) $2\sqrt{2}+1$ and $2\sqrt{2}-1$

(c)(i) $\big||z_0| - r\big| \le |z| \le |z_0| + r$

(ii) $\big||z_0 - z_1| - r\big| \le |z - z_1| \le |z_0 - z_1| + r$

19(a) straight line external to z_1 and z_2 **(b)** major arc **(c)** semi-circle **(d)** minor arc **(e)** interval between z_1 and z_2

20 It is the circle with the interval joining z_1 and z_2 as diameter.

21(b) The graph is the perpendicular bisector of the line joining z_1 and z_2.

Exercise 1G (Page 51)

1(a) $(x-2)(x+1-\sqrt{3})(x+1+\sqrt{3})$

(b) $(x-1)(x+2-\sqrt{2})(x+2+\sqrt{2})$

(c) $(x-1)(x-1-\sqrt{5})(x-1+\sqrt{5})$

2(a) The coefficients of $P(x)$ are real, so complex zeroes occur in conjugate pairs. **(b)** 6

3(a) $1 + 2i$; the coefficients of $P(x)$ are real, so complex zeroes occur in conjugate pairs.

(c) $P(x) = (x+2)(x^2 - 2x + 5)$

4(a) $3i$; the coefficients of $P(z)$ are real, so complex zeroes occur in conjugate pairs. **(b)** $z^2 + 9$

(c) $P(z) = (2z+3)(z^2+9)$

5(b) 0; the coefficients of $P(z)$ are real, so complex zeroes occur in conjugate pairs.

(c)(i) $P(z) = (2z-1)(z-3-i)(z-3+i)$

(ii) $P(z) = (2z-1)(z^2 - 6z + 10)$

6(a) The coefficients of $Q(x)$ are real, so complex zeroes occur in conjugate pairs. **(b)** $3+\sqrt{5}$, $3-\sqrt{5}$

(c)(i) $(x-2i)(x+2i)(x-3-\sqrt{5}\,)(x-3+\sqrt{5}\,)$

(ii) $(x^2+4)(x-3-\sqrt{5}\,)(x-3+\sqrt{5}\,)$

(iii) $(x^2+4)(x^2-6x+4)$

7(a) $x=1\pm 3i$, 3 or -2 **(b)** $x=1\pm i$ or $2\pm i$

8(b) $x=3,\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i,\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

9(a) $a=3$ **(b)** $b=1$

(c) $(z^2-6z+10)(z^2-6z+13)$

10(b) $k=3$

11(b) $m=7,\ n=-4$

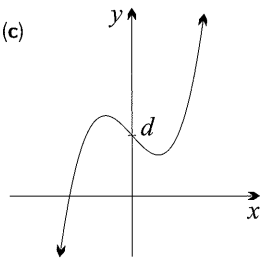
12(a) $-7-4i$ **(b)(i)** $-7+4i$ **(ii)** $2x-7$

13(b) $P(z)=\frac{1}{2}(z^4-2)(2z^4-1)$
so one root is $z=\sqrt[4]{2}$.

(c) $\sqrt[4]{2},\ \frac{1}{\sqrt[4]{2}},\ -\sqrt[4]{2},\ -\frac{1}{\sqrt[4]{2}}$,
and $i\sqrt[4]{2},\ \frac{1}{\sqrt[4]{2}}i,\ -i\sqrt[4]{2},\ -\frac{1}{\sqrt[4]{2}}i$

14(a) $P(x)$ has minimum value B, when $x=0$. Since $B>0$, it follows that $P(x)>0$ for all real values of x. **(b)** $-ic$, $-id$; the coefficients of $P(x)$ are real, so complex zeroes occur in conjugate pairs.

15(a) They form a conjugate pair, since $P(x)$ has real coefficients. **(c)**



16(a) The minimum stationary point is at $x=1$. $f(1)=k-2>0$. Hence the graph of $f(x)$ has only one x-intercept which lies to the left of the maximum stationary point at $x=-1$.

(b) $f(x)$ has real coefficients **(d)** $-14,\ 7\pm 12i$

17 HINT: consider $P(x)-P'(x)$

18(b) $-1+2i$ is a double zero of $P(z)$ **(c)** The coefficients of $P(z)$ are real and $-1+2i$ counts as two of the zeroes of $P(z)$, so its conjugate $-1-2i$ must also count as two zeroes.

(d) $P(z)=(z+1-2i)^2(z+1+2i)^2=(z^2+2z+5)^2$

22(b) $(z-\alpha)^2(z-\overline{\alpha})^2$ is a factor. **(c)** HINT: Begin by writing: $P(z)=\big(z-2\,\mathrm{Re}(\alpha)+|\alpha|^2\big)^2\times Q(z)$

Review Exercise **1H** (Page 53)

1(a) $1-5i$ **(b)** $18-26i$ **(c)** $5+2i$

2(a) $(z+10i)(z-10i)$ **(b)** $(z+5-3i)(z+5+3i)$

3(a) $z=4+3i$ or $4-3i$ **(b)** $z=-\frac{1}{2}+\frac{3}{4}i$ or $-\frac{1}{2}-\frac{3}{4}i$

4(a) $\pm(3-2i)$ **(b)** $\pm(3+\sqrt{2}\,i)$

5(a) $z=2+i$ or $3-i$ **(b)** $z=2+3i$ or $4-2i$

6 $\overline{3i}=-3i$ is also a zero, so $(z-3i)(z+3i)=z^2+9$ is a factor.

7(a) The coefficients of $P(z)$ are real. **(b)** 4

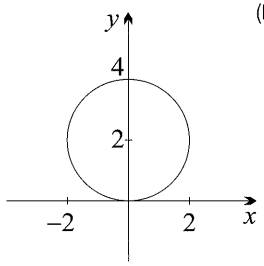
(c) $P(z)=(z-4)(z^2-4z+29)$

8(a) $\sqrt{2}\,\mathrm{cis}(-\frac{\pi}{4})$ **(b)** $6\,\mathrm{cis}\,\frac{5\pi}{6}$

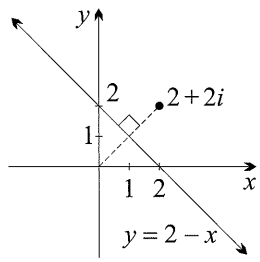
9(a) $4i$ **(b)** $-\sqrt{3}-\sqrt{3}\,i$

10(a) $6\,\mathrm{cis}\,\frac{5\pi}{6}$ **(b)** $2\,\mathrm{cis}\,5\theta$ **(c)** $9\,\mathrm{cis}\,6\alpha$

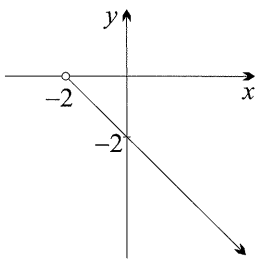
11(a)



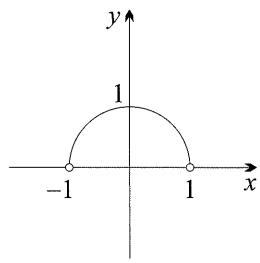
(b)



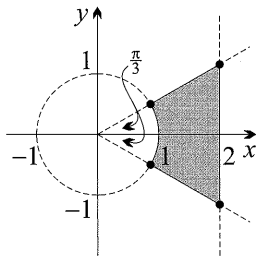
(c)



(d)

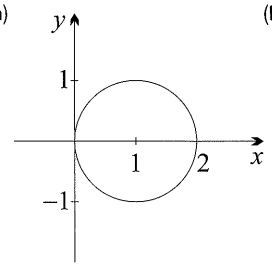


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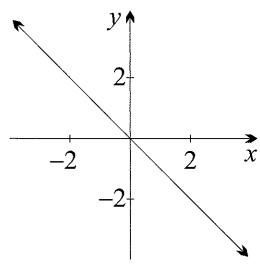


13(a) $\frac{1}{2}(\sqrt{3}-1)+\frac{1}{2}(\sqrt{3}+1)i$ **(b)** $z=2\,\mathrm{cis}\,\frac{2\pi}{3}$ and $w=\sqrt{2}\,\mathrm{cis}\,\frac{\pi}{4}$ **(c)** $\sqrt{2}\,\mathrm{cis}\,\frac{5\pi}{12}$

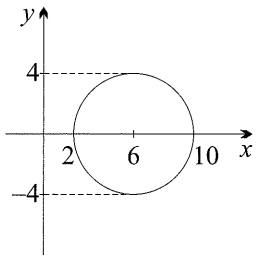
14(a)



(b)



(c)



15(a) $3+5i$ **(b)** $-5+3i$ **(c)** $-1+i$

16(a) $1-5i,\ 7+3i$ **(b)** $3+6i,\ -3-2i$

18(a) $\frac{7\pi}{24}$ **(b)** $\frac{19\pi}{24}$

21 Use similar triangles.

22(b) $\sqrt{5}+1$

Chapter Two

Exercise **2A** (Page 64)

1(a) equality **(b)** implication **(c)** equivalence **(d)** for all **(e)** there exists

2(a) If a triangle has two equal angles, then it has two equal sides. True. **(b)** If the square of a number is odd, then the number is odd. True. **(c)** If I have four legs, then I am a horse. False. **(d)** If a number is even, then it ends with the digit 6. False. **(e)** Every rhombus is a square. False. **(f)** If $n \geq 0$, then $\sqrt{n} \in \mathbf{R}$. True.

3(a) True **(b)** False **(c)** False **(d)** False **(e)** True **(f)** True

4(a) Not all cars are red. (Alternatively, some cars are not red.) **(b)** $a \leq b$ or $a \not> b$ **(c)** Hillary does not like steak or she does not like pizza. **(d)** Bill and Dave are both wrong. **(e)** I live in Tasmania and I don't live in Australia. **(f)** Nikhil doesn't study and he passes.
(g) $x < -3$ or $x > 8$ **(h)** $-5 \leq x < 0$

5(a) If my plants do not grow, then I haven't watered them. **(b)** If you live in Melbourne, then you live in Australia. **(c)** If a triangle does not have three equal angles, then it does not have three equal sides. **(d)** If I like motorists, then I do not like cycling. **(e)** If a number is even, then the previous number is odd. **(f)** If $\frac{1}{a} \geq \frac{1}{b}$, then $a \leq b$ or a and b are not both positive.

6(a) If a number is divisible by both 3 and 5, then it is divisible by 15. Conversely, if a number is divisible by 15, then it is divisible by both 3 and 5. **(b)** If a triangle has two equal sides, then it has two equal angles. Conversely, if a triangle has two equal angles, then it has two equal sides. **(c)** If the only divisors of the integer n, where $n > 1$, are 1 and n, then n is prime. Conversely, if n is prime, then its only divisors are 1 and n. **(d)** If a quadrilateral has a pair of opposite sides that are equal and parallel, then it is a parallelogram. Conversely, if a quadrilateral is a parallelogram, then it has a pair of opposite sides that are equal and parallel.

7(a) true **(b)** false, $3 \times (-1) < -1$ **(c)** true **(d)** false, $\frac{1}{2} > \left(\frac{1}{2}\right)^2$ **(e)** false, $|-(-1)| \neq -1$ **(f)** true

8(a) false, $2 > -3$ but $2^2 < (-3)^2$ **(b)** false, $(-3)^2 > 2^2$ but $-3 < 2$ **(c)** true **(d)** true **(e)** false, $|2+(-1)| < |2| + |-1|$ **(f)** true

9(a) $\Rightarrow$ **(b)** $\Leftrightarrow$ **(c)** $\Leftrightarrow$ **(d)** $\Rightarrow$ **(e)** $\Leftrightarrow$ **(f)** $\Rightarrow$

10(a) false **(b)** false **(c)** true **(d)** true

11(a) If Jack does Extension 2 Mathematics then he is crazy. **(b)** Jack does Extension 2 Mathematics and he is not crazy. **(c)** If Jack does not do Extension 2 Mathematics then he is crazy. **(d)** If Jack is not crazy then he will do Extension 2 Mathematics. **(e)** If Jack does not do Extension 2 Mathematics then he is not crazy. **(f)** If Jack does Extension 2 Mathematics then he is not crazy.

12(a) For each integer there always exists a larger integer. **(b)** The sum of any positive real number and its reciprocal is greater than or equal to two.

13(a) true **(b)** false **(c)** true **(d)** true

14(a) They are both false.

(b) If $1 < 0$ then 1 is a negative number. True

15(a) yes — consider the contrapositive of (1)

(b) unknown — studying hard is a sufficient condition for passing, but it is not a necessary condition

16 If either Anna or Bryan passed, then Chris passed. So since the statement is negated, Chris failed.

17 Pender is the driver.

Exercise **2B** (Page 68)

1(a) [HINT: An even number has the form $2n$, where $n \in \mathbf{Z}$.] **(b)** [HINT: An odd number has the form $2n+1$, where $n \in \mathbf{Z}$.]

4(a) [HINT: If b is divisible by a, then $b = ka$ for some $k \in \mathbf{Z}$.]

6(a) [HINT: Let the consecutive integers be $n-1$, n, $n+1$ and $n+2$.]

8 [HINT: Find a pair of simultaneous equations.]

10 [HINT: Factorise the expression, then explain why it is divisible by both 2 and 3.]

13 [HINT: Let the consecutive integers range from $n-3$ to $n+3$.]

14(b) No. There will always be a remainder of $\frac{n}{2}$.

15 [HINT: A 4-digit number with digits a, b, c, d has value $1000a + 100b + 10c + d$.]

