9.

INVESTMENTS, ANNUITIES AND LOANS

Series and sequences have many applications. Financial mathematics is an important part of everyday living as we put money in the bank, pay off credit cards, take out superannuation, buy houses and cars, and many other things. In this chapter you will study the finances of investments, annuities and loans and see how they relate to series.

CHAPTER OUTLINE

9.01 Arithmetic growth and decay
9.02 Geometric growth and decay
9.03 Compound interest
9.04 Compound interest formula
9.05 Annuities
9.06 Annuities and geometric series
9.07 Reducing balance loans
9.08 Loans and geometric series



IN THIS CHAPTER YOU WILL:

- identify arithmetic and geometric growth and decay
- solve practical problems of growth and decay
- solve problems involving compound interest investments using repeated calculations, tables and formulas
- solve problems involving annuities using repeated calculations, tables and geometric series
- solve problems involving reducing balance loans using repeated calculations, tables and geometric series

Shutterstock.com/ITIGallery

TERMINOLOGY

annuity: An investment for a fixed period of time where payments are made or received regularly.
compound interest: The interest earned on both the principal and previous interest payments of an investment.
future value: The total value at the close of an investment including all payments and interest earned.
present value: A single payment (called the principal) that will produce a future value over a given time.
reducing balance loan: A loan that is repaid by making regular payments with interest calculated on the amount still owing (the reducing balance of the loan) after each payment.
superannuation: A fixed portion of income that is invested regularly to provide a lump sum or pension when a person retires from the paid workforce; an example of an annuity.

WS
Arithmetic growth and decay

9.01 Arithmetic growth and decay

We can use arithmetic sequences and series to describe **growth** (increase) and **decay** (decrease) in practical problems. This is sometimes called **discrete linear growth and decay**.

EXAMPLE 1

A stack of cans on a display at a supermarket has 5 cans on the top row. The next row down has 2 more cans and the next one has 2 more cans and so on.

a Calculate the number of cans in the 11th row down.

b If there are 320 cans in the display altogether, how many rows are there?

Solution

a The first row has 5 cans, the 2nd row has 7 cans, the 3rd row 9 cans and so on. This forms an arithmetic sequence with $a = 5$ and $d = 2$.

For the 11th row, we want $n = 11$:

$$T_n = a + (n-1)d$$
$$T_{11} = 5 + (11-1) \times 2$$
$$= 5 + 10 \times 2$$
$$= 25$$

So there are 25 cans in the 11th row.

ISBN 9780170413220

b If there are 320 cans altogether, this is the sum of cans in all rows.

$S_n = 320$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$320 = \frac{n}{2}[2 \times 5 + (n-1) \times 2]$

$= \frac{n}{2}(10 + 2n - 2)$

$= \frac{n}{2}(2n + 8)$

$= n^2 + 4n$

$0 = n^2 + 4n - 320$

$= (n - 16)(n + 20)$

$n - 16 = 0, \quad n + 20 = 0$

$n = 16, \qquad n = -20$

Since n must be a positive integer, then $n = 16$.

There are 16 rows of cans.

We could substitute $n = 16$ into S_n to check this.

Exercise 9.01 Arithmetic growth and decay

1 A market gardener plants daffodil bulbs in rows, starting with a row of 45 bulbs.
Each successive row has 5 more bulbs than the row before.

a Calculate the number of bulbs in the 34th row.

b Which row would be the first to have more than 100 bulbs in it?

c The market gardener plants 10 545 bulbs altogether. How many rows are there?



Photo courtesy Margaret Grove

2 A stack of logs has 1 on the top, then 3 on the next row down, and each successive row has 2 more logs than the one on top of it.

a How many logs are in the 20th row?

b Which row has 57 logs?

c If there are 1024 logs altogether, how many rows are in the stack?

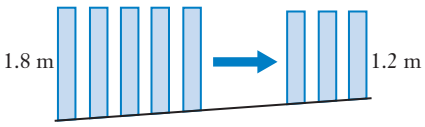
ISBN 9780170413220

3 A set of books is stacked in layers, where each layer contains 3 books fewer than the layer below. There are 6 books in the top layer, 9 in the next layer, 12 in the next and so on. There are n layers altogether.

a Write down the number of books in the bottom layer.

b Show that there are $\frac{3}{2}n(n+3)$ books in the stack altogether.

4 A timber fence is to be built on sloping land, with the shortest piece of timber 1.2 m and the longest 1.8 m. There are 61 pieces of timber in the fence.



a What is the difference in height between each piece of timber?

b Assuming no wastage, what length of timber is needed for the fence altogether?

5 A sculpture consists of a set of poles set in a row, with the tallest pole 2.4 m high, the next pole 2.1, the next one 1.8 and so on, down to the last pole which is 0.6 m high.

a How many poles are in the sculpture?

b The poles are made of timber. What length of timber is there altogether in the poles?

6 Johanna has $2000 in a term deposit that earns simple interest of 2.5% p.a. How much money does she have, including interest, after:

a 1 year? **b** 2 years? **c** 3 years?

d 10 years? **e** 30 years?

7 Each house in a row of terraced houses is to have a new fence. The houses are on a hill so the first fence will be 1 m high, the second will be 1.05 m high, the third 1.1 m high and so on.

a How high will the fence need to be for the 6th house?

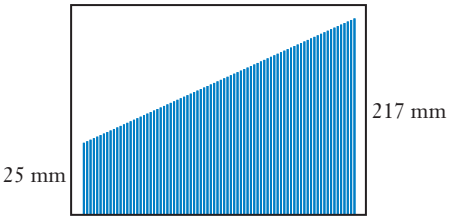
b If the height of the last fence is 1.35 m, how many houses are there?

8 At a courier company, there are different price categories for different weights of parcels. The 1st category is parcels in the range 0–0.5 kg, then 0.5–1 kg, then 1–1.5 kg and so on.

a What is the 10th weight category?

b Which category is 8.5–9 kg?

9 A logo is made with vertical lines equally spaced as shown. The shortest line is 25 mm, the longest is 217 mm, and the sum of the lengths of all the lines is 5929 mm.

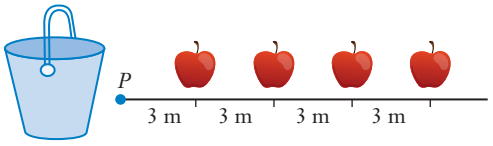


a How many lines are in the logo?

b Find the difference in length between adjacent lines.

ISBN 9780170413220

10 In a game, a child starts at point P and runs and picks up an apple 3 m away. She then runs back to P and puts the apple in a bucket. The child then runs to get the next apple 6 m away, and runs back to P to place it in the bucket. This continues until she has all the apples in the bucket.



a How far does the child run from P to pick up the kth apple?

b How far does the child run to fetch all k apples, including return trips to P?

c The child runs 270 m to fetch all the apples and return them to the bucket. How many apples are there?

9.02 Geometric growth and decay

WS Geometric growth and decay

WS Practical applications of series

We can use geometric sequences and series to describe growth and decay in practical problems. This is called **geometric growth and decay**. It is also called exponential growth and decay because $T_n = ar^{n-1}$ is an exponential function. You studied exponential growth and decay involving e^x in the Year 11 course, Chapter 8, *Exponential and logarithmic functions*.

EXAMPLE 2

A layer of tinting for a car window lets in 95% of light.

a What percentage of light is let in by:

i 2 layers of tinting? **ii** 3 layers of tinting? **iii** 10 layers of tinting?

b How many layers will let in 40% of light?

Solution

a **i** 1 layer lets in 95% of light.

So 2 layers lets in 95% × 95% of light.

