Chapter 8 worked solutions – Vectors

Solutions to Exercise 8A

Solutions for Enrichment questions are in preparation.

1a Magnitude of $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 100 \text{ km} - 40 \text{ km} = 60 \text{ km}$

$\overrightarrow{BC}$ is towards west direction which means the magnitude to be considered negative as per the quadrant rule.

Direction of $\overrightarrow{AC}$ is $90°$ towards east direction as west to east is $90°$ in the quadrants.

1b Magnitude of $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

In this case, using Pythagoras theorem of triangles,

$$AC^2 = AB^2 + BC^2$$
$$= 6^2 + 4^2$$
$$= 36 + 16$$
$$= 52$$

$$AC = \sqrt{52} = 7.21 \div 7$$

Direction of $\overrightarrow{AC}$ is

$$\tan\theta = \frac{AB}{BC} = \frac{4}{6}$$

$$\theta = \tan^{-1}\frac{4}{6} = 33.69 \div 34$$

The tangent is in fourth quadrant where it is negative,

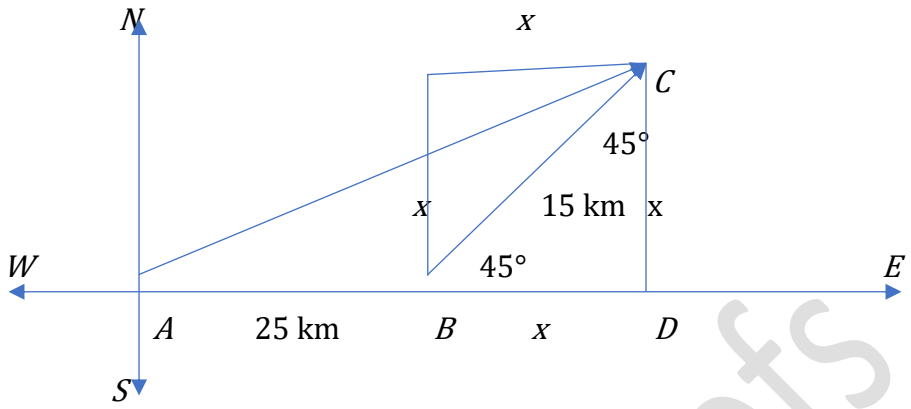
therefore, bearing from A to C is $180° - 34° = 146°$

Direction of $\overrightarrow{AC}$ is $146°$.

Uncorrected proofs

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1c



According to Pythagoras' Theorem

$BC^2 = BD^2 + DC^2$

$15^2 = x^2 + x^2$

$2x^2 = 15^2$

$x = 15\frac{\sqrt{2}}{2} = 10.606 \div 10.6$

Magnitude of $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{CD}$

In this case, using Pythagoras' Theorem,

$AC^2 = AD^2 + CD^2$

$= (25 + 10.6)^2 + 10.6^2$

$= 1267.36 + 112.36$

$= 1379.72$

$AC = \sqrt{1379.72} = 37.14 \div 37$

Direction of $\overrightarrow{AC}$ is

$\tan\theta = \frac{AD}{DC} = \frac{35.6}{10.6}$

$\theta = \tan^{-1}\frac{35.6}{10.6} = 73.42 \div 73$

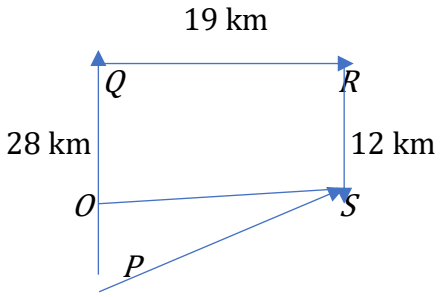
The tangent is in first quadrant where it is positive.

Direction of $\overrightarrow{AC}$ is 73°.

Uncorrected proofs

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2a



2b According to the picture, QR is parallel to OS which means

$QR = OS = 19$ km

Similarly, OQ is parallel to RS, therefore

$OQ = RS = 12$ km

Hence, $OP = PQ - OQ = 28 - 12 = 16$ km

Using Pythagoras theorem

$PS^2 = OP^2 + OS^2$

$= 16^2 + 19^2$

$= 256 + 361$

$= 617$

$PS = \sqrt{617} = 24.8$

Magnitude of $\overrightarrow{PS} = 24.8$ km

Direction of $\overrightarrow{PS}$ is

$\tan\theta = \frac{OS}{OP} = \frac{19}{16}$

$\theta = \tan^{-1}\frac{19}{16} = 49.90 \div 50\,°$

The tangent is in first quadrant where it is positive,

Direction of $\overrightarrow{AC}$ is 50°

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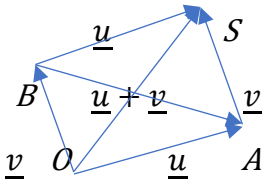
3 The opposite sides WX and ZY are parallel and equal hence $WXYZ$ is a parallelogram.

4 The opposite sides BA and CD are parallel and equal, and $\angle BAD = 90°$. Hence, $ABCD$ is a parallelogram with an angle being 90°, therefore, it is a rectangle.

5a All the sides of the quadrilateral are equal therefore it is a rhombus.

5b The opposite sides of a rhombus are parallel, so $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ have opposite directions.

6



As per the above diagram, $OBSA$ is a parallelogram where $OB \parallel AS$ and $BS \parallel OA$

Hence, $OB = AS = \underline{u}$ and $BS = OA = \underline{v}$

$\underline{u} + \underline{v} = \overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AS} = \overrightarrow{OS}$

And $\underline{v} + \underline{u} = \overrightarrow{OB} + \overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BS} = \overrightarrow{OS}$

So, $\underline{u} + \underline{v} = \underline{v} + \underline{u}$ as required

7 $\overrightarrow{UV} + \overrightarrow{VW} + \overrightarrow{WU}$

$= \overrightarrow{UW} + \overrightarrow{WU}$

$= \overrightarrow{UU}$

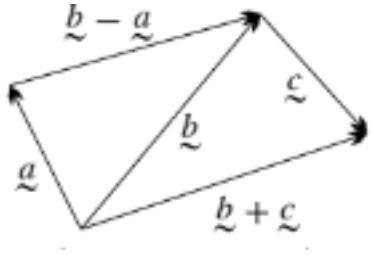
$= \underline{0}$

It gives a zero vector.

Uncorrected proofs

Chapter 8 worked solutions – Vectors

8



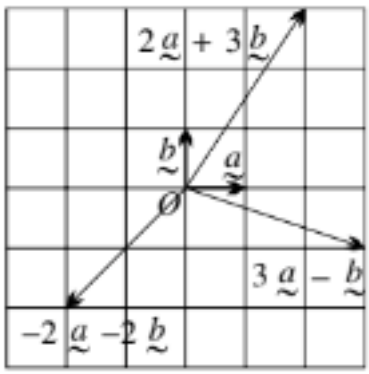
9a $\overrightarrow{AC}+\overrightarrow{AD}=\overrightarrow{AD}$

9b $\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{BA}$

9c $\overrightarrow{AD}-\overrightarrow{AB}=\overrightarrow{AD}+\overrightarrow{BA}=\overrightarrow{BA}+\overrightarrow{AD}=\overrightarrow{BD}$

9d $\overrightarrow{AC}-\overrightarrow{BC}=\overrightarrow{AC}+\overrightarrow{CB}=\overrightarrow{AB}$

10



11a $\underline{f}$

11b $\underline{d}$

11c $\underline{h}$

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11d $\underline{b}$

11e $\underline{c}$

11f $\underline{a}$

11g $\underline{e}$

11h $\underline{g}$

12a $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

$= \overrightarrow{AC} - \overrightarrow{BC}$

$= \underline{u} - \underline{v}$

12b $\overrightarrow{AM} = \overrightarrow{AC} + \overrightarrow{CM}$

$= \overrightarrow{AC} - \overrightarrow{MC}$

$= \overrightarrow{AC} - \frac{1}{2}\overrightarrow{BC}$

$= \underline{u} - \frac{1}{2}\underline{v}$

13a $\overrightarrow{AM}$ and $\overrightarrow{MB}$ have the same length and direction. Similarly, $\overrightarrow{PN}$ and $\overrightarrow{NQ}$ have the same length and direction.

13b $\overrightarrow{AN} = \overrightarrow{AP} + \overrightarrow{PN} = \underline{a} + \underline{v}$

and $\overrightarrow{AN} = \overrightarrow{AM} + \overrightarrow{MN} = \underline{u} + \underline{p}$

Hence, $\underline{a} + \underline{v} = \underline{u} + \underline{p}$

$a = \underline{p} + \underline{u} - \underline{v}$

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Similarly, $\overrightarrow{MQ} = \overrightarrow{MB} + \overrightarrow{BQ} = \underline{u} + \underline{b}$

and $\overrightarrow{MQ} = \overrightarrow{MN} + \overrightarrow{NQ} = \underline{p} + \underline{v}$

Hence, $\underline{u} + \underline{b} = \underline{p} + \underline{v}$

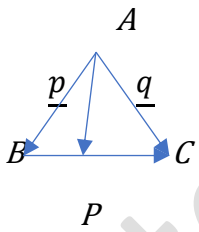
$\underline{b} = \underline{p} + \underline{v} - \underline{u}$

13c LHS $= \underline{a} + \underline{b}$

$= (\underline{p} + \underline{u} - \underline{v}) + (\underline{p} + \underline{v} - \underline{u})$

$= 2\underline{p} =$ RHS

14a



$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -\overrightarrow{AB} + \overrightarrow{AC}$

$= \overrightarrow{AC} - \overrightarrow{AB}$

$= \underline{q} - \underline{p}$

Given, $BP:PC = 1:2$

Hence, $\overrightarrow{BP} = \frac{1}{3}\overrightarrow{BC} = \frac{1}{3}(\underline{q} - \underline{p})$

14b $\overrightarrow{AP} = \overrightarrow{AC} + \overrightarrow{CP}$

$= \overrightarrow{AC} - \overrightarrow{PC}$

$= \underline{q} - \frac{2}{3}(\underline{q} - \underline{p})$

$= \underline{q} - \frac{2}{3}\underline{q} + \frac{2}{3}\underline{p}$

$= \frac{1}{3}\underline{q} + \frac{2}{3}\underline{p}$

Hence, proved.

Uncorrected proofs

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15a $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

$= \overrightarrow{AD} - \overrightarrow{CD}$

$= \underline{w} - \underline{u}$

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = \underline{v} + \underline{w} - \underline{u}$

As M is the mid-point of $\overrightarrow{BC}$

So, $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}(\underline{v} + \underline{w} - \underline{u})$

$\overrightarrow{MB}$ is in opposite direction of $\overrightarrow{BM}$

Hence, $\overrightarrow{MB} = -\frac{1}{2}(\underline{v} + \underline{w} - \underline{u}) = \frac{1}{2}(\underline{u} - \underline{v} - \underline{w})$

15b $\overrightarrow{MA} = \overrightarrow{MB} + \overrightarrow{BA}$

$= \frac{1}{2}(\underline{u} - \underline{v} - \underline{w}) + \underline{v}$

$= \frac{1}{2}(\underline{u} + \underline{v} - \underline{w})$

16a $\overrightarrow{WX} = \overrightarrow{WR} + \overrightarrow{RX}$

$= \overrightarrow{RX} - \overrightarrow{RW}$

$= \underline{x} - \underline{w}$

$\overrightarrow{WP} = \frac{1}{2}\overrightarrow{WX} = \frac{1}{2}(\underline{x} - \underline{w})$

16b $\overrightarrow{RP} = \overrightarrow{RX} + \overrightarrow{XP}$

$= \overrightarrow{RX} - \overrightarrow{PX}$

$= \underline{x} - \frac{1}{2}(\underline{x} - \underline{w})$

$= \underline{x} - \frac{1}{2}\underline{x} + \frac{1}{2}\underline{w}$

$= \frac{1}{2}\underline{x} + \frac{1}{2}\underline{w}$

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$= \frac{1}{2}(\underline{x} + \underline{w})$

16c $\overrightarrow{YZ} = \overrightarrow{YR} + \overrightarrow{RZ}$

$= \overrightarrow{RZ} - \overrightarrow{RY}$

$= \underline{z} - \underline{y}$

$\overrightarrow{RQ} = \overrightarrow{RZ} + \overrightarrow{ZQ}$

$= \overrightarrow{RZ} - \overrightarrow{QZ}$

$= \overrightarrow{RZ} - \frac{1}{2}\overrightarrow{YZ}$

$= \underline{z} - \frac{1}{2}(\underline{z} - \underline{y})$

$= \underline{z} - \frac{1}{2}\underline{z} + \frac{1}{2}\underline{y}$

$= \frac{1}{2}\underline{z} + \frac{1}{2}\underline{y}$

$= \frac{1}{2}(\underline{y} + \underline{z})$

16d $\underline{w} + \underline{x} + \underline{y} + \underline{z}$

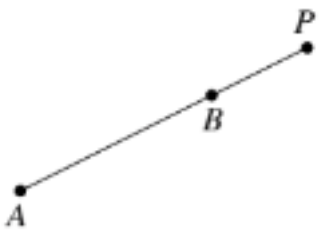
$= \overrightarrow{RW} + \overrightarrow{RX} + \overrightarrow{RY} + \overrightarrow{RZ}$

$= \overrightarrow{RR} + \overrightarrow{RR}$

$= \overrightarrow{RR}$

$= \underline{0}$

17a i



The magnitude of $\overrightarrow{AP}$ is more than $\overrightarrow{AB}$ as given $\overrightarrow{AP} = k\overrightarrow{AB}$ and $k > 1$.

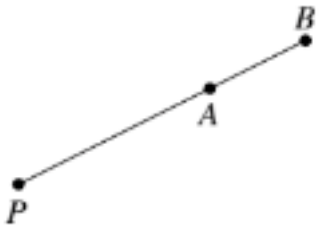
Uncorrected proofs

17a ii



The magnitude of $\overrightarrow{AP}$ is less than $\overrightarrow{AB}$ but more than zero vector as given $\overrightarrow{AP} = k\overrightarrow{AB}$ and $0 < k < 1$.

17a iii



The magnitude of $\overrightarrow{AP}$ is less than $\overrightarrow{AB}$ as given $\overrightarrow{AP} = k\overrightarrow{AB}$ and $k < 0$.

17b i $\overrightarrow{AP} = \frac{3}{2}\overrightarrow{PB}$

$\overrightarrow{PB} = \frac{2}{3}\overrightarrow{AP}$

And, $\overrightarrow{AB} = \overrightarrow{AP} + \overrightarrow{PB}$

$= \overrightarrow{AP} + \frac{2}{3}\overrightarrow{AP}$

$= \frac{5}{3}\overrightarrow{AP}$

$\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB}$

Given, $\overrightarrow{AP} = k\overrightarrow{AB}$

Hence, $k = \frac{3}{5}$

Uncorrected proofs

17b ii $\overrightarrow{AP} = -\frac{3}{2}\overrightarrow{PB}$

$\overrightarrow{PB} = -\frac{2}{3}\overrightarrow{AP}$

And, $\overrightarrow{AB} = \overrightarrow{AP} + \overrightarrow{PB}$

$= \overrightarrow{AP} + \left(-\frac{2}{3}\overrightarrow{AP}\right)$

$= \overrightarrow{AP} - \frac{2}{3}\overrightarrow{AP}$

$= \frac{1}{3}\overrightarrow{AP}$

$\overrightarrow{AP} = 3\overrightarrow{AB}$

Given, $\overrightarrow{AP} = k\overrightarrow{AB}$

Hence, $k = 3$

17b iii $\overrightarrow{AP} = -\frac{2}{3}\overrightarrow{PB}$

$\overrightarrow{PB} = -\frac{3}{2}\overrightarrow{AP}$

And, $\overrightarrow{AB} = \overrightarrow{AP} + \overrightarrow{PB}$

$= \overrightarrow{AP} + \left(-\frac{3}{2}\overrightarrow{AP}\right)$

$= \overrightarrow{AP} - \frac{3}{2}\overrightarrow{AP}$

$= -\frac{1}{2}\overrightarrow{AP}$

$\overrightarrow{AP} = -2\overrightarrow{AB}$

Given, $\overrightarrow{AP} = k\overrightarrow{AB}$

Hence, $k = -2$

17c $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{OB} - \overrightarrow{OA} = \underline{b} - \underline{a}$

And, $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP} = \overrightarrow{OP} - \overrightarrow{OA} = \underline{p} - \underline{a}$

Given, $\overrightarrow{AP} = k\overrightarrow{AB}$

$\underline{p} - \underline{a} = k(\underline{b} - \underline{a})$

$\underline{p} - \underline{a} = k\underline{b} - k\underline{a}$

Uncorrected proofs

$\underline{p} = \underline{a} - k\underline{a} + k\underline{b} = (1-k)\underline{a} + k\underline{b}$

18 The triangles are similar by the SAS similarity test — the angles between $\underline{a}$ and $\underline{b}$, and between $\lambda\underline{a}$ and $\lambda\underline{b}$ are equal, and the matching sides are in ratio1:λ. It now follows that the head of the vector $\lambda\underline{b}$ is the head of the vector $\lambda(\underline{a} + \underline{b})$.

19a Two zero vectors each have zero length and no direction, and so are equal.

19b Rome for administration (in the distant past), Greenwich UK for longitude, Jerusalem and Mecca for religious ceremonies, the North and South Poles for maps. The obelisk in Macquarie Place, Sydney, remains the origin for road distances in NSW. It is inscribed on the front,

'This Obelisk was erected in Macquarie Place A.D. 1818, to Record that all the public roads leading to the interior of the colony are measured from it. L. Macquarie Esq Governor'.

Solutions for Enrichment questions 20 and 21 are in preparation.

Uncorrected proofs

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Solutions to Exercise 8B

Solutions for Enrichment questions are in preparation.