16(c) $\Big[10x + y = 13m\Big] \Leftrightarrow \Big[x + 4y = 13(4m - 3x)\Big]$

18(a) A factor of n is of the form $p^c q^d$, where $c \in \{0, 1, 2, \ldots, a\}$ and $d \in \{0, 1, 2, \ldots, b\}$. So there are $a+1$ possible values for c and $b+1$ possible values for d. So by the multiplication principle, there are $(a+1)(b+1)$ possible factors of n. **(b)** 40

19 [HINT: Consider the expression $(a-c)(b-d)$.]

Exercise **2C** (Page 72)

3(a) If a is even then a^2 is even.

10(a) An odd number lies between two consecutive multiples of 4. It is one more than the smaller multiple of 4 or one less than the larger multiple of 4.

16(a) If p is not prime then $\exists\ a, b \in \mathbf{Z}^+$ such that $p|ab \Rightarrow p \nmid a$ and $p \nmid b$.

Exercise **2D** (Page 78)

6(b) Use part (a) three times.

8(a) Use Question 6(b) with $p = a^2$ and so on.

(b) In part (a) replace a^2 with ab and so on.

(c) Use parts (a) and (b).

9(c) Use part (b).

(d) Use part (c) with $a^3 = x$ and so on.

11(a) $\frac{a^4}{b^4} - \frac{4a^2}{b^2} + 6 - \frac{4b^2}{a^2} + \frac{b^4}{a^4}$

12(a) Use the given AM/GM inequality twice on the RHS.

13(d)(i) The triangle inequality: the length of any side is less than (or equal to if the points are collinear) the sum of the other two sides.

(ii) Use part (i) three times then add.

14(b) Expand the LHS and use part (a).

(c)(i) Begin with LHS – RHS.

16(a) Begin with LHS – RHS.

18(c) When $z = kw$, with $k > 0$, or when either $z = 0$ or $w = 0$.

Exercise **2E** (Page 83)

8(a) $x > 1 + \sqrt{2}$ or $x < 1 - \sqrt{2}$

25(a) Ben may pair $(n-1)$ other players. In each case there are $(n-2)$ remaining players. In each case, the number of derangements for those players is D_{n-2}. Hence multiply to get $(n-1)D_{n-2}$.

(d) $D_1 = 0$, $D_2 = 1$

Exercise **2F** (Page 88)

1(a) $|\triangle OAB| = \frac{1}{4}$ cm^2 , 3 cm^2

(b)(ii) $|\triangle OGH| = (2-\sqrt{3})$ cm^2,
dodecagon area $= 12(2-\sqrt{3})$ cm^2

2(a) $\frac{\pi}{36}(4+\sqrt{3})$

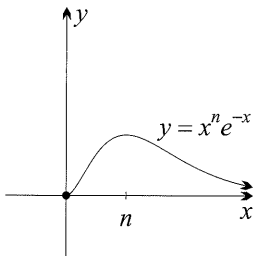
3(a) $\frac{3}{4}$ and $\frac{2}{3}$ square units.

4(a) $(1-e^{-1})$ sq. units **(b)** $\frac{1}{2}(1+e^{-1})$ sq. units
(c) $e^{-\frac{1}{2}}$ sq. units

6(b) It diverges to infinity.

7(c) $(\frac{a+2b}{3}, \frac{\ln a + 2\ln b}{3})$

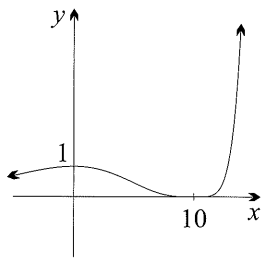
8(b)



10(a) $(0,1)$ is a maximum turning point, $(10,0)$ is a minimum turning point.

(b) $y \to \infty$ as $x \to \infty$, and $y \to 0$ as $x \to -\infty$.

(c)



11(c) When $z = kw$, with $k < 0$, or when either $z = 0$ or $w = 0$.

12(a)(i) $6^6 = 46\,656$, $3 \times 5^6 = 46\,875$

(ii) $5 \times 6^6 = 233\,280$, $2 \times 7^6 = 235\,298$

13(f) 0·693

16(c) $n = 9$

18(b) In part (a), put $f(x) = x^{-2}$, $a = (n-1)$ and $b = n$.

20(a) Put $f(x) = x - 1 - \log x$.
Show that $y = f(x)$ is concave up for all $x > 0$.
Show that $f(x)$ has a global minimum at $x = 1$.

(c) Put $p_r = \frac{x_r}{x_1+x_2+\ldots+x_n}$ so that $\sum_{r=1}^{n} p_r = 1$.
Also $np_r = \frac{x_r}{\mu}$, where $\mu = \frac{x_1+x_2+\ldots+x_n}{n}$.

Review Exercise **2G** (Page 93)

1(a) If the opposite angles of a quadrilateral are supplementary, then it is cyclic. True. **(b)** If two numbers have an even sum, then they are both odd. False. **(c)** Every parallelogram is a rhombus. False.

2(a) Not all mathematicians are intelligent.

(b) Suzie does not like Physics or she does not like Chemistry. **(c)** I am on vacation and I am working.

3(a) If I don't have two wheels, then I am not a bicycle. **(b)** If the last digit of a number is 6, then the number is even. **(c)** If a quadrilateral does not have four equal sides, then it is not a square.

4(a) If a number is divisible by 2, then it is even. Conversely, if a number is even, then it is divisible by 2. **(b)** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. Conversely, the diagonals of a parallelogram bisect each other. **(c)** If $\exists\ c \in \mathbf{Z}$ such that $a = bc$, then a is divisible by b. Conversely, if a is divisible by b, then $\exists\ c \in \mathbf{Z}$ such that $a = bc$.

16(b) $\frac{1}{2}$

23(c) From part (b), $\sqrt[n]{n}$ is not an integer, so it is not rational.

Chapter Three

Exercise **3A** (Page 100)

1(a) $\operatorname{cis} 5\theta$ **(b)** $\operatorname{cis}(-3\theta)$ **(c)** $\operatorname{cis} 8\theta$ **(d)** $\operatorname{cis}(-\theta)$ **(e)** $\operatorname{cis} 7\theta$ **(f)** $\operatorname{cis}(-6\theta)$

2(a) $\operatorname{cis} 7\theta$ **(b)** $\operatorname{cis}(-5\theta)$

3(a) -1 **(b)** $-i$ **(c)** $-\frac{\sqrt{3}}{2}+\frac{1}{2}i$ **(d)** $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ **(e)** $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$ **(f)** $\frac{1}{2}+\frac{\sqrt{3}}{2}i$

4(a) $\sqrt{2}\operatorname{cis}\frac{\pi}{4}$ **(b)** $256+256i$

5(a) $2\operatorname{cis}\frac{\pi}{3}$ **(b)** $1024-1024\sqrt{3}\,i$

6(a) $2,\ \frac{5\pi}{6}$

7(a) $2\operatorname{cis}\left(-\frac{\pi}{6}\right)$ **(b)** $128\operatorname{cis}\frac{5\pi}{6}$ **(c)** $-64\sqrt{3}+64i$

8(a) $2\operatorname{cis}\left(-\frac{2\pi}{3}\right)$ **(b)** $32\operatorname{cis}\frac{2\pi}{3}$ **(c)** $-16+16\sqrt{3}\,i$

9(a) $2\operatorname{cis}\left(-\frac{\pi}{4}\right)$ **(b)** $2^{22}i$

12(a)(i) 6 **(ii)** 3 **(b)** $-64,\ 8i$

13(b) $n=2,\ 6,\ 10,\ \ldots$

15(b) -2^{2n}

Exercise **3B** (Page 102)

5(b) $\frac{8}{15}$

8(c) $b=2,\ c=-1$

(d) No, since $\sin\frac{\pi}{10}=\sin\frac{9\pi}{10}$ and $\sin\frac{13\pi}{10}=\sin\frac{17\pi}{10}$

(e) $\sin\frac{\pi}{10}=\frac{\sqrt{5}-1}{4}$, $\sin\frac{3\pi}{10}=\frac{\sqrt{5}+1}{4}$

9(a) $64\cos^7\theta-112\cos^5\theta+56\cos^3\theta-7\cos\theta$

(c)(i) $\frac{7}{4}$ **(ii)** $\frac{21}{16}$

10(b) $\theta=0,\ \frac{\pi}{6},\ \frac{5\pi}{6},\ \pi,\ \frac{7\pi}{6},\ \frac{11\pi}{6}$

13(b) $z=\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$ or $\frac{3}{5}\pm\frac{4}{5}i$

14(a) $8(1-10s^2+24s^4-16s^6)$

(b) $x=2\sin\frac{n\pi}{8}$ for $n=\pm1,\pm2,\pm3$

Exercise **3C** (Page 108)

1(a) $\operatorname{cis}0=1$, $\operatorname{cis}\frac{2\pi}{3}=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$,

$\operatorname{cis}\left(-\frac{2\pi}{3}\right)=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ **(d)(i)** 1 **(ii)** 0

2(a) $z=\pm1,\ \frac{1}{2}+\frac{\sqrt{3}}{2}i,\ \frac{1}{2}-\frac{\sqrt{3}}{2}i,\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i,$

$-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ **(e)** $(z^2-z+1)(z^2+z+1)$

3(a) $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i,\ \frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i,\ -\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i,\ -\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$

(b) $(z^2-\sqrt{2}\,z+1)(z^2+\sqrt{2}\,z+1)$

4(a) $i,\ -i,\ \frac{\sqrt{3}}{2}+\frac{1}{2}i,\ \frac{\sqrt{3}}{2}-\frac{1}{2}i,\ -\frac{\sqrt{3}}{2}+\frac{1}{2}i,\ -\frac{\sqrt{3}}{2}-\frac{1}{2}i$

5(a) $\operatorname{cis}\left(-\frac{7\pi}{10}\right),\ \operatorname{cis}\left(-\frac{3\pi}{10}\right),\ \operatorname{cis}\frac{\pi}{10},\ \operatorname{cis}\frac{\pi}{2}=i,\ \operatorname{cis}\frac{9\pi}{10}$

(b) $\operatorname{cis}\left(-\frac{5\pi}{8}\right),\ \operatorname{cis}\left(-\frac{\pi}{8}\right),\ \operatorname{cis}\frac{3\pi}{8},\ \operatorname{cis}\frac{7\pi}{8}$

(c) $1+\sqrt{3}\,i,\ -1-\sqrt{3}\,i,\ \sqrt{3}-i,\ -\sqrt{3}+i$

(d) $2\operatorname{cis}\left(-\frac{17\pi}{20}\right),\ 2\operatorname{cis}\left(-\frac{9\pi}{20}\right),\ 2\operatorname{cis}\left(-\frac{\pi}{20}\right),\ 2\operatorname{cis}\frac{7\pi}{20},$

$2\operatorname{cis}\frac{3\pi}{4}$

6(a) $-1,\ \operatorname{cis}\frac{\pi}{5},\ \operatorname{cis}\left(-\frac{\pi}{5}\right),\ \operatorname{cis}\frac{3\pi}{5},\ \operatorname{cis}\left(-\frac{3\pi}{5}\right)$

7(a) $1,\ \operatorname{cis}\left(\pm\frac{2\pi}{7}\right),\ \operatorname{cis}\left(\pm\frac{4\pi}{7}\right),\ \operatorname{cis}\left(\pm\frac{6\pi}{7}\right)$

(c) $(z-1)\times(z^2-2\cos\frac{2\pi}{7}\,z+1)\times$

$(z^2-2\cos\frac{4\pi}{7}\,z+1)\times(z^2-2\cos\frac{6\pi}{7}\,z+1)$

8(a)(i) $1,\ \operatorname{cis}\frac{2\pi}{5},\ \operatorname{cis}\left(-\frac{2\pi}{5}\right),\ \operatorname{cis}\frac{4\pi}{5},\ \operatorname{cis}\left(-\frac{4\pi}{5}\right)$

9(a) $\operatorname{cis}\frac{2k\pi}{9}$ for $k=-4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4$

11(b) $\beta=\rho^3+\rho^5+\rho^6$ **(c)** $a=1, b=2$

12(a)(i) $\cos4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$,

$\sin4\theta=4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta$

(b) $\pm\frac{1}{\sqrt{2}}(3+i)\,,\pm\frac{1}{\sqrt{2}}(1-3i)$

14(a) 3, when k is a multiple of 3, 0 otherwise.

(b) $(1+\omega)^n=\sum_{r=0}^{n}\binom{n}{r}\omega^r$ and

$(1+\omega^2)^n=\sum_{r=0}^{n}\binom{n}{r}\omega^{2r}$

15(a) The roots are $-i\cot\frac{(2k-1)\pi}{4n}$ for $k=1,\ 2,\ 3,\ \ldots,\ 2n$.