$95\% \times 95\% = 0.95 \times 0.95$

$= 0.9025$

$= 90.25\%$

So 2 layers lets in 90.25% of light.

ii 1 layer lets in 95% or 0.95 of light.

2 layers lets in $0.95 \times 0.95 = 0.95^2$ of light.

3 layers lets in $0.95^2 \times 0.95 = 0.95^3$ of light.

$0.95^3 \approx 0.857$

$= 85.7\%$

So 3 layers lets in 85.7% of light.

ISBN 9780170413220

iii The number of layers forms the geometric sequence $0.95, 0.95^2, 0.95^3, \ldots$ with $a = 0.95, r = 0.95$.

For 10 layers, $n = 10$.

$$T_n = ar^{n-1}$$

$$T_{10} = 0.95(0.95)^{10-1}$$

$$= 0.95(0.95)^9$$

$$= 0.95^{10}$$

$$\approx 0.5987$$

$$= 59.87\%$$

So 10 layers lets in 59.87% of light.

b We want to find n when the nth term is 40% or 0.4.

$$T_n = ar^{n-1}$$

$$0.4 = 0.95(0.95)^{n-1}$$

$$= 0.95^n$$

$$\log 0.4 = \log 0.95^n$$

$$= n \log 0.95$$

$$\frac{\log 0.4}{\log 0.95} = n$$

$$17.9 \approx n$$

So around 18 layers of tinting will let in 40% of light.

EXAMPLE 3

A car bought for \$35 000 depreciates (loses value) by 12% p.a.

a Find its value after:

i 1 year **ii** 2 years **iii** 3 years

b Write the value of the car as a sequence.

c Find what the car is worth after 10 years.

d When will the value of the car drop below \$15 000? Answer to the nearest year.

ISBN 9780170413220

Solution

a **i** After 1 year the car is worth \$35 000 – 12% of \$35 000.

\$35 000 – 12% of \$35 000 = \$35 000(1 – 12%)

= \$35 000(1 – 0.12)

= \$35 000(0.88)

= \$30 800

So the car is worth \$30 800 after 1 year.

ii After 2 years the car is worth \$30 800 – 12% of \$30 800.

\$30 800 – 12% of \$30 800 = \$30 800(1 – 12%)

= \$30 800(0.88)

= \$27 104

So the car is worth \$27 104 after 2 years.

iii After 3 years the car is worth \$27 104 – 12% of \$27 104.

\$27 104 – 12% of \$27 104 = \$27 104(0.88)

= \$23 851.52

So the car is worth \$23 851.52 after 3 years.

b 1st year = \$30 800, 2nd year = \$27 104, 3rd year = \$23 851.52, …

So 30 800, 27 104, 23 851.52, … is a geometric sequence with $a = 30\,800$ and $r = 0.88$.

c When $n = 10$:

$T_n = ar^{n-1}$

$T_{10} = 30\,800\,(0.88)^{10-1}$

$= 30\,800\,(0.88)^9$

$= 9747.53$

So the car is worth \$9747.53 after 10 years.

d We want $T_n < 15\,000$.

$$30\,800(0.88)^{n-1} < 15\,000$$

$$0.88^{n-1} < 0.487$$

$$\log 0.88^{n-1} < \log 0.487$$

$$(n-1)\log 0.88 < \log 0.487$$

$$(n-1) > \frac{\log 0.487}{\log 0.88} \quad \text{(The inequality reverses because } \log x < 0 \text{ for } 0 < x < 1\text{)}$$

$$n > \frac{\log 0.487}{\log 0.88} + 1$$

$$> 6.63$$

We can substitute $n = 7$ into T_n to check this.

So it will take approximately 7 years for the car to be worth less than \$15 000.

We can also use the limiting sum to model some types of problems.

EXAMPLE 4

a Write $0.\dot{5}$ as a fraction.

b A ball is dropped from a height of 1 metre and bounces up to $\frac{1}{3}$ of its height.

It continues bouncing, rising $\frac{1}{3}$ of its height on each bounce.

i Draw a diagram showing the motion.

ii What is the total distance through which the ball travels?

Solution

a $0.\dot{5} = 0.5555555\ldots$

$$= \frac{5}{10} + \frac{5}{100} + \frac{5}{1000} + \ldots$$

This is a geometric series with $a = \frac{5}{10} = \frac{1}{2}$ and $r = \frac{1}{10}$.

$$S = \frac{a}{1-r}$$

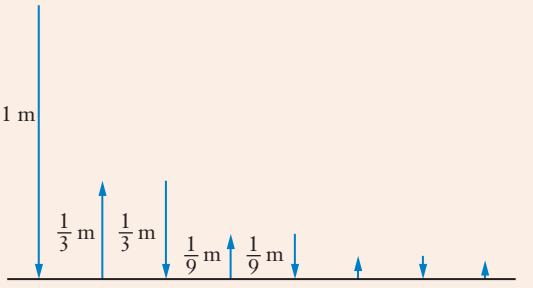
$$= \frac{\frac{1}{2}}{1-\frac{1}{10}}$$

$$= \frac{\frac{1}{2}}{\frac{9}{10}}$$

$$= \frac{5}{9}$$

ISBN 9780170413220

b i



ii Notice that there is a series for the ball coming downwards and another series upwards. There is more than one way of calculating the total distance. Here is one way of solving it.

$$\text{Total distance} = 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{9} + \frac{1}{9} + \frac{1}{27} + \frac{1}{27} + \ldots$$

$$= 1 + 2\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\right)$$

$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ is a geometric series with $a = \frac{1}{3}$ and $r = \frac{1}{3}$

$$S = \frac{a}{1-r}$$

$$= \frac{\frac{1}{3}}{1-\frac{1}{3}}$$

$$= \frac{1}{2}$$

$$\text{Total distance} = 1 + 2\left(\frac{1}{2}\right)$$

$$= 2$$

So the ball travels 2 metres altogether.

INVESTIGATION

LIMITING SUM APPLICATIONS

1 In the above example, in theory will the ball ever stop?

2 Kim owes $1000 on her credit card. If she pays back 10% of the amount owing each month, she will never finish paying it off. Is this true or false?

ISBN 9780170413220

Exercise 9.02 Geometric growth and decay

1 Water evaporates from a pond at an average rate of 7% each week.

a What percentage of water is left in the pond after:

i 1 week? **ii** 2 weeks? **iii** 3 weeks?

b What percentage is left after 15 weeks?

c If there was no rain, approximately how long (to the nearest week) would it take for the pond to only have 25% of its water left?

2 The price of shares in a particular company is falling by an average of 2% each day.

a What percentage of their initial value do they have after 2 days?

b Approximately how many days will it take for the shares to halve in value?

c After how many days will the shares be worth 10% of their initial value?

3 A painting appreciates (increases its value) by 16% p.a. It is currently worth $20 000.

a How much will it be worth in:

i 1 year? **ii** 2 years? **iii** 3 years?

b How much will it be worth in 11 years?

c How long will it take for it to be worth $50 000?

4 A southern brown bandicoot population in Western Australia is decreasing by 5% each year.

a What percentage of the population is left after 5 years?

b After how many years will the population be only 50% of its current level?

c How many years will it take for the population to decrease by 80%?

5 Write each recurring decimal as a fraction.

a $0.\dot{4}$ **b** $0.\dot{7}$ **c** $1.\dot{2}$ **d** $0.\dot{2}\dot{5}$ **e** $2.\dot{8}\dot{1}$

f $0.2\dot{3}$ **g** $1.4\dot{7}$ **h** $1.01\dot{5}$ **i** $0.1\dot{3}\dot{2}$ **j** $2.\dot{3}6\dot{1}$

6 A frog jumps 0.5 metres. It then jumps 0.1 m and on each subsequent jump it travels 0.2 of the previous distance. Find the total distance through which the frog jumps.