1a $\underline{a} = 8\underline{i} + 6\underline{j}$

$x = 8$ and $y = 6$

Hence, $|\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100} = 10$

1b $2\underline{a} = 2 \times \left(8\underline{i} + 6\underline{j}\right) = 16\underline{i} + 12\underline{j}$

1c $2\underline{a} = 16\underline{i} + 12\underline{j}$

$x = 16$ and $y = 12$

Hence, $|2\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{16^2 + 12^2}$

$= \sqrt{256 + 144}$

$= \sqrt{400} = 20$

1d $-5\underline{a} = -5 \times \left(8\underline{i} + 6\underline{j}\right) = -40\underline{i} - 30\underline{j}$

1e $-5\underline{a} = -40\underline{i} - 30\underline{j}$

$x = -40$ and $y = -30$

Hence, $|-5\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{(-40)^2 + (-30)^2}$

$= \sqrt{1600 + 900}$

$= \sqrt{2500} = 50$

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Chapter 8 worked solutions – Vectors

2a $\underline{a} = 2\underline{i} + 3\underline{j}$ and $\underline{b} = \underline{i} - 4\underline{j}$

$$\underline{a} + \underline{b} = \left(2\underline{i} + 3\underline{j}\right) + \left(\underline{i} - 4\underline{j}\right)$$
$$= (2 + 1)\underline{i} + (3 - 4)\underline{j}$$
$$= 3\underline{i} - \underline{j}$$

2b $\underline{a} + \underline{b} = 3\underline{i} - \underline{j}$

$x = 3$ and $y = -1$

Hence, $|\underline{a} + \underline{b}| = \sqrt{x^2 + y^2}$
$$= \sqrt{3^2 + (-1)^2}$$
$$= \sqrt{9 + 1}$$
$$= \sqrt{10}$$

2c $\underline{a} = 2\underline{i} + 3\underline{j}$ and $\underline{b} = \underline{i} - 4\underline{j}$

$$\underline{a} - \underline{b} = \left(2\underline{i} + 3\underline{j}\right) - \left(\underline{i} - 4\underline{j}\right)$$
$$= (2 - 1)\underline{i} + (3 - (-4))\underline{j}$$
$$= (2 - 1)\underline{i} + (3 + 4)\underline{j}$$
$$= \underline{i} + 7\underline{j}$$

2d $\underline{a} - \underline{b} = \underline{i} + 7\underline{j}$

$x = 1$ and $y = 7$

Hence, $|\underline{a} - \underline{b}| = \sqrt{x^2 + y^2}$
$$= \sqrt{1^2 + 7^2}$$
$$= \sqrt{1 + 49}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

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Chapter 8 worked solutions – Vectors

2e $\underline{a} = 2\underline{i} + 3\underline{j}$ and $\underline{b} = \underline{i} - 4\underline{j}$

$-3\underline{a} = -6\underline{i} - 9\underline{j}$

$-2\underline{b} = -2\underline{i} + 8\underline{j}$

$-3\underline{a} - 2\underline{b} = \left(-6\underline{i} - 9\underline{j}\right) - \left(-2\underline{i} + 8\underline{j}\right)$

$= (-6 + (-2))\underline{i} + (-9 + 8)\underline{j}$

$= (-6 - 2)\underline{i} + (-9 + 8)\underline{j}$

$= -8\underline{i} - \underline{j}$

2f $-3\underline{a} - 2\underline{b} = -8\underline{i} - \underline{j}$

$x = -8$ and $y = -1$

Hence, $\left|-3\underline{a} - 2\underline{b}\right| = \sqrt{x^2 + y^2}$

$= \sqrt{(-8)^2 + (-1)^2}$

$= \sqrt{64 + 1}$

$= \sqrt{65}$

3a $\underline{a} = \begin{bmatrix} -17 \\ 3 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 5 \\ -11 \end{bmatrix}$ $\underline{c} = \begin{bmatrix} -7 \\ -13 \end{bmatrix}$

$\underline{a} + \underline{b} - \underline{c} = \begin{bmatrix} -17 + 5 - (-7) \\ 3 + (-11) - (-13) \end{bmatrix}$

$= \begin{bmatrix} -17 + 5 + 7 \\ 3 - 11 + 13 \end{bmatrix}$

$= \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

3b $\underline{a} + \underline{b} - \underline{c} = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

$x = -5$ and $y = 5$

$\left|\underline{a} + \underline{b} - \underline{c}\right| = \sqrt{x^2 + y^2}$

$= \sqrt{(-5)^2 + 5^2}$

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$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= 5\sqrt{2}$

3c $\underline{a} = \begin{bmatrix} -17 \\ 3 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 5 \\ -11 \end{bmatrix}$ $\underline{c} = \begin{bmatrix} -7 \\ -13 \end{bmatrix}$

$-3\underline{a} = \begin{bmatrix} 51 \\ -9 \end{bmatrix}$

$-5\underline{b} = \begin{bmatrix} -25 \\ 55 \end{bmatrix}$

$2\underline{c} = \begin{bmatrix} -14 \\ -26 \end{bmatrix}$

$-3\underline{a} - 5\underline{b} + 2\underline{c} = \begin{bmatrix} 51 - 25 - 14 \\ -9 + 55 - 26 \end{bmatrix}$

$= \begin{bmatrix} 12 \\ 20 \end{bmatrix}$

3d $-3\underline{a} - 5\underline{b} + 2\underline{c} = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$

$x = 12$ and $y = 20$

$|-3\underline{a} - 5\underline{b} + 2\underline{c}| = \sqrt{x^2 + y^2}$

$= \sqrt{12^2 + 20^2}$

$= \sqrt{144 + 400}$

$= \sqrt{544}$

$= 4\sqrt{34}$

4a $\underline{u} = 2\underline{i} + \underline{j}$ and $\underline{v} = -\underline{i} + 2\underline{j}$

$\underline{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\underline{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\underline{u} + \underline{v} = \begin{bmatrix} 2 + (-1) \\ 1 + 2 \end{bmatrix}$

$= \begin{bmatrix} 2 - 1 \\ 1 + 2 \end{bmatrix}$

Uncorrected proofs

$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

4b $\underline{a} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\underline{a} = 3\underline{i} + 2\underline{j}$ and $\underline{b} = 4\underline{i} + \underline{j}$

$\underline{a} - \underline{b} = \left(3\underline{i} + 2\underline{j}\right) - \left(4\underline{i} + \underline{j}\right)$

$= (3 - 4)\underline{i} + (2 - 1)\underline{j}$

$= -\underline{i} + \underline{j}$

5a $\underline{u} = \underline{i} + 2\underline{j}$

$x = 1$ and $y = 2$

Hence, $|\underline{u}| = \sqrt{x^2 + y^2}$

$= \sqrt{1^2 + 2^2}$

$= \sqrt{1 + 4}$

$= \sqrt{5}$

$\hat{\underline{u}} = \frac{\underline{u}}{|\underline{u}|} = \frac{\underline{i} + 2\underline{j}}{\sqrt{5}} = \frac{1}{\sqrt{5}}\underline{i} + \frac{2}{\sqrt{5}}\underline{j}$

5b $\underline{v} = -4\underline{i} + 3\underline{j}$

$x = -4$ and $y = 3$

Hence, $|\underline{v}| = \sqrt{x^2 + y^2}$

$= \sqrt{(-4)^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{-4\underline{i} + 3\underline{j}}{5} = -\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}$

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5c $\underline{w} = \underline{u} + \underline{v}$

$\underline{u} + \underline{v} = \left(\underline{i} + 2\underline{j}\right) + \left(-4\underline{i} + 3\underline{j}\right)$

$= (1 + (-4))\underline{i} + (2 + 3)\underline{j}$

$= (1 - 4)\underline{i} + (2 + 3)\underline{j}$

$= -3\underline{i} - 5\underline{j}$

$\underline{w} = -3\underline{i} + 5\underline{j}$

$x = -3$ and $y = 5$

Hence, $|\underline{w}| = \sqrt{x^2 + y^2}$

$= \sqrt{(-3)^2 + 5^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

$\hat{\underline{w}} = \frac{\underline{w}}{|\underline{w}|} = \frac{-3\underline{i} + 5\underline{j}}{\sqrt{34}} = -\frac{3}{\sqrt{34}}\underline{i} + \frac{5}{\sqrt{34}}\underline{j}$

6a $\underline{a} = \begin{bmatrix} 5 \\ -12 \end{bmatrix}$

$2\underline{a} = \begin{bmatrix} 10 \\ -24 \end{bmatrix}$

$|2\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{10^2 + (-24)^2}$

$= \sqrt{100 + 576}$

$= \sqrt{676}$

$= 26$

$|\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{5^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

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LHS: $|2\underline{a}| = 26$

RHS: $2|\underline{a}| = 2 \times 13 = 26$

Hence, LHS = RHS.

6b $\underline{b} = \begin{bmatrix} -15 \\ -8 \end{bmatrix}$

$-\underline{b} = \begin{bmatrix} 15 \\ 8 \end{bmatrix}$

$|-\underline{b}| = \sqrt{x^2 + y^2}$

$= \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289} = 17$

$|\underline{b}| = \sqrt{x^2 + y^2}$

$= \sqrt{(-15)^2 + (-8)^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289}$

$= 17$

LHS: $|-\underline{b}| = 17$

RHS: $|\underline{b}| = 17$

Hence, LHS = RHS.

6c $\underline{a} = \begin{bmatrix} 5 \\ -12 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} -15 \\ -8 \end{bmatrix}$

$\underline{a} + \underline{b} = \begin{bmatrix} 5 + (-15) \\ (-12) + (-8) \end{bmatrix}$

$= \begin{bmatrix} 5 - 15 \\ -12 - 8 \end{bmatrix}$

$= \begin{bmatrix} -10 \\ -20 \end{bmatrix}$

$|\underline{a} + \underline{b}| = \sqrt{x^2 + y^2}$

Uncorrected proofs

$= \sqrt{(-10)^2 + (-20)^2}$

$= \sqrt{100 + 400}$

$= \sqrt{500}$

$= 5\sqrt{10}$

$|\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{5^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

$|\underline{b}| = \sqrt{x^2 + y^2}$

$= \sqrt{(-15)^2 + (-8)^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289}$

$= 17$

LHS: $|\underline{a} + \underline{b}| = 5\sqrt{10}$

RHS: $|\underline{a}| + |\underline{b}| = 13 + 17 = 30$

Hence, LHS $<$ RHS

6d LHS: $|\underline{a} + \underline{b}| = 5\sqrt{10}$

RHS: $|\underline{a}| - |\underline{b}| = 13 - 17 = -4$

Hence, LHS $>$ RHS

7a $\overrightarrow{OP} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$ $\overrightarrow{OQ} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\overrightarrow{OR} = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \begin{bmatrix} 3 - (-1) \\ 2 - 6 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

Uncorrected proofs

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$$\text{Gradient} = \frac{y}{x} = \frac{-4}{4} = -1$$

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$$

$$= \begin{bmatrix} 8-3 \\ (-3)-2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

$$\text{Gradient} = \frac{y}{x} = \frac{-5}{5} = -1$$

P, Q and R are collinear as the gradient is -1

7b $\overrightarrow{OA} = 3\underline{i} + 8\underline{j}$ $\overrightarrow{OB} = -\underline{i} + 3\underline{j}$ and $\overrightarrow{OC} = -4\underline{i} - \underline{j}$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= \left(-\underline{i} + 3\underline{j}\right) - \left(3\underline{i} + 8\underline{j}\right)$$

$$= \left((-1) - 3\right)\underline{i} + (3-8)\underline{j}$$

$$= -4\underline{i} - 5\underline{j}$$

$$\text{Gradient} = \frac{y}{x} = \frac{-5}{-4} = \frac{5}{4}$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$

$$= \left(-4\underline{i} - \underline{j}\right) - \left(-\underline{i} + 3\underline{j}\right)$$

$$= \left((-4) - (-1)\right)\underline{i} + \left((-1) - 3\right)\underline{j}$$

$$= -3\underline{i} - 4\underline{j}$$

$$\text{Gradient} = \frac{y}{x} = \frac{-3}{-4} = \frac{3}{4}$$

A, B and C are not collinear as the gradient is not same.

8a $\underline{a} = \overrightarrow{OA} = 4\underline{i} - 7\underline{j}$

Consider O to be the origin and $\overrightarrow{OA}$ represented as the position vector.

Uncorrected proofs

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Chapter 8 worked solutions – Vectors

8b $\underline{b} = \overrightarrow{OB} = 6\underline{i} + 3\underline{j}$

Consider O to be the origin and $\overrightarrow{OB}$ represented as the position vector.

8c $M = \overrightarrow{OM}$ is the mid-point of $\overrightarrow{AB}$

$$\overrightarrow{AB} = \underline{a} + \underline{b}$$
$$= \left(6\underline{i} + 3\underline{j}\right) + \left(4\underline{i} - 7\underline{j}\right)$$
$$= (6 + 4)\underline{i} + (3 - 7)\underline{j}$$
$$= 10\underline{i} - 4\underline{j}$$
$$M = \overrightarrow{OM} = \frac{1}{2}\overrightarrow{AB}$$
$$= \frac{1}{2}(10\underline{i} - 4\underline{j})$$
$$= 5\underline{i} - 2\underline{j}$$

9a $\overrightarrow{OP} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

9b $2\overrightarrow{OP} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$\overrightarrow{OQ} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$

$2\overrightarrow{OP} - \overrightarrow{OQ} = \begin{bmatrix} 8 - (-3) \\ -2 - 5 \end{bmatrix} = \begin{bmatrix} 11 \\ -7 \end{bmatrix}$

9c $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$$= \begin{bmatrix} -3 - 4 \\ 5 - (-1) \end{bmatrix}$$
$$= \begin{bmatrix} -7 \\ 6 \end{bmatrix}$$

Uncorrected proofs

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9d $\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ}$

$= \begin{bmatrix} 4-(-3) \\ (-1)-5 \end{bmatrix}$

$= \begin{bmatrix} 7 \\ -6 \end{bmatrix}$

10a $\underline{a} = \overrightarrow{OA} = \underline{i} - \underline{j}$

Consider O to be the origin and $\overrightarrow{OA}$ represented as the position vector.

$\underline{b} = \overrightarrow{OB} = 7\underline{i} + 3\underline{j}$

Consider O to be the origin and $\overrightarrow{OB}$ represented as the position vector.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \left(7\underline{i} + 3\underline{j}\right) - \left(\underline{i} - \underline{j}\right)$

$= (7-1)\underline{i} + (3-(-1))\underline{j}$

$= 6\underline{i} + 4\underline{j}$

10b $|AB| = \sqrt{x^2 + y^2}$

$= \sqrt{6^2 + 4^2}$

$= \sqrt{36 + 16}$

$= \sqrt{52}$

$= 2\sqrt{13}$

10c $\widehat{AB} = \frac{\overrightarrow{AB}}{|AB|}$

$= \frac{6\underline{i}+4\underline{j}}{2\sqrt{13}}$

$= \frac{6}{2\sqrt{13}}\underline{i} + \frac{4}{2\sqrt{13}}\underline{j}$

$= \frac{3}{\sqrt{13}}\underline{i} + \frac{2}{\sqrt{13}}\underline{j}$

Uncorrected proofs

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11a $\underline{a} = 2\underline{i} + 2\underline{j}$

$|\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{2^2 + 2^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$\tan\theta = \frac{y}{x}$

$= \frac{2}{2}$

$\tan\theta = 1$

x and y are both positive in 1st quadrant hence,

$\theta = \frac{\pi}{4}$

11b $\underline{b} = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$

$|\underline{b}| = \sqrt{x^2 + y^2}$

$= \sqrt{1^2 + (-\sqrt{3})^2}$

$= \sqrt{1 + 3}$

$= \sqrt{4} = 2$

$\tan\theta = \frac{y}{x}$

$= \frac{-\sqrt{3}}{1}$

$\tan\theta = -\sqrt{3}$

x is positive and y is negative hence θ in 4th quadrant

$\theta = -\frac{\pi}{3}$

11c $\underline{c} = -3\sqrt{3}\underline{i} + 3\underline{j}$

$|\underline{c}| = \sqrt{x^2 + y^2}$

Uncorrected proofs

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$= \sqrt{(-3\sqrt{3})^2 + 3^2}$

$= \sqrt{27 + 9}$

$= \sqrt{36} = 6$

$\tan\theta = \frac{y}{x}$

$= \frac{3}{-3\sqrt{3}}$

$\tan\theta = -\frac{1}{\sqrt{3}}$

x is negative and y is positive hence θ in 2nd quadrant

$\theta = \frac{5\pi}{6}$

11d $\underline{d} = \begin{bmatrix} -\sqrt{6} \\ -\sqrt{6} \end{bmatrix}$

$|\underline{d}| = \sqrt{x^2 + y^2}$

$= \sqrt{(-\sqrt{6})^2 + (-\sqrt{6})^2}$

$= \sqrt{6 + 6}$

$= \sqrt{12}$

$= 2\sqrt{3}$

$\tan\theta = \frac{y}{x}$

$= \frac{-\sqrt{6}}{-\sqrt{6}}$

$\tan\theta = 1$

x is negative and y is negative hence θ in 3rd quadrant

$\theta = -\frac{3\pi}{4}$

12a Let $\underline{a}$ be the vector

$|\underline{a}| = 4$

$\sqrt{x^2 + y^2} = 4$

Uncorrected proofs

$x^2 + y^2 = 16 \quad (1)$

$\theta = -\frac{\pi}{4}$

$\tan\theta = -1$

θ in 4th quadrant so x is positive and y is negative.

$\frac{y}{x} = -1$

$y = -x$

Substituting the value of y in equation (1)

$x^2 + y^2 = 16$

$x^2 + (-x)^2 = 16$

$x^2 + x^2 = 16$

$2x^2 = 16$

$x^2 = 8$

$x = 2\sqrt{2}$

$y = -x = -2\sqrt{2}$

$\underline{a} = 2\sqrt{2}\underline{i} - 2\sqrt{2}\underline{j}$

12b Let $\underline{a}$ be the vector

$|\underline{a}| = 2\sqrt{6}$

$\sqrt{x^2 + y^2} = 2\sqrt{6}$

$x^2 + y^2 = 24 \quad (1)$

$\theta = \frac{2\pi}{3}$

$\tan\theta = \tan\frac{2\pi}{3} = -\sqrt{3}$

θ in 2nd quadrant so x is negative and y is positive.

$\frac{y}{x} = -\sqrt{3}$

$y = -\sqrt{3}x$

Substituting the value of y in equation (1)

$x^2 + y^2 = 24$

Uncorrected proofs

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$x^2 + (-\sqrt{3}x)^2 = 24$

$x^2 + 3x^2 = 24$

$4x^2 = 24$

$x^2 = 6$

$x = -\sqrt{6}$

$y = -\sqrt{3}x = -\sqrt{6} \times -\sqrt{3} = 3\sqrt{2}$

$\underline{a} = -\sqrt{6}\underline{i} + 3\sqrt{2}\underline{j}$

12c Let $\underline{a}$ be the vector

$|\underline{a}| = 2$

$\sqrt{x^2 + y^2} = 2$

$x^2 + y^2 = 4 \quad (1)$

$\theta = -\frac{5\pi}{6}$

$\tan\theta = \tan\left(-\frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}}$

θ in 3rd quadrant so x is negative and y is negative.

$\frac{y}{x} = \frac{1}{\sqrt{3}}$

$y = \frac{1}{\sqrt{3}}x$

Substituting the value of y in equation (1)

$x^2 + y^2 = 4$

$x^2 + (\frac{1}{\sqrt{3}}x)^2 = 4$

$x^2 + \frac{1}{3}x^2 = 4$

$\frac{4}{3}x^2 = 4$

$x^2 = 3$

$x = -\sqrt{3}$

$y = \frac{1}{\sqrt{3}}x = \frac{1}{\sqrt{3}} \times -\sqrt{3} = -1$

Uncorrected proofs

$\underline{a} = -\sqrt{3}\underline{i} - \underline{j}$

12d Let $\underline{a}$ be the vector

$|\underline{a}| = 2\sqrt{2}$

$\sqrt{x^2 + y^2} = 2\sqrt{2}$

$x^2 + y^2 = 8 \quad (1)$

$\theta = \frac{5\pi}{12}$

$\tan\theta = \tan\frac{5\pi}{12} = 2 + \sqrt{3}$

θ in 1st quadrant so x is positive and y is positive.

$\frac{y}{x} = 2 + \sqrt{3}$

$y = (2 + \sqrt{3})\, x$

Substituting the value of y in equation (1)

$x^2 + y^2 = 8$

$x^2 + ((2 + \sqrt{3})x)^2 = 8$

$x = \sqrt{3} - 1$

$y = (2 + \sqrt{3})x = (2 + \sqrt{3}) \times (\sqrt{3} - 1) = \sqrt{3} + 1$

$\underline{a} = -(\sqrt{3} - 1)\underline{i} + (\sqrt{3} + 1)\,\underline{j}$

13 $\underline{a} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} -3 \\ -5 \end{bmatrix} \quad \underline{c} = \begin{bmatrix} 24 \\ 8 \end{bmatrix}$

$\underline{c} = \lambda_1\underline{a} + \lambda_2\underline{b}$

$\begin{bmatrix} 24 \\ 8 \end{bmatrix} = \lambda_1 \begin{bmatrix} 2 \\ -2 \end{bmatrix} + \lambda_2 \begin{bmatrix} -3 \\ -5 \end{bmatrix}$

$\begin{bmatrix} 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 2\lambda_1 \\ -2\lambda_1 \end{bmatrix} + \begin{bmatrix} -3\lambda_2 \\ -5\lambda_2 \end{bmatrix}$

$\begin{bmatrix} 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 2\lambda_1 - 3\lambda_2 \\ -2\lambda_1 - 5\lambda_2 \end{bmatrix}$

Hence:

$2\lambda_1 - 3\lambda_2 = 24 \quad (1)$

Uncorrected proofs

$-2\lambda_1 - 5\lambda_2 = 8 \quad (2)$

Add the equations (1) and (2) we get,

$-8\lambda_2 = 32$

$\lambda_2 = -4$

Substituting the value of λ_2 in equation (1) we get,

$2\lambda_1 - 3\lambda_2 = 24$

$2\lambda_1 - 3 \times -4 = 24$

$2\lambda_1 + 12 = 24$

$2\lambda_1 = 12$

$\lambda_1 = 6$

14a Let O be the origin.