Exercise **3D** (Page 115)

1(a) $e^{3i\theta}$ **(b)** $e^{-6i\theta}$ **(c)** $e^{8i\theta}$ **(d)** $e^{10i\theta}$

2(a) $e^{-i\theta}$ **(b)** $e^{3i\theta}$ **(c)** $e^{2i\theta}$ **(d)** $e^{20i\theta}$

3(a) $2e^{i\pi/2}$ **(b)** $\sqrt{2}\,e^{i\pi/4}$ **(c)** $6e^{i\pi}$ **(d)** $2e^{2i\pi/3}$ **(e)** $3\sqrt{2}\,e^{-3i\pi/4}$ **(f)** $4e^{-i\pi/6}$

4(a) -5 **(b)** $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ **(c)** $-4i$ **(d)** $-\sqrt{3}+i$ **(e)** $2-2i$ **(f)** $-2\sqrt{3}-6i$

5(a) $2\sqrt{2}\,e^{i\pi/12}$ **(b)** $\frac{1}{\sqrt{2}}e^{-7i\pi/12}$ **(c)** $8\sqrt{2}\,e^{3i\pi/4}$ **(d)** $2\sqrt{2}\,e^{11i\pi/12}$

6(a) -64 **(b)** $4-4i$ **(c)** $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ **(d)** $-648-648\sqrt{3}\,i$

11(a) n is divisible by 4 **(b)** $n=2,6,10,\ldots$ **(c)** n is divisible by 6 **(d)** $n=\frac{3}{2},\frac{9}{2},\frac{15}{2},\ldots$

12(b)(i) $2i\sin3\theta$ **(ii)** $4\cos^2\theta$ **(iii)** $-8i\sin^3\theta$ **(iv)** $2\cos\theta\,(2\cos\theta+1)$ **(v)** $2\,(\sin3\theta-\sin\theta)\,i$

13(a) $\cos\theta=\frac{1}{2}\left(e^{i\theta}+e^{-i\theta}\right)$ **(d)** $\tan\theta$, $\cot\theta$ and $\operatorname{cosec}\theta$ are odd, $\sec\theta$ is even

14(a) z^2+4 **(b)** z^2-z+1 **(c)** z^3+8 **(d)** z^4+4

15(b) $e^{i(\phi-\theta)}=1$ with $-2\pi<\phi-\theta<2\pi$.
This has only one solution, which is $\phi-\theta=0$.
(c) If two complex numbers are equal, then they represent the same point in the Argand diagram. Hence the moduli are equal and the principal arguments are equal.

Exercise **3E** (Page 120)

1(a) $2i = 2e^{i\pi/2}$ **(b)** $2i = 2e^{i\left(\frac{\pi}{2}+2k\pi\right)}$, $k \in \mathbf{Z}$
(d) $z = \sqrt{2}e^{-3i\pi/4}, \sqrt{2}e^{i\pi/4}$
(e) $z = -1-i, 1+i$
2(a) $-1 = e^{i\pi}$ **(b)** $-1 = e^{i(\pi+2k\pi)}$, $k \in \mathbf{Z}$
(d) $z = e^{-3i\pi/4}, e^{-i\pi/4}, e^{i\pi/4}, e^{3i\pi/4}$
(e) $z = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i, \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$
3(a) $i = e^{-i\pi/2}$ **(b)** $-i = e^{-i(\pi/2+2k\pi)}$, $k \in \mathbf{Z}$
(d) $z = e^{i\pi/2}, e^{-i\pi/6}, e^{-5i\pi/6}$
(e) $z = i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i$
7(c) $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}, \cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}$
9(a) $\cos^6\theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$
10(a) $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$,
$\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$ **(c)** $\frac{47}{480}$
11(c) $z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ or $\frac{3}{5} \pm \frac{4}{5}i$
12 $a = \frac{1}{2}\ln 2, b = -\frac{\pi}{4}$
13(a) $2\cos A\cos B$
14(b) $\tan\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{i(e^{i\theta} + e^{-i\theta})}$
15(a) $\frac{z^{n+1}-z}{z-1}$
17(a) $z = e^{\frac{2k\pi}{2n+1}i}$ for $k = 0, 1, 2, \ldots, 2n$

Review Exercise **3F** (Page 123)

1(a) $\operatorname{cis} 7\theta$ **(b)** $\operatorname{cis} 6\theta$
2 -1
3(a) $\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$ **(b)** $-64 + 64i$
4(a) 2^{13} **(b)(ii)** n is even or a multiple of 3
5(a) $\cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$,
$6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$
6(a) $z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$,
$z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4}$
7(b) -43
8 $z = 2e^{-i\pi/2}, 2e^{i\pi/6}, 2e^{5i\pi/6}$
9(a) $\sqrt{2}\operatorname{cis}\frac{k\pi}{12}$ for $k = -7, 1, 9$
(b) $\operatorname{cis}\frac{k\pi}{12}$ for $k = -11, -7, -3, 1, 5, 9$
10(a) $8e^{i\pi/6}$ **(c)** $2e^{-11i\pi/18}, 2e^{i\pi/18}, 2e^{13i\pi/18}$
11(d) $-7\cos 3\theta + \frac{7}{5}\cos 5\theta - \frac{1}{7}\cos 7\theta + C$
12(b) $x = \cos\frac{\pi}{10}, \cos\frac{3\pi}{10}, \cos\frac{7\pi}{10}, \cos\frac{9\pi}{10}$ **(e)** $\frac{\sqrt{5}+1}{4}$
14(a) $\operatorname{cis}\frac{k\pi}{7}$ for $k = -5, -3, -1, 1, 3, 5, 7$
15(a) $e^{ki\pi/5}$ for $k = -4, -2, 0, 2, 4$ **(b)(i)** -1 and $\sqrt{5}$
18(c) $-\frac{1}{2}$ **(e)** $\frac{1+\sqrt{5}}{4}$

Chapter Four

Exercise **4A** (Page 126)

1(a) $\frac{1}{4}e^{4x} + C$ **(b)** $-\frac{1}{5}\cos 5x + C$ **(c)** $2\tan\frac{1}{2}x + C$
(d) $\frac{1}{3}\ln|3x-4| + C$ **(e)** $4\sqrt{x} + C$ **(f)** $\frac{3^x}{\ln 3} + C$
2(a) $-\frac{1}{2(2x-1)} + C$ **(b)** $\sin^{-1}\frac{x}{5} + C$
(c) $\frac{1}{3}e^{x^3} + C$ **(d)** $\frac{1}{3}\tan^{-1}\frac{x}{3} + C$
(e) $2\ln(x^2+x+1) + C$ **(f)** $\frac{1}{5}(x^2+1)^5 + C$
3(a) $2(e^2-1)$ **(b)** $\frac{\sqrt{3}}{2}$ **(c)** $\frac{\pi}{8}$ **(d)** $\frac{\pi}{4}$ **(e)** $\ln\frac{8}{5}$ **(f)** $\frac{1}{16}$
4(a) $e^{\frac{1}{x}} + C$ **(b)** $\frac{1}{3}\ln(1+\sin 3x) + C$
(c) $\frac{1}{2}\tan x^2 + C$ **(d)** $\frac{5^{2x}}{2\ln 5} + C$
(e) $\ln|x + \tan x| + C$ **(f)** $\sin^{-1}e^x + C$
5(a) -20 **(b)** $\frac{1}{3}\ln 2$ **(c)** $\frac{\pi}{3\sqrt{3}}$ **(d)** $\frac{1}{2}\ln\frac{e^2+1}{2}$ **(e)** $\frac{\pi}{18}$
(f) $\ln 2$
6 $\ln 2$
7 $-\frac{1+\ln x}{x} + C$
9 $\frac{\pi}{4} - \frac{2}{3}$

Exercise **4B** (Page 128)

1(a) $x + \ln|x-1| + C$ **(b)** $x - 2\ln|x+1| + C$
(c) $x + 2\ln|x-1| + C$
2(a) $1 - \ln 4$ **(b)** $1 - \frac{1}{4}\ln 5$ **(c)** $\pi - 1$
3(a) $\frac{\pi}{3} - \frac{1}{2}$ **(b)** $\frac{\pi}{4} + \ln 2$ **(c)** $\frac{1}{4}(\pi - \ln 4)$
(d) $\frac{\pi}{8} + \frac{1}{2}\ln 2$
4(a) $\dfrac{1}{\sqrt{x^2+a^2}}$ **(b)** $\log(x + \sqrt{x^2+a^2}) + C$
(c)(i) $\log\left(x + \sqrt{x^2+3}\right) + C$ **(ii)** $2\log 3$
5(a) $\frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1) + C$
(b) $\frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1| + C$
(c)(i) $\frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + C$
(ii) $\frac{x^3}{3} - x + \tan^{-1}x + C$ **(iii)** $x - \ln(1+e^x) + C$
(iv) $\frac{2}{3}(x-4)\sqrt{2+x} + C$ **(v)** $-\frac{2}{3}(2+x)\sqrt{1-x} + C$
(vi) $\frac{1}{2}x^2 - 2\ln(x^2+4) + C$
6(a) $\ln(e + e^{-1})$ **(b)** $\frac{1}{2}\ln\frac{e^2+1}{2}$ **(c)** $\frac{\pi}{12} + \ln 2$
7(a) $\frac{1}{2}x^2 + \ln|x+1| + C$ **(b)** $\frac{1}{3}x^3 + 3\ln|x-2| + C$
(c) $x + \ln(1+x^2) + C$
8(a) $\dfrac{1}{\sqrt{x^2-a^2}}$ **(b)** $\log(x + \sqrt{x^2-a^2}) + C$
(c)(i) $\log\left(x + \sqrt{x^2-5}\right) + C$
(ii) $\log\left(\frac{3+\sqrt{5}}{1+\sqrt{5}}\right) = \log\left(\frac{1+\sqrt{5}}{2}\right)$
9 $2\ln(1+\sqrt{x}) + C$

Exercise **4C** (Page 132)

1(a) $\frac{1}{5}(x^2+1)^5+C$ **(b)** $\frac{1}{7}(1+x^3)^7+C$
(c) $-\frac{2}{1+x^3}+C$ **(d)** $\frac{1}{2(3-x^2)^4}+C$
(e) $\sqrt{x^2-2}+C$ **(f)** $\frac{1}{2}\sqrt{1+x^4}+C$
2(a) $\frac{-1}{2\sin^2 x}+C$ **(b)** $\frac{-1}{1+\tan x}+C$ **(c)** $\frac{1}{3}(\ln x)^3+C$
(d) $2\sin\sqrt{x}+C$ **(e)** $\frac{1}{2}\tan^{-1}x^2+C$
(f) $\frac{1}{3}\sin^{-1}x^3+C$
3(a) $\frac{7}{4}$ **(b)** $2-\sqrt{3}$ **(c)** $3(\sqrt{3}-\sqrt{2})$
(d) $\frac{1}{5}$ **(e)** $\frac{1}{3}$ **(f)** 2
4(a) $-\frac{1}{42}$ **(b)** Begin by writing $x=(x-1)+1$.
5(a) $\frac{2}{15}(3x-2)(1+x)\sqrt{1+x}+C$
(b) $2\left(1+\sqrt{x}-\ln(1+\sqrt{x})\right)+C$
(c) $4\left(x^{\frac{1}{4}}-\frac{1}{2}\sqrt{x}+\frac{1}{3}x^{\frac{3}{4}}-\ln(1+x^{\frac{1}{4}})\right)+C$
(d) $\tan^{-1}\sqrt{e^{2x}-1}+C$
6(a) $\frac{1}{9}$ **(b)** $\frac{128}{15}$ **(c)** $4+10\ln\frac{5}{7}$ **(d)** $\frac{\pi}{12}$
7(a) $2\tan^{-1}\left(\sqrt{x}\right)+C$ **(b)** $\frac{2}{3}(x-2)\sqrt{x+1}+C$
8(a) $\frac{x}{\sqrt{1+x^2}}+C$ **(b)** $2\sin^{-1}\frac{x}{2}-\frac{1}{2}x\sqrt{4-x^2}+C$
(c) $-\frac{\sqrt{25-x^2}}{25x}+C$ **(d)** $-\frac{1}{x}\sqrt{1+x^2}+C$
9(a) $\frac{2}{3}$ **(b)** Begin by writing $x^3=x(x^2+1)-x$.
10(b) The region is half a segment.
11(b) $\frac{\pi}{4}$
12(b)(ii) $\frac{\pi^2}{4}$
13(b) Begin by writing $x^2=1-(1-x^2)$.
14(a) $\tan^{-1}\sqrt{x^2-1}+C_1$ **(b)** $\tan^{-1}\sqrt{x^2-1}+C_2$
15(a) $\frac{\sqrt{3}}{8}-\frac{\sqrt{\epsilon(4+\epsilon)}}{4(2+\epsilon)}$ **(b)** $\frac{\sqrt{3}}{8}$

Exercise **4D** (Page 139)