7 A tree grows by $\frac{4}{5}$ of each previous year's growth.

If it was initially 3 m high, find the ultimate height of the tree.



Photo courtesy Margaret Grove

ISBN 9780170413220

8 An 8 cm seedling grows by 4.8 cm in the first week, and then keeps growing by 0.6 of its previous week's growth. How tall will it grow?

9 An object rolls 0.5 m in the first second. Then each second after, it rolls by $\frac{5}{6}$ of its previous roll. Find how far it will roll altogether.

10 A 100 m cliff erodes by $\frac{2}{7}$ of its height each year.

a What will the height of the cliff be after 10 years?

b After how many years will the cliff be less than 50 m high?

11 A lamb grows by $\frac{2}{5}$ of its previous growth each month. If a lamb is 45 cm tall:

a how tall will it be after 6 months?

b what will its final height be?



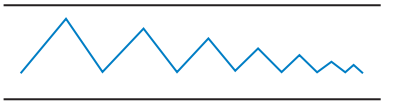
Photo courtesy Margaret Grove

12 A weight on an elastic string drops down 60 cm and then bounces back to $\frac{2}{3}$ of its initial height. It keeps bouncing, each time rising back to $\frac{2}{3}$ of its previous height.

What is the total distance through which the weight travels?

13 Mary bounces a ball, dropping it from 1.5 m on its first bounce. It then rises up to $\frac{2}{5}$ of its height on each bounce. Find the distance through which the ball travels.

14 A roadside wall has a zigzag pattern on it as shown. The two longest lines are each 2 m long, then the next two lines are $1\frac{3}{4}$ m long and the lines in each subsequent pair are $\frac{7}{8}$ of the length of the previous pair. Find the total length of the lines.



15 Frankie receives a text message that she is asked to send to 8 friends. Frankie forwards this text on to 8 friends and each of them sends it on to 8 friends, and so on.

a Describe the number of people receiving the text as a sequence (including Frankie's text).

b How many people would receive the message in the 9th round of texts?

c How many people would have received the text altogether if it is sent 9 times?

ISBN 9780170413220

9.03 Compound interest

An **investment** is money that is put into the bank or used to pay for something that will increase in value, or **appreciate**, in the future. An investment can include real estate, art, jewellery or antiques.

The amount invested is called the **present value** or **principal**. The amount the investment is worth after a period of time is called the **future value**.

Future value of investments (FV)

Money in the bank earns interest. Some investments earn simple interest, but most earn **compound interest**. For example, a term deposit or investment account gives the option of adding the interest back into the account (compound interest) or taking the interest as cash or another investment (simple interest).

EXAMPLE 5

Mahmoud invests \$7000 into a term deposit account for 3 years, where it earns 5% p.a.

a How much interest does he earn over the 3 years if the interest is paid into the term deposit account at the end of each year?

b What is the future value of the investment?

Solution

a Interest is $5\% = 0.05$

The interest is added to the principal each time it is paid.

$$\begin{aligned}\text{Amount after 1 year} &= \$7000 + 0.05 \times \$7000\\ &= \$7000(1 + 0.05)\\ &= \$7000(1.05)\\ &= \$7350\end{aligned}$$

$$\begin{aligned}\text{Amount after 2 years} &= \$7350 + 0.05 \times \$7350\\ &= \$7350(1.05)\\ &= \$7717.50\end{aligned}$$

$$\begin{aligned}\text{Amount after 3 years} &= \$7717.50 + 0.05 \times \$7717.50\\ &= \$7717.50(1.05)\\ &\approx \$8103.38\end{aligned}$$

$$\begin{aligned}\text{Amount of interest} &= \$8103.38 - \$7000\\ &= \$1103.38\end{aligned}$$

b The future value is \$8103.38.

ISBN 9780170413220

We also use compound interest to calculate the future value of other investments.

EXAMPLE 6

Rachel and Wade buy a house in Sydney for $1 250 000. House prices in that area go up by an average of 11.5% p.a. What is their house worth after 2 years?

Solution

Interest is $11.5\% = 0.115$

Amount after 1 year $= \$1\,250\,000 + 0.115 \times \$1\,250\,000$

$= \$1\,250\,000(1 + 0.115)$

$= \$1\,250\,000(1.115)$

$= \$1\,393\,750$

Amount after 2 years $= \$1\,393\,750(1.115)$

$= \$1\,554\,031.25$

So value after 2 years is $1 554 031.25



Shutterstock.com/MagSpace

Compound interest tables simplify calculations. The values in the table next page are called **future value interest factors** as they give the future values of an investment of $1 at a certain interest rate and time.

Future value interest factors on $1							
Periods	**1%**	**2%**	**5%**	**8%**	**10%**	**15%**	**20%**
1	1.0100	1.0200	1.0500	1.0800	1.1000	1.1500	1.2000
2	1.0201	1.0404	1.1025	1.1664	1.2100	1.3225	1.4400
3	1.0303	1.0612	1.1576	1.2597	1.3310	1.5209	1.7280
4	1.0406	1.0824	1.2155	1.3605	1.4641	1.7490	2.0736
5	1.0510	1.1041	1.2763	1.4693	1.6105	2.0114	2.4883
6	1.0615	1.1262	1.3401	1.5869	1.7716	2.3131	2.9860
7	1.0721	1.1487	1.4071	1.7138	1.9487	2.6600	3.5832
8	1.0829	1.1717	1.4775	1.8509	2.1436	3.0590	4.2998
9	1.0937	1.1951	1.5513	1.9990	2.3579	3.5179	5.1598
10	1.1046	1.2190	1.6289	2.1589	2.5937	4.0456	6.1917
11	1.1157	1.2434	1.7103	2.3316	2.8531	4.6524	7.4301
12	1.1268	1.2682	1.7959	2.5182	3.1384	5.3503	8.9161
13	1.1381	1.2936	1.8856	2.7196	3.4523	6.1528	10.6993
14	1.1495	1.3195	1.9799	2.9372	3.7975	7.0757	12.8392
15	1.1610	1.3459	2.0789	3.1722	4.1772	8.1371	15.4070
16	1.1726	1.3728	2.1829	3.4259	4.5950	9.3576	18.4884
17	1.1843	1.4002	2.2920	3.7000	5.0545	10.7613	22.1861
18	1.1961	1.4282	2.4066	3.9960	5.5599	12.3755	26.6233

To see how the table works, we can use the example of Mahmoud's term deposit.

EXAMPLE 7

Mahmoud invests $7000 into a term deposit account for 3 years where it earns 5% p.a. Use the table to find the future value of the investment.

Solution

From the table:

For 3 years, $n = 3$ and interest is 5%.

Finding the column for 3 years at 5% gives 1.1576.

1.1576 is the future value on $1.

So future value on $7000 = $7000 × 1.1576

= $8103.20

The table gives a slightly different answer from Example 6 because the future value interest factors are rounded to 4 decimal places. So using a table is quicker but not as accurate!

ISBN 9780170413220

We can use the table for investments of less than a year. The value of n stands for time periods, not years.

EXAMPLE 8

Stephanie invests $2000 into a term deposit account for 5 months where it earns 12% p.a., paid monthly. Use the table to find the future value of the investment.

Solution

Interest is 12% p.a.

p.a. or per annum means each year

So interest per month = 12% ÷ 12 = 1%

The value across from $n = 5$ months in the 1% column is 1.0510

Future value on $2000 = $2000 × 1.0510

= $2102

Present value of investments (PV)

Sometimes you want to know how much you would need to invest now to end up with a certain amount in the future. For example, you may be saving up for a holiday or a deposit for a house. This value you need to invest now to achieve a future value is called the **present value**.