$\overrightarrow{OA} = \begin{bmatrix} 2\sqrt{3} \\ 3 \end{bmatrix}$

$\overrightarrow{OB} = \begin{bmatrix} 3\sqrt{3} \\ 4 \end{bmatrix}$

$\overrightarrow{OC} = \begin{bmatrix} 2\sqrt{3} \\ 5 \end{bmatrix}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{bmatrix} 3\sqrt{3} - 2\sqrt{3} \\ 4 - 3 \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$

$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \begin{bmatrix} 3\sqrt{3} - 2\sqrt{3} \\ 4 - 5 \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ -1 \end{bmatrix}$

14b $\left|\overrightarrow{AB}\right| = \sqrt{x^2 + y^2}$

$= \sqrt{(\sqrt{3})^2 + 1^2}$

$= \sqrt{3 + 1}$

$= \sqrt{4}$

$= 2$

Uncorrected proofs

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$$|\overrightarrow{CB}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$= \sqrt{3 + 1}$$

$$= \sqrt{4}$$

$$= 2$$

14c $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{bmatrix} 2\sqrt{3} - 2\sqrt{3} \\ 5 - 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$$|\overrightarrow{AC}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{0^2 + 2^2}$$

$$= \sqrt{0 + 4}$$

$$= \sqrt{4}$$

$$= 2$$

$|\overrightarrow{AB}| = |\overrightarrow{CB}| = |\overrightarrow{AC}|$

Hence, ABC is a triangle.

15 Let O be the origin.

$\overrightarrow{OA} = \begin{bmatrix} -7 \\ -5 \end{bmatrix}$

$\overrightarrow{OB} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$\overrightarrow{OC} = \begin{bmatrix} 10 \\ 9 \end{bmatrix}$

$\overrightarrow{OD} = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{bmatrix} -5 - (-7) \\ 6 - (-5) \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{bmatrix} 10 - 8 \\ 9 - (-2) \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$

Hence, $\overrightarrow{AD} = \overrightarrow{BC}$

Therefore, $ABCD$ is a parallelogram as opposite sides are equal and parallel.

Uncorrected proofs

16a Let O be the origin.

$\overrightarrow{OP} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$

$\overrightarrow{OQ} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\overrightarrow{OR} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$\overrightarrow{OS} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{bmatrix} 2-(-3) \\ -2-(-4) \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$\overrightarrow{SR} = \overrightarrow{OR} - \overrightarrow{OS} = \begin{bmatrix} 4-(-1) \\ 3-1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

Hence, $\overrightarrow{PQ} = \overrightarrow{SR}$

16b $|\overrightarrow{PQ}| = \sqrt{x^2 + y^2}$

$= \sqrt{5^2 + 2^2}$

$= \sqrt{25 + 4}$

$= \sqrt{29}$

$|\overrightarrow{SR}| = \sqrt{x^2 + y^2}$

$= \sqrt{5^2 + 2^2}$

$= \sqrt{25 + 4}$

$= \sqrt{29}$

Hence, $|\overrightarrow{PQ}| = |\overrightarrow{SR}|$

16c $PQRS$ is a rhombus as adjacent sides are equal $|\overrightarrow{PQ}| = |\overrightarrow{SR}|$.

17a $\overrightarrow{OA} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

$\overrightarrow{OB} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

Uncorrected proofs

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$\overrightarrow{OC} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{bmatrix} 2-5 \\ 7-(-3) \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$

$\frac{1}{2}\overrightarrow{AC} = \begin{bmatrix} -\frac{3}{2} \\ 5 \end{bmatrix}$

LHS: $\overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$

$= \begin{bmatrix} 5 \\ -3 \end{bmatrix} + \begin{bmatrix} -\frac{3}{2} \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 5 + (-\frac{3}{2}) \\ -3 + 5 \end{bmatrix}$

$= \begin{bmatrix} \frac{7}{2} \\ 2 \end{bmatrix}$

RHS: $\overrightarrow{OB} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

$\frac{1}{2}\overrightarrow{OB} = \begin{bmatrix} \frac{7}{2} \\ 2 \end{bmatrix}$

Hence, LHS = RHS.

17b $OABC$ is a parallelogram as diagonals bisect each other.

18 Let O be the origin hence,

$\overrightarrow{OW} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$

$\overrightarrow{OX} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

$\overrightarrow{OY} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$

$\overrightarrow{OZ} = \begin{bmatrix} a \\ b \end{bmatrix}$

$\overrightarrow{WZ} = \overrightarrow{OZ} - \overrightarrow{OW} = \begin{bmatrix} a-(-6) \\ b-4 \end{bmatrix} = \begin{bmatrix} a+6 \\ b-4 \end{bmatrix}$

$\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX} = \begin{bmatrix} 4-6 \\ 9-2 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$

As $WXYZ$ is a parallelogram, opposite sides are equal hence,

Uncorrected proofs

$\overrightarrow{WZ} = \overrightarrow{XY}$

$\begin{bmatrix} a+6 \\ b-4 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$

$a + 6 = -2$

$a = -8$

$b - 4 = 7$

$b = 11$

Solutions for Enrichment question 19 are in preparation.

Uncorrected proofs

Solutions to Exercise 8C

Solutions for Enrichment questions are in preparation.

1a $\underline{a} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\underline{a} \cdot \underline{b} = 3 \times 2 + 1 \times 4 = 6 + 4 = 10$

1b $\underline{a} = \begin{bmatrix} -8 \\ -5 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 6 \\ -14 \end{bmatrix}$

$\underline{a} \cdot \underline{b} = (-8) \times 6 + (-5) \times (-14) = -48 + 70 = 22$

1c $\underline{a} = \begin{bmatrix} 6u \\ -2v \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 3v \\ 9u \end{bmatrix}$

$\underline{a} \cdot \underline{b} = 6u \times 3v + (-2v) \times 9u = 18uv - 18uv = 0$

1d $\underline{a} = \begin{bmatrix} x-1 \\ x-2 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} x-1 \\ x+2 \end{bmatrix}$

$\underline{a} \cdot \underline{b} = (x-1) \times (x-1) + (x-2) \times (x+2)$

$= (x-1)^2 + x^2 - 4$

$= x^2 + 1 - 2x + x^2 - 4$

$= 2x^2 - 2x - 3$

2a $|\underline{a}| = 6$ $|\underline{b}| = 5$ $\theta = 60°$

$\underline{a} \cdot \underline{b} = |\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = 6 \times 5 \cos 60°$

$\underline{a} \cdot \underline{b} = 6 \times 5 \times \frac{1}{2}$

$\underline{a} \cdot \underline{b} = 6 \times \frac{5}{2}$

$\underline{a} \cdot \underline{b} = 15$

Uncorrected proofs

Chapter 8 worked solutions – Vectors

2b $|\underline{a}| = 4 \qquad |\underline{b}| = 3 \qquad \theta = 45°$

$\underline{a} \cdot \underline{b} = |\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = 4 \times 3 \cos 45°$

$\underline{a} \cdot \underline{b} = 4 \times 3 \times \frac{1}{\sqrt{2}}$

$\underline{a} \cdot \underline{b} = 12 \times \frac{1}{\sqrt{2}}$

$\underline{a} \cdot \underline{b} = 6\sqrt{2}$

3a $|\underline{u}| = 4 \qquad |\underline{v}| = 5$

$\underline{u} \cdot \underline{v} = |\underline{u}| \times |\underline{v}| \cos\theta$

$-10 = 4 \times 5 \cos\theta$

$20 \cos\theta = -10$

$\cos\theta = -\frac{1}{2}$

$\theta = 120°$

3b $|\underline{u}| = 3 \qquad |\underline{v}| = 5$

$\underline{u} \cdot \underline{v} = |\underline{u}| \times |\underline{v}| \cos\theta$

$12 = 3 \times 5 \cos\theta$

$15 \cos\theta = 12$

$\cos\theta = \frac{4}{5} = 0.8$

$\theta \div 37°$

4a $4\underline{i} \cdot 2\underline{j} = 8 \times \underline{i} \cdot \underline{j} = 8 \times 0 = 0$

4b $-5\underline{i} \cdot 3\underline{j} = -15 \times \underline{i} \cdot \underline{j} = -15 \times 0 = 0$

4c $4\underline{i} \cdot 2\underline{i} = 8 \times \underline{i} \cdot \underline{i} = 8 \times 1 = 8$

Uncorrected proofs

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4d $-5\underline{j} \cdot 3\underline{j} = -15 \times \underline{j} \cdot \underline{j} = -15 \times 1 = -15$

5a $(4\underline{i} + 2\underline{j}) \cdot (4\underline{i} + 2\underline{j})$

$= 4\underline{i} \cdot 4\underline{i} + 4\underline{i} \cdot 2\underline{j} + 2\underline{j} \cdot 4\underline{i} + 2\underline{j} \cdot 2\underline{j}$

$= 16\underline{i} \cdot \underline{i} + 8\underline{i} \cdot \underline{j} + 8\underline{j} \cdot \underline{i} + 4\underline{j} \cdot \underline{j}$

$= 16 \times 1 + 8 \times 0 + 8 \times 0 + 4 \times 1$

$= 16 + 4$

$= 20$

5b $(-5\underline{i} + 3\underline{j}) \cdot (-5\underline{i} + 3\underline{j})$

$= (-5)\underline{i} \cdot (-5)\underline{i} + (-5)\underline{i} \cdot 3\underline{j} + 3\underline{j} \cdot (-5)\underline{i} + 3\underline{j} \cdot 3\underline{j}$

$= 25\underline{i} \cdot \underline{i} - 15\underline{i} \cdot \underline{j} - 15\underline{j} \cdot \underline{i} + 9\underline{j} \cdot \underline{j}$

$= 25 \times 1 - 15 \times 0 - 15 \times 0 + 9 \times 1$

$= 25 + 9$

$= 34$

5c $(4\underline{i} + 2\underline{j}) \cdot (-5\underline{i} + 3\underline{j})$

$= 4\underline{i} \cdot (-5)\underline{i} + 4\underline{i} \cdot 3\underline{j} + 2\underline{j} \cdot (-5)\underline{i} + 2\underline{j} \cdot 3\underline{j}$

$= -20\underline{i} \cdot \underline{i} + 12\underline{i} \cdot \underline{j} - 10\underline{j} \cdot \underline{i} + 6\underline{j} \cdot \underline{j}$

$= -20 \times 1 + 12 \times 0 - 10 \times 0 + 6 \times 1$

$= -20 + 6$

$= -14$

6a $\underline{u} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ $\underline{v} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\underline{u} \cdot \underline{v} = (-4) \times 7 + 5 \times 6$

$= -28 + 30$

$= 2$

Uncorrected proofs

$\underline{u}$ and $\underline{v}$ are not perpendicular as $\underline{u} \cdot \underline{v} \neq 0$.

6b $\underline{u} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$ $\underline{v} = \begin{bmatrix} 18 \\ -12 \end{bmatrix}$

$\underline{u} \cdot \underline{v} = (-4) \times 18 + (-6) \times (-12)$

$= -72 + 72$

$= 0$

$\underline{u}$ and $\underline{v}$ are perpendicular as $\underline{u} \cdot \underline{v} = 0$.

6c $\underline{u} = \begin{bmatrix} -1 \\ a^{-2} \end{bmatrix}$ $\underline{v} = \begin{bmatrix} a^{-1} \\ a \end{bmatrix}$

$\underline{u} \cdot \underline{v} = (-1) \times a^{-1} + a^{-2} \times a$

$= -a^{-1} + a^{-1}$

$= 0$

$\underline{u}$ and $\underline{v}$ are perpendicular as $\underline{u} \cdot \underline{v} = 0$.

7a Consider O to be the origin and $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ represent position vectors.

Hence,

$\overrightarrow{OA} = 2\underline{i} + 5\underline{j}$ $\overrightarrow{OB} = 5\underline{i} + 14\underline{j}$ and $\overrightarrow{OC} = -2\underline{i} + 13\underline{j}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \left(5\underline{i} + 14\underline{j}\right) - \left(2\underline{i} + 5\underline{j}\right)$

$= (5 - 2)\underline{i} + (14 - 5)\underline{j}$

$= 3\underline{i} + 9\underline{j}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= \left(-2\underline{i} + 13\underline{j}\right) - \left(2\underline{i} + 5\underline{j}\right)$

$= ((-2) - 2)\underline{i} + (13 - 5)\underline{j}$

$= -4\underline{i} + 8\underline{j}$

Uncorrected proofs

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7b $\overrightarrow{AB} \cdot \overrightarrow{AC} = 3 \times (-4) + 9 \times 8$

$= -12 + 72$

$= 60$

7c $|\overrightarrow{AB}| = \sqrt{x^2 + y^2}$

$= \sqrt{3^2 + 9^2}$

$= \sqrt{9 + 81}$

$= \sqrt{90}$

$= 3\sqrt{10}$

$|\overrightarrow{AC}| = \sqrt{x^2 + y^2}$

$= \sqrt{(-4)^2 + 8^2}$

$= \sqrt{16 + 64}$

$= \sqrt{80}$

$= 4\sqrt{5}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{AB}| \times |\overrightarrow{AC}| \cos\theta$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 3\sqrt{10} \times 4\sqrt{5} \cos 45°$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 3\sqrt{10} \times 4\sqrt{5} \times \frac{1}{\sqrt{2}}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 60\sqrt{2} \times \frac{1}{\sqrt{2}} = 60$

Hence, it is confirmed with part b above.

8a Consider O to be the origin and $\overrightarrow{OP}$, $\overrightarrow{OQ}$ and $\overrightarrow{OR}$ represent position vectors.

Hence,

$\overrightarrow{OP} = \sqrt{3}\underset{\sim}{i} + 8\underset{\sim}{j}$ $\overrightarrow{OQ} = 3\sqrt{3}\underset{\sim}{i} + 14\underset{\sim}{j}$ and $\overrightarrow{OR} = 5\sqrt{3}\underset{\sim}{i} + 12\underset{\sim}{j}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \left(3\sqrt{3}\underset{\sim}{i} + 14\underset{\sim}{j}\right) - \left(\sqrt{3}\underset{\sim}{i} + 8\underset{\sim}{j}\right)$

Uncorrected proofs

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$= (3\sqrt{3} - \sqrt{3})\underline{i} + (14 - 8)\underline{j}$

$= 2\sqrt{3}\underline{i} + 6\underline{j}$

$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$= \left(5\sqrt{3}\underline{i} + 12\underline{j}\right) - \left(\sqrt{3}\underline{i} + 8\underline{j}\right)$

$= (5\sqrt{3} - \sqrt{3})\underline{i} + (12 - 8)\underline{j}$

$= 4\sqrt{3}\underline{i} + 4\underline{j}$

8b $\overrightarrow{PQ} \cdot \overrightarrow{PR} = 2\sqrt{3} \times 4\sqrt{3} + 6 \times 4$

$= 24 + 24$

$= 48$

8c $\left|\overrightarrow{PQ}\right| = \sqrt{x^2 + y^2}$

$= \sqrt{(2\sqrt{3})^2 + 6^2}$

$= \sqrt{12 + 36}$

$= \sqrt{48}$

$= 4\sqrt{3}$

$\left|\overrightarrow{PR}\right| = \sqrt{x^2 + y^2}$

$= \sqrt{(4\sqrt{3})^2 + 4^2}$

$= \sqrt{48 + 16}$

$= \sqrt{64}$

$= 8$

$\overrightarrow{PQ} \cdot \overrightarrow{PR} = \left|\overrightarrow{PQ}\right| \times \left|\overrightarrow{PR}\right| \cos\theta$

$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 4\sqrt{3} \times 8 \cos 30°$

$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 4\sqrt{3} \times 8 \times \frac{\sqrt{3}}{2}$

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$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 32\sqrt{3} \times \frac{\sqrt{3}}{2} = 48$

Hence, it is confirmed with part b above.

9a $\underline{a} = 4\underline{i} + 3\underline{j} \quad \underline{b} = 5\underline{j}$

$\underline{a} \cdot \underline{b} = (4\underline{i} + 3\underline{j}) \cdot (5\underline{j})$

$= 4\underline{i} \cdot 0 + 4\underline{i} \cdot 5\underline{j} + 3\underline{j} \cdot 0 + 3\underline{j} \cdot 5\underline{j}$

$= 0 + 20\underline{i} \cdot \underline{j} + 0 + 15\underline{j} \cdot \underline{j}$

$= 0 + 20 \times 0 + 0 + 15 \times 1$

$= 15$

$|\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

$|\underline{b}| = \sqrt{x^2 + y^2}$

$= \sqrt{5^2}$

$= \sqrt{25}$

$= 5$

$\underline{a} \cdot \underline{b} = |\underline{a}| \times |b| \cos\theta$

$15 = 5 \times 5 \cos\theta$

$25 \cos\theta = 15$

$\cos\theta = \frac{15}{25} = \frac{3}{5}$

9b $\underline{a} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \underline{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\underline{a} \cdot \underline{b} = 2 \times 3 + 2 \times (-1)$

Uncorrected proofs

$= 6 - 2$

$= 4$

$|\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{2^2 + 2^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$|\underline{b}| = \sqrt{x^2 + y^2}$

$= \sqrt{3^2 + (-1)^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

$\underline{a} \cdot \underline{b} = |\underline{a}| \times |b| \cos\theta$

$4 = 2\sqrt{2} \times \sqrt{10} \cos\theta$

$4\sqrt{5} \cos\theta = 4$

$\cos\theta = \frac{4}{4\sqrt{5}} = \frac{1}{\sqrt{5}}$

9c $\underline{a} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$ $\underline{v} = \begin{bmatrix} -8 \\ -2 \end{bmatrix}$

$\underline{a} \cdot \underline{b} = (-6) \times (-8) + 4 \times (-2)$

$= 48 - 8$

$= 40$

$|\underline{a}| = \sqrt{x^2 + y^2}$

$= \sqrt{(-6)^2 + 4^2}$

$= \sqrt{36 + 16}$

$= \sqrt{52}$

$= 2\sqrt{13}$

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$$|\underline{b}| = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-8)^2 + (-2)^2}$$
$$= \sqrt{64 + 4}$$
$$= \sqrt{68}$$
$$= 2\sqrt{17}$$

$$\underline{a} \cdot \underline{b} = |\underline{a}| \times |b| \cos\theta$$
$$40 = 2\sqrt{13} \times 2\sqrt{17} \cos\theta$$
$$4\sqrt{221} \cos\theta = 40$$
$$\cos\theta = \frac{40}{4\sqrt{221}} = \frac{10}{\sqrt{221}}$$

10 $\underline{u} = \lambda^2\underline{i} + 2\underline{j} \quad \underline{v} = 3\underline{i} - (2 + 2\lambda)\underline{j}$