1(a) $\frac{1}{x-1}-\frac{1}{x+1}$ **(b)** $\frac{1}{3(x-4)}-\frac{1}{3(x-1)}$ **(c)** $\frac{2}{x-3}+\frac{2}{x+3}$
(d) $\frac{2}{x-2}-\frac{1}{x-1}$ **(e)** $\frac{1}{5(x-2)}+\frac{4}{5(x+3)}$ **(f)** $\frac{1}{x-1}+\frac{2-x}{x^2+3}$
2(a) $\ln|x-4|-\ln|x-2|+C$
(b) $2\ln|x+1|-2\ln|x+3|+C$
(c) $4\ln|x-2|-\ln|x-1|+C$
(d) $3\ln|x-1|-\ln|x+3|+C$
(e) $\ln|x+1|+\ln|2x+3|+C$
(f) $2\ln|x+1|+3\ln|2x-3|+C$
3(a) $\frac{1}{4}\ln\frac{3}{2}$ **(b)** $\ln 2$ **(c)** $\ln\frac{14}{3}$ **(d)** $\frac{1}{2}\ln 2$
4(a) $\ln|x-2|-2\tan^{-1}x+C$
(b) $\ln|2x+1|-\frac{1}{2}\ln(x^2+3)+C$
(c) $\tan^{-1}x+3\ln|x|-\ln(x^2+1)+C$
5(a) $\frac{\pi}{4}-\ln\frac{3}{2}$ **(b)** $\pi+\ln 2$ **(c)** $\ln 4-\frac{3}{2}\ln 3$
6(a) $5\ln|x-1|+7\ln|x-2|-12\ln|2x-3|+C$
(b) $\frac{3}{2}\ln|x|-5\ln|x-2|+\frac{7}{2}\ln|x-4|+C$
7(a) $\frac{5}{3}\ln 3-\ln 2$ **(b)** $2\ln 3-8\ln 2$
8(a)(i) $A=2,\ B=1,\ C=-3$
(ii) $2x+\ln|x-1|-3\ln|x+2|+C$
(b)(i) $x+\ln|x-2|-2\ln|x+1|+C$
(ii) $3x+2\ln|x+4|+\ln|x-5|+C$
9(a)(i) $A=1,\ B=-1,\ C=2,\ D=-1$
(ii) $\ln 3+\ln 2-\frac{1}{2}$ **(b)** $12+\ln 2$
10(a)(i) $A=3,\ B=12,\ C=2$
(ii) $3x+12\ln|x-2|-\frac{2}{x-2}+C$
(b)(i) $A=23,\ B=10,\ C=-23,\ D=13$
(ii) $23\ln|\frac{x-1}{x-2}|-\frac{10}{x-1}-\frac{13}{x-2}+C$
12(a) $A=0,\ B=-1,\ C=0,\ D=2$
13(a) $x+\ln|x-1|-\ln|x+1|+C$
(b) $x+2\ln|x-1|-\ln|x|+C$
(c) $x-\tan^{-1}x+\ln|x|-\frac{1}{2}\ln(x^2+1)+C$
(d) $x+9\ln|x-3|-4\ln|x-2|+C$
(e) $\frac{1}{2}x^2-x+5\ln|x|-4\ln|x+1|+C$
(f) $\frac{1}{3}x^3+\frac{3}{2}x^2+7x+16\ln|x-2|-\ln|x-1|+C$

Exercise **4E** (Page 143)

1(a) $\frac{1}{3}\tan^{-1}\frac{x}{3}+C$ **(b)** $\sin^{-1}\frac{x}{3}+C$
(c) $\frac{1}{6}\ln\left|\frac{x-3}{x+3}\right|+C$ **(d)** $\frac{1}{6}\ln\left|\frac{3+x}{3-x}\right|+C$
(e) $\ln\left|x+\sqrt{9+x^2}\right|+C$ **(f)** $\ln\left|x+\sqrt{x^2-9}\right|+C$
2(a) $\tan^{-1}(x+2)+C$ **(b)** $\frac{1}{4}\tan^{-1}\left(\frac{x-2}{4}\right)+C$
(c) $\sin^{-1}\frac{x-4}{5}+C$ **(d)** $\sin^{-1}\frac{x+4}{6}+C$
(e) $\ln\left|x-3+\sqrt{x^2-6x+13}\right|+C$
(f) $\frac{1}{2}\ln\left|x+1+\sqrt{x^2+2x+\frac{3}{2}}\right|+C$
3(a) $\frac{\pi}{8}$ **(b)** π **(c)** $\frac{\pi}{6}$ **(d)** $\frac{\pi}{2}$ **(e)** $\ln 3$ **(f)** $\ln 3$
4(a) $\ln(x^2+2x+2)-\tan^{-1}(x+1)+C$
(b) $\frac{1}{2}\ln(x^2+2x+10)-\frac{1}{3}\tan^{-1}\frac{x+1}{3}+C$
(c) $-\sqrt{6x-x^2}+3\sin^{-1}\frac{x-3}{3}+C$
(d) $-\sqrt{4-2x-x^2}+2\sin^{-1}\frac{x+1}{\sqrt{5}}+C$
(e) $\sqrt{x^2+2x+10}-\ln\left|x+1+\sqrt{x^2+2x+10}\right|+C$
(f) $\sqrt{x^2-2x-4}+4\ln\left|x-1+\sqrt{x^2-2x-4}\right|+C$
5(a) $\frac{1}{2}\ln 2+\frac{\pi}{8}$ **(b)** $\frac{1}{4}(3\pi-\ln 4)$ **(c)** $\ln 2-\frac{\pi}{4}$ **(d)** $2-\sqrt{3}-\frac{\pi}{6}$ **(e)** $3\ln(3+2\sqrt{2})-4\sqrt{2}$
(f) $\ln\left(1+\sqrt{\frac{2}{3}}\right)+\sqrt{6}-1$
6(a) $\sin^{-1}x-\sqrt{1-x^2}+C$
(b) $\sqrt{6+x-x^2}+\frac{5}{2}\sin^{-1}\frac{2x-1}{5}+C$
(c) $\sqrt{x^2-1}-\ln\left|x+\sqrt{x^2-1}\right|+C$
7(a) $\frac{\pi}{3}+\sqrt{3}-2$ **(b)** $3\sin^{-1}\frac{1}{3}$
(c) $2\sqrt{2}-\sqrt{3}+\ln\left(\frac{2+\sqrt{3}}{3+2\sqrt{2}}\right)$
8(a) $\frac{x}{\sqrt{4x-x^2}}$ is undefined at $x=0$.

Exercise **4F** (Page 148)

1(a) $e^x(x-1)+C$ **(b)** $-e^{-x}(x+1)+C$
(c) $\frac{1}{9}e^{3x}(3x+2)+C$ **(d)** $x\sin x+\cos x+C$
(e) $-\frac{1}{2}(x-1)\cos 2x+\frac{1}{4}\sin 2x+C$
(f) $(2x-3)\tan x+2\ln(\cos x)+C$
2(a) π **(b)** $\frac{\pi}{2}-1$ **(c)** $\frac{\pi}{4}-\frac{1}{2}\ln 2$ **(d)** $\frac{1}{4}(e^2+1)$
(e) e^{-1} **(f)** $1+e^{-2}$
3(a) $x(\ln x-1)+C$ **(b)** $2x(\ln x-1)+C$
(c) $x\cos^{-1}x-\sqrt{1-x^2}+C$
4(a) $\frac{\pi}{4}-\frac{1}{2}\ln 2$ **(b)** 1 **(c)** $\frac{1}{2}$
5(a) $\frac{1}{4}x^2(2\ln x-1)+C$ **(b)** $\frac{1}{9}x^3(3\ln x-1)+C$
(c) $-\frac{1}{x}(\ln x+1)+C$
6(a) $(2-2x+x^2)e^x+C$
(b) $x^2\sin x+2x\cos x-2\sin x+C$
(c) $x(\ln x)^2-2x\ln x+2x+C$
7(a) $-\frac{1}{42}$ **(b)** $\frac{4}{15}(1+\sqrt{2})$ **(c)** $\frac{128}{15}$
8(a) $\frac{1}{2}e^x(\cos x+\sin x)+C$
(b) $-\frac{1}{2}e^{-x}(\cos x+\sin x)+C$
9(a) $\frac{1}{5}(e^\pi-2)$ **(b)** $\frac{1}{5}(e^{\frac{\pi}{4}}+2)$
10(a) $\frac{1}{2\sqrt{3}}(\pi-\sqrt{3})$ **(b)** $\frac{\sqrt{3}\pi}{2}$ **(c)** $\pi-2$
12(a) $\frac{1}{4}x^2(2\ln x-1)+C$
(b) $\frac{1}{4}x^2\left(2(\ln x)^2-2\ln x+1\right)+C$
14(a) $\frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}(\frac{x}{a})\right)+C$
(b) $x\ln\left|x+\sqrt{x^2+a^2}\right|-\sqrt{x^2+a^2}+C$
(c) $x\ln\left|x+\sqrt{x^2-a^2}\right|-\sqrt{x^2-a^2}+C$
15(a) $\frac{1}{32}(\sin 4x-4x\cos 4x+8x\cos 2x-4\sin 2x)+C$
(b) $\frac{1}{18}(3x\sin 3x+\cos 3x+9x\sin x+9\cos x)+C$
(c) $\frac{1}{20}e^x(\sin 3x-3\cos 3x+5\sin x-5\cos x)+C$
16(a) $\frac{1}{48}(3\sqrt{3}-\pi)$ **(b)** $\frac{1}{12}(\pi+2\ln 2-2)$

Exercise **4G** (Page 154)

1(a) $\sin x+C$ **(b)** $-\cos x+C$ **(c)** $-\ln|\cos x|+C$
(d) $\ln|\sin x|+C$
2(a) $\frac{1}{3}\sin^3 x+C$ **(b)** $-\frac{1}{3}\cos^3 x+C$
(c) $\frac{1}{3}\cos^3 x-\cos x+C$ **(d)** $\sin x-\frac{1}{3}\sin^3 x+C$
(e) $\frac{1}{5}\sin^5 x-\frac{2}{3}\sin^3 x+\sin x+C$
(f) $\frac{1}{4}\sin^4 x-\frac{1}{6}\sin^6 x+C$
3(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{12}$ **(c)** $\frac{\pi}{8}$
4(a) $\tan x+C$ **(b)** $\tan x-x+C$
(c) $\frac{1}{3}\tan^3 x+\tan x+C$ **(d)** $\frac{1}{3}\tan^3 x-\tan x+x+C$
5(a) $\frac{1}{4}$ **(b)** $\frac{11}{24}$ **(c)** $\frac{9}{64}$ **(d)** $\frac{53}{480}$ **(e)** $\frac{4}{15}$ **(f)** $\frac{7}{60\sqrt{2}}$
6(a) $\frac{1}{32}(\sin 4x+8\sin 2x+12x)+C$
(b) $\frac{1}{32}(\sin 4x-8\sin 2x+12x)+C$
(c) $\frac{1}{1024}(24x-8\sin 4x+\sin 8x)+C$
8(a) 1 **(b)** $\frac{1}{3}\ln 2$ **(c)** $\frac{1}{2}(\tan^{-1}2+\tan^{-1}\frac{1}{2})=\frac{\pi}{4}$
9(a) $\frac{\pi}{4}$ **(b)** $\frac{2}{15}(1+\sqrt{2})$ **(c)** $\frac{\pi}{16}$
10(a) $\frac{1}{2}\sin^2 x+C_1$ **(b)** $-\frac{1}{4}\cos 2x+C_2$
11(a) $\frac{1}{2}$ **(b)** $\frac{\sqrt{3}}{4}$
12(a) $\frac{1}{2}$ **(b)** $\frac{4}{3}$
13(a) $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x+C$
(b) $-\frac{1}{8}\cos 4x+\frac{1}{4}\cos 2x+C$
(c) $\frac{1}{16}\sin 8x+\frac{1}{8}\sin 4x+C$
14(a) $\frac{1}{4}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{3}{8}$
15(a) $\tan\frac{x}{2}+C$ **(b)** $\ln\left|\frac{\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right|+C$
(c) $\frac{1}{5}\ln\left|\frac{1+2\tan\frac{x}{2}}{2-\tan\frac{x}{2}}\right|+C$
17 $\frac{1}{3}\sin 3\theta+\sin^3\theta+C$

Exercise **4H** (Page 158)

3(b) $\frac{1}{2}(e^2-1)$
4(c) $\frac{8}{15}$
6(b) $(\frac{\pi}{2})^6-30(\frac{\pi}{2})^4+360(\frac{\pi}{2})^2-720$
7(b) $\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2}\ln(1+x^2)+C$
8(b) $I_0=1,\ I_4=\frac{128}{315}$
9(b) $u_4=\frac{243}{1540}$
10(b) $J_2=-\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x+C$
12(d) $\frac{1}{15}(14\sqrt{2}-16)$
13(d) $\frac{1}{9}\Big((1+x^2)^4+\frac{8}{7}(1+x^2)^3+\frac{48}{35}(1+x^2)^2$
$+\frac{192}{105}(1+x^2)+\frac{384}{105}\Big)$
15(d) $J_n=\dfrac{2n}{2n+3}J_{n-1}$
16(d) $I_5=\frac{1}{4}(2\ln 2-1)$
17(b) $\frac{\pi}{3}-\frac{3\sqrt{3}}{4}$

Exercise **4I** (Page 161)

1(a) $\frac{1}{36}$ **(b)** π **(c)** $\ln\frac{12}{5}$ **(d)** $2-2\ln 3$ **(e)** $2\sqrt{2}-1$
(f) $\frac{\pi}{18}$
2(a) $\sqrt{1+x^2}+C$ **(b)** $\tan^{-1}x+\frac{1}{2}\ln(1+x^2)+C$
(c) $-\frac{1}{5}\cos^5 x+C$ **(d)** $\ln\left|\frac{2x+1}{x+1}\right|+C$
(e) $\frac{1}{4}x^4\ln x-\frac{1}{16}x^4+C$ **(f)** $\frac{1}{6}\cos^3 2x-\frac{1}{2}\cos 2x+C$
(g) $\frac{1}{4}\tan^{-1}\frac{x+3}{4}+C$ **(h)** $x\sin 3x+\frac{1}{3}\cos 3x+C$
(i) $\frac{2}{3}(x-8)\sqrt{4+x}+C$
4(a) $A=-\frac{2}{3},\ B=\frac{2}{3},\ C=-\frac{1}{3}$
6(a) $2\sqrt{3}$ **(b)** $\frac{24\sqrt{3}}{5}$
8(a) $A=0,\ B=-2,\ C=0,\ D=2$ **(b)** $\frac{\pi}{2}-1$
10 $\frac{1}{2}a^2\sin^{-1}\frac{x}{a}+\frac{1}{2}x\sqrt{a^2-x^2}+C$
11(b) $\frac{1}{10}(\pi+\ln\frac{27}{16})$
12(a) $P=2,\ Q=-1$
(b) $2x-\ln|3\sin x+2\cos x-1|+C$
14(b) $6-2e$
17(c) If $pq\le 0$, then $0\in[p,q]$ and u is undefined at $t=0$.