EXAMPLE 9

Geordie wants to invest enough money now so that he will have $5000 in 4 years' time to buy a car. Use the table of future value interest factors to calculate how much present value he would need to invest if the interest rate is 5% p.a.

Solution

We use $n = 4$ and 5%. We know FV = 5000 and we want to find the present value.

Let PV = x

From the table, the value across from $n = 4$ in the 5% column is 1.2155.

This is the future value on $1.

So FV = $x \times 1.2155$ or $1.2155x$

But FV = 5000

So $1.2155x = 5000$

$$x = \frac{5000}{1.2155}$$

$$= 4113.53$$

So Geordie needs to invest a present value of $4113.53 to have $5000 in 4 years' time.

Exercise 9.03 Compound interest

1 Calculate the future value if $6500 is invested for:

a 2 years at 3% p.a.
b 3 years at 2.5% p.a.
c 4 years at 4.1% p.a.
d 3 years at 1.8% p.a.
e 2 years at 5.3% p.a.

2 Calculate the future value of each investment.

a $2500 for 3 years at 4.5% p.a.
b $10 000 for 4 years at 6.2% p.a.
c $3400 for 5 years at 3.5% p.a.
d $5000 for 3 years at 6% p.a.
e $80 000 for 2 years at 4.5% p.a.

3 Christian and Kate buy a house for $750 000. What is the house worth after 3 years if its value increases by 6% p.a.?

4 Aparna bought a diamond ring for $3000. How much was it worth 3 years later if it appreciated by 5.8% p.a.?

5 A painting bought for $15 000 appreciates by 9% p.a. What is its future value after 4 years?

6 The present value of a necklace is $950. What is its future value after 4 years if it appreciates at 3% p.a.?

7 Hien deposits $4500 into a tour fund where it earns interest of 2.9% p.a. What will be the future value of the tour fund after 3 years?

8 Use the table of future value interest factors on page 380 to calculate the future value of each investment.

a $800 for 7 years at 5% p.a.
b $2000 for 10 years at 1% p.a.
c $5000 for 6 years at 20% p.a.
d $60 000 for 5 years at 10% p.a.
e $100 000 for 8 years at 15% p.a.
f $673.25 for 6 years at 5% p.a.
g $1249.53 for 4 years at 1% p.a.
h $3000 for 3 months at 12% p.a., paid monthly
i $1000 for 6 months at 12% p.a. paid monthly
j $3500 for 10 months at 12% p.a. paid monthly

9 Use the table of future value interest factors on page 380 to calculate the present value if the future value is $10 000 after:

a 7 years at 2% p.a.
b 5 years at 15% p.a.
c 10 years at 8% p.a.
d 3 years at 1% p.a.
e 4 years at 5% p.a.

ISBN 9780170413220

9.04 Compound interest formula

The calculations on compound interest follow a pattern called a **recurrence relation**.

WS
Compound interest

EXAMPLE 10

Patrick invests \$2000 at the beginning of the year at 6% p.a. Find a formula for the amount in the bank at the end of n years.

Solution

$6\% = 0.06$

Amount after 1 year:

$A_1 = \$2000 + 0.06 \text{ of } \$2000$

$= \$2000(1 + 0.06)$

$= \$2000(1.06)$

Amount after 2 years:

$A_2 = A_1 + 0.06A_1$

$= [\$2000(1.06)] + 0.06 \times [\$2000(1.06)]$

$= [\$2000(1.06)](1 + 0.06)$

$= [\$2000(1.06)](1.06)$

$= \$2000(1.06)^2$

Amount after 3 years:

$A_3 = A_2 + 0.06A_2$

$= [\$2000(1.06)^2] + 0.06 \times [\$2000(1.06)^2]$

$= [\$2000(1.06)^2](1 + 0.06)$

$= [\$2000(1.06)^2](1.06)$

$= \$2000(1.06)^3$

The recurrence relation is $A_{n+1} = A_n + 0.06A_n$

$A_1, A_2, A_3, \ldots$ is a geometric sequence with $a = 2000(1.06)$ and $r = 1.06$.

$T_n = ar^{n-1}$

$= 2000(1.06)(1.06)^{n-1}$

$= 2000(1.06)^n$

So the amount after n years is $\$2000(1.06)^n$.

Compound interest

$A = P(1 + r)^n$

where P = principal (present value)

r = interest rate per period, as a decimal

n = number of periods

A = future value

EXAMPLE 11

Find the amount that will be in the bank after 6 years if \$2000 is invested at 12% p.a. with interest paid:

a yearly **b** quarterly **c** monthly

Solution

$P = 2000$

a $r = 12\% = 0.12, n = 6$

$A = P(1 + r)^n$

$= 2000(1 + 0.12)^6$

$= 2000(1.12)^6$

$= 3947.65$

So the amount is \$3947.65.

b For quarterly interest, the annual interest rate is divided by 4.

$r = 0.12 \div 4 = 0.03$

Interest is paid 4 times a year.

$n = 6 \times 4 = 24$

$A = P(1 + r)^n$

$= 2000(1 + 0.03)^{24}$

$= 2000(1.03)^{24}$

$= 4065.59$

So the amount is \$4065.59.

ISBN 9780170413220

c For monthly interest, the annual interest rate is divided by 12.

$r = 0.12 \div 12 = 0.01$

Interest is paid 12 times a year.

$n = 6 \times 12 = 72$

$A = P(1 + r)^n$

$= 2000(1 + 0.01)^{72}$

$= 2000(1.01)^{72}$

$= 4094.20$

So the amount is $4094.20.

We can find the present value using the compound interest formula.

EXAMPLE 12

Geoff wants to invest enough money to pay for a $10 000 holiday in 7 years' time. If interest is 2.5% p.a., what present value does Geoff need to invest now?

Solution

$A = 10\,000, r = 2.5\%$ or $0.025, n = 7$

We want to find the present value P.

$A = P(1 + r)^n$

$10\,000 = P(1 + 0.025)^7$

$= P(1.025)^7$

$\frac{10000}{1.025^7} = P$

$P \approx 8412.65$

The present value to invest is $8412.65.



Shutterstock.com/Steve Photography

We can use the compound interest formula to find the interest rate or time period by rearranging the formula.

EXAMPLE 13

a Silvana invested \$1800 at 6% p.a. interest and it grew to \$2722.66. For how many years was the money invested if interest was paid twice a year?

b Find the interest rate if a \$1500 investment is worth \$1738.91 after 5 years.

Solution

a $P = 1800$ and $A = 2722.66$.

Interest is paid twice a year:

$r = 0.06 \div 2 = 0.03$

$$A = P(1 + r)^n$$
$$2722.66 = 1800(1 + 0.03)^n$$
$$= 1800(1.03)^n$$
$$\frac{2722.66}{1800} = 1.03^n$$
$$1.51259 = 1.03^n$$
$$\log(1.51259) = \log(1.03)^n$$
$$= n \log(1.03)$$
$$\frac{\log(1.51259)}{\log(1.03)} = n$$
$$14 \approx n$$

Since interest is paid in twice a year, the number of years will be $14 \div 2 = 7$.

So the money was invested for 7 years.

b $P = 1500, A = 1738.91, n = 5$

$$A = P(1 + r)^n$$
$$1738.91 = 1500(1 + r)^5$$
$$\frac{1738.91}{1500} = (1 + r)^5$$
$$1.15927 = (1 + r)^5$$
$$\sqrt[5]{1.15927} = 1 + r$$
$$1.03 = 1 + r$$
$$0.03 = r$$
$$r = 3\%$$

So the interest rate is 3% p.a.

ISBN 9780170413220

Exercise 9.04 Compound interest formula

1 Find the amount of money in the bank after 10 years if:

a $500 is invested at 4% p.a.
b $7500 is invested at 7% p.a.
c $8000 is invested at 8% p.a.
d $5000 is invested at 6.5% p.a.
e $2500 is invested at 7.8% p.a.