$\underline{u}$ and $\underline{v}$ are perpendicular

Hence,

$$\underline{u} \cdot \underline{v} = 0$$
$$\left(\lambda^2\underline{i} + 2\underline{j}\right) \cdot \left(3\underline{i} - (2 + 2\lambda)\underline{j}\right) = 0$$
$$\lambda^2\underline{i} \cdot 3\underline{i} - \lambda^2\underline{i} \cdot (2 + 2\lambda)\underline{j} + 2\underline{j} \cdot 3\underline{i} - 2\underline{j} \cdot (2 + 2\lambda)\underline{j} = 0$$
$$3\lambda^2\underline{i} \cdot \underline{i} - (2 + 2\lambda)\lambda^2\underline{i} \cdot \underline{j} + 6\underline{j} \cdot \underline{i} - (2 + 2\lambda)2\underline{j} \cdot \underline{j} = 0$$
$$3\lambda^2 \times 1 + (2 + 2\lambda)\lambda^2 \times 0 + 6 \times 0 - (2 + 2\lambda)2 \times 1 = 0$$
$$3\lambda^2 - (2 + 2\lambda)2 = 0$$
$$3\lambda^2 - 4 - 4\lambda = 0$$
$$3\lambda^2 - 6\lambda + 2\lambda - 4 = 0$$
$$3\lambda(\lambda - 2) + 2(\lambda - 2) = 0$$
$$(3\lambda + 2)(\lambda - 2) = 0$$
$$(3\lambda + 2) = 0 \; or \; (\lambda - 2) = 0$$
$$\lambda = -\frac{2}{3} \; 0r \; \lambda = 2$$

Uncorrected proofs

Chapter 8 worked solutions – Vectors

11a i $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{3}$

$\underline{a} \cdot \underline{b} = |\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = 6 \times 2 \cos\frac{\pi}{3}$

$\underline{a} \cdot \underline{b} = 12 \times \frac{1}{2}$

$\underline{a} \cdot \underline{b} = 6$

11a ii $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{3}$

$2\underline{a} \cdot (-5)\underline{b} = -10|\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = -10 \times 6 \times 2 \cos\frac{\pi}{3}$

$\underline{a} \cdot \underline{b} = -120 \times \frac{1}{2}$

$\underline{a} \cdot \underline{b} = -60$

11a iii $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{3}$

$4\underline{a} \cdot 0\underline{b} = 0|\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = 0 \times 6 \times 2 \cos\frac{\pi}{3}$

$\underline{a} \cdot \underline{b} = 0$

11a iv $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{3}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = |\underline{a}|^2 + |\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 6^2 + 6 \times 2 \cos\frac{\pi}{3}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 36 + 6 \times 2 \times \frac{1}{2}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 36 + 6 = 42$

Uncorrected proofs

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11a v $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{3}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = |\underline{b}|^2 + |\underline{b}| \times |\underline{a}| \cos\theta$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 2^2 + 2 \times 6 \cos\frac{\pi}{3}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 4 + 2 \times 6 \times \frac{1}{2}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 4 + 6 = 10$

11a vi $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{3}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} - \underline{b} \cdot \underline{a} - \underline{b} \cdot \underline{b}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} - \underline{b} \cdot \underline{b}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = |\underline{a}|^2 - |\underline{b}|^2$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = 6^2 - 2^2$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = 36 - 4 = 32$

11b i $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{2\pi}{3}$

$\underline{a} \cdot \underline{b} = |\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = 6 \times 2 \cos\frac{2\pi}{3}$

$\underline{a} \cdot \underline{b} = 12 \times -\frac{1}{2}$

$\underline{a} \cdot \underline{b} = -6$

11b ii $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{2\pi}{3}$

$2\underline{a} \cdot (-5)\underline{b} = -10|\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = -10 \times 6 \times 2 \cos\frac{2\pi}{3}$

$\underline{a} \cdot \underline{b} = -120 \times -\frac{1}{2}$

$\underline{a} \cdot \underline{b} = 60$

Uncorrected proofs

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11b iii $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{2\pi}{3}$

$4\underline{a} \cdot 0\underline{b} = 0|\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = 0 \times 6 \times 2 \cos\frac{2\pi}{3}$

$\underline{a} \cdot \underline{b} = 0$

11b iv $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{2\pi}{3}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = |\underline{a}|^2 + |\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 6^2 + 6 \times 2 \cos\frac{2\pi}{3}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 36 + 6 \times 2 \times -\frac{1}{2}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 36 - 6 = 30$

11b v $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{2\pi}{3}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = |\underline{b}|^2 + |\underline{b}| \times |\underline{a}| \cos\theta$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 2^2 + 2 \times 6 \cos\frac{2\pi}{3}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 4 + 2 \times 6 \times -\frac{1}{2}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 4 - 6 = -2$

11b vi $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{2\pi}{3}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} - \underline{b} \cdot \underline{a} - \underline{b} \cdot \underline{b}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} - \underline{b} \cdot \underline{b}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = |\underline{a}|^2 - |\underline{b}|^2$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = 6^2 - 2^2$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = 36 - 4 = 32$

Uncorrected proofs

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11c i $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{2}$

$\underline{a} \cdot \underline{b} = |\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = 6 \times 2 \cos\frac{\pi}{2}$

$\underline{a} \cdot \underline{b} = 12 \times 0$

$\underline{a} \cdot \underline{b} = 0$

11c ii $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{2}$

$2\underline{a} \cdot (-5)\underline{b} = -10|\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = -10 \times 6 \times 2 \cos\frac{\pi}{2}$

$\underline{a} \cdot \underline{b} = -120 \times 0$

$\underline{a} \cdot \underline{b} = 0$

11c iii $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{2}$

$4\underline{a} \cdot 0\underline{b} = 0|\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot \underline{b} = 0 \times 6 \times 2 \cos\frac{\pi}{2}$

$\underline{a} \cdot \underline{b} = 0$

11c iv $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{2}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = |\underline{a}|^2 + |\underline{a}| \times |\underline{b}| \cos\theta$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 6^2 + 6 \times 2 \cos\frac{\pi}{2}$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 36 + 6 \times 2 \times 0$

$\underline{a} \cdot (\underline{a} + \underline{b}) = 36 + 0 = 36$

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11c v $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{2}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = |\underline{b}|^2 + |\underline{b}| \times |\underline{a}| \cos\theta$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 2^2 + 2 \times 6 \cos\frac{\pi}{2}$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 4 + 2 \times 6 \times 0$

$\underline{b} \cdot (\underline{a} + \underline{b}) = 4 + 0 = 4$

11c vi $|\underline{a}| = 6 \qquad |\underline{b}| = 2 \qquad \theta = \frac{\pi}{3}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} - \underline{b} \cdot \underline{a} - \underline{b} \cdot \underline{b}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = \underline{a} \cdot \underline{a} - \underline{b} \cdot \underline{b}$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = |\underline{a}|^2 - |\underline{b}|^2$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = 6^2 - 2^2$

$(\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) = 36 - 4 = 32$

12a Consider O to be the origin and $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ represent position vectors.

Hence,

$\overrightarrow{OA} = -3\underline{i} - 6\underline{j}$ $\overrightarrow{OB} = \underline{i} - 4\underline{j}$ $\overrightarrow{OC} = -2\underline{i} + 2\underline{j}$ and $\overrightarrow{OD} = -6\underline{i}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (\underline{i} - 4\underline{j}) - (-3\underline{i} - 6\underline{j})$

$= (1 - (-3))\underline{i} + ((-4) - (-6))\underline{j}$

$= (1 + 3)\underline{i} + (-4 + 6)\underline{j}$

$= 4\underline{i} + 2\underline{j}$

$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$

$= (-2\underline{i} + 2\underline{j}) - (-6\underline{i})$

$= ((-2) - (-6))\underline{i} + 2\underline{j}$

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$= (-2 + 6)\underline{i} + 2\underline{j}$

$= 4\underline{i} + 2\underline{j}$

Hence, $\overrightarrow{AB} = \overrightarrow{DC}$

12b $\overrightarrow{AB} = 4\underline{i} + 2\underline{j}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \left(-6\underline{i}\right) - \left(-3\underline{i} - 6\underline{j}\right)$

$= ((-6) - (-3))\underline{i} - (-6)\underline{j}$

$= (-6 + 3)\underline{i} + 6\underline{j}$

$= -3\underline{i} + 6\underline{j}$

By using the method, $\underline{u} \cdot \underline{v} = x_1 \times x_2 + y_1 \times y_2$

$\overrightarrow{AB} \cdot \overrightarrow{AD} = 4 \times (-3) + 2 \times 6$

$\overrightarrow{AB} \cdot \overrightarrow{AD} = -12 + 12 = 0$

12c From part a, $\overrightarrow{AB} = \overrightarrow{DC}$

From part b, $\overrightarrow{AB} \cdot \overrightarrow{AD} = 0$

This means two sides of the quadrilateral are equal and parallel and two sides are perpendicular to each other hence, a right-angled parallelogram or rectangle.

13a Consider O to be the origin and $\overrightarrow{OP}$, $\overrightarrow{OQ}$, $\overrightarrow{OR}$ and $\overrightarrow{OS}$ represent position vectors.

Hence,

$\overrightarrow{OP} = -8\underline{i} + 3\underline{j}$ $\overrightarrow{OQ} = 3\underline{i} + 7\underline{j}$ $\overrightarrow{OR} = 7\underline{i} + 18\underline{j}$ and $\overrightarrow{OS} = -4\underline{i} + 14\underline{j}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \left(3\underline{i} + 7\underline{j}\right) - \left(-8\underline{i} + 3\underline{j}\right)$

$= (3 - (-8))\underline{i} + (7 - 3)\underline{j}$

$= (3 + 8)\underline{i} + (7 - 3)\underline{j}$

Uncorrected proofs

$= 11\underset{\sim}{i} + 4\underset{\sim}{j}$

$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$= \left(7\underset{\sim}{i} + 18\underset{\sim}{j}\right) - \left(-8\underset{\sim}{i} + 3\underset{\sim}{j}\right)$

$= (7 - (-8))\underset{\sim}{i} + (18 - 3)\underset{\sim}{j}$

$= (7 + 8)\underset{\sim}{i} + (18 - 3)\underset{\sim}{j}$

$= 15\underset{\sim}{i} + 15\underset{\sim}{j}$

$\frac{1}{2}\overrightarrow{PR} = \frac{1}{2}\left(15\underset{\sim}{i} + 15\underset{\sim}{j}\right) = \frac{15}{2}\underset{\sim}{i} + \frac{15}{2}\underset{\sim}{j}$

$\overrightarrow{QS} = \overrightarrow{OS} - \overrightarrow{OQ}$

$= \left(-4\underset{\sim}{i} + 14\underset{\sim}{j}\right) - \left(3\underset{\sim}{i} + 7\underset{\sim}{j}\right)$

$= (-4 - 3)\underset{\sim}{i} + (14 - 7)\underset{\sim}{j}$

$= -7\underset{\sim}{i} + 7\underset{\sim}{j}$

$\frac{1}{2}\overrightarrow{QS} = \frac{1}{2}\left(-7\underset{\sim}{i} + 7\underset{\sim}{j}\right) = -\frac{7}{2}\underset{\sim}{i} + \frac{7}{2}\underset{\sim}{j}$

LHS $= \frac{1}{2}\overrightarrow{PR} = \frac{15}{2}\underset{\sim}{i} + \frac{15}{2}\underset{\sim}{j}$

RHS $= \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QS}$

$= \left(11\underset{\sim}{i} + 4\underset{\sim}{j}\right) + \left(-\frac{7}{2}\underset{\sim}{i} + \frac{7}{2}\underset{\sim}{j}\right)$

$= \left(11 - \frac{7}{2}\right)\underset{\sim}{i} + (4 + \frac{7}{2})\underset{\sim}{j}$

$= \frac{15}{2}\underset{\sim}{i} + \frac{15}{2}\underset{\sim}{j}$

Hence, LHS = RHS.

13b $\overrightarrow{PR} = 15\underset{\sim}{i} + 15\underset{\sim}{j}$

$\overrightarrow{QS} = -7\underset{\sim}{i} + 7\underset{\sim}{j}$

By using the method, $\underset{\sim}{u} \cdot \underset{\sim}{v} = x_1 \times x_2 + y_1 \times y_2$

Uncorrected proofs

$\overrightarrow{PR} \cdot \overrightarrow{QS} = 15 \times (-7) + 15 \times 7$

$\overrightarrow{PR} \cdot \overrightarrow{QS} = -105 + 105 = 0$

13c From part a, $\frac{1}{2}\overrightarrow{PR} = \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QS}$

From part b, $\overrightarrow{PR} \cdot \overrightarrow{QS} = 0$

This means the diagonals of the quadrilateral bisect each other at 90° hence, a rhombus.

14a $\overrightarrow{OA} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}, \overrightarrow{OP} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}, \overrightarrow{OQ} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

$= \begin{bmatrix} 2 - (-3) \\ 9 - 3 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

$\overrightarrow{AQ} = \overrightarrow{OQ} - \overrightarrow{OA}$

$= \begin{bmatrix} 10 - (-3) \\ 0 - 3 \end{bmatrix}$

$= \begin{bmatrix} 13 \\ -3 \end{bmatrix}$

14b $|\overrightarrow{AP}| = \sqrt{x^2 + y^2}$

$= \sqrt{5^2 + 6^2}$

$= \sqrt{25 + 36}$

$= \sqrt{61}$

$|\overrightarrow{AQ}| = \sqrt{x^2 + y^2}$

$= \sqrt{13^2 + (-3)^2}$

$= \sqrt{169 + 9}$

$= \sqrt{178}$

By using the method, $\underline{u} \cdot \underline{v} = x_1 \times x_2 + y_1 \times y_2$

Uncorrected proofs

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$\overrightarrow{AP} \cdot \overrightarrow{AQ} = 5 \times 13 + 6 \times (-3)$

$\overrightarrow{AP} \cdot \overrightarrow{AQ} = 65 - 18 = 47$

$\left|\overrightarrow{AP}\right| \times \left|\overrightarrow{AQ}\right| \cos\theta = \overrightarrow{AP} \cdot \overrightarrow{AQ}$

$\sqrt{61} \times \sqrt{178} \cos\theta = 47$

$\cos\theta = \frac{47}{\sqrt{61} \times \sqrt{178}}$

$\cos\theta = \frac{47}{\sqrt{61} \times \sqrt{178}} = \frac{47 \times 47}{61 \times 178} = \cos 63°$

$\theta = 63°$

15 Consider O to be the origin and $\overrightarrow{OP}$, $\overrightarrow{OQ}$, $\overrightarrow{OR}$ and $\overrightarrow{OS}$ represent position vectors.

Hence,

$\overrightarrow{OP} = \underline{i} + 2\underline{j}$, $\overrightarrow{OQ} = 8\underline{i} + 3\underline{j}$, $\overrightarrow{OR} = 6\underline{i} + 13\underline{j}$ and $\overrightarrow{OS} = 4\underline{i} + 9\underline{j}$

$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$= \left(6\underline{i} + 13\underline{j}\right) - \left(\underline{i} + 2\underline{j}\right)$

$= (6 - 1)\underline{i} + (13 - 2)\underline{j}$

$= 5\underline{i} + 11\underline{j}$

$\overrightarrow{QS} = \overrightarrow{OS} - \overrightarrow{OQ}$

$= \left(4\underline{i} + 9\underline{j}\right) - \left(8\underline{i} + 3\underline{j}\right)$

$= (4 - 8)\underline{i} + (9 - 3)\underline{j}$

$= -5\underline{i} + 6\underline{j}$

$\left|\overrightarrow{PR}\right| = \sqrt{x^2 + y^2}$

$= \sqrt{5^2 + 11^2}$

$= \sqrt{25 + 121}$

$= \sqrt{146}$

$\left|\overrightarrow{QS}\right| = \sqrt{x^2 + y^2}$

Uncorrected proofs

$= \sqrt{(-5)^2 + 6^2}$

$= \sqrt{25 + 36}$

$= \sqrt{61}$

By using the method, $\underline{u} \cdot \underline{v} = x_1 \times x_2 + y_1 \times y_2$

$\overrightarrow{PR} \cdot \overrightarrow{QS} = 5 \times (-5) + 11 \times 6$

$\overrightarrow{PR} \cdot \overrightarrow{QS} = -25 + 66 = 41$

$\left|\overrightarrow{PR}\right| \times \left|\overrightarrow{QS}\right| \cos\theta = \overrightarrow{PR} \cdot \overrightarrow{QS}$

$\sqrt{146} \times \sqrt{61} \cos\theta = 41$

$\cos\theta = \frac{41}{\sqrt{146} \times \sqrt{61}}$

$\cos\theta = \frac{41}{\sqrt{146} \times \sqrt{61}} = \frac{41 \times 41}{146 \times 61}$

$\theta = 58°8'$

16 Consider O to be the centre of the circle, $\overrightarrow{OP} = r\cos\theta\, \underline{i} + r\sin\theta\, \underline{j}$

$\overrightarrow{OA} = -r\underline{i}$ $\overrightarrow{OB} = r\underline{i}$

Hence,

$\overrightarrow{PA} = \overrightarrow{OA} - \overrightarrow{OP}$

$= \left(-r\underline{i}\right) - \left(r\cos\theta\, \underline{i} + r\sin\theta\, \underline{j}\right)$

$= \left((-r) - r\cos\theta\right)\underline{i} - r\sin\theta\, \underline{j}$

$= (-r - r\cos\theta)\underline{i} - r\sin\theta\, \underline{j}$

$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP}$

$= \left(r\underline{i}\right) - \left(r\cos\theta\, \underline{i} + r\sin\theta\, \underline{j}\right)$

$= (r - r\cos\theta)\underline{i} - r\sin\theta\, \underline{j}$

$\left|\overrightarrow{PA}\right| = \sqrt{(-r - r\cos\theta)^2 + (-r\sin\theta)^2}$

$= \sqrt{r^2 + r^2\cos^2\theta + 2r^2\cos\theta + r^2\sin^2\theta}$

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$$= \sqrt{r^2 + r^2\cos^2\theta + r^2\sin^2\theta + 2r^2\cos\theta}$$

$$= \sqrt{r^2 + r^2(\cos^2\theta + \sin^2\theta) + 2r^2\cos\theta}$$

$$= \sqrt{r^2 + r^2 \times 1 + 2r^2\cos\theta}$$

$$= \sqrt{2r^2 + 2r^2\cos\theta}$$

$$= \sqrt{2r^2(1 + \cos\theta)}$$

$$= \sqrt{2r^2 \times 2\cos^2\frac{\theta}{2}}$$

$$= \sqrt{4r^2\cos^2\frac{\theta}{2}}$$

$$= 2r\cos\frac{\theta}{2}$$

$$\left|\overrightarrow{PA}\right| = \sqrt{(r - r\cos\theta)^2 + (-r\sin\theta)^2}$$

$$= \sqrt{r^2 + r^2\cos^2\theta - 2r^2\cos\theta + r^2\sin^2\theta}$$

$$= \sqrt{r^2 + r^2\cos^2\theta + r^2\sin^2\theta - 2r^2\cos\theta}$$

$$= \sqrt{r^2 + r^2(\cos^2\theta + \sin^2\theta) - 2r^2\cos\theta}$$

$$= \sqrt{r^2 + r^2 \times 1 - 2r^2\cos\theta}$$

$$= \sqrt{2r^2 - 2r^2\cos\theta}$$

$$= \sqrt{2r^2(1 - \cos\theta)}$$

$$= \sqrt{2r^2 \times 2\sin^2\frac{\theta}{2}}$$

$$= \sqrt{4r^2\sin^2\frac{\theta}{2}}$$

$$= 2r\sin\frac{\theta}{2}$$

By using the method, $\underline{u} \cdot \underline{v} = x_1 \times x_2 + y_1 \times y_2$

$$\overrightarrow{PA} \cdot \overrightarrow{PB} = (-r - r\cos\theta) \times (r - r\cos\theta) + (-r\sin\theta) \times (-r\sin\theta)$$

$$\overrightarrow{PA} \cdot \overrightarrow{PB} = -(r^2 - r^2\cos^2\theta) + r^2\sin^2\theta$$

$$\overrightarrow{PA} \cdot \overrightarrow{PB} = -r^2 + r^2\cos^2\theta + r^2\sin^2\theta$$

$$\overrightarrow{PA} \cdot \overrightarrow{PB} = -r^2 + r^2(\cos^2\theta + \sin^2\theta)$$

Uncorrected proofs

$\overrightarrow{PA} \cdot \overrightarrow{PB} = -r^2 + r^2 \times 1 = 0$

The dot product is 0, hence angle between $\overrightarrow{PA}$ and $\overrightarrow{PB}$ $i.e., \angle APB$ *is* 90°.