Review Exercise **4J** (Page 164)

1(a) $\frac{1}{2}e^{x^2}+C$ **(b)** $\frac{3}{2}\ln(x^2+1)+C$ **(c)** $\frac{1}{12}(1+x^2)^6+C$ **(d)** $-\frac{1}{4}\cos^4 x+C$ **(e)** $3\ln|x-3|+\ln|x+1|+C$ **(f)** $-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+C$

2(a) $\tan x-x+C$ **(b)** $\frac{2}{3}(3+x)^{\frac{3}{2}}-6(3+x)^{\frac{1}{2}}+C$ **(c)** $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$ **(d)** $3x\sin\frac{1}{3}x+9\cos\frac{1}{3}x+C$ **(e)** $x+\ln|x+1|+C$ **(f)** $2\ln|x|+\frac{1}{2}\ln(x^2+1)+C$

3(a) $\frac{x}{4\sqrt{4-x^2}}+C$ **(b)** $\frac{1}{2}\ln\left(\frac{e^x-1}{e^x+1}\right)+C$ **(c)** $2\sqrt{x}-4\ln(2+\sqrt{x})+C$ **(d)** $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{1}{2}x\right)+C$

4(a) 6 **(b)** $\ln 6$ **(c)** $\frac{5}{24}$ **(d)** $2-\frac{3}{2}\ln\frac{7}{3}$ **(e)** $\frac{16}{105}$ **(f)** $\frac{1}{4}$

5(a) $\frac{\pi}{9}$ **(b)** $14-\frac{11\pi}{2}$ **(c)** $\frac{2}{15}$ **(d)** $\frac{\pi}{12}+\ln\frac{4}{3}$ **(e)** $\frac{8\pi}{3}-2\sqrt{3}$ **(f)** $\frac{2e+\pi}{4+\pi^2}$

6(a) $\frac{1}{2}\ln\frac{5}{3}$ **(b)** $\frac{\pi}{18}$ **(c)** π **(d)** $\ln(2+\sqrt{3})$

8(a) $I_n=\frac{1}{4}x^4(\ln x)^n-\frac{1}{4}nI_{n-1}$

9(b) $\frac{5\pi}{32}$

10(b) $(2n-1)J_n=2^{n-1}+2(n-1)J_{n-1}$ **(c)** $\frac{28}{15}$

11(b) $\frac{26}{15}$

Chapter Five

Exercise **5A** (Page 171)

1 The xy-plane with equation $z=0$, the xz-plane with equation $y=0$ and the yz-plane with equation $x=0$.

2(a) 2nd **(b)** 5th **(c)** 4th **(d)** 6th **(e)** 8th **(f)** 3rd

3(a) $(3,2,-1)$ **(b)** $(-5,2,5)$ **(c)** $(3,12,5)$ **(d)** $(8,2,12)$ **(e)** $(3,-1,1)$ **(f)** $(3,2,-5)$ **(g)** $(-3,2,5)$ **(h)** $(3,-2,5)$ **(i)** $(3,-2,-5)$

4(a) $A(2,0,0), B(2,2,0), C(2,2,2), D(2,0,2), O(0,0,0), P(0,2,0), Q(0,2,2), R(0,0,2)$ **(b)** $2\sqrt{2}$ **(c)** $2\sqrt{3}$ **(d)** $x=0, y=0, z=0, x=2, y=2, z=2$

5(a) $A(2,0,0), B(2,4,0), D(2,0,3), P(0,4,0), Q(0,4,3), R(0,0,3)$ **(b)** $2\sqrt{5}$ **(c)** $\sqrt{29}$ **(d)** $x=0, y=0, z=0, x=2, y=4, z=3$

6(a) $6\,\text{u}^2$ **(b)** $10\,\text{u}^3$

7(a) $OA=7, OB=7\sqrt{2}, AB=7\sqrt{3}$ **(b)** $OA^2+OB^2=AB^2$, so $\angle AOB=90°$.

8 $BC=2\sqrt{21}, MN=\sqrt{21}=\frac{1}{2}BC$

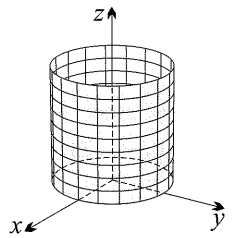
9(a) $M=(-8,6,18)$ **(b)** $X=(-7,-1,16)$ and $Y=(-9,13,20)$

11 $x=-1$

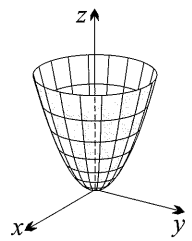
12(a) $AB=AC=2\sqrt{14}$

13(a) $3x+4y=12$ **(b)** $3x+6z=12$ with the xz-plane, $4y+6z=12$ with the yz-plane **(c)** a line **(d)** They intersect in a line.

15(a) z can take any real value. **(b)** circle $x^2+y^2=4$ **(c)** It is the curved surface of a cylinder with radius 2. The diagram shows the portion from $z=0$ to $z=4$. **(d)**



16(a) A sum of squares can never be negative. **(b)** circle $x^2+y^2=k$. **(c)** parabola $z=x^2$. **(d)**



Exercise **5B** (Page 177)

1(a) $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$, $2\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$ **(b)** $\begin{bmatrix} -4 \\ 0 \\ 13 \end{bmatrix}$, $-4\underset{\sim}{i} + 13\underset{\sim}{k}$

(c) $\begin{bmatrix} a \\ -2a \\ -3a \end{bmatrix}$, $a\underset{\sim}{i} - 2a\underset{\sim}{j} - 3a\underset{\sim}{k}$

2(a) $|\underset{\sim}{a}| = 5$, $\hat{\underset{\sim}{a}} = \frac{4}{5}\underset{\sim}{i} - \frac{3}{5}\underset{\sim}{k}$

(b) $|\underset{\sim}{a}| = 3$, $\hat{\underset{\sim}{a}} = \frac{1}{3}\underset{\sim}{i} + \frac{2}{3}\underset{\sim}{j} - \frac{2}{3}\underset{\sim}{k}$

3(a) $|\underset{\sim}{v}| = 3\sqrt{2}$, $\hat{\underset{\sim}{v}} = \frac{1}{6}\begin{bmatrix} -\sqrt{2} \\ -4\sqrt{2} \\ \sqrt{2} \end{bmatrix}$

(b) $|\underset{\sim}{v}| = 5\sqrt{2}$, $\hat{\underset{\sim}{v}} = \frac{1}{10}\begin{bmatrix} 5\sqrt{2} \\ 3\sqrt{2} \\ -4\sqrt{2} \end{bmatrix}$

4(a) $\begin{bmatrix} 5 \\ -10 \\ 23 \end{bmatrix}$ **(b)** $\sqrt{654}$ **(c)** $\begin{bmatrix} 19 \\ 28 \\ -38 \end{bmatrix}$ **(d)** $\sqrt{2589}$

5(a) $3\underset{\sim}{i} - 12\underset{\sim}{j} + 4\underset{\sim}{k}$ **(b)** $-3\underset{\sim}{i} + 12\underset{\sim}{j} - 4\underset{\sim}{k}$ **(c)** 13

6(a) $\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$ **(b)** $\begin{bmatrix} -8 \\ -3 \\ 2 \end{bmatrix}$ **(c)** $\sqrt{77}$

7 $\lambda_1 = 2, \lambda_2 = -4$

8 $\lambda_1 = -1, \lambda_2 = 3, \lambda_3 = -2$

9(a) $\overrightarrow{CD} = -3\overrightarrow{AB}$ **(b)** They are not parallel.

10 $\overrightarrow{BC} = 2\overrightarrow{AB}$, so $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel.

12 $-\underset{\sim}{i} - 14\underset{\sim}{j} - 6\underset{\sim}{k}$

13 $58°, 74°, 37°$

14(a) $\begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$ **(b)** $\begin{bmatrix} -1 \\ -7 \\ 4 \end{bmatrix}$

15(a) $-5\underset{\sim}{j} + 7\underset{\sim}{k}$ **(b)** $-24\underset{\sim}{i} + 7\underset{\sim}{j} - 5\underset{\sim}{k}$

16(a) $\overrightarrow{AG} = -\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ **(b)** $\sqrt{3}$ **(c)** $\frac{\sqrt{6}}{2}$

18(a) No, since $-\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = \underset{\sim}{0}$ **(b)** Yes

19 $(-10, -1, 3), (4, 9, -1), (6, 5, 7)$

Exercise **5C** (Page 183)

1(a) $12\sqrt{2}$ **(b)** -20

2(a) 5 **(b)** $x_1x_2 + y_1y_2 + z_1z_2$ **(c)** $a_1b_1 + a_2b_2 + a_3b_3$

4(a) 0 **(b)** They are perpendicular.

6(a) 126 **(b)** 360 **(c)** -48 **(d)** 78

7(a) $-21 \leq 8 \leq 21$ **(b)** $-20 \leq -18 \leq 20$

8(a) $4 \leq \sqrt{74} \leq 10$ **(b)** $\sqrt{10} \leq \sqrt{14} \leq 3\sqrt{10}$

10(a) $\lambda = 8$ **(b)** $\lambda = -\frac{5}{2}$ or 3

11 One such vector is $\underset{\sim}{i} + 7\underset{\sim}{j} + 3\underset{\sim}{k}$.

18 $\sqrt{23}$

19 $2\sqrt{7}$

Exercise **5D** (Page 188)

1(a) 3 **(b)** Both $\sqrt{6}$. **(c)** $\frac{\pi}{3}$

2(a) $\frac{2}{3}$ **(b)** $\frac{\sqrt{2}}{3}$

4(a) $44°$ **(b)** $87°$

6(a) $\frac{2}{9}\underset{\sim}{i} - \frac{2}{9}\underset{\sim}{j} - \frac{1}{9}\underset{\sim}{k}$ **(b)** $\begin{bmatrix} \frac{8}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}$

7(a) $\frac{8}{3\sqrt{5}}$ **(b)** $\frac{5}{3}$

8(a) $\overrightarrow{BA} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - 7\underset{\sim}{k}$, $\overrightarrow{BC} = 5\underset{\sim}{i} - 4\underset{\sim}{j} + \underset{\sim}{k}$

(c) $AB = \sqrt{62}$, $BC = \sqrt{42}$, $AC = \sqrt{104} = 2\sqrt{26}$

9(a) $\overrightarrow{AB} = \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ **(b)** $78°$

10 $117°49'$

11(b) $\frac{3}{2}$ u^2

12(a) $\overrightarrow{AP} = \begin{bmatrix} -7 \\ 1 \\ -2 \end{bmatrix}$, $\overrightarrow{AB} = \begin{bmatrix} -3 \\ -6 \\ 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$

(c) 7 units

13(a) 3 units **(b)** $\frac{5\sqrt{70}}{14}$ units

15 $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

17(a) $90°$ **(c)** $\frac{220}{3}$ u^2

18(a) $\overrightarrow{OP} = \overrightarrow{OA} + \lambda\overrightarrow{AB}$ **(b)** $\lambda = \frac{1}{12}$ **(c)** $\lambda = \frac{3}{10}$

19 $\lambda = 4$ or $-\frac{44}{65}$

20(b) $\lambda = \frac{2}{3}$ or $\frac{4}{3}$

21(a) $12\sqrt{10}$ u^2 **(b)** $\overrightarrow{OD} = 7\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$, $\overrightarrow{OE} = 6\underset{\sim}{i} - 2\underset{\sim}{j}$

(d) 80 u^3

Exercise **5E** (Page 191)

2(a) 0 **(b)** $\overrightarrow{OB} = \underset{\sim}{a} + \underset{\sim}{c}$, $\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

3(a) equal radii

5(a) equal chords

6(b) $\overrightarrow{CB} \parallel \overrightarrow{OA}$

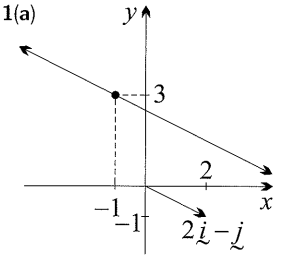
8(a) $60°$ **(b)** equilateral **(c)** regular tetrahedron (all 4 faces are equilateral triangles)

(d) $\cos^{-1}\left(-\frac{1}{3}\right) \doteq 109°28'$

9(a) $\frac{1}{2}\underset{\sim}{v} + \underset{\sim}{w}$

Exercise **5F** (Page 201)

1(a)



(b) Using basis vectors: $\underset{\sim}{r} = (-\underset{\sim}{i} + 3\underset{\sim}{j}) + \lambda(2\underset{\sim}{i} - \underset{\sim}{j})$, $\lambda \in \mathbf{R}$

(c) $-\frac{1}{2}$

(d) $y = -\frac{1}{2}x + \frac{5}{2}$

2(a) $3\underset{\sim}{i} - 2\underset{\sim}{j}$ **(b)** $3\underset{\sim}{i} + 2\underset{\sim}{j}$

(c) $\underset{\sim}{r} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \lambda(3\underset{\sim}{i} + 2\underset{\sim}{j}), \lambda \in \mathbf{R}$

3(a)(i) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(ii) The x- and y-intercepts are $\begin{bmatrix} -12 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$.