2 Sam banks $1500 where it earns interest at the rate of 6% p.a. Find the amount after 5 years if interest is paid:

a annually **b** twice a year **c** quarterly

3 Chantelle banks $3000 in an account that earns 5% p.a. Find the amount in the bank after 10 years if interest is paid:

a quarterly **b** monthly

4 Reza put $350 in the bank where it earns interest of 8% p.a. Find the amount there will be in the account after 2 years if interest is paid:

a annually **b** monthly

5 How much money will there be in an investment account after 3 years if the present value is $850 and interest of 4.5% p.a. is paid:

a twice a year? **b** quarterly?

6 Find the amount of money there will be in a bank after 8 years if $1000 earns interest of 7% p.a. with interest paid:

a twice a year **b** quarterly **c** monthly

7 Tanya left $2500 in a credit union account for 4 years, with interest of 5.5% p.a. paid yearly.

a How much money did she have in the account at the end of that time?
b What would be the difference in the future value if interest was paid quarterly?

8 **a** Find the amount of money there will be after 15 years if Hannah banks $6000 and it earns 9% p.a. interest, paid quarterly.
b How much more money will Hannah have than if interest was paid annually?

9 How much money will be in a bank account after 5 years if $500 earns 6.5% p.a. with interest paid monthly?

10 Find the amount of interest earned over 4 years if $1400 earns 6% p.a. paid quarterly.

11 How much money will be in a credit union account after 8 years if $8000 earns 7.5% p.a. interest paid monthly?

12 Elva wins a lottery and invests $500 000 in an account that earns 8% p.a. with interest paid monthly. How much will be in the account after 12 years?

ISBN 9780170413220

13 Calculate the principal invested for 4 years at 5% p.a. to achieve a future value of:

a \$5000 **b** \$675 **c** \$12 000 **d** \$289.50 **e** \$12 800

14 What present value is required to accumulate to \$5400 in 3 years with interest of 5.8% p.a. paid quarterly?

15 How many years ago was an investment made if \$5000 was invested at 6% p.a. paid monthly and it is now worth \$6352.45?

16 Find the number of years that \$10 000 was invested at 8% p.a. with interest paid twice a year if there is now \$18 729.81 in the bank.

17 Jude invested \$4500 five years ago at x% p.a. Evaluate x if the amount in his bank account is now:

a \$6311.48 **b** \$5743.27 **c** \$6611.98 **d** \$6165.39 **e** \$6766.46

18 Hamish is given the choice of a bank account in which interest is paid annually or quarterly. If he deposits \$1200, find the difference in the amount of interest paid over 3 years if interest is 7% p.a.

19 Kate has \$4000 in a bank account that pays 5% p.a. with interest paid annually, and Rachel has \$4000 in a different account paying 4% quarterly. Which person will receive more interest over 5 years, and by how much?

20 A bank offers investment account A at 8% p.a. with interest paid twice a year and account B with interest paid at 6% p.a. at monthly intervals. If Georgia invests \$5000 over 6 years, which account pays more interest? How much more does it pay?

21 A hairdresser earns \$36 400 for the first year of work. His salary increases each year by 2%.

a What is his salary in his:

i 5th year of work? **ii** 8th year of work?

b When will his salary reach \$60 000?

22 Yuron earns \$120 000 in his 1st year, then his salary goes up by 3.5% each year after that.

a How much does Yuron earn in his:

i 3rd year? **ii** 12th year? **iii** 20th year?

b What is the first year in which Yuron earns over \$300 000?

23 Masae invests \$5000 in a bank account.

a How many years at 2% p.a. interest will it take for her investment to grow to:

i \$5410? **ii** \$7000?

b At what interest rate would the investment grow to \$6000 after:

i 6 years? **ii** 10 years?

ISBN 9780170413220

24 a Use the table of future value interest factors on page 380 to find the interest factor for an investment on $1 over 9 years at 8% p.a.

b Prove this interest factor is correct by using the compound interest formula.

25 Show that the future value interest factor of 1.5209 is true for an investment over 3 years at 15% p.a.

9.05 Annuities

A better way to build up money faster is to make regular contributions to an investment. This is called an **annuity**, a name that comes from the same Latin word 'annus' as annual, meaning yearly. However contributions to an annuity could be made more frequently than this. For example, payments into **superannuation** can be made every week or fortnight when an employee is paid.

WS Investments and annuities

WS Annuity table problems

WS Annuity: spreadsheet

Future value of an annuity

EXAMPLE 14

Stevie's grandparents put $100 into a bank account for her on her first birthday. They deposit $100 into the account on each birthday until Stevie is 18 and give her the total amount for her 18th birthday.

How much is in the account at the end of 3 years if interest is 2% p.a.?

Solution

Amount at the end of the 1st year:

Since $100 is deposited at the end of the 1st year on Stevie's 1st birthday, it earns no interest in that year.

$A_1 = \$100$

Amount at the end of the 2nd year:

$A_2 = \$100 + 0.02 \times \$100$

$= \$100(1 + 0.02)$

$= \$100(1.02)$

$= \$102$

But *another* $100 is deposited at the end of the 2nd year on Stevie's 2nd birthday.

So amount $= \$102 + \$100$

$= \$202$

Amount at the end of the 3rd year:

$A_3 = \$202(1.02)$

$= \$206.04$

But *another* $100 is deposited at the end of the 3rd year on Stevie's 3rd birthday.

So amount $= \$206.04 + \$100$

$= \$306.04$

So after 3 years the annuity is worth $306.04.

This is the $300 put in by Stevie's grandparents plus interest of $6.04.

ISBN 9780170413220

Future value of an annuity with a contribution of $1 at the end of each period															
Periods	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200	2.1300	2.1400	2.1500
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744	3.4069	3.4396	3.4725
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793	4.8498	4.9211	4.9934
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528	6.4803	6.6101	6.7424
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152	8.3227	8.5355	8.7537
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890	10.4047	10.7305	11.0668
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997	12.7573	13.2328	13.7268
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757	15.4157	16.0853	16.7858
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487	18.4197	19.3373	20.3037
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546	21.8143	23.0445	24.3493
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331	25.6502	27.2707	29.0017
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291	29.9847	32.0887	34.3519
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926	34.8827	37.5811	40.5047
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797	40.4175	43.8424	47.5804
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533	46.6717	50.9804	55.7175
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837	53.7391	59.1176	65.0751
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497	61.7251	68.3941	75.8364
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397	70.7494	78.9692	88.2118
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524	80.9468	91.0249	102.4436
21	23.2392	25.7833	28.6765	31.9692	35.7193	39.9927	44.8652	50.4229	56.7645	64.0025	72.2651	81.6987	92.4699	104.7684	118.8101
22	24.4716	27.2990	30.5368	34.2480	38.5052	43.3923	49.0057	55.4568	62.8733	71.4027	81.2143	92.5026	105.4910	120.4360	137.6316
23	25.7163	28.8450	32.4529	36.6179	41.4305	46.9958	53.4361	60.8933	69.5319	79.5430	91.1479	104.6029	120.2048	138.2970	159.2764
24	26.9735	30.4219	34.4265	39.0826	44.5020	50.8156	58.1767	66.7648	76.7898	88.4973	102.1742	118.1552	136.8315	158.6586	184.1678
25	28.2432	32.0303	36.4593	41.6459	47.7271	54.8645	63.2490	73.1059	84.7009	98.3471	114.4133	133.3339	155.6196	181.8708	212.7930
26	29.5256	33.6709	38.5530	44.3117	51.1135	59.1564	68.6765	79.9544	93.3240	109.1818	127.9988	150.3339	176.8501	208.3327	245.7120
27	30.8209	35.3443	40.7096	47.0842	54.6691	63.7058	74.4838	87.3508	102.7231	121.0999	143.0786	169.3740	200.8406	238.4993	283.5688
28	32.1291	37.0512	42.9309	49.9676	58.4026	68.5281	80.6977	95.3388	112.9682	134.2099	159.8173	190.6989	227.9499	272.8892	327.1041
29	33.4504	38.7922	45.2189	52.9663	62.3227	73.6398	87.3465	103.9659	124.1354	148.6309	178.3972	214.5828	258.5834	312.0937	377.1697
30	34.7849	40.5681	47.5754	56.0849	66.4388	79.0582	94.4608	113.2832	136.3075	164.4940	199.0209	241.3327	293.1992	356.7868	434.7451

ISBN 9780170413220

It is assumed that annuity payments are made at the *end* of each period, unless stated otherwise. If they are made at the *beginning* of each period, then the calculations would be different.