17a Consider O to be the origin hence, $\overrightarrow{OA} = 2\underline{i} + \underline{j}$

$\overrightarrow{OB} = 10\underline{i} + 4\underline{j}$

$\overrightarrow{OC} = 5\underline{i} + 13\underline{j}$

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$= \left(2\underline{i} + \underline{j}\right) - \left(10\underline{i} + 4\underline{j}\right)$

$= (2 - 10)\underline{i} + (1 - 4)\underline{j}$

$= -8\underline{i} - 3\underline{j}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \left(5\underline{i} + 13\underline{j}\right) - \left(10\underline{i} + 4\underline{j}\right)$

$= (5 - 10)\underline{i} + (13 - 4)\underline{j}$

$= -5\underline{i} + 9\underline{j}$

$\left|\overrightarrow{BA}\right| = \sqrt{x^2 + y^2}$

$= \sqrt{(-8)^2 + (-3)^2}$

$= \sqrt{64 + 9}$

$= \sqrt{73}$

$\left|\overrightarrow{BC}\right| = \sqrt{x^2 + y^2}$

$= \sqrt{(-5)^2 + 9^2}$

$= \sqrt{25 + 81}$

$= \sqrt{106}$

By using the method, $\underline{u} \cdot \underline{v} = x_1 \times x_2 + y_1 \times y_2$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = (-8) \times (-5) + (-3) \times 9$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = 40 - 27 = 13$

Uncorrected proofs

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$|\overrightarrow{BA}| \times |\overrightarrow{BC}| \cos\theta = \overrightarrow{BA} \cdot \overrightarrow{BC}$

$\sqrt{73} \times \sqrt{106} \cos\theta = 13$

$\cos\theta = \frac{13}{\sqrt{73} \times \sqrt{106}}$

$\cos\theta = \frac{13}{\sqrt{7738}}$

17b $\sin\theta = \sqrt{1 - \cos^2\theta}$

$\sin\theta = \sqrt{1 - \left(\frac{13}{\sqrt{7738}}\right)^2}$

$\sin\theta = \sqrt{1 - \frac{169}{7738}} = \sqrt{\frac{7569}{7738}} = \frac{87}{\sqrt{7738}}$

17c Area of triangle

$A = \frac{1}{2}|\overrightarrow{BA}| \times |\overrightarrow{BC}| \sin\theta$

$A = \frac{1}{2}\sqrt{73} \times \sqrt{106} \times \frac{87}{\sqrt{7738}}$

$A = \frac{1}{2} \times \sqrt{7738} \times \frac{87}{\sqrt{7738}} = \frac{87}{2} = 43.5$

Area of triangle is 43.5 square units

18a $\overrightarrow{PA} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\overrightarrow{PB} = \begin{bmatrix} a \\ b \end{bmatrix}$

$|\overrightarrow{PB}| = 4\sqrt{5}$

$|\overrightarrow{PA}| = \sqrt{x^2 + y^2}$

$= \sqrt{3^2 + 1^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

Area of triangle is 10 square units.

$\frac{1}{2}|\overrightarrow{PA}| \times |\overrightarrow{PB}| \sin\theta = A$

$\frac{1}{2}\sqrt{10} \times 4\sqrt{5} \sin\theta = 10$

Uncorrected proofs

$\frac{1}{2} \times 20\sqrt{2} \sin\theta = 10$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$\cos\theta = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2}$

$\cos\theta = \sqrt{1 - \frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$\overrightarrow{PA} \cdot \overrightarrow{PB} = \left|\overrightarrow{PA}\right| \times \left|\overrightarrow{PB}\right| \cos\theta$

$\overrightarrow{PA} \cdot \overrightarrow{PB} = \sqrt{10} \times 4\sqrt{5} \frac{1}{\sqrt{2}} = 20$

By using the method, $\underline{u} \cdot \underline{v} = x_1 \times x_2 + y_1 \times y_2$

$\overrightarrow{PA} \cdot \overrightarrow{PB} = 3 \times a + 1 \times b$

$20 = 3a + b$

18b All the possibilities can be

$\overrightarrow{PB} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \begin{bmatrix} -8 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ -8 \end{bmatrix}$

19a $\overrightarrow{OA} = \underline{u} = x_1\underline{i} + y_1\underline{j}$, $\overrightarrow{OB} = \underline{v} = x_2\underline{i} + y_2\underline{j}$ and $\angle AOB = \theta$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = \underline{u} - \underline{v}$

$\left|\underline{u}\right|^2 = {x_1}^2 + {y_1}^2$

$\left|\underline{v}\right|^2 = {x_2}^2 + {y_2}^2$

$\underline{u} \cdot \underline{v} = x_1 \times x_2 + y_1 \times y_2$

Using the cosine rule,

$|AB|^2 = \left|\underline{u}\right|^2 + \left|\underline{v}\right|^2 - 2\underline{u} \cdot \underline{v}$

$|AB|^2 = \left|\underline{u}\right|^2 + \left|\underline{v}\right|^2 - 2\left|\underline{u}\right| \times \left|\underline{v}\right| \cos\theta$

Uncorrected proofs

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19b $|AB|^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

19c $(x_1 - x_2)^2 + (y_1 - y_2)^2 = |\underline{u}|^2 + |\underline{v}|^2 - 2|\underline{u}| \times |\underline{v}| \cos\theta$

$(x_1 - x_2)^2 + (y_1 - y_2)^2 = |\underline{u}|^2 + |\underline{v}|^2 - 2|\underline{u}| \times |\underline{v}| \cos\theta$

$x_1{}^2 + x_2{}^2 - 2(x_1 \times x_2) + y_1{}^2 + y_2{}^2 - 2(y_1 \times y_2) = |\underline{u}|^2 + |\underline{v}|^2 - 2|\underline{u}| \times |\underline{v}| \cos\theta$

$x_1{}^2 + y_1{}^2 + x_2{}^2 + y_2{}^2 - 2(x_1 \times x_2) - 2(y_1 \times y_2) = |\underline{u}|^2 + |\underline{v}|^2 - 2|\underline{u}| \times |\underline{v}| \cos\theta$

$|\underline{u}|^2 + |\underline{v}|^2 - 2(x_1 \times x_2 + y_1 \times y_2) = |\underline{u}|^2 + |\underline{v}|^2 - 2|\underline{u}| \times |\underline{v}| \cos\theta$

$-2(x_1 \times x_2 + y_1 \times y_2) = -2|\underline{u}| \times |\underline{v}| \cos\theta$

$x_1 \times x_2 + y_1 \times y_2 = |\underline{u}| \times |\underline{v}| \cos\theta$

Hence, proved.

Solutions for Enrichment questions 20-23 are in preparation.

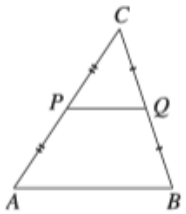
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Solutions to Exercise 8D

Solutions for Enrichment questions are in preparation.

1a



$\overrightarrow{AC} = \underline{a}$ and $\overrightarrow{CB} = \underline{b}$

P is the mid-point of AC and Q is the mid-point of BC

Hence, $AP = PC$ and $QC = QB$

$\overrightarrow{AB} = \overrightarrow{CB} - \overrightarrow{CA}$

$\overrightarrow{AB} = \overrightarrow{CB} - (-\overrightarrow{AC})$

$\overrightarrow{AB} = \overrightarrow{CB} + \overrightarrow{AC} = \underline{b} + \underline{a} = \underline{a} + \underline{b}$

1b $\overrightarrow{PC} = \frac{1}{2}(\overrightarrow{AC}) = \frac{1}{2}\underline{a}$

$\overrightarrow{CQ} = \frac{1}{2}(\overrightarrow{CB}) = \frac{1}{2}\underline{b}$

$\overrightarrow{PQ} = \overrightarrow{CQ} - \overrightarrow{CP}$

$\overrightarrow{PQ} = \overrightarrow{CQ} - (-\overrightarrow{PC})$

$\overrightarrow{PQ} = \overrightarrow{CQ} + \overrightarrow{PC} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} = \frac{1}{2}(\underline{a} + \underline{b})$

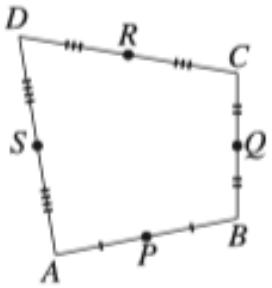
1c $\overrightarrow{PQ} = \frac{1}{2}(\underline{a} + \underline{b}) = \frac{1}{2}\overrightarrow{AB}$

Hence, $PQ \parallel AB$ and $PQ = \frac{1}{2}AB$

Uncorrected proofs

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2a



$\overrightarrow{AB} = \underline{a}$ $\overrightarrow{BC} = \underline{b}$ $\overrightarrow{AD} = \underline{d}$ and $\overrightarrow{DC} = \underline{c}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \underline{a} + \underline{b}$

$\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC} = \underline{d} + \underline{c}$

As, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

Hence, $\overrightarrow{AC} = \underline{a} + \underline{b} = \underline{d} + \underline{c}$

2b P is the mid-point of AB and Q is the mid-point of BC.

Hence, $AP = PB$ and $BQ = QC$.

$\overrightarrow{PB} = \frac{1}{2}(\overrightarrow{AB}) = \frac{1}{2}\underline{a}$

$\overrightarrow{BQ} = \frac{1}{2}(\overrightarrow{BC}) = \frac{1}{2}\underline{b}$

$\overrightarrow{PQ} = \overrightarrow{BQ} - \overrightarrow{BP}$

$\overrightarrow{PQ} = \overrightarrow{BQ} - (-\overrightarrow{PB})$

$\overrightarrow{PQ} = \overrightarrow{BQ} + \overrightarrow{PB} = \frac{1}{2}\underline{b} + \frac{1}{2}\underline{a} = \frac{1}{2}(\underline{a} + \underline{b})$

2c S is the mid-point of CD and R is the mid-point of DA.

Hence, $AS = SD$ and $DR = RC$

$\overrightarrow{DR} = \frac{1}{2}(\overrightarrow{DC}) = \frac{1}{2}\underline{c}$

$\overrightarrow{SD} = \frac{1}{2}(\overrightarrow{AD}) = \frac{1}{2}\underline{d}$

$\overrightarrow{SR} = \overrightarrow{DR} - \overrightarrow{DS}$

$\overrightarrow{SR} = \overrightarrow{DR} - (-\overrightarrow{SD})$

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$\overrightarrow{SR} = \overrightarrow{DR} + \overrightarrow{SD} = \frac{1}{2}\underline{c} + \frac{1}{2}\underline{d} = \frac{1}{2}(\underline{d} + \underline{c})$

2d From 2a, $\overrightarrow{AC} = \underline{a} + \underline{b} = \underline{d} + \underline{c}$

$\overrightarrow{PQ} = \frac{1}{2}(\underline{a} + \underline{b}) = \frac{1}{2}\overrightarrow{AC}$

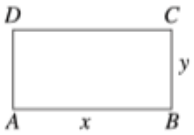
$\overrightarrow{SR} = \frac{1}{2}(\underline{d} + \underline{c}) = \frac{1}{2}\overrightarrow{AC}$

Hence, $\overrightarrow{PQ} = \overrightarrow{SR}$

2e As, $\overrightarrow{PQ} = \overrightarrow{SR}$

This means, line joining mid-points adjacent sides of the quadrilateral are parallel and equal to each other, hence, $ABCD$ is a parallelogram.

3a



$\overrightarrow{AB} = \underline{a}$ and $\overrightarrow{BC} = \underline{b}$ $\overrightarrow{CD} = \underline{c}$

$\overrightarrow{AC} = \overrightarrow{BC} - \overrightarrow{BA}$

$\overrightarrow{AC} = \overrightarrow{BC} - (-\overrightarrow{AB})$

$\overrightarrow{AC} = \overrightarrow{BC} + \overrightarrow{AB} = \underline{b} + \underline{a} = \underline{a} + \underline{b}$

$\overrightarrow{BD} = \overrightarrow{CD} - \overrightarrow{CB}$

$\overrightarrow{BD} = \overrightarrow{CD} - (-\overrightarrow{BC})$

$\overrightarrow{BD} = \overrightarrow{CD} + \overrightarrow{BC} = \underline{c} + \underline{b} = \underline{b} + \underline{c}$

Uncorrected proofs

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3b $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos\theta$

θ is the angle between $\underline{a}$ and $\underline{b}$

$ABCD$ is a rectangle hence, $\theta = 90°$

$|\underline{a}| = x \; |\underline{b}| = y$

$\underline{a} \cdot \underline{b} = x \times y \times \cos 90° = x \times y \times 0 = 0$

3c $\underline{a} \cdot \underline{a} = |\underline{a}|^2 = x^2$

3d $|\underline{a} + \underline{b}|^2 = |\overrightarrow{AC}|^2 = x^2 + y^2$

$|\overrightarrow{AC}| = \sqrt{x^2 + y^2}$

$(\underline{a} + \underline{b}) \cdot (\underline{a} + \underline{b})$

$= |\underline{a} + \underline{b}||\underline{a} + \underline{b}| \cos\theta$

$= \sqrt{x^2 + y^2} \times \sqrt{x^2 + y^2} \times \cos 0°$

$= (x^2 + y^2) \times 1 = x^2 + y^2$

$|\underline{b} + \underline{c}|^2 = |\overrightarrow{BD}|^2 = x^2 + y^2$

As $ABCD$ is a rectangle, $|\overrightarrow{AB}| = |\overrightarrow{CD}|$

$|\overrightarrow{BD}| = \sqrt{x^2 + y^2}$

$(\underline{b} + \underline{c}) \cdot (\underline{b} + \underline{c})$

$= |\underline{b} + \underline{c}||\underline{b} + \underline{c}| \cos\theta$

$= \sqrt{x^2 + y^2} \times \sqrt{x^2 + y^2} \times \cos 0°$

$= (x^2 + y^2) \times 1 = x^2 + y^2$

3e The diagonals of a rectangle have equal length.

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Chapter 8 worked solutions – Vectors

4a



$\overrightarrow{AB} = \underline{a}$ and $\overrightarrow{BC} = \underline{b}$

$\overrightarrow{AC} = \overrightarrow{BC} - \overrightarrow{BA}$

$\overrightarrow{AC} = \overrightarrow{BC} - (-\overrightarrow{AB})$

$\overrightarrow{AC} = \overrightarrow{BC} + \overrightarrow{AB} = \underline{b} + \underline{a} = \underline{a} + \underline{b}$

$\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

$\overrightarrow{BD} = \underline{b} - \underline{a}$

4b $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos\theta$

θ is the angle between $\underline{a}$ and $\underline{b}$

$ABCD$ is a square hence, $\theta = 90°$

$|\underline{a}| = l \; |\underline{b}| = l$

$\underline{a} \cdot \underline{b} = l \times l \times \cos 90° = x \times y \times 0 = 0$

4c $\underline{a} \cdot \underline{a} = |\underline{a}|^2 = l^2$

4d $|\underline{a} + \underline{b}|^2 = |\overrightarrow{AC}|^2 = l^2 + l^2 = 2l^2$

$|\overrightarrow{AC}| = \sqrt{2l^2} = \sqrt{2}l$

$|\underline{b} - \underline{a}|^2 = |\overrightarrow{BD}|^2 = l^2 + l^2$

$|\overrightarrow{BD}| = \sqrt{2l^2} = \sqrt{2}l$

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$(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{b} - \underset{\sim}{a})$

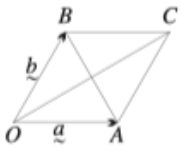
$= |\underset{\sim}{a} + \underset{\sim}{b}||\underset{\sim}{b} - \underset{\sim}{a}| \cos\theta$

$= \sqrt{2}l \times \sqrt{2}l \times \cos 90°$

$= 2l^2 \times 0 = 0$

4e The diagonals of the square meet at right angles.

5a



$\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$

$|\underset{\sim}{a}| = |\underset{\sim}{b}|$ because the sides of rhombus are equal.

5b $\underset{\sim}{a} \cdot \underset{\sim}{a} = |\underset{\sim}{a}|^2$

$\underset{\sim}{b} \cdot \underset{\sim}{b} = |\underset{\sim}{b}|^2 = |\underset{\sim}{a}|^2$ as $|\underset{\sim}{a}| = |\underset{\sim}{b}|$

Hence, $\underset{\sim}{a} \cdot \underset{\sim}{a} = \underset{\sim}{b} \cdot \underset{\sim}{b}$

5c $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OA}$

$\overrightarrow{OC} = \underset{\sim}{b} + \underset{\sim}{a} = \underset{\sim}{a} + \underset{\sim}{b}$

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$\overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$

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5d $\overrightarrow{OC} \cdot \overrightarrow{BA} = (\underline{a} + \underline{b}) \cdot (\underline{a} - \underline{b})$

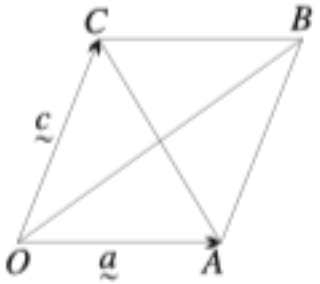
$= |\underline{a} + \underline{b}||\underline{b} - \underline{a}| \cos\theta$

$= |\underline{a} + \underline{b}||\underline{b} - \underline{a}| \cos 90°$ as diagonals of the rhombus meet at right angles

$= |\underline{a} + \underline{b}||\underline{b} - \underline{a}| \times 0 = 0$

5e Diagonals of a rhombus are perpendicular.

6a



The opposite sides of a parallelogram are equal.

6b $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{OA}$

$\overrightarrow{OB} = \underline{c} + \underline{a}$

6c $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$\overrightarrow{AC} = \underline{c} - \underline{a}$

6d $|\underline{c} + \underline{a}| = |\underline{c} - \underline{a}|$ because $\overrightarrow{OB} = \overrightarrow{AC}$ (diagonals of the parallelogram are equal).

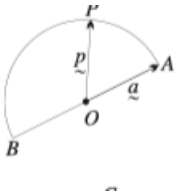
6e <Solution to come>

6f It is a rectangle as $\underline{a} \cdot \underline{c} = 0$

Uncorrected proofs

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7a



$\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OP} = \underset{\sim}{p}$

$\overrightarrow{OB} = -\underset{\sim}{a}$

As $\overrightarrow{AB}$ is the diameter and $\overrightarrow{OB}$ is the radius in opposite direction to $\overrightarrow{OA}$

7b $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

$\overrightarrow{AP} = \underset{\sim}{p} - \underset{\sim}{a}$

$\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB}$

$\overrightarrow{BP} = \underset{\sim}{p} - (-\underset{\sim}{a}) = \underset{\sim}{p} + \underset{\sim}{a}$

7c $\overrightarrow{AP} \cdot \overrightarrow{BP} = (\underset{\sim}{p} + \underset{\sim}{a}) \cdot (\underset{\sim}{p} - \underset{\sim}{a})$

$= \underset{\sim}{p} \cdot \underset{\sim}{p} - \underset{\sim}{p} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{p} - \underset{\sim}{a} \cdot \underset{\sim}{a}$

$= \underset{\sim}{p} \cdot \underset{\sim}{p} - \underset{\sim}{a} \cdot \underset{\sim}{a}$

$= \left|\underset{\sim}{p}\right|^2 - \left|\underset{\sim}{a}\right|^2$

$\overrightarrow{OA} = \overrightarrow{OP}$ as radius of the circle.