(iii) Using the y-intercept, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

(b)(i) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(ii) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(iii) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

4(a) $y = 4x + 17$ **(b)** $3x + 2y = 19$

5(a) yes **(b)** yes **(c)** no

6(a) $\underset{\sim}{r} = 7\underset{\sim}{i} - 5\underset{\sim}{k} + \lambda(-4\underset{\sim}{i} - 6\underset{\sim}{j} + 9\underset{\sim}{k})$

(b) $\underset{\sim}{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} -6 \\ -7 \\ -8 \end{bmatrix}$

7(a) $\underset{\sim}{r} = 3\underset{\sim}{i} - 2\underset{\sim}{j} - 4\underset{\sim}{k} + \lambda(5\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k})$

(b) $\underset{\sim}{r} = -\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k} + \lambda(\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k})$

8(a) yes **(b)** yes

9(a)(i) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (iii) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

11(a) $\underset{\sim}{r} = 4\underset{\sim}{i} + 3\underset{\sim}{j} + \lambda(2\underset{\sim}{i} - 3\underset{\sim}{j})$

(b) $\underset{\sim}{r} = -7\underset{\sim}{i} + 5\underset{\sim}{j} + \lambda(6\underset{\sim}{i} + 13\underset{\sim}{j})$

Note that there are many possible answers.

12(a) $\underset{\sim}{r} = -\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k} + \lambda(3\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k})$

(b) $\underset{\sim}{r} = 7\underset{\sim}{i} - 11\underset{\sim}{j} + 14\underset{\sim}{k} + \lambda(10\underset{\sim}{i} + 20\underset{\sim}{j} - 30\underset{\sim}{k})$

13(a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \quad 0 \le \lambda \le 1$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad 0 \le \lambda \le 1$

15(a) $(1, 2, 0)$ **(b)** $(3, -2, 6)$

17(a) $-\underset{\sim}{i} + \underset{\sim}{k}$ **(b)** skew

18(a) $(1, 1, 2)$ **(b)** $70.5°$

19 $a = -3$

20(d) $\frac{11\sqrt{5}}{5}$ units

21 $\frac{\sqrt{145}}{5}$ units

22(a) $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_2$ have the same direction vector.

(b) $\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ **(c)** $\sqrt{5}$

23(b) For $\angle BAD$: $\underset{\sim}{r} = (-\underset{\sim}{i} - 2\underset{\sim}{j}) + \lambda(\underset{\sim}{i} + \underset{\sim}{j})$, for $\angle ABC$: $\underset{\sim}{r} = 2\underset{\sim}{i} + \mu(-\underset{\sim}{i} + \underset{\sim}{j})$

24(a) $\overrightarrow{OM} = \begin{bmatrix} 2 \\ \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$ **(b)** $\underset{\sim}{r} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$

(d) $3 : 1$

25(a) The interval or line segment AB. **(b)** The ray with endpoint B in the direction of $\underset{\sim}{b} - \underset{\sim}{a}$. **(c)** The ray with endpoint A in the direction of $\underset{\sim}{a} - \underset{\sim}{b}$.

26(a) $\sqrt{2\lambda^2 - 8\lambda + 10}$ **(b)** $\sqrt{2}$

Exercise 5G (Page 210)

1(a) $(x - 6)^2 + (y + 9)^2 = 28$

(b) $\left| \underset{\sim}{r} - \begin{bmatrix} 6 \\ -9 \end{bmatrix} \right| = 2\sqrt{7}$

(c) $x = 6 + 2\sqrt{7}\cos\theta$, $y = -9 + 2\sqrt{7}\sin\theta$

2(a) $(x + 2)^2 + (y - 7)^2 + (z + 4)^2 = 81$

(b) $\left| \underset{\sim}{r} - \begin{bmatrix} -2 \\ 7 \\ -4 \end{bmatrix} \right| = 9$

3(a) $(x + 5)^2 + (y + 10)^2 = 45$

(b) $(x - 3)^2 + (y + 1)^2 + (z - 8)^2 = 121$

4 $\left| \underset{\sim}{r} - \begin{bmatrix} 5 \\ -3 \end{bmatrix} \right| = 2\sqrt{2}$, $(x - 5)^2 + (y + 3)^2 = 8$

5(a) $\left| \underset{\sim}{r} - \begin{bmatrix} 3 \\ -4 \end{bmatrix} \right| = 5$ **(b)** $\left| \underset{\sim}{r} - \begin{bmatrix} -\frac{1}{2} \\ 1 \\ \frac{5}{2} \end{bmatrix} \right| = \frac{\sqrt{30}}{2}$

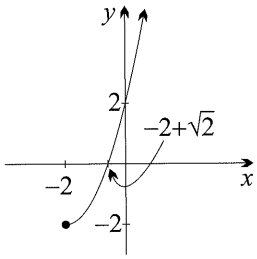
7 inside

8 centre $(2, 1, -1)$, radius $2\sqrt{5}$

9(a) $x = 2\cos t + 1, y = 2\sin t - 1$ **(b)** $(x - 1)^2 + (y + 1)^2 = 4$

10(a) $y = (x + 2)^2 - 2$ **(c)**

(b) $[-2, \infty)$



11(a) $|\underset{\sim}{r}| = \sqrt{10}$ **(b)** The radius and tangent are perpendicular. **(c)** $y = 3x - 10$

12(a) $x = 3\lambda + 1$, $y = 2\lambda - 1$

(b) $(-2, -3)$ and $(4, 1)$

14(a) Both spheres have centres on the z-axis.

(b) centre $(0, 0, \frac{9}{5})$, radius $\frac{12}{5}$

15 $(4, 4, -6)$ and $(11, -8, -3)$

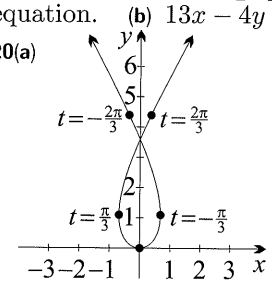
16(a) $x = 3\lambda - 2$, $y = 4\lambda + 3$, $z = 5\lambda + 4$

(b) $(7, 15, 19)$

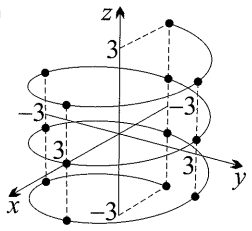
17(a) $x^2 - y^2 = 1$ **(b)** $y = \pm \frac{x^2}{\sqrt{4 - x^2}}$

18(a) $\underset{\sim}{r} = \begin{bmatrix}1\\-1\\0\end{bmatrix} + \lambda\begin{bmatrix}1\\4\\1\end{bmatrix} + \mu\begin{bmatrix}2\\5\\-2\end{bmatrix}$ is a possible equation. **(b)** $13x - 4y + 3z = 17$

20(a)



(b)



$t < 0$: blue, $t \geq 0$: black

Review Exercise **5H** (Page 213)

1 $|\underset{\sim}{a}| = 7$, $\hat{\underset{\sim}{a}} = \frac{6}{7}\underset{\sim}{i} - \frac{3}{7}\underset{\sim}{j} + \frac{2}{7}\underset{\sim}{k}$

2(a) $-5\underset{\sim}{i} - 4\underset{\sim}{j} + 7\underset{\sim}{k}$ **(b)** $5\underset{\sim}{i} + 4\underset{\sim}{j} - 7\underset{\sim}{k}$ **(c)** $3\sqrt{10}$

5(a) 50 **(b)** 44 **(c)** 8 **(d)** 110

7 $\lambda = -1$

8 $\cos\theta = \frac{6}{\sqrt{102}}$

9 $\frac{44}{29}\underset{\sim}{i} - \frac{33}{29}\underset{\sim}{j} - \frac{22}{29}\underset{\sim}{k}$

10(a) $\overrightarrow{AP} = \begin{bmatrix}1\\3\\3\end{bmatrix}$, $\overrightarrow{AB} = \begin{bmatrix}-1\\-1\\3\end{bmatrix}$ **(b)** $\frac{5}{11}\begin{bmatrix}-1\\-1\\3\end{bmatrix}$

(c) $\frac{2\sqrt{506}}{11}$ units

11 81°

13 $\underset{\sim}{r} = \begin{bmatrix}0\\3\end{bmatrix} + \lambda\begin{bmatrix}1\\2\end{bmatrix}$

14 $x - 3y = 14$

15 $(0, 7, -3), (-14, 0, 11), (-3, \frac{11}{2}, 0)$

16(a) no **(b)** yes

17 $\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k} + \lambda(\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k})$

18(a) $(2, 3, -5)$ **(b)** $(-3, 5, -1)$

19(a) $(x-3)^2 + (y+4)^2 + (z-2)^2 = 7$

(b) $\left|\underset{\sim}{r} - \begin{bmatrix}3\\-4\\2\end{bmatrix}\right| = \sqrt{7}$

21 $\left|\underset{\sim}{r} - \begin{bmatrix}2\\5\\-6\end{bmatrix}\right| = 2\sqrt{6}$

22 $(4, -5, 1)$ and $(6, 8, -10)$

23(a) $y = \frac{8}{4+x^2}$ **(b)** $x^2 + y^2 = 1$, where $y \neq -1$.

(c) $y = \pm 2x\sqrt{1-x^2}$

Chapter Six

Exercise **6A** (Page 223)

1(a) $t = \frac{1}{6}(x-1)$, $x = 6t + 1$

(b) $t = \frac{1}{18}(1 - x^3)$, $x = (1 - 18t)^{\frac{1}{3}}$

(c) $t = \frac{1}{12}(x^{-2} - 1)$, $x = (12t + 1)^{-\frac{1}{2}}$

(d) $t = \frac{1}{2}(e^{2x} - e^2)$, $x = \frac{1}{2}\ln(2t + e^2)$

(e) $t = \tan^{-1}x - \frac{\pi}{4}$, $x = \tan(t + \frac{\pi}{4})$

(f) $t = \tan x - \tan 1$, $x = \tan^{-1}(t + \tan 1)$

2(a) $\ddot{x} = 0$ **(b)** $\ddot{x} = -72x^{-5}$ **(c)** $\ddot{x} = 108x^5$

(d) $\ddot{x} = -2e^{-4x}$ **(e)** $\ddot{x} = 2x(1 + x^2)$

(f) $\ddot{x} = -2\cos^3 x \sin x$

3(a) $v^2 = 4x^3$ **(b)** $v^2 = 2(1 - e^{-x})$ **(c)** $v^2 = \ln|2x + 1|$ **(d)** $v^2 = \tan^{-1}\frac{1}{2}x$

4(a) $t = \frac{v^3}{6}$ **(b)** $t = 2 - \frac{1}{v}$

(c) $t = \ln\left|\frac{2+v}{3}\right|$

5(a) $x = 4\ln|v|$ **(b)** $x = \frac{1}{9}v^3 - 24$

(c) $x = v + 2\ln\left|\frac{2}{2+v}\right|$

6(a) 16 **(b)** 4 **(c)** 4 **(d)** 9

7 $17\,\text{m/s}^2$ at an angle of $\tan^{-1}\frac{15}{8}$ above the horizontal

8(a) $\overrightarrow{OA} = (20\cos 32°)\underset{\sim}{i} + (20\sin 32°)\underset{\sim}{j}$, $\overrightarrow{OB} = (-15\cos 54°)\underset{\sim}{i} + (15\sin 54°)\underset{\sim}{j}$

(b) 24 N **(c)** 70° above the positive horizontal direction or 020° T

9(a) $(6 - 2\sqrt{2})\underset{\sim}{i} + (20 - 2\sqrt{2})\underset{\sim}{j}$ **(b)** 17·5 N, 79·5° above the horizontal

11(a) 20 metres **(b)** Upwards is positive, so while the stone is rising its velocity is positive.

(c) $t = 2 - \frac{\sqrt{400-20x}}{10}$, $x = 20t - 5t^2$, 2 seconds

12(a) $x = 150 - \dfrac{50\ln|v|}{\ln 10}$, $x = 150$ metres

(b) $t = \dfrac{1000}{99}\left(\dfrac{1}{v} - \dfrac{1}{1000}\right)$, $t = 10\frac{1}{11}$ s

13 $v^2 = 6 - 2e^{-x}$. The acceleration is always positive, and the velocity is initially 2. Hence the velocity is always increasing with minimum 2. The particle continues to accelerate, but with limiting velocity $\sqrt{6}$.

14(a) $\ddot{x} = -12$ **(b)** $x = 3(1 - e^{-2t})$ **(c)** As $t \to \infty$, the particle moves to the limiting position $x = 3$.

15(a) 1·52 s **(b)** 8·48 m

16(a) $\frac{m}{k}$ **(b)(i)** $2ku$ **(ii)** $k\ln 3$

17(a) $v^2 = 2(x-5)(x+4)$. v^2 cannot be negative.

(b) $x = 6$ m ($x = -5$ is impossible, because the

particle can never pass through the origin). The particle moves forwards with increasing velocity.

18(a) $v^2 = 2x^3$ **(b)** $x = \dfrac{2}{(t+\sqrt{2})^2}$. The particle starts at $x = 1$ and moves backwards towards the origin, its speed having limit zero, and position having limit the origin.

19(a) $v^2 = 6x - 2x^3 + 16$ **(b)** Yes. (Sketch the graph of v^2 against x.)