You can use the table on the previous page to calculate annuities. You can also download a copy from NelsonNet.

EXAMPLE 15

Stevie's grandparents put $100 into a bank account for her on each birthday, with the final deposit on her 18th birthday. They give Stevie the total amount of the money for her 18th birthday.

a Use the table of future value of annuities factors to calculate how much is in the account after 3 years if interest is 2% p.a.

b How much will Stevie receive on her 18th birthday?

Solution

a From the table:

The value across from $n = 3$ years in the 2% column is 3.0604

This is the future value on $1.

Future value on $100 = $100 × 3.0604

= $306.04

b The value across from $n = 18$ years in the 2% column is 21.4123.

Future value on $100 = $100 × 21.4123

= $2141.23

So Stevie will receive $2141.23 on her 18th birthday.

We can use the table of future values of an annuity to calculate how much to contribute regularly to achieve a particular future value.

EXAMPLE 16

a Christopher wants to save a certain amount at the end of each year for 5 years until he has $20 000 to buy a car. If the interest rate is 3% p.a., find the amount of each annual contribution Christopher needs to make.

b Alexis wants to save up a $50 000 deposit for a home over 7 years. She wants to make contributions at the end of each quarter. Interest is 8% p.a., paid quarterly. What size contribution would she make?

Solution

a From the future value for annuities table on the previous page, the value across from $n = 5$ in the 3% column is 5.3091.

If we call the contribution x:

Future value = $x \times 5.3091$

ISBN 9780170413220

So $5.3091x = 20\,000$

$$x = \frac{20000}{5.3091}$$

$$= 3767.12$$

So each contribution is \$3767.12

b The contribution is quarterly, or 4 times a year.

Interest rate = $8\% \div 4 = 2\%$

Alexis makes 4 contributions each year for 7 years.

Number of periods = $7 \times 4 = 28$

From the table, the value across from $n = 28$ in the 2% column is 37.0512.

If we call the contribution x:

Future value = $x \times 37.0512$

So $37.0512x = 50\,000$

$$x = \frac{50000}{37.0512}$$

$$= 1349.48$$

So each contribution is \$1349.48.

Annuities with regular withdrawals

Another type of annuity is a sum of money earning compound interest that has regular withdrawals or payouts coming out of it.

EXAMPLE 17

Yasmin retires with a lump sum superannuation payment of \$145 000. She puts the money into a financial management company that guarantees 12% p.a. on her annuity, with interest paid monthly. Yasmin withdraws \$1800 at the end of each month as a pension.

Find what Yasmin's annuity is worth after 3 months.

Solution

Monthly interest = $12\% \div 12 = 1\%$

Amount at the end of 1st month = $\$145\,000(1 + 0.01)^1$

$= \$145\,000(1.01)$

$= \$146\,450$

ISBN 9780170413220

But Yasmin withdraws \$1800

So amount = \$146 450 – \$1800

= \$144 650

Amount at the end of 2nd month = $\$144\,650(1.01)^1$

= \$146 096.50

But Yasmin withdraws \$1800

So amount = \$146 096.50 – \$1800

= \$144 296.50

Amount at the end of 3rd month = $\$144\,296.50(1.01)^1$

= \$145 739.47

But Yasmin withdraws \$1800

So amount = \$144 739.47 – \$1800

= \$143 939.47

So after 3 months Yasmin's annuity is worth \$143 939.47.

Notice that Yasmin's annuity is gradually decreasing. If she took a little less money out each month, she could keep the value of her annuity at around \$145 000 or increase its value a little. Try doing the above example with different values to see if Yasmin could draw a pension while keeping her lump sum the same.

TECHNOLOGY

Annuities and spreadsheets

1 The formula for the future value of an annuity is: $FV = a\left[\frac{(1+r)^n - 1}{r}\right]$ where a = regular contribution, r = interest rate and n = number of periods.

You can use this formula to draw up a spreadsheet for future values of an annuity.

Does this formula look familiar? It comes from the sum of a geometric series.

We can use this formula to write a table of future values in a spreadsheet.

Using rows 1 and 2 for headings, we can put 1 in A3 and the formula **=1+A3** in A4. Drag this formula down the column for the periods 1, 2, 3, …

We will use $a = 1$ and $r = 0.05$ (5% interest)

In B3 put the formula **=((1+0.05)^A3–1)/0.05** and drag it down the column.

This gives a set of values for the future value of an annuity at 5% p.a.

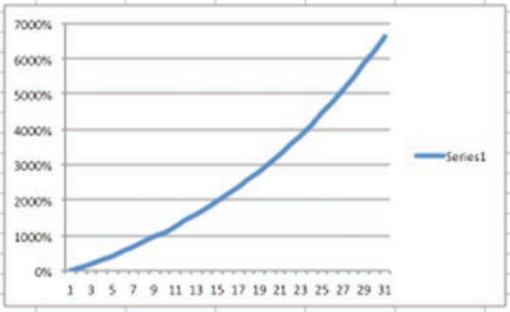
ISBN 9780170413220

Now highlight the column of future values and select the line graph from Charts.

Can you find future values from the graph?

Change the formula to a different interest rate. For example, use 0.08 instead of 0.05 in the formula and drag it down the column. How does this change the graph? Try other interest rate changes and look at how the graph changes.

	A	B
1	ture value of annuities	
2	Periods	5%
3	1	1.0000
4	2	2.0500
5	3	3.1525
6	4	4.3101
7	5	5.5256
8	6	6.8019
9	7	8.1420
10	8	9.5491
11	9	11.0266
12	10	12.5779
13	11	14.2068
14	12	15.9171
15	13	17.7130
16	14	19.5986
17	15	21.5786
18	16	23.6575
19	17	25.8404
20	18	28.1324
21	19	30.5390
22	20	33.0660
23	21	35.7193
24	22	38.5052
25	23	41.4305
26	24	44.5020
27	25	47.7271
28	26	51.1135
29	27	54.6691
30	28	58.4026
31	29	62.3227
32	30	66.4388



2 Use the formula $PV = a\left[\dfrac{(1+r)^n - 1}{r(1+r)^n}\right]$ for the present value of an annuity to draw up a spreadsheet and graph for present value interest factors using similar steps.

For example, for 5% interest, in B3 use the formula **=((1.05)^A3–1)/(0.05*(1.05)^A3)** for the value of 1 in A3, then drag the formula down the column.

How does the graph change if you change the interest rate?

Exercise 9.05 Annuities

1 Calculate the future value of an annuity with a yearly contribution (at the end of each year) of:

- **a** $5000 for 2 years at 2.5% p.a.
- **b** $1200 for 3 years at 4% p.a.
- **c** $875 for 3 years at 3.6% p.a.
- **d** $10 000 for 2 years at 4.1% p.a.
- **e** $2000 for 3 years at 3.2% p.a.