Hence,

$\overrightarrow{AP} \cdot \overrightarrow{BP} = \left|\underset{\sim}{p}\right|^2 - \left|\underset{\sim}{a}\right|^2$

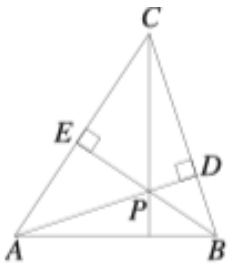
$= \left|\underset{\sim}{p}\right|^2 - \left|\underset{\sim}{p}\right|^2 = 0$

Therefore, $\angle APB = 90°$.

7d An angle in a semi-circle is a right angle.

Uncorrected proofs

8a



$\overrightarrow{OA} = \underline{a} \quad \overrightarrow{OB} = \underline{b}\ \overrightarrow{OC} = \underline{c}$ and $\overrightarrow{OP} = \underline{p}$

$(\underline{p} - \underline{a}) \cdot (\underline{c} - \underline{b}) = 0$

$(\overrightarrow{OP} - \overrightarrow{OA}) \cdot (\overrightarrow{OC} - \overrightarrow{OB}) = 0$

$\overrightarrow{AP} \cdot \overrightarrow{BC} = 0$

P lies on the altitude from A to BC.

8b $(\underline{p} - \underline{b}) \cdot (\underline{a} - \underline{c}) = 0$

$(\overrightarrow{OP} - \overrightarrow{OB}) \cdot (\overrightarrow{OA} - \overrightarrow{OC}) = 0$

$\overrightarrow{AP} \cdot \overrightarrow{AC} = 0$

P lies on the altitude from B to CA.

8c $(\underline{p} - \underline{a}) \cdot (\underline{c} - \underline{b}) = 0$ and $(\underline{p} - \underline{b}) \cdot (\underline{a} - \underline{c}) = 0$

$(\underline{p} - \underline{a}) \cdot (\underline{c} - \underline{b}) = (\underline{p} - \underline{b}) \cdot (\underline{a} - \underline{c})$

$\underline{p} \cdot \underline{c} - \underline{p} \cdot \underline{b} - \underline{a} \cdot \underline{c} + \underline{a} \cdot \underline{b} = \underline{p} \cdot \underline{a} - \underline{p} \cdot \underline{c} - \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{c}$

$\underline{p} \cdot \underline{c} - \underline{p} \cdot \underline{b} - \underline{a} \cdot \underline{c} + \underline{a} \cdot \underline{b} - \underline{p} \cdot \underline{a} + \underline{p} \cdot \underline{c} + \underline{b} \cdot \underline{a} - \underline{b} \cdot \underline{c} = 0$

$-\underline{p} \cdot \underline{b} - \underline{a} \cdot \underline{c} - \underline{p} \cdot \underline{a} - \underline{b} \cdot \underline{c} = 0$

$\underline{p} \cdot \underline{b} + \underline{a} \cdot \underline{c} + \underline{p} \cdot \underline{a} + \underline{b} \cdot \underline{c} = 0$

$\underline{p} \cdot \underline{a} - \underline{b} \cdot \underline{p} - \underline{c} \cdot \underline{a} + \underline{b} \cdot \underline{c} = 0$

Uncorrected proofs

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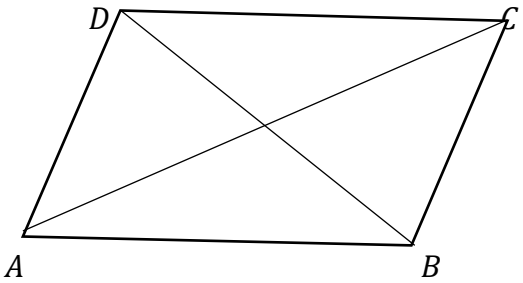
$\underline{p}(\underline{a} - \underline{b}) - \underline{c}(\underline{a} - \underline{b}) = 0$

$(\underline{p} - \underline{c}) \cdot (\underline{a} - \underline{b}) = 0$

8d From 8c, we conclude that P lies on the altitude from C to BA.

Hence, three altitudes are concurrent.

9



Let $ABCD$ be a parallelogram.

And $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ be the respective position vectors of A, B, C and D.

As $ABCD$ is a parallelogram, then opposite sides of the parallelogram are equal.

$\overrightarrow{AB} = \overrightarrow{DC}$

$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OD}$

$\underline{b} - \underline{a} = \underline{c} - \underline{d}$

$\underline{b} + \underline{d} = \underline{a} + \underline{c}$

We can also write,

$\frac{1}{2}(\underline{b} + \underline{d}) = \frac{1}{2}(\underline{a} + \underline{c})$

$\frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OD}) = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OC})$

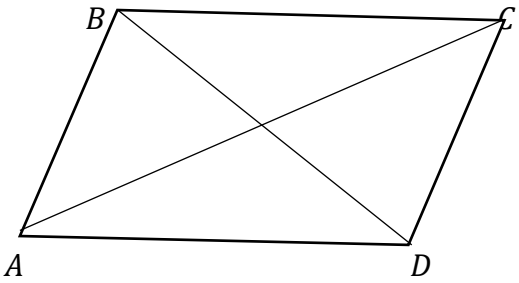
$\frac{1}{2}\overrightarrow{BD} = \frac{1}{2}\overrightarrow{AC}$

This shows that the diagonals of the parallelogram bisect each other.

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10



Let $ABCD$ be a parallelogram.

And $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$ be the respective position vectors of A, B, C and D.

AC and BD are the two diagonals that intersect at P.

Hence,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ and $\overrightarrow{BD} = \overrightarrow{BC} - \overrightarrow{AB}$

Now,

$$\left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{BD}\right|^2 = \left|\overrightarrow{AB} + \overrightarrow{BC}\right|^2 + \left|\overrightarrow{BC} - \overrightarrow{AB}\right|^2$$

We know that, $\underline{a} \cdot \underline{a} = \left|\underline{a}\right|^2$

$$\left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{BD}\right|^2 = \left|\overrightarrow{AB} + \overrightarrow{BC}\right| \cdot \left|\overrightarrow{AB} + \overrightarrow{BC}\right| + \left|\overrightarrow{BC} - \overrightarrow{AB}\right| \cdot \left|\overrightarrow{BC} - \overrightarrow{AB}\right|$$

$$\left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{BD}\right|^2 = \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{BC}\right|^2 + 2\overrightarrow{AB} \cdot \overrightarrow{BC} + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{AB}\right|^2 - 2\overrightarrow{AB} \cdot \overrightarrow{BC}$$

$$\left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{BD}\right|^2 = \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{AB}\right|^2$$

$ABCD$ be a parallelogram. So, $\left|\overrightarrow{BC}\right| = \left|\overrightarrow{AD}\right|$ and $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{CD}\right|$

Therefore, we can write

$$\left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{BD}\right|^2 = \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CD}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{AD}\right|^2$$

Hence, proved.

11 <Solution to come>

Solutions for Enrichment question 12 are in preparation.

Uncorrected proofs

Solutions to Exercise 8E

Solutions for Enrichment questions are in preparation.

1a $\underline{a} = \underline{i} + \underline{j}$, $\underline{b} = \underline{i}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{1\times1+1\times0}{1\times1+0\times0} \times \underline{i}$

$Proj_{\underline{b}}\underline{a} = \frac{1}{1} \times \underline{i} = \underline{i}$

1b $\underline{a} = \underline{i} + 2\underline{j}$, $\underline{b} = \underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{1\times0+2\times1}{0\times0+1\times1} \times \underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{2}{1} \times \underline{j} = 2\underline{j}$

1c $\underline{a} = -3\underline{i} + 2\underline{j}$, $\underline{b} = \underline{i}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{-3\times1+2\times0}{1\times1+0\times0} \times \underline{i}$

$Proj_{\underline{b}}\underline{a} = \frac{-3}{1} \times \underline{i} = -3\underline{i}$

2a $\underline{a} = 2\underline{i} + 3\underline{j}$, $\underline{b} = \underline{i}$

Length of $Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|}$

$Proj_{\underline{b}}\underline{a} = \frac{2\times1+3\times0}{\sqrt{1^2+0^2}}$

$Proj_{\underline{b}}\underline{a} = \frac{2}{1} = 2$

Uncorrected proofs

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Chapter 8 worked solutions – Vectors

2b $\underline{a} = -2\underline{i} - 4\underline{j}$, $\underline{b} = \underline{j}$

Length of $Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|}$

$Proj_{\underline{b}}\underline{a} = \frac{-2\times0-4\times1}{\sqrt{0^2+1^2}}$

$Proj_{\underline{b}}\underline{a} = \frac{-4}{1} = 4$

2c $\underline{a} = -6\sqrt{2}\underline{i} + 8\sqrt{2}\underline{j}$, $\underline{b} = \underline{i}$

Length of $Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|}$

$Proj_{\underline{b}}\underline{a} = \frac{-6\sqrt{2}\times1+8\sqrt{2}\times0}{\sqrt{1^2+0^2}}$

$Proj_{\underline{b}}\underline{a} = \frac{-6\sqrt{2}}{1} = 6\sqrt{2}$

3a $\underline{a} = \begin{bmatrix}2\\1\end{bmatrix}$, $\underline{b} = \begin{bmatrix}4\\0\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{2\times4+1\times0}{4\times4+0\times0} \times \begin{bmatrix}4\\0\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{8}{16} \times \begin{bmatrix}4\\0\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{1}{2} \times \begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$

3b $\underline{a} = 3\underline{i} + 3\underline{j}$, $\underline{b} = 2\underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{3\times0+3\times2}{0\times0+2\times2} \times 2\underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{6}{4} \times 2\underline{j} = 3\underline{j}$

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3c $\underline{a} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$, $\underline{b} = \begin{bmatrix} -6 \\ 0 \end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{5\times(-6)+(-3)\times 0}{(-6)\times(-6)+0\times 0} \times \begin{bmatrix} -6 \\ 0 \end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{-30}{36} \times \begin{bmatrix} -6 \\ 0 \end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{-5}{6} \times \begin{bmatrix} -6 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

4a $|\overrightarrow{OA}| = 6, \quad \angle AOB = 30°$

$Proj_{\overrightarrow{OB}}\overrightarrow{OA} = |\overrightarrow{OA}| \cos 30°$

$Proj_{\overrightarrow{OB}}\overrightarrow{OA} = 6 \times \cos 30°$

$Proj_{\overrightarrow{OB}}\overrightarrow{OA} = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

4b $|\overrightarrow{OA}| = 6\sqrt{6} \quad \angle AOB = 45°$

$Proj_{\overrightarrow{OB}}\overrightarrow{OA} = |\overrightarrow{OA}| \cos 45°$

$Proj_{\overrightarrow{OB}}\overrightarrow{OA} = 6\sqrt{6} \times \cos 45°$

$Proj_{\overrightarrow{OB}}\overrightarrow{OA} = 6\sqrt{6} \times \frac{1}{\sqrt{2}} = 6\sqrt{3}$

5 $\underline{a} = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 1 \\ -7 \end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{10\times 1+(-2)\times(-7)}{1\times 1+(-7)\times(-7)} \times \begin{bmatrix} 1 \\ -7 \end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{24}{49} \times \begin{bmatrix} 1 \\ -7 \end{bmatrix} = \begin{bmatrix} 24 \\ -343 \end{bmatrix}$

Length of $Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|} = \frac{24}{5\sqrt{2}} = \frac{12\sqrt{2}}{5}$

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6a $\underline{a} = \begin{bmatrix}1\\2\end{bmatrix}$ $\underline{b} = \begin{bmatrix}2\\2\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{1\times2+2\times2}{2\times2+2\times2} \times \begin{bmatrix}2\\2\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{6}{8} \times \begin{bmatrix}2\\2\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{3}{4} \times \begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}\frac{3}{2}\\\frac{3}{2}\end{bmatrix}$

6b $\underline{a} = \underline{i} + \underline{j}$, $\underline{b} = 3\underline{i} - \underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{1\times3+1\times(-1)}{3\times3+(-1)\times(-1)} \times 3\underline{i} - \underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{2}{10} \times (3\underline{i} - \underline{j})$

$Proj_{\underline{b}}\underline{a} = \frac{1}{5} \times (3\underline{i} - \underline{j})$

$Proj_{\underline{b}}\underline{a} = \frac{3}{5}\underline{i} - \frac{1}{5}\underline{j}$

6c $\underline{a} = \begin{bmatrix}-5\\5\end{bmatrix}$ $\underline{b} = \begin{bmatrix}-6\\8\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{(-5)\times(-6)+5\times8}{(-6)\times(-6)+8\times8} \times \begin{bmatrix}-6\\8\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{70}{100} \times \begin{bmatrix}-6\\8\end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{7}{10} \times \begin{bmatrix}-6\\8\end{bmatrix} = \begin{bmatrix}-\frac{21}{5}\\\frac{28}{5}\end{bmatrix}$

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Chapter 8 worked solutions – Vectors

7a $\underline{a} = \underline{i} + \underline{j}\,, \underline{b} = 3\underline{i} + \underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{1\times3+1\times1}{3\times3+1\times1} \times 3\underline{i} + \underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{4}{10} \times (3\underline{i} + \underline{j})$

$Proj_{\underline{b}}\underline{a} = \frac{2}{5} \times (3\underline{i} + \underline{j})$

$Proj_{\underline{b}}\underline{a} = \frac{6}{5}\underline{i} + \frac{2}{5}\underline{j}$

7b $\underline{a} = 4\underline{i} - 3\underline{j}\,, \underline{b} = 6\underline{i} + 2\underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{4\times6+(-3)\times2}{6\times6+2\times2} \times 6\underline{i} + 2\underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{18}{40} \times (6\underline{i} + 2\underline{j})$

$Proj_{\underline{b}}\underline{a} = \frac{9}{20} \times (6\underline{i} + 2\underline{j})$

$Proj_{\underline{b}}\underline{a} = \frac{27}{10}\underline{i} + \frac{9}{10}\underline{j}$

8a $\underline{a} = -2\underline{i}\,, \underline{b} = -3\underline{i} - 2\underline{j}$

Length of $Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|}$

$Proj_{\underline{b}}\underline{a} = \frac{(-2)\times(-3)+0\times(-2)}{\sqrt{(-3)^2+(-2)^2}}$

$Proj_{\underline{b}}\underline{a} = \frac{6}{\sqrt{13}}$

8b $\underline{a} = 6\underline{i} - 4\underline{j}\,, \underline{b} = -3\underline{i} + 6\underline{j}$

Length of $Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|}$

$Proj_{\underline{b}}\underline{a} = \frac{6\times(-3)+(-4)\times6}{\sqrt{(-3)^2+6^2}}$

$Proj_{\underline{b}}\underline{a} = \frac{-18-24}{\sqrt{(-3)^2+6^2}}$

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$Proj_{\underline{b}}\underline{a} = \frac{42}{\sqrt{5}}$

9 Let O be the origin and $\overrightarrow{OA} = -3\underline{i} - 7\underline{j}$ and $\overrightarrow{OB} = \underline{i} + 5\underline{j}$ be the position vectors

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = = \left(\underline{i} + 5\underline{j}\right) - \left(-3\underline{i} - 7\underline{j}\right)$

$\overrightarrow{AB} = = \underline{i} + 5\underline{j} + 3\underline{i} + 7\underline{j}$

$\overrightarrow{AB} = = 4\underline{i} + 12\underline{j}$

Let $\underline{b} = -6\underline{i} + 4\underline{j}$

$Proj_{\underline{b}}\overrightarrow{AB} = \frac{\overrightarrow{AB}\cdot\underline{b}}{\underline{b}\cdot\underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{4\times(-6)+12\times4}{(-6)\times(-6)+4\times4} \times -6\underline{i} + 4\underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{24}{52} \times (-6\underline{i} + 4\underline{j})$

$Proj_{\underline{b}}\underline{a} = \frac{6}{13} \times (6\underline{i} + 4\underline{j})$

$Proj_{\underline{b}}\underline{a} = -\frac{36}{13}\underline{i} + \frac{24}{13}\underline{j}$

10 Let O be the origin and let $\overrightarrow{OA} = \underline{i} + 3\underline{j}, \overrightarrow{OB} = 6\underline{i} + 18\underline{j}, \overrightarrow{OC} = 9\underline{i} + 4\underline{j}$ and $\overrightarrow{OD} = 19\underline{i} + 24\underline{j}$ be the position vectors.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = = \left(6\underline{i} + 18\underline{j}\right) - \left(\underline{i} + 3\underline{j}\right)$

$\overrightarrow{AB} = = 6\underline{i} + 18\underline{j} - \underline{i} - 3\underline{j}$

$\overrightarrow{AB} = = 5\underline{i} + 15\underline{j}$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

$\overrightarrow{CD} = = \left(19\underline{i} + 24\underline{j}\right) - \left(9\underline{i} + 4\underline{j}\right)$

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$\overrightarrow{CD} = = 19\underline{i} + 24\underline{j} - 9\underline{i} - 4\underline{j}$

$\overrightarrow{CD} = = 10\underline{i} + 20\underline{j}$

Length of $Proj_{\overrightarrow{CD}}\overrightarrow{AB} = \frac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{CD}|}$

$Proj_{\overrightarrow{CD}}\overrightarrow{AB} = \frac{5\times10+15\times20}{\sqrt{10^2+20^2}}$

$Proj_{\overrightarrow{CD}}\overrightarrow{AB} = \frac{50+300}{\sqrt{100+400}}$

$Proj_{\overrightarrow{CD}}\overrightarrow{AB} = \frac{350}{\sqrt{500}} = \frac{350}{10\sqrt{5}} = 7\sqrt{5}$

11 Let $\underline{a} = \lambda\underline{i} + 4\underline{j}$ and $\underline{b} = 12\underline{i} - 5\underline{j}$

Length of $Proj_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|}$

$\frac{\underline{a}\cdot\underline{b}}{|\underline{b}|} = \frac{140}{13}$

$\frac{\lambda\times12+4\times(-5)}{\sqrt{12^2+(-5)^2}} = \frac{140}{13}$

$\frac{12\lambda-20}{\sqrt{144+25}} = \frac{140}{13}$

$\frac{12\lambda-20}{\sqrt{169}} = \frac{140}{13}$

$\frac{12\lambda-20}{\pm13} = \frac{140}{13}$

$\frac{12\lambda-20}{13} = \frac{140}{13}$ or $\frac{12\lambda-20}{-13} = \frac{140}{13}$

$12\lambda - 20 = 140$ or $-(12\lambda - 20) = 140$

$12\lambda - 20 = 140$ or $-12\lambda + 20 = 140$

$12\lambda = 160$ or $-12\lambda = 120$

$\lambda = \frac{160}{12} = \frac{40}{3}$ or $\lambda = \frac{120}{-12} = -10$

12 Let $\underline{u} = \underline{i} + \underline{j}$ and $\underline{x} = \underline{i}$

$Proj_{\underline{x}}\underline{u} = \frac{\underline{u}\cdot\underline{x}}{\underline{x}\cdot\underline{x}} \times \underline{x}$

$Proj_{\underline{x}}\underline{u} = \frac{1\times1+1\times0}{1\times1+0\times0} \times \underline{i}$

Uncorrected proofs

$Proj_{\underline{x}}\underline{u} = \frac{1}{1} \times \underline{i}$

$Proj_{\underline{x}}\underline{u} = \underline{i}$

Let, $\lambda = 2$

$2\underline{u} = 2\left(\underline{i} + \underline{j}\right) = 2\underline{i} + 2\underline{j}$

$Proj_{\underline{x}}2\underline{u} = \frac{\underline{u}\cdot\underline{x}}{\underline{x}\cdot\underline{x}} \times \underline{x}$

$Proj_{\underline{x}}2\underline{u} = \frac{2\times1+1\times0}{1\times1+0\times0} \times \underline{i}$

$Proj_{\underline{x}}2\underline{u} = \frac{2}{1} \times \underline{i}$

$Proj_{\underline{x}}2\underline{u} = 2\underline{i} = 2(Proj_{\underline{x}}\underline{u})$

Hence, proved.