21(b) $v^2 = V^2 + 2gR^2\,(1/x - 1/R)$, $H = 2gR^2/(2gR - V^2)$ **(c)** 11·2 km/s

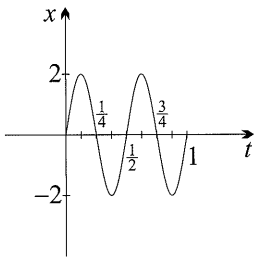
22(b) $12\ln 2 - 3{\cdot}5 \div 4{\cdot}82$ m

Exercise **6B** (Page 232)

1(a) 12 cm, 4 seconds

(b) $v = -6\pi \sin \frac{\pi}{2}t$, $\ddot{x} = -3\pi^2 \cos \frac{\pi}{2}t$ **(c)** 12 cm, 0 cm/s **(d)** After 1 second **(e)** 2 seconds

2(a) 2 m, $\frac{1}{2}$ s **(b)**



(c) $v = 8\pi \cos 4\pi t$, $\ddot{x} = -32\pi^2 \sin 4\pi t$

(d) $\ddot{x} = -16\pi^2 x$

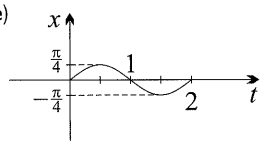
(e) $\ddot{x} = -32\pi^2$ at $t = \frac{1}{8}$ and $\ddot{x} = 32\pi^2$ at $t = \frac{3}{8}$

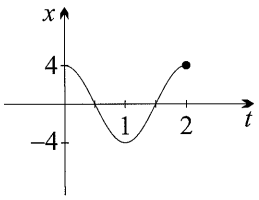
(f) 8π m/s

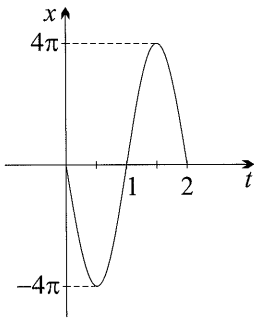
3(b) $v = 4\cos \pi t$, $\ddot{x} = -4\pi \sin \pi t$

(c) $a = \frac{4}{\pi}$, $T = 2$ seconds **(d)** $\frac{4}{\pi}$ m, 4 m/s

(e)







(f) $t = 1$ (when $v = -4$ m/s) and $t = 2$ (when $v = 4$ m/s) **(g)** $t = \frac{1}{2}$ (when $\ddot{x} = -4\pi$ m/s^2) and $t = \frac{3}{2}$ (when $\ddot{x} = 4\pi$ m/s^2)

4(a) $x = 2\sin 2t$

(b) $x = 6\sin \frac{2}{3}t$

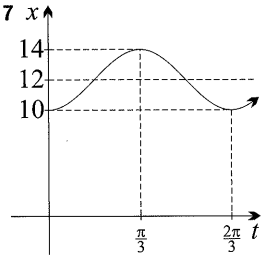
5(a) $v = bn\cos nt - cn\sin nt$, $\ddot{x} = -bn^2 \sin nt - cn^2 \cos nt = -n^2 x$

(b)(i) $c = 3$ and $b = 0$, so $x = 3\cos nt$.

(ii) $x = 5\cos 2\pi t$, $\frac{1}{4}$ s

6(b) $a = 4$ **(c)** $\frac{1}{3}$ s, $\frac{2}{3}$ s

7



(a) $v = 6\sin 3t$, $\ddot{x} = 18\cos 3t$

(b) $a = 2$, $T = \frac{2\pi}{3}$ seconds, centre $x = 12$

(c) $10 \le x \le 14$, $\frac{\pi}{3}$ seconds

(d) $t = \frac{2\pi}{3}$ and $t = \frac{4\pi}{3}$. At both times, $|v| = 0$ and $\ddot{x} = 18$ cm/s^2. **(e)** $t = \frac{\pi}{6}$ and $t = \frac{\pi}{2}$. At both times, $|v| = 6$ cm/s and $\ddot{x} = 0$ cm/s^2.

8(a) amplitude: 6, period: π, initial phase: $\frac{\pi}{2}$

(b) $\dot{x} = 12\cos(2t + \frac{\pi}{2})$, $\ddot{x} = -24\sin(2t + \frac{\pi}{2})$, $\ddot{x} = -4x$, so $n = 2$.

(c) $t = \frac{\pi}{4}$ when $v = -12$, $t = \frac{3\pi}{4}$ when $v = 12$

(d) $t = \frac{3\pi}{4}$ and $t = \frac{7\pi}{4}$, when $x = 0$

(e) $t = \pi$ and $t = 2\pi$, when $v = 0$ and $\ddot{x} = -24$

9(a) $x = 120\sin \frac{\pi}{12}t$, $v = 10\pi\cos \frac{\pi}{12}t$, 10π m/s

(b)(i) $\frac{12}{\pi}\sin^{-1}\frac{1}{4} \div 0{\cdot}9652$ seconds

(ii) $12 + \frac{12}{\pi}\sin^{-1}\frac{1}{4} \div 12{\cdot}97$ seconds **(c)** 4 seconds and 8 seconds

10(a) $x = 4\cos 4t$, $v = -16\sin 4t$ **(b)(i)** $\frac{\pi}{12}$ s **(ii)** $\frac{\pi}{6}$ s

(c) $\frac{\pi}{24}$ seconds and $\frac{5\pi}{24}$ seconds

11 $x = \frac{1}{2} - \frac{1}{2}\cos 2t$, $x_0 = \frac{1}{2}$, $\frac{1}{2}$, $0 \le x \le 1$, π

12(a) $x = 2 - \cos 4t$ **(b)** $x_0 = 2$, 1 cm, $1 \le x \le 3$, $\frac{\pi}{2}$ s **(c)** 4 cm/s when $t = \frac{\pi}{8}$

13(b) $x = 2$ **(c)** 2π s **(d)** $6\cos(t - \frac{\pi}{3})$ **(e)** 6, $-\frac{\pi}{3}$

(f) $-4 \le x \le 8$

14 $v = bn\cos nt - cn\sin nt$

(a) $n = \frac{1}{2}, c = 6, b = 6$, $\frac{3\pi}{2}$ s and $\frac{7\pi}{2}$ s

(b) $n = \frac{\pi}{3}, c = -2, b = \frac{9}{\pi}$, about 0·582 s and 3·582 s

15 $v = an\cos(nt + \alpha)$ **(a)** $n = \frac{\pi}{3}$, $\alpha = 0$, $a = \frac{15}{\pi}$ **(b)** $n = \frac{2}{3}$, $\alpha = \frac{3\pi}{2}$, $a = 5$ **(c)** $n = 1$, $\alpha = \frac{3\pi}{4}$, $a = \sqrt{2}$

16(a) $a = 3, \alpha = -\frac{\pi}{2}$

(b) $a = 2, \alpha = \frac{\pi}{3}$

17 $a = \frac{32\sqrt{2}}{\pi}$, $\alpha = \frac{\pi}{4}$

18 $a = 5$, $\alpha \div 2{\cdot}248$

19(a) $x = 4\sin 4t + 3\cos 4t$ **(b)** $x = 5\cos(4t - \varepsilon)$, where $\varepsilon = \tan^{-1}\frac{4}{3}$ **(c)** 5 m, 20 m/s

(d) $t = \frac{\pi}{4} - \frac{1}{4}\tan^{-1}\frac{3}{4}$, $t = \frac{\pi}{8} + \frac{1}{4}\tan^{-1}\frac{4}{3}$

20(a) 10:00 am **(b)** 7:33 am **(c)** 12:27 pm

21 11:45 am to 8:15 pm

23(b) $2\sqrt{5 + 2\sqrt{3}}$

Exercise **6C** (Page 238)

1(a) $v = -6\sin 2t$, $\ddot{x} = -12\cos 2t$, $\ddot{x} = -4x$, $v^2 = 4(9 - x^2)$ **(b)** $\pm 2\sqrt{5}\,\text{m/s}$, $-8\,\text{m/s}^2$

2(a) $v^2 = 9(25 - x^2)$

(b) $v = \pm 12\,\text{m/s}$, $\ddot{x} = -27\,\text{m/s}^2$ **(c)** $15\,\text{m/s}$

3(a) $v^2 = 16(36 - x^2)$ **(b)** $6\,\text{cm}$, $\frac{\pi}{2}$ seconds

(c) $|v| = 16\sqrt{2}\,\text{cm/s}$, $\ddot{x} = -32\,\text{cm/s}^2$

4(a) $v^2 = 4(36 - x^2)$, π seconds, $12\,\text{m/s}$

(b)(i) $x = 6\cos 2t$ **(ii)** $x = -6\cos 2t$

(iii) $x = 6\sin 2t$ **(iv)** $x = -6\sin 2t$

5(a) $32\,\text{cm/s}$ **(b)** $8\,\text{cm}$

6(a) $a = 1$ metre **(b)** $\frac{4\pi}{3}\,\text{m/s}$

7(a) $\frac{5\pi}{2}\,\text{cm/s}$, $\frac{5\pi^2}{8}\,\text{cm/s}^2$

(b) $\pm 2\pi\,\text{cm/s}$, $\pm\frac{3\pi^2}{8}\,\text{cm/s}^2$

8 $5\sqrt{2}\,\text{m/s}$, $3\sqrt{2}\,\text{m/s}$

9 4, $2\sqrt{7}\,\text{m/s}$

10 $v^2 = -\frac{5}{3}(x^2 - 16^2)$, so the amplitude $= 16$.

11 $15\,\text{cm/s}$

13(b)(i) When $x = 0$, $|v| = an$.

(ii) When $x = \frac{1}{2}a$, $|v| = \frac{1}{2}\sqrt{3}\,an$ and $\ddot{x} = -\frac{1}{2}an^2$.

14(a) $\ddot{x} = -9(x - 1)$, centre: $x = 1$, period: $\frac{2\pi}{3}$, amplitude: 2 **(b)(i)** $\ddot{x} = -16(x - 2)$, centre: $x = 2$, period: $\frac{\pi}{2}$, amplitude: 3 **(ii)** $\ddot{x} = -9(x - 6)$, centre: $x = 6$, period: $\frac{2\pi}{3}$, amplitude: 4

(iii) $\ddot{x} = -2(x + 2)$, centre: $x = -2$, period: $\pi\sqrt{2}$, amplitude: 1 **(iv)** $\ddot{x} = -3(x + \frac{5}{3})$, centre: $x = -\frac{5}{3}$, period: $2\pi/\sqrt{3}$, amplitude: $2\frac{1}{3}$

15(a)(i) $\ddot{x} = 50\cos 10t = -100(x - \frac{1}{2})$

(ii) $\ddot{x} = -50(2\sin^2 5t - 1) = -100(x - \frac{1}{2})$

(b) centre: $x = \frac{1}{2}$, range: $0 \le x \le 1$, period: $\frac{\pi}{5}$ minutes, $t = \frac{\pi}{5}$

16(a) centre: $x = 7$. Since the amplitude is 7, the extremes of motion are $x = 0$ and $x = 14$, and the particle is stationary there.

(b) $v^2 = 9\big(49 - (x - 7)^2\big)$, $21\,\text{cm/s}$

(c) Although the particle is stationary for an instant, its acceleration at that time is positive (it is actually $63\,\text{m/s}^2$), and so the velocity immediately changes and the particle moves away.

17(a) $\ddot{x} = -9x$ **(b)** period: $\frac{2\pi}{3}$, amplitude: $2\sqrt{13}$, maximum speed $6\sqrt{13}$, $|\ddot{x}| = 9\sqrt{13}$

18(a) $x = 3$, $\frac{\pi}{2}$ **(b)** $x = 3 + 2\sin(4t + \frac{\pi}{3})$

(c) $t = \frac{\pi}{6}, \frac{5\pi}{12}, \frac{2\pi}{3}$, $|v| = 8$

19(a) $\ddot{x} = -4(x - 10)$, centre: $x = 10$, period: π, amplitude: 10

(b) $\frac{3\pi}{4} - \frac{1}{2}\tan^{-1}\frac{3}{4}$ $(= \pi - \tan^{-1} 2 \div 2{\cdot}034)$

20(b) $\dot{x} = -16\pi\sin 2\pi t$, $\dot{y} = 16\pi\cos 2\pi t$, $\ddot{x} = -32\pi^2\cos 2\pi t$, $\ddot{y} = -32\pi^2\sin 2\pi t$

(c)(i) $\frac{\pi}{6}$ or $\frac{7\pi}{6}$ **(ii)** $\frac{\pi}{3}$ or $\frac{4\pi}{3}$ **(iii)** $\frac{3\pi}{4}$ or $\frac{7\pi}{4}$

21(b) $a = \sqrt{\dfrac{{v_2}^2{x_1}^2 - {v_1}^2{x_2}^2}{{v_1}^2 - {v_2}^2}}$

(c) $5\,\text{cm}$, π seconds, $10\,\text{cm/s}$

22 $v = \frac{1}{2}V\sqrt{3}$ or $v = -\frac{1}{2}V\sqrt{3}$, $x = \frac{1}{2}a\sqrt{3}$ or $x = -\frac{1}{2}a\sqrt{3}$

23(b) When $\alpha = \pi$, $A = 3$ and $x = 3\sin t$. When $\alpha = 0$, $A = 1$ and $x = -\sin t$.

(c) twice **(d)** When $\alpha = \frac{\pi}{3}$, $x = \sqrt{3}\cos t$. When $\alpha = \frac{5\pi}{3}$, $x = -\sqrt{3}\cos t$.