ISBN 9780170413220

Use the future value table for annuities on page 390 to answer Questions 2 to 7.

2 Find the future value of an annuity with annual contributions of:

a $6300 for 7 years at 4% p.a.
b $980 for 5 years at 6% p.a.
c $7500 for 10 years at 5% p.a.
d $495.75 for 4 years at 3% p.a.
e $20 500 for 12 years at 2% p.a.
f $647.12 for 6 years at 1% p.a.
g $800 for 15 years at 7% p.a.
h $598 for 14 years at 9% p.a.
i $15 000 for 11 years at 13% p.a.
j $160 000 for 8 years at 4% p.a.

3 Find the future value of an annuity with contributions of:

a $400 a month for 2 years at 12% p.a. paid monthly
b $940 a quarter for 5 years at 8% p.a. paid quarterly
c $2500 twice a year for 8 years at 14% p.a. paid every 6 months
d $550 three times a year for 5 years at 6% p.a. paid every 4 months
e $587 a month for 18 months at 12% p.a. paid monthly

4 At the end of each year, Alicia puts $3500 into a superannuation fund where it earns 9% p.a. How much will she have in superannuation after 30 years?

5 The future value of an annuity is $35 000 after 12 years. If interest is 6% p.a., find the amount of each yearly contribution.

6 Find the amount of each annual contribution needed to give a future value of:

a $8450 after 5 years at 7% p.a.
b $25 000 after 8 years at 3% p.a.
c $10 000 after 7 years at 4% p.a.
d $3200 after 5 years at 2% p.a.
e $1 000 000 after 20 years at 5% p.a.

7 Emlynn wants to put aside a regular amount of money each month for 2 years at 12% p.a., paid monthly, so she will have $8000 to pay for a film-making course. How much will she need to contribute?

8 Ilona wins $50 000 in a lottery and invests it in a holiday fund annuity where she withdraws $5000 at the end of each year. The annuity pays interest of 4% p.a.

a What is the value of her annuity after:
i 1 year? **ii** 2 years? **iii** 3 years?
b What will the annuity be worth after 3 years if Ilona decides to withdraw $4000 each year instead?
c What will the annuity be worth after 3 years if the interest is 2.7% and Ilona withdraws $4000 each year?

ISBN 9780170413220

9 Dave puts his $125 000 superannuation payout into an annuity and takes out a pension of $500 a month. The annuity pays interest of 12% p.a., paid monthly.

What is the value of the annuity after:

a 1 month? **b** 2 months? **c** 3 months?

e 3 months if interest is 6% p.a.?

d 3 months if Dave decides to take a pension of $1000 each month?

10 Graph A shows an annuity of $15 000 earning 1% interest per month, with a regular withdrawal of $500 per month. Graph B shows the same $15 000 annuity paying interest of 2% per month with a regular withdrawal of $500 each month.

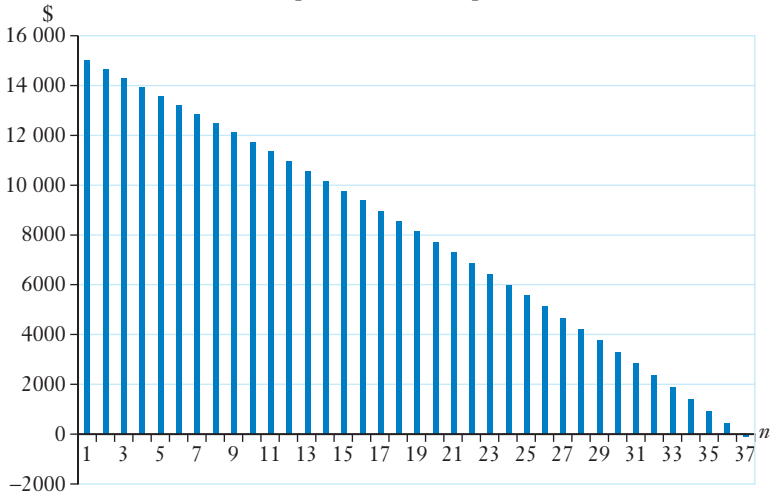
For each graph, determine:

a after how many months the annuity will be worth $8000

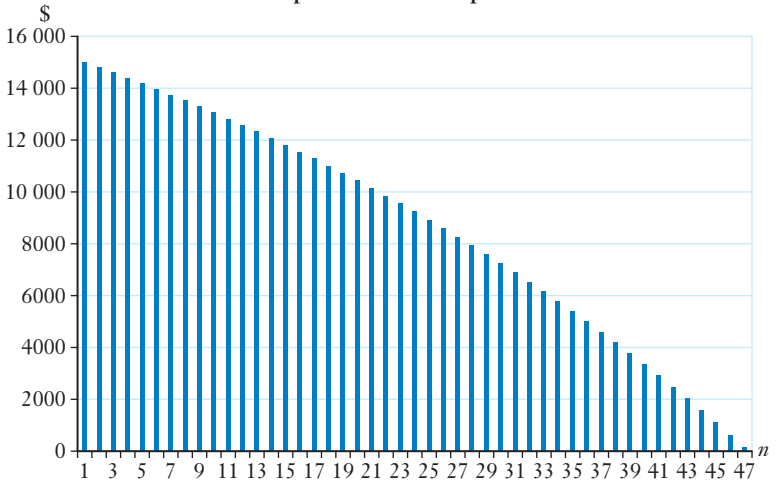
b what the annuity will be worth after a year

c how long it will take for the annuity to run out

Graph A: 1% interest per month



Graph B: 2% interest per month



ISBN 9780170413220

Use the future value table on p. 390 to answer Questions 11 and 12.

11 Ryuji pays $2000 into a superannuation fund at the end of each year at 6% p.a. interest. How many payments would Ryuji make for his superannuation to be greater than:

a $50 000? **b** $80 000?

12 Find the interest rate if $1500 is invested at the end of each year grows to:

a $11 284.95 after 6 years **b** $17 195.85 after 10 years

9.06 Annuities and geometric series



Annuities and geometric series


Annuities


Annuities

EXAMPLE 18

A sum of $1500 is invested at the end of each year in a superannuation fund.
If interest is paid at 6% p.a., how much money will be available at the end of 25 years?

Solution

It is easier to keep track of each annual contribution separately.

Use $A = P(1 + r)^n$ with $P = 1500$ and $r = 0.06$.

The 1st contribution goes in at the end of the 1st year, so it only earns interest for 24 years.

$A_1 = 1500(1 + 0.06)^{24}$

$= 1500(1.06)^{24}$

The 2nd contribution goes in at the end of the 2nd year, so it earns interest for 23 years.

$A_2 = 1500(1.06)^{23}$

Similarly, the 3rd contribution earns interest for 22 years.

$A_3 = 1500(1.06)^{22}$

This pattern continues until the final contribution.

The 25th contribution goes in at the end of the 25th year, so it earns interest for 0 years.

$A_{25} = 1500(1.06)^0$

The future value is the total of all these contributions together with their interest.

$\text{FV} = A_1 + A_2 + A_3 + \ldots + A_{25}$

$= 1500(1.06)^{24} + 1500(1.06)^{23} + 1500(1.06)^{22} + \ldots + 1500(1.06)^0$

$= 1500(1.06)^0 + 1500(1.06)^1 + 1500(1.06)^2 + \ldots + 1500(1.06)^{24}$

$= 1500(1.06^0 + 1.06^1 + 1.06^2 + \ldots + 1.06^{24})$ (factorising)

$1.06^0 + 1.06^1 + 1.06^2 + \ldots + 1.06^{24}$ is a geometric series with $a = 1.06^0 = 1$, $r = 1.06$ and $n = 25$.