Let $\underline{u} = \underline{i} + \underline{j}$ $\underline{v} = 2\underline{i} + 3\underline{j}$ and $\underline{x} = \underline{i}$

$Proj_{\underline{x}}\underline{u} = \frac{\underline{u}\cdot\underline{x}}{\underline{x}\cdot\underline{x}} \times \underline{x}$

$Proj_{\underline{x}}\underline{u} = \frac{1\times1+1\times0}{1\times1+0\times0} \times \underline{i}$

$Proj_{\underline{x}}\underline{u} = \frac{1}{1} \times \underline{i}$

$Proj_{\underline{x}}\underline{u} = \underline{i}$

$Proj_{\underline{x}}\underline{v} = \frac{\underline{v}\cdot\underline{x}}{\underline{x}\cdot\underline{x}} \times \underline{x}$

$Proj_{\underline{x}}\underline{v} = \frac{2\times1+3\times0}{1\times1+0\times0} \times \underline{i}$

$Proj_{\underline{x}}\underline{v} = \frac{2}{1} \times \underline{i}$

$Proj_{\underline{x}}\underline{v} = 2\underline{i}$

$\underline{u} + \underline{v} = \underline{i} + \underline{j} + 2\underline{i} + 3\underline{j} = 3\underline{i} + 4\underline{j}$

$Proj_{\underline{x}}(\underline{u} + \underline{v}) = \frac{(\underline{u}+\underline{v})\cdot\underline{x}}{\underline{x}\cdot\underline{x}} \times \underline{x}$

$Proj_{\underline{x}}(\underline{u} + \underline{v}) = \frac{3\times1+4\times0}{1\times1+0\times0} \times \underline{i}$

Uncorrected proofs

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$Proj_{\underline{x}}(\underline{u}+\underline{v}) = \frac{3}{1} \times \underline{i}$

$Proj_{\underline{x}}(\underline{u}+\underline{v}) = 3\underline{i} = \underline{i} + 2\underline{i} = Proj_{\underline{x}}\underline{u} + Proj_{\underline{x}}\underline{v}$

Hence, proved.

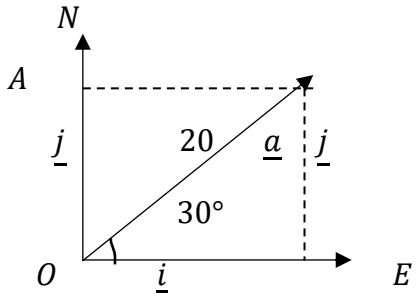
Solutions for Enrichment questions 13 and 14 are in preparation.

Uncorrected proofs

Solutions to Exercise 8F

Solutions for Enrichment questions are in preparation.

1



Using trigonometry,

$\cos 30° = \frac{i}{a}$

$\frac{\sqrt{3}}{2} = \frac{i}{20}$

$2i = 20\sqrt{3}$

$i = 10\sqrt{3}$

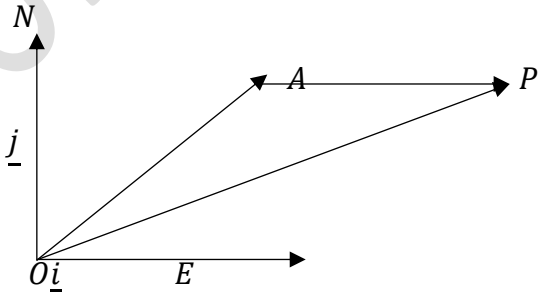
$\tan 30° = \frac{j}{i}$

$\frac{1}{\sqrt{3}} = \frac{j}{10\sqrt{3}}$

$\sqrt{3}j = 10\sqrt{3}$

$j = 10$

2



Let $\overrightarrow{OA} = 4i + 5j$ be the position vector and $\overrightarrow{AP} = 3i - 2j$ be the velocity vector.

Uncorrected proofs

Let's say, in t seconds, it moves from $\overrightarrow{AP}$ hence,

$\overrightarrow{OP} = \overrightarrow{OA} + t \times \overrightarrow{AP}$

$\overrightarrow{OP} = 4\underline{i} + 5\underline{j} + 7 \times (3\underline{i} - 2\underline{j})$

$\overrightarrow{OP} = 4\underline{i} + 5\underline{j} + 21\underline{i} - 14\underline{j}$

$\overrightarrow{OP} = 25\underline{i} - 9\underline{j}$

Hence, position vector after 7 seconds is $25\underline{i} - 9\underline{j}$.

3 Let $\underline{u} = \left(2\underline{i} - 3\underline{j}\right)$ N, $\underline{v} = \left(4\underline{i} + \underline{j}\right)$ N and $\underline{w} = \left(-3\underline{i} + 3\underline{j}\right)$ N

Resultant vector $\underline{a} = (\underline{u} + \underline{v}) + \underline{w}$

$\underline{a} = \left((2\underline{i} - 3\underline{j}) + (4\underline{i} + \underline{j})\right) + (-3\underline{i} + 3\underline{j})$

$\underline{a} = \left(2\underline{i} - 3\underline{j} + 4\underline{i} + \underline{j}\right) + (-3\underline{i} + 3\underline{j})$

$\underline{a} = \left(6\underline{i} - 2\underline{j}\right) + (-3\underline{i} + 3\underline{j})$

$\underline{a} = \left(6\underline{i} - 2\underline{j} - 3\underline{i} + 3\underline{j}\right)$

$\underline{a} = \left(3\underline{i} + \underline{j}\right)$

The magnitude of resultant vector is

$|\underline{a}| = \sqrt{3^2 + 1^2}$

$|\underline{a}| = \sqrt{9 + 1} = \sqrt{10}$ N

4 The resultant force is

$\underline{F} = \left(30\underline{i} + 16\underline{j}\right)$

The magnitude of resultant force is

$|\underline{F}| = \sqrt{30^2 + 16^2}$

$|\underline{F}| = \sqrt{900 + 256} = 34$ N

The direction of resultant force, if θ is the angle between $\underline{i}$ and $\underline{F}$

Uncorrected proofs

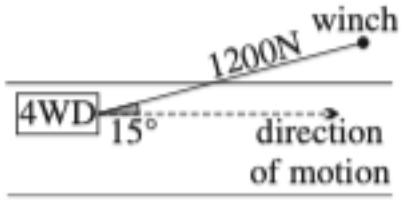
and using simple trigonometry:

$\tan\theta = \frac{16}{30}$

$\theta = 28.07 \div 28°$

The resultant force is about 34 N at 28° to the 30 N force.

5a



Magnitude of the component of the force in the direction of motion

$= 2000\cos 15°$

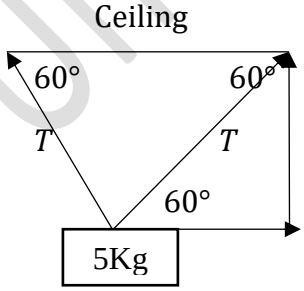
$= 2000 \times 0.966 \div 1932\ N$

5b Magnitude of the component of the force in the perpendicular direction of motion

$= 2000\sin 15°$

$= 2000 \times 0.259 \div 518\ N$

6



Regard resultant vector to be the Tension T and weight W to be normal to plane making angle of 60° with each other.

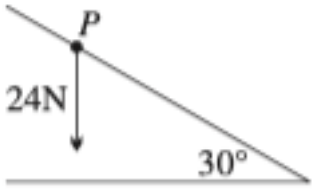
$|\underline{T}| = |\underline{W}|\cos 60°$

Uncorrected proofs

$|\underline{T}| = 5 \times \frac{1}{2} = 2.5$

In newtons, $T = 2.5 \times 9.8 \frac{m}{s} = 24.5$

7a



Component of the weight down the plane

$= 24 \sin 30°$

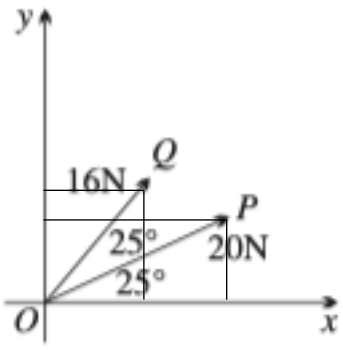
$= 24 \times \frac{1}{2} = 12\ N$

7b Component of the weight perpendicular to the plane

$= 24 \cos 30°$

$= 24 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}\ N$

8a



Let $\underline{i}$ and $\underline{j}$ be the unit vectors of horizontal and vertical plane respectively.

For $\overrightarrow{OP}$, horizontal position vector is $\underline{i}$ 20 cos 25° while vertical vector is $\underline{j}$ 20 sin 25°

Uncorrected proofs

Hence, $\overrightarrow{OP} = 20\cos 25°\underline{i} + 20\sin 25°\,\underline{j}$

The angle between $\overrightarrow{OQ}$ and plane will be $25° + 25° = 50°$.

For $\overrightarrow{OQ}$, horizontal position vector be $\underline{i}$ $16\cos 50°$ while vertical vector be $\underline{j}$ $16\sin 50°$

Hence, $\overrightarrow{OQ} = 16\cos 50°\underline{i} + 16\sin 50°\,\underline{j}$

8b The resultant vector of two forces is

$\overrightarrow{PQ} = \overrightarrow{OP} + \overrightarrow{OQ}$

$\overrightarrow{PQ} = 20\cos 25°\underline{i} + 20\sin 25°\,\underline{j} + 16\cos 50°\underline{i} + 16\sin 50°\,\underline{j}$

$\overrightarrow{PQ} = 20 \times 0.906\underline{i} + 20 \times 0.423\underline{j} + 16 \times 0.643\underline{i} + 16 \times 0.766\underline{j}$

$\overrightarrow{PQ} = 18\underline{i} + 9\underline{j} + 10\underline{i} + 12\underline{j}$

$\overrightarrow{PQ} = 28\underline{i} + 21\underline{j}$

The magnitude of resultant force is

$\left|\overrightarrow{PQ}\right| = \sqrt{28^2 + 21^2}$

$\left|\overrightarrow{PQ}\right| = \sqrt{784 + 441} = 35N$

Let θ be the angle between $\underline{i}$ and $\overrightarrow{PQ}$.

The direction of resultant force is

$Proj_{\underline{i}}\overrightarrow{PQ} = 28\underline{i}$

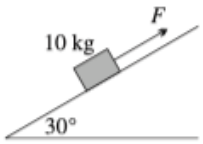
$(35\cos\theta)\underline{i} = 28\underline{i}$

$\cos\theta = \frac{28}{35} \div 36°$

The resultant force is 35 N in a horizontal direction of 36°.

Uncorrected proofs

9



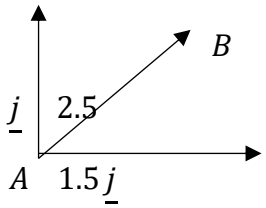
Magnitude of the component of the force F acting parallel to the plane

$= 10 \sin 30°$

$= 10 \times \frac{1}{2} = 5$

$F = m \times a = 5 \times 9.8 = 49$ N

10

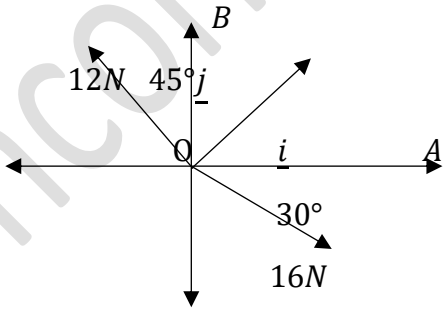


$\cos\theta = \frac{1.5}{2.5}$

$\theta = 53°$

Sam should row in the direction of N37°E.

11



Using trigonometry,

$\cos 30° = \frac{OA}{16}$

$\frac{\sqrt{3}}{2} = \frac{OA}{16}$

$OA = 8\sqrt{3}$

Uncorrected proofs

Similarly,

$\cos 45° = \frac{OB}{16}$

$\frac{1}{\sqrt{2}} = \frac{OB}{12}$

$OB = 6\sqrt{2}$

The resultant force is $\vec{F} = 8\sqrt{3}\underline{i} + 6\sqrt{2}\underline{j}$

The magnitude of the resultant force is

$|\vec{F}| = \sqrt{(8\sqrt{3})^2 + (6\sqrt{2})^2}$

$|\vec{F}| = \sqrt{192 + 72} \div 16N$

Direction of the resultant force is

$(16\cos\theta)\,\underline{i} = 8\sqrt{3}\underline{i}$

$\cos\theta = \frac{8\sqrt{3}}{16} = \frac{\sqrt{3}}{2}$

$\theta = 30°$

12 Let the vectors be $\underline{a} = 9\underline{i} - 2\underline{j}$, $\underline{b} = -3\underline{i} + 10\underline{j}$ and $\underline{c} = 18\underline{i} - \underline{j}$.

and the resultant force be $\underline{F} = (\underline{a} + \underline{b}) + \underline{c}$

$\underline{F} = \left((9\underline{i} - 2\underline{j}) + (-3\underline{i} + 10\underline{j})\right) + (18\underline{i} - \underline{j})$

$\underline{F} = \left(9\underline{i} - 2\underline{j} - 3\underline{i} + 10\underline{j}\right) + (18\underline{i} - \underline{j})$

$\underline{F} = \left(6\underline{i} + 8\underline{j}\right) + (18\underline{i} - \underline{j})$

$\underline{F} = \left(6\underline{i} + 8\underline{j} + 18\underline{i} - \underline{j}\right)$

$\underline{F} = \left(24\underline{i} + 7\underline{j}\right)$

The direction of acceleration of the object is in direction of the resultant force i.e.,

$\tan\theta = \frac{7}{24}$

$\theta = \tan^{-1}\frac{7}{24}$ above the horizontal

Uncorrected proofs

The magnitude of resultant force is

$|\underline{F}| = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = 25$

The magnitude of acceleration of the object is

$|a| = \frac{|\underline{F}|}{m} = \frac{25}{5} = 5 \text{ m/s}$

13a At 12 noon, the position vector is $\overrightarrow{OA} = 40\underline{i} + 16\underline{j}$

After 5 minutes, the position vector is $\overrightarrow{OB} = 33\underline{i} + 40\underline{j}$

The resultant vector be $\overrightarrow{AB}$ which is

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = (33\underline{i} + 40\underline{j}) - (40\underline{i} + 16\underline{j})$

$\overrightarrow{AB} = 33\underline{i} + 40\underline{j} - 40\underline{i} - 16\underline{j}$

$\overrightarrow{AB} = -7\underline{i} + 24\underline{j}$

At 12.15 pm, the resultant vector of the plane be

$3\overrightarrow{AB} = 3\left(-7\underline{i} + 24\underline{j}\right) = -21\underline{i} + 72\underline{j}$

Hence, the position vector will be

$3\overrightarrow{AB} + \overrightarrow{OA} = -21\underline{i} + 72\underline{j} + 40\underline{i} + 16\underline{j} = 19\underline{i} + 88\underline{j}$

13b After 1 hour i. e. 60 minutes, the velocity of the plane will be

$12\overrightarrow{AB} = 12\left(-7\underline{i} + 24\underline{j}\right) = -84\underline{i} + 288\underline{j}$

14-19 <Solutions to come>

Solutions for Enrichment question 20 are in preparation.

Solutions to Chapter review

Uncorrected proofs

Chapter 8 worked solutions – Vectors

1 $|\overrightarrow{AB}| = 133$ km and angle is 68°

$|\overrightarrow{BC}| = 98$ km and angle is 116°

Hence,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

The magnitude of the vectors can be

$|\overrightarrow{AC}| \times |\overrightarrow{AC}| = |\overrightarrow{AB} \cdot \overrightarrow{BC}| \times |\overrightarrow{AB} \cdot \overrightarrow{BC}|$

$|\overrightarrow{AC}|^2 = |\overrightarrow{AB}|^2 \times |\overrightarrow{BC}|^2 \times 2|\overrightarrow{AB}||\overrightarrow{BC}| \cos\theta$

Let θ be the angle of $\overrightarrow{AC}$ which is

$\theta = 116° - 48° = 48°$

$|\overrightarrow{AC}|^2 = 133^2 \times 68^2 \times 2 \times 133 \times 68 \times \cos 48°$

$|\overrightarrow{AC}|^2 = 44733$

$|\overrightarrow{AC}| = \sqrt{44733} \div 211.5$ km

The direction of $\overrightarrow{AC}$ is

$(90° - 68°) + (180° - 116°) = 22° + 64° = 86°$

2a $\overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}$

2b $\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}$

2c $\overrightarrow{DA} + \overrightarrow{AC} = \overrightarrow{DC}$

2d $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$

$= \overrightarrow{AC} + \overrightarrow{CE} + \overrightarrow{EA}$

$= \overrightarrow{AE} + \overrightarrow{EA}$

$= \overrightarrow{AA}$

$= 0$

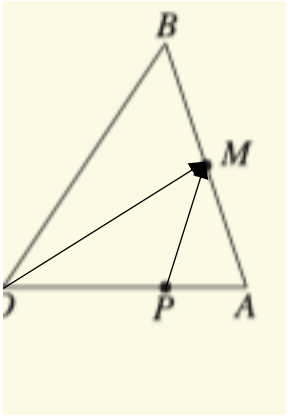
Uncorrected proofs

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2e $\overrightarrow{AD} - \overrightarrow{AC} = \overrightarrow{CD}$

2f $\overrightarrow{EB} - \overrightarrow{ED} = \overrightarrow{DB}$

3a



$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

3b $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{AM} = \frac{1}{2}(\underline{b} - \underline{a})$

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$

$\overrightarrow{OM} = \underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$\overrightarrow{OM} = \underline{a} + \frac{1}{2}\underline{b} - \frac{1}{2}\underline{a}$

$\overrightarrow{OM} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} = \frac{1}{2}(\underline{a} + \underline{b})$

Uncorrected proofs

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3c $\frac{\overrightarrow{OP}}{\overrightarrow{PA}} = \frac{2}{1}$

$\overrightarrow{OP} = 2\overrightarrow{PA}$

$\overrightarrow{OP} = 2(\overrightarrow{OA} - \overrightarrow{OP})$

$\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA} = \frac{2}{3}\underline{a}$

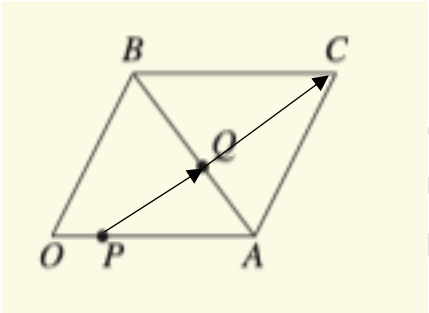
$\overrightarrow{PM} = \overrightarrow{OM} - \overrightarrow{OP}$

$\overrightarrow{PM} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} - \frac{2}{3}\underline{a}$

$\overrightarrow{PM} = \frac{1}{2}\underline{b} - \frac{1}{6}\underline{a}$

$\overrightarrow{PM} = \frac{1}{6}(3\underline{b} - \underline{a})$

4a



$\frac{\overrightarrow{OP}}{\overrightarrow{PA}} = \frac{1}{3}$

$\overrightarrow{PA} = 3\overrightarrow{OP}$

$\overrightarrow{PA} = 3(\overrightarrow{OA} - \overrightarrow{PA})$

$\overrightarrow{PA} = \frac{3}{4}\overrightarrow{OA} = \frac{3}{4}\underline{a}$

4b $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$\frac{\overrightarrow{AQ}}{\overrightarrow{BQ}} = \frac{3}{4}$

$\overrightarrow{AQ} = \frac{3}{4}\overrightarrow{BQ}$

Uncorrected proofs

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$\overrightarrow{AQ} = \frac{3}{4}(\overrightarrow{AB} - \overrightarrow{AQ})$

$\overrightarrow{AQ} = \frac{3}{7}\overrightarrow{AB} = \frac{3}{7}(\underline{b} - \underline{a})$

4c $\overrightarrow{PQ} = \overrightarrow{PA} - \overrightarrow{AQ}$

$\overrightarrow{PQ} = \frac{3}{4}\underline{a} + \frac{3}{7}(\underline{b} - \underline{a})$

$\overrightarrow{PQ} = \frac{3}{4}\underline{a} + \frac{3}{7}\underline{b} - \frac{3}{7}\underline{a}$

$\overrightarrow{PQ} = \frac{9}{28}\underline{a} + \frac{3}{7}\underline{b}$

4d $\overrightarrow{QC} = \overrightarrow{AC} - \overrightarrow{AQ}$

$OABC$ is a parallelogram, hence, $\overrightarrow{AC} = \overrightarrow{OB} = \underline{b}$

$\overrightarrow{QC} = \underline{b} + \frac{3}{7}(\underline{b} - \underline{a})$

$\overrightarrow{QC} = \underline{b} + \frac{3}{7}\underline{b} - \frac{3}{7}\underline{a}$

$\overrightarrow{QC} = \frac{3}{7}\underline{a} + \frac{4}{7}\underline{b}$

4e $\overrightarrow{PC} = \overrightarrow{PA} + \overrightarrow{AC}$

$\overrightarrow{PC} = \frac{3}{4}\underline{a} + \underline{b}$

and

$\overrightarrow{PQ} + \overrightarrow{QC}$

$= \frac{9}{28}\underline{a} + \frac{3}{7}\underline{b} + \frac{3}{7}\underline{a} + \frac{4}{7}\underline{b}$

$= \frac{3}{4}\underline{a} + \underline{b} = \overrightarrow{PC}$

Hence, P, Q and C are collinear.