Exercise **6D** (Page 245)

1(b) 360 metres

2(c) 80%

4(a) The force of the spring is directed towards the origin.The resistive force is in the opposite direction to the velocity. **(b)** $\dddot{y} + 3\dot{y} + 2y = 0$

(e) $A = -1$, $B = 1$

6(b) $v^2 = \frac{Q^2}{(1+Q^2)e^{2t} - Q^2}$ **(c)** $\lim\limits_{t\to\infty} v = 0$. Since $v \neq 0$, it can never change sign. **(d)** $\tan^{-1} Q$

(e) Yes. Since Q and v have the same sign, $\tan^{-1} Q - \tan^{-1} v = \tan^{-1}\frac{Q - v}{1 + Qv}$. In contrast, evaluate each side when $Q = \sqrt{3}$ and $v = -\sqrt{3}$.

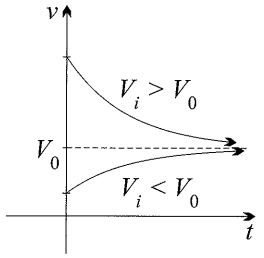
7(a) $A = B = \frac{1}{5}$ **(b)(i)** 1 **(ii)** $t = \frac{m}{5\times 10^4}\log\left(\frac{6+2v}{6-3v}\right)$

8(c) 878 metres **(d)** 1190 metres

9(a) $F = mP - mkv$

(b) $v = \frac{P}{k}$

(d) $\frac{1}{k}\log 2$



10(a) At $x = 1$ the acceleration is positive.

(c) The velocity approaches 1 from above.

11(a)(ii) $0 \le x \le \frac{2\sqrt{V_0}}{k}$, and $v = 0$ when $x = \frac{2\sqrt{V_0}}{k}$.

(b)(i) $x = \frac{2\sqrt{V_0}}{k}\left(1 - \frac{2}{2 + kt\sqrt{V_0}}\right)$

(ii) $\lim\limits_{t\to\infty} x = \frac{2\sqrt{V_0}}{k}$, and $\lim\limits_{t\to\infty} v = 0$.

Exercise **6E** (Page 254)

1(b) 1·9 seconds **(c)** 25 metres

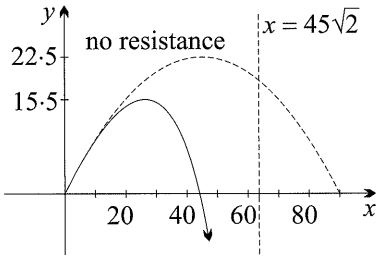
4(b)(i) $v\dfrac{dv}{dy} = \dfrac{400 - v^2}{40}$ **(ii)** $10\sqrt{2}\,\text{m/s}$

5(d) 40 metres

9(a)(i) 0 **(iii)** $f'(x) > 0$ for $x > g$. That is, f is increasing for $x > g$. **(b)(ii)** $T = \frac{1}{k}\ln\left(\dfrac{g + kV_0}{g}\right)$

10(b)(i) $\ddot{x} = 10 - \frac{1}{10}v^2$ **(ii)** 10 m/s **(iv)** $t \doteqdot 1{\cdot}446$

11(b) $y = 45(2+\sqrt{2})(1 - e^{-t/3}) - 30t$ **(d)** $R \doteqdot 44$



Here is the trajectory for $0 \le t \le 4$. The dotted line is the trajectory for no air resistance.

13(a)(ii) $x(t + \delta t) \doteqdot x(t) + \delta t\, u(t)$

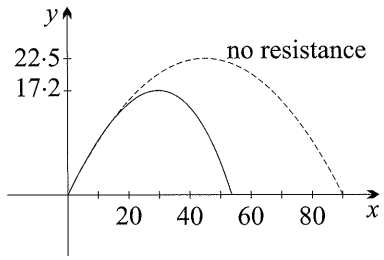
$y(t + \delta t) \doteqdot y(t) + \delta t\, v(t)$

$v(t + \delta t) \doteqdot v(t)\left(1 - k\,\delta t\left(u(t)^2 + v(t)^2\right)^{\frac{1}{2}}\right) - g\delta t$

(b)(i) $u(0{\cdot}1) \doteqdot 20{\cdot}51$

(ii) $v(0{\cdot}1) \doteqdot 19{\cdot}51$, $x(0{\cdot}1) \doteqdot 2{\cdot}12$, $y(0{\cdot}1) \doteqdot 2{\cdot}12$

(c) $u(0{\cdot}2) \doteqdot 19{\cdot}86$, $v(0{\cdot}2) \doteqdot 17{\cdot}89$, $x(0{\cdot}2) \doteqdot 4{\cdot}17$, $y(0{\cdot}2) \doteqdot 4{\cdot}07$ **(d)** $R \doteqdot 54$



Exercise **6F** (Page 259)

1(a) $x = 20t$, $y = -5t^2 + 20\sqrt{3}t$ **(b)** $4\sqrt{3}$ seconds, $80\sqrt{3}$ metres **(c)** $2\sqrt{3}$ seconds, 60 metres

(d) It is false. The horizontal range would not have changed, although the flight time would have been 4 seconds and the maximum height would have been 20 metres.

2(a) $\dot{x} = 15, \dot{y} = -10t + 36, x = 15t, y = -5t^2 + 36t$

(b) 52 m **(c)** 21.9 m/s **(d)** after

3(a) $x = 10\sqrt{3}t$, $y = -5t^2 + 10t$

(b) 5 s, $50\sqrt{3}$ metres **(c)** 80 metres

(d) 44 m/s, 67° **(e)** $y = -\frac{1}{60}x^2 + \frac{1}{\sqrt{3}}x$

4(a) 101 m/s **(b)** $x = 101t$, $y = -5t^2$ **(d)** 149 m/s, $\tan^{-1}\frac{20\sqrt{30}}{101} \doteqdot 47°19'$ below the horizontal

(e) 1·106 km

6(c) $V = 36$, $\theta \doteqdot 41°49'$ **(d)** 129 metres

7(c)(i) $\alpha = 15°$ or 75°

(ii) It will if $\alpha = 75°$, but not if $\alpha = 15°$.

8(b) range: 38·4 metres, height: 12·8 metres

(c)(ii) 33·3 metres

9(d) 60°15′ or 72°54′

10(b) 0·36 s **(c)** 12° **(d)** No, it lands 4·72 metres in front of him.

11(b)(ii) 16 metres **(iii)** 112°

12(d) 27 m

13(d) 15 metres **(e)** 10 m/s, 63°26′

14(c) $T = 4$, $\theta = 30°$

15(b)(ii) Yes. The vertical components of their initial velocities are equal, and they are both subject to the same force (gravity) acting in the vertical direction.

18(c) For $0° < \alpha < 45°$, $0 < \tan\alpha < 1$. Hence if α_1 and α_2 are both less than 45°, then the two roots of the quadratic both lie between 0 and 1. But the product of these roots is greater than 1, so α_1 and α_2 cannot both be less than 45°.

19(a) $(d\cos\beta, d\sin\beta)$

20(b)(iv) 52°

Exercise **6G** (Page 269)

1(b) 6·05 m

2(a) $1\frac{3}{5}$ s, $12\frac{4}{5}$ m **(c)** $24\frac{1}{3}$ m/s

3(a) $B - Mg = -Md$, $B - (M-m)g = (M-m)a$

4(a) $v^2 = {V_0}^2 + \dfrac{2gR^2}{r} - 2gR$

(b) $V_0 = \sqrt{gR} \doteqdot 1680\,\text{m/s}$

5(a) $T = mg\cos\theta$, $\dfrac{d^2\theta}{dt^2} = -\dfrac{g\sin\theta}{L}$

6 5:09 am to 9:36 am

7(b) $\dfrac{gx^2}{2V^2}\tan^2\beta - x\tan\beta + \left(y + \dfrac{gx^2}{2V^2}\right) = 0$

(c)(iii) $\tan\beta = \dfrac{V^2}{gh\cot\alpha}$

8(c) $x^2 = -\frac{2V^2}{g}(y - \frac{V^2}{2g})$ with focal length $\frac{V^2}{2g}$ and vertex $(0, \frac{V^2}{2g})$

9 centre $= (0, -\frac{1}{2}gt^2)$, radius $= Vt$

10(b)(iii) 2ℓ

11(a) The acceleration is downwards.

(b) $h = \frac{R{V_0}^2}{20R - {V_0}^2}$ **(c)** 12 524 m

12(b) $C = 112$ **(c)** $B = 4(16 + 14e^{-7kt/4})$

(d) $78°\text{C}$ **(e)** $64°\text{C}$

13(b) 2100 **(c)** 3000 **(d)** 10%

14(b)(i) $\left(\frac{V^2\sin 2\alpha}{2g}, \frac{V^2\sin^2\alpha}{2g}\right)$ **(ii)** $\frac{V^2\cos^2\alpha}{2g}$

(iii) $y = \frac{V^2}{2g}$, $S = \left(\frac{V^2\sin 2\alpha}{2g}, -\frac{V^2\cos 2\alpha}{2g}\right)$

Notice that the directrix is independent of α.

15(b) $15(2+\sqrt{2})\,\text{s}$, which is about 51·2s.

16(a) $\frac{3}{2}mg$ **(c)** $\dot{y}^2 = 3g$, and since y is decreasing to that point on the curve, $\dot{y} = -\sqrt{3g}$. $\dot{x} = 0$.

(d) $\frac{\sqrt{3}}{2\sqrt{g}}\left(\sin^{-1}\frac{1}{4} + 1 - \frac{\sqrt{15}}{4}\right)$

(e) The integral is improper.

Review Exercise **6H** (Page 274)

1(a) $t = \frac{1}{2}\ln|2x-1|$, $x = \frac{1}{2}(e^{2t}+1)$

(b) $t = \frac{1}{6}(x^{-1}-1)$, $x = (1+6t)^{-1}$

2(a) $\ddot{x} = 2(2x-1)$ **(b)** $\ddot{x} = 72x^3$

3(a) $v^2 = 12x$

(b) $v^2 = \frac{1}{3}(1-\cos 6x)$

4 $t = \frac{1}{4}\ln\frac{v}{2}$

5 $x = v^3 - 1$

6(a) 8 **(b)** $2\sqrt{5}$ **(c)** 12 **(d)** $\ln 3$

7 $41{\cdot}8\,\text{m/s}^2$, $035{\cdot}3°\text{T}$

8(a) $\ddot{x} = -\frac{5}{2}\,\text{m/s}$, $v^2 = 10\,000 - 5x$

(b)(i) $v = 50\sqrt{2}\,\text{m/s}$ **(ii)** $x = 1500$ metres

(c) The plane is still moving forwards while it is braking. **(d)** $x = 100t - \frac{5}{4}t^2$, 40 seconds

9(a) $v = 500 - 5x$, $x = 100\left(1 - e^{-5t}\right)$

(b) The torpedo moves to a limiting position of $x = 100$ as the velocity decreases to zero.

10(a) $v^2 = e^{-x}$ **(b)** v is initially positive, and is never zero. $x = 2\ln\frac{1}{2}(t+2)$ **(c)** As $t \to \infty$, $x \to \infty$ (slowly) and $v \to 0$.

12(b) $\sqrt{2}$ **(c)** $x = \sqrt{2}\cos\left(4t + \frac{\pi}{4}\right)$

13(b) $2\sin\left(3t - \frac{\pi}{6}\right)$ **(c)** the amplitude is 2, the centre is $x = 5$ **(d)** $t = \frac{\pi}{18}$

14 $\frac{2}{3}\,\text{m}$, $\frac{2\pi}{3}\,\text{s}$

15(a) $18{\cdot}8\,\text{m/s}$ **(b)** 3553 N

16(c) $0{\cdot}34\,\text{s}$

18(b) $12\cdot1\,\text{m/s}$

19(a) $\ddot{x} = 10 - 20v$ **(d)** $0{\cdot}035\,\text{s}$

20(b) $\frac{V}{g}\ln 2$

22(a) $\dot{x} = 48, \dot{y} = -10t + 36, x = 48t, y = -5t^2 + 36t$ **(b)** $64{\cdot}8\,\text{m}$ **(c)** $345{\cdot}6\,\text{m}$ **(d)** $48\underset{\sim}{i} + 20\underset{\sim}{j}$

23(b) $R = 21{\cdot}6$ metres, $H = 4{\cdot}05$ metres

(c) $\tan^{-1}\frac{3}{4}$ **(d)** $15\,\text{m/s}$

(e) $t = 0{\cdot}8$, when $x = 9{\cdot}6$, and $t = 1$, when $x = 12$

24(a) Initially, $\dot{x} = \sqrt{5}$ and $\dot{y} = 2\sqrt{5}$. **(b)** $\dot{x} = \sqrt{5}$, $x = t\sqrt{5}$, $\dot{y} = -10t + 2\sqrt{5}$, $y = -5t^2 + 2t\sqrt{5}$

(c) 1 metre **(d)** 2 metres **(e)** $\dot{x} = \sqrt{5}$, $\dot{y} = -2\sqrt{5}$, $v = 5\,\text{m/s}$, $\theta = -\tan^{-1}2$ **(f)** $y = 2x - x^2$

25(a) $\dot{x} = 200$, $\dot{y} = 0$ **(b)** $\dot{x} = 200$, $x = 200t$, $\dot{y} = -10t$, $y = -5t^2$, $y = -\frac{1}{8000}x^2$ **(c)** 600 metres

(d) $8°32'$

26(b) $y = x - \frac{1}{90}x^2$

27(b) $62°22'$ or $37°5'$

28(c) $7{\cdot}5\,\text{m}$ **(d)** $0{\cdot}8 \le m \le 1{\cdot}2$ or $2{\cdot}8 \le m \le 3{\cdot}2$

29(c) $\frac{1}{k}\ln\left(1 + \frac{ku}{g}\sin\alpha\right)$ **(d)** $\frac{u}{k}\cos\alpha$

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