Uncorrected proofs

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5a Let O be the origin hence, $\overrightarrow{OA} = -4\underline{i} + 2\underline{j}$ and $\overrightarrow{OB} = 2\underline{i} + 10\underline{j}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = (2\underline{i} + 10\underline{j}) - (-4\underline{i} + 2\underline{j})$

$\overrightarrow{AB} = 2\underline{i} + 10\underline{j} + 4\underline{i} - 2\underline{j}$

$\overrightarrow{AB} = 6\underline{i} + 8\underline{j}$

5b $\left|\overrightarrow{AB}\right| = \sqrt{x^2 + y^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{6^2 + 8^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{100} = 10$

5c $\widehat{AB} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|}$

$\widehat{AB} = \frac{6\underline{i}+8\underline{j}}{10} = \frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$

6a $\underline{v} = \begin{bmatrix} 2a \\ a \end{bmatrix}$

$|\underline{v}| = \sqrt{x^2 + y^2}$

$|\underline{v}| = \sqrt{(2a)^2 + (a)^2}$

$|\underline{v}| = \sqrt{5a^2} = \sqrt{5}a$

6b $\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|}$

$\hat{\underline{v}} = \frac{1}{\sqrt{5}a}\begin{bmatrix} 2a \\ a \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}$

6c $\underline{v} + \underline{v} = \begin{bmatrix} 2a \\ a \end{bmatrix} + \begin{bmatrix} 2a \\ a \end{bmatrix} = \begin{bmatrix} 4a \\ 2a \end{bmatrix}$

Uncorrected proofs

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6d $\underline{v} \cdot \underline{v} = |\underline{v}|^2$

$\underline{v} \cdot \underline{v} = (\sqrt{5}a)^2 = 5a^2$

7a $\underline{a} = \begin{bmatrix} x-1 \\ 1-x \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} x+1 \\ 1+x \end{bmatrix}$

$\underline{a} \cdot \underline{b} = x_1x_2 + y_1y_2$

$\underline{a} \cdot \underline{b} = (x-1)(x+1) + (1-x)(1+x)$

$\underline{a} \cdot \underline{b} = x^2 - 1 + 1 - x^2 = 0$

As, $\underline{a} \cdot \underline{b} = 0$, vectors are perpendicular.

7b $\underline{a} = \begin{bmatrix} 5x \\ 5x-1 \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} 1-2x \\ 2x \end{bmatrix}$

$\underline{a} \cdot \underline{b} = x_1x_2 + y_1y_2$

$\underline{a} \cdot \underline{b} = 5x(1-2x) + (5x-1)2x$

$\underline{a} \cdot \underline{b} = 5x - 10x^2 + 10x^2 - 2x = 3x$

As, $\underline{a} \cdot \underline{b} \neq 0$, vectors are not perpendicular.

8 $\underline{a} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

$\underline{a} \cdot \underline{b} = x_1x_2 + y_1y_2$

$\underline{a} \cdot \underline{b} = -5 \times 10 + 3 \times 2$

$\underline{a} \cdot \underline{b} = -50 + 6 = -44$

Also, where θ is the angle between $\underline{a}$ and $\underline{b}$

$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos\theta$

$|\underline{a}| = \sqrt{x^2 + y^2}$

$|\underline{a}| = \sqrt{(-5)^2 + 3^2}$

$|\underline{a}| = \sqrt{25 + 9} = \sqrt{34}$

$|\underline{b}| = \sqrt{x^2 + y^2}$

Uncorrected proofs

$|\underline{b}| = \sqrt{10^2 + 2^2}$

$|\underline{b}| = \sqrt{100 + 4} = \sqrt{104}$

$\underline{a} \cdot \underline{b} = \sqrt{34} \times \sqrt{104} \cos\theta$

$\sqrt{34} \times \sqrt{104} \cos\theta = -44$

$\cos\theta = \frac{-44}{\sqrt{3536}} = \frac{-11}{15}$

$\theta = 137°44'$

9a Let O be the origin hence, $\overrightarrow{OP} = -4\underline{i} - 5\underline{j}$, $\overrightarrow{OQ} = 10\underline{i} + 5\underline{j}$, $\overrightarrow{OR} = 5\underline{i} + 12\underline{j}$ and $\overrightarrow{OS} = -9\underline{i} + 2\underline{j}$.

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$\overrightarrow{PQ} = (10\underline{i} + 5\underline{j}) - (-4\underline{i} - 5\underline{j})$

$\overrightarrow{PQ} = 10\underline{i} + 5\underline{j} + 4\underline{i} + 5\underline{j}$

$\overrightarrow{PQ} = 14\underline{i} + 10\underline{j}$

$\overrightarrow{SR} = \overrightarrow{OR} - \overrightarrow{OS}$

$\overrightarrow{SR} = (5\underline{i} + 12\underline{j}) - (-9\underline{i} + 2\underline{j})$

$\overrightarrow{SR} = 5\underline{i} + 12\underline{j} + 9\underline{i} - 2\underline{j}$

$\overrightarrow{SR} = 14\underline{i} + 10\underline{j}$

Hence, $\overrightarrow{PQ} = \overrightarrow{SR}$

9b $\overrightarrow{PS} = \overrightarrow{OS} - \overrightarrow{OP}$

$\overrightarrow{PS} = (-9\underline{i} + 2\underline{j}) - (-4\underline{i} - 5\underline{j})$

$\overrightarrow{PS} = -9\underline{i} + 2\underline{j} + 4\underline{i} + 5\underline{j}$

$\overrightarrow{PS} = -5\underline{i} + 7\underline{j}$

Uncorrected proofs

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$\overrightarrow{PQ} \cdot \overrightarrow{PS} = (14\underline{i} + 10\underline{j}) \cdot (-5\underline{i} + 7\underline{j})$

$\overrightarrow{PQ} \cdot \overrightarrow{PS} = 14 \times (-5) + 10 \times 7$

$\overrightarrow{PQ} \cdot \overrightarrow{PS} = -70 + 70 = 0$

9c $\overrightarrow{PQ} = \overrightarrow{SR}$ and $\overrightarrow{PQ} \cdot \overrightarrow{PS} = 0$

This means opposite sides are equal and adjacent sides are 90° with each other.

Hence, $PQRS$ is a rectangle.

10a $\underline{a} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a} \cdot \underline{b}}{\underline{b} \cdot \underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{5 \times (-3) + (-2) \times (-3)}{(-3) \times (-3) + (-3) \times (-3)} \times \begin{bmatrix} -3 \\ -3 \end{bmatrix}$

$Proj_{\underline{b}}\underline{a} = -\frac{1}{2} \times \begin{bmatrix} -3 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$

10b $\underline{a} = 4\underline{i} - \underline{j}$ and $\underline{b} = 6\underline{i} + 2\underline{j}$

$Proj_{\underline{b}}\underline{a} = \frac{\underline{a} \cdot \underline{b}}{\underline{b} \cdot \underline{b}} \times \underline{b}$

$Proj_{\underline{b}}\underline{a} = \frac{4 \times 6 + (-1) \times 2}{6 \times 6 + 2 \times 2} \times (6\underline{i} + 2\underline{j})$

$Proj_{\underline{b}}\underline{a} = \frac{11}{20} \times (6\underline{i} + 2\underline{j}) = (\frac{33}{10}\underline{i} + \frac{11}{10}\underline{j})$

11 $\underline{a} = \begin{bmatrix} -8 \\ 9 \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} 3 \\ 12 \end{bmatrix}$

Length of $Proj_{\underline{b}}\underline{a} = \frac{\underline{a} \cdot \underline{b}}{|\underline{b}|}$

$Proj_{\underline{b}}\underline{a} = \frac{(-8) \times 3 + 9 \times 12}{\sqrt{9^2 + 12^2}}$

$Proj_{\underline{b}}\underline{a} = \frac{84}{\sqrt{153}}$

Uncorrected proofs

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12 Let O be the origin hence $\overrightarrow{OA} = -3\underline{i} + \underline{j}$, $\overrightarrow{OB} = 4\underline{i} + 8\underline{j}$ and $\overrightarrow{OC} = 2\underline{i} - 5\underline{j}$.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = (4\underline{i} + 8\underline{j}) - (-3\underline{i} + \underline{j})$

$\overrightarrow{AB} = 4\underline{i} + 8\underline{j} + 3\underline{i} - \underline{j}$

$\overrightarrow{AB} = 7\underline{i} + 7\underline{j}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$\overrightarrow{BC} = (2\underline{i} - 5\underline{j}) - (4\underline{i} + 8\underline{j})$

$\overrightarrow{BC} = 2\underline{i} - 5\underline{j} - 4\underline{i} - 8\underline{j}$

$\overrightarrow{BC} = -2\underline{i} - 13\underline{j}$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = (7\underline{i} + 7\underline{j}) \cdot (-2\underline{i} - 13\underline{j})$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = 7 \times (-2) + 7 \times (-13) = -105$

Also, $\overrightarrow{AB} \cdot \overrightarrow{BC} = \left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right| \cos\theta$

Let θ be $\angle ABC$.

$\left|\overrightarrow{AB}\right| = \sqrt{x^2 + y^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{7^2 + 7^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{49 + 49} = 7\sqrt{2}$

$\left|\overrightarrow{BC}\right| = \sqrt{x^2 + y^2}$

$\left|\overrightarrow{BC}\right| = \sqrt{(-2)^2 + (-13)^2}$

$\left|\overrightarrow{BC}\right| = \sqrt{4 + 169} = \sqrt{173}$

$\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right| \cos\theta = \overrightarrow{AB} \cdot \overrightarrow{BC}$

$7\sqrt{2} \times \sqrt{173} \cos\theta = -105$

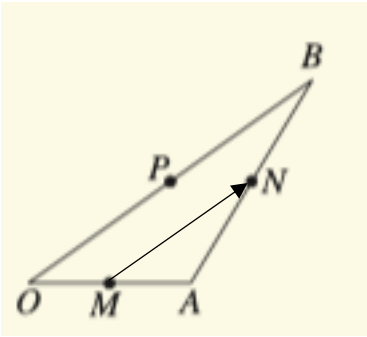
Uncorrected proofs

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$\cos\theta = \frac{-105}{7\sqrt{2}\times\sqrt{173}} = \frac{-15}{18.6}$

$\theta \div 36°$

13a



$\overrightarrow{MA} = \frac{1}{2}\overrightarrow{OA}$

$\overrightarrow{MA} = \frac{1}{2}\underline{a}$

13b $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$\overrightarrow{AN} = \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{AN} = \frac{1}{2}(\underline{b} - \underline{a})$

13c $\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$

$\overrightarrow{MN} = \frac{1}{2}\underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$\overrightarrow{MN} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} - \frac{1}{2}\underline{a}$

$\overrightarrow{MN} = \frac{1}{2}\underline{b}$

13d $\overrightarrow{MN} = \frac{1}{2}\underline{b} = \overrightarrow{PB}$

This means a pair of opposite sides are equal and parallel. Hence, $MNBP$ is a parallelogram.

Uncorrected proofs

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14a $\underset{\sim}{p} = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\underset{\sim}{p} = \overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{OM} = \overrightarrow{OB} - \overrightarrow{BM}$

$\overrightarrow{OM} = \underset{\sim}{b} - \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

$\overrightarrow{OM} = \underset{\sim}{b} - \frac{1}{2}\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$

$\underset{\sim}{m} = \overrightarrow{OM} = \frac{1}{2}\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a} = \frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{a})$

14b $LHS: \left|\underset{\sim}{p}\right|^2 + 4\left|\underset{\sim}{m}\right|^2$

$= (\underset{\sim}{b} - \underset{\sim}{a}) \cdot (\underset{\sim}{b} - \underset{\sim}{a}) + 4\left(\frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{a}) \cdot \frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{a})\right)$

$= \underset{\sim}{b} \cdot \underset{\sim}{b} - \underset{\sim}{b} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a} + 4\left(\frac{1}{4}\underset{\sim}{b} \cdot \underset{\sim}{b} + \frac{1}{4}\underset{\sim}{b} \cdot \underset{\sim}{a} + \frac{1}{4}\underset{\sim}{a} \cdot \underset{\sim}{b} + \frac{1}{4}\underset{\sim}{a} \cdot \underset{\sim}{a}\right)$

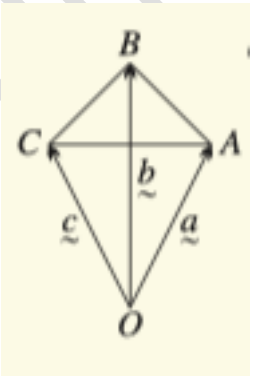
$= \underset{\sim}{b} \cdot \underset{\sim}{b} - \underset{\sim}{b} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{b} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a}$

$= \underset{\sim}{b} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{b} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a}$

$= 2(\underset{\sim}{b} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a})$

$= 2\left(\left|\underset{\sim}{b}\right|^2 + \left|\underset{\sim}{a}\right|^2\right)$

15a



$\underset{\sim}{a} \cdot \underset{\sim}{a} = \underset{\sim}{c} \cdot \underset{\sim}{c}$

Uncorrected proofs

$|\underline{a}|^2 = |\underline{c}|^2$

Adjacent sides of a kite (rhombus) are equal.

15b $(\underline{b} - \underline{a}) \cdot (\underline{b} - \underline{a}) = (\underline{b} - \underline{c}) \cdot (\underline{b} - \underline{c})$

$|\overrightarrow{AB}|^2 = |\overrightarrow{CB}|^2$

Adjacent sides of a kite (rhombus) are equal.

15c $\overrightarrow{AC} = \overrightarrow{OA} - \overrightarrow{OC}$

$\overrightarrow{AC} = \underline{a} - \underline{c}$

$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$

$\overrightarrow{OB} = \underline{c} + \underline{b} - \underline{c} = \underline{b}$

$\overrightarrow{AC} \cdot \overrightarrow{OB} = (\underline{a} - \underline{c}) \cdot \underline{b}$

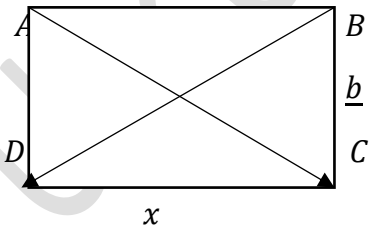
$\overrightarrow{AC} \cdot \overrightarrow{OB} = \underline{a} \cdot \underline{b} - \underline{c} \cdot \underline{b}$

As, $|\underline{a}|^2 = |\underline{c}|^2$

$\overrightarrow{AC} \cdot \overrightarrow{OB} = \underline{a} \cdot \underline{b} - \underline{a} \cdot \underline{b} = 0$

Hence, diagonals of a kite are perpendicular.

16a $\underline{a}$ x



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AC} = \underline{a} + \underline{b}$

$\overrightarrow{BD} = \overrightarrow{BC} - \overrightarrow{CD}$

$\overrightarrow{BD} = \underline{b} - \underline{a}$

Uncorrected proofs

As, $ABCD$ is a parallelogram, $\overrightarrow{AC} = \overrightarrow{CD} = \underline{a}$

16b $(\underline{a} + \underline{b}) \cdot (\underline{a} + \underline{b})$

$= \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b}$

$= \underline{a} \cdot \underline{a} + 2\underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{b}$

$= |\underline{a}|^2 + |\underline{b}|^2 + 2\underline{a} \cdot \underline{b}$

$= x^2 + y^2 + 2\underline{a} \cdot \underline{b}$

$(\underline{b} - \underline{a}) \cdot (\underline{b} - \underline{a})$

$= \underline{b} \cdot \underline{b} - \underline{b} \cdot \underline{a} - \underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{a}$

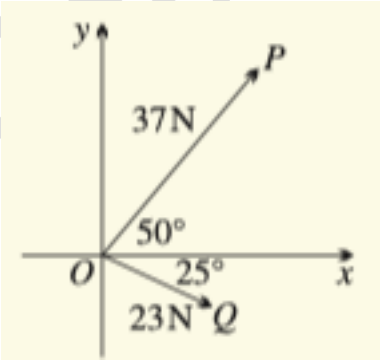
$= \underline{a} \cdot \underline{a} - 2\underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{b}$

$= |\underline{a}|^2 + |\underline{b}|^2 - 2\underline{a} \cdot \underline{b}$

$= x^2 + y^2 - 2\underline{a} \cdot \underline{b}$

16c if the parallelogram is a rectangle, then $\underline{a} \cdot \underline{b} = 0$, so the diagonals are equal. Conversely, if the diagonals are equal, then $\underline{a} \cdot \underline{b} = 0$, so the parallelogram is a rectangle.

17a



$\overrightarrow{OP} = 37\cos 50° + 37\sin 50°$

$\overrightarrow{OQ} = 23\cos 25° - 23\sin 25°$

Uncorrected proofs

17b $\overrightarrow{OP} = 37\cos 50° + 37\sin 50° = 23.78\underline{i} + 28.34\underline{j}$

$\overrightarrow{OQ} = 23\cos 25° - 23\sin 25° = 20.84\underline{i} - 9.72\underline{j}$

$\overrightarrow{PQ} = \overrightarrow{OP} - \overrightarrow{OQ}$

$\overrightarrow{PQ} = 23.78\underline{i} + 28.34\underline{j} + 20.84\underline{i} - 9.72\underline{j} = 44.62\underline{i} + 18.62\underline{j}$

$\left|\overrightarrow{PQ}\right| = \sqrt{x^2 + y^2}$

$\left|\overrightarrow{PQ}\right| = \sqrt{44.62^2 + 18.62^2}$

$\left|\overrightarrow{PQ}\right| = \sqrt{2337.64} = 48.4$ N

$(\left|\overrightarrow{PQ}\right|\cos\theta)\underline{i} = 44.62\underline{i}$

$\cos\theta = \frac{44.62}{48.4}$

$\theta = 22.7°$

18-20 <Solutions to come>

Uncorrected proofs