ISBN 9780170413220

$$S_n = \frac{a(r^n - 1)}{(r - 1)}$$

$$S_{25} = \frac{1(1.06^{25} - 1)}{(1.06 - 1)}$$

$$\approx 54.86$$

So FV $\approx 1500(54.86)$

$$= 82\ 296.77$$

So the total amount of superannuation after 25 years is \$82 296.77.

In the previous example, the contributions were made at the *end* of each year. If they were made at the *beginning* of each year, they would all earn an extra year of interest. The 1st contribution would be invested for 25 years, the 2nd for 24 years, and so on until the last contribution for 1 year.

EXAMPLE 19

An amount of \$50 is put into an investment account at the end of each month. If interest is paid at 12% p.a. paid monthly, how much is in the account at the end of 10 years?

Solution

We use the compound interest formula where $P = 50$.

$r = 0.12 \div 12 = 0.01$, $n = 10 \times 12 = 120$

The 1st contribution goes in at the end of the 1st month, so it only earns interest for 119 months.

$A_1 = 50(1 + 0.01)^{119}$

$= 50(1.01)^{119}$

The 2nd contribution goes in at the end of the 2nd month, so it earns interest for 118 months.

$A_2 = 50(1.01)^{118}$

The 3rd contribution earns interest for 117 months.

$A_3 = 50(1.01)^{117}$

This pattern continues until the final contribution.

The 120th contribution earns interest for 0 months.

$A_{120} = 50(1.01)^0$

$FV = A_1 + A_2 + A_3 + \ldots + A_{120}$

$= 50(1.01)^{119} + 50(1.01)^{118} + 50(1.01)^{117} + \ldots + 50(1.01)^0$

$= 50(1.01)^0 + 50(1.01)^1 + 50(1.01)^2 + \ldots + 50(1.01)^{119}$

$= 50(1.01^0 + 1.01^1 + 1.01^2 + \ldots + 1.01^{119})$ (factorising)

ISBN 9780170413220

$1.01^0 + 1.01^1 + 1.01^2 + \ldots + 1.01^{119}$ is a geometric series with $a = 1.01^0$ or 1, $r = 1.01$ and $n = 120$.

$$S_n = \frac{a(r^n - 1)}{(r - 1)}$$

$$S_{120} = \frac{1(1.01^{120} - 1)}{(1.01 - 1)}$$

$$\approx 230.04$$

So FV $\approx 50(230.04)$

$= 11\ 501.93$

So the total amount after 10 years is \$11 501.93.

Exercise 9.06 Annuities and geometric series

1 A sum of \$1500 is invested at the end of each year for 15 years at 8% p.a. Find the amount of superannuation available at the end of the 15 years.

2 Liam wants to save up \$15 000 for a car in 5 years' time. He invests \$2000 at the end of each year in an account that pays 7.5% p.a. interest. How much more will Liam have to pay at the end of 5 years to make up the \$15 000?

3 A school invests \$5000 at the end of each year at 6% p.a. to go towards a new library. How much will the school have after 10 years?

4 Jacqueline puts aside \$500 at the end of each year for 5 years. If the money is invested at 6.5% p.a., how much will she have at the end of the 5 years?

5 Miguel's mother invests \$200 for him each birthday up to and including his 18th birthday. The money earns 6% p.a. How much money will Miguel have on his 18th birthday?

6 Xuan is saving up for a holiday. She invests \$800 at the end of each year at 7.5% p.a. How much will she have for her holiday after 5 years' time?

7 A couple saves \$3000 at the *beginning* of each year towards a deposit on a house. If the interest rate is 5% p.a., how much will the couple have saved after 6 years?

8 Lucia saves up \$2000 each year and at the end of the year she invests it at 6% p.a.

a She does this for 10 years. What is her investment worth?

b Lucia continues investing \$2000 a year for 5 more years. What is the future value of her investment?

9 Jodie starts work in 2019 and puts $1000 in a superannuation fund at the end of the year. She keeps putting in this same amount at the end of every year until she retires at the end of 2036. If interest is paid at 10% p.a., calculate how much Jodie will have when she retires.

10 Bol invests $1000 at the *beginning* of each year. The interest rate is 8% p.a.

a How much will her investment be worth after 6 years?

b How much more would Bol's investment be worth after 6 years if she had invested $1200 each year?

11 Asam cannot decide whether to invest $1000 at the end of each year for 15 years or $500 for 30 years in a superannuation fund. If the interest rate is 5% p.a., which would be the better investment for Asam?

12 Pooja is saving up to go overseas in 8 years' time. She invests $1000 at the end of each year at 7% p.a. and estimates that the trip will cost her around $10 000. Will she have enough? If so, how much over will it be? If she doesn't have enough, how much will she need to add to this money to make it up to the $10 000?

13 Mila puts aside $20 at the *beginning* of each month for 3 years. How much will she have then if the investment earns 8.2% p.a., paid monthly?

14 **a** Find the future value on an investment of $1 at the end of each year for 19 years at 7% p.a. using the table of future values of an annuity on page 390.

b Prove that this table value is correct.

Reducing balance loans

Reducing balance loan: spreadsheet

Reducible loans

Loan repayment tables

9.07 Reducing balance loans

People take out loans for many reasons – to buy items such as a car, boat or furniture, to consolidate debts, to buy a home, and for home renovations. A home loan is called a **mortgage**.

An investment or annuity increases in value over time while a loan decreases as the loan is paid off. This is called a **reducing balance loan**. The amount of time taken to pay off the loan is called the term of the loan. A reducing balance loan is similar to an annuity with regular withdrawals.

EXAMPLE 20

Trang borrows $8000 over 3 years to buy furniture. Interest on the loan is 1.25% per month and monthly repayments are $277.32. Find the amount owing after:

a 1 month **b** 2 months **c** 3 months

Solution

Use $A = P(1 + r)^n$ with $P = 8000$ and $r = 1.25\% = 0.0125$.
Let A_n be the amount owing after n months.

ISBN 9780170413220

a 1st month: Amount owing is \$8000 plus interest less the repayment.

$A_1 = 8000(1 + 0.0125)^1 - 277.32$

$= 8000(1.0125) - 277.32$

$= 7822.68$

Amount owing = \$7822.68

b 2nd month: Amount owing is \$7822.68 plus interest less the repayment.

$A_2 = 7822.68(1.0125)^1 - 277.32$

$= 7643.14$

Amount owing = \$7643.14

c 3rd month: Amount owing is \$7643.14 plus interest less the repayment.

$A_3 = 7643.14(1.0125)^1 - 277.32$

$= 7461.36$

Amount owing = \$7461.36

If you know the term of the loan and the amount of the regular contributions, you can calculate the amount of interest owing.

You can use a loan repayments table to calculate the amount you need to contribute to pay off a loan. Here is a table that gives the monthly loan repayments on a \$1000 loan.

	Term (years)					
Interest rate (% p.a.)	**5**	**10**	**15**	**20**	**25**	**30**
2	\$17.53	\$9.20	\$6.44	\$5.06	\$4.24	\$3.70
2.5	\$17.75	\$9.43	\$6.67	\$5.30	\$4.49	\$3.95
3	\$17.97	\$9.66	\$6.91	\$5.55	\$4.74	\$4.22
3.5	\$18.19	\$9.89	\$7.15	\$5.80	\$5.01	\$4.49
4	\$18.42	\$10.12	\$7.40	\$6.06	\$5.28	\$4.77
4.5	\$18.64	\$10.36	\$7.65	\$6.33	\$5.56	\$5.07
5	\$18.87	\$10.61	\$7.91	\$6.60	\$5.85	\$5.37
5.5	\$19.10	\$10.85	\$8.17	\$6.88	\$6.14	\$5.68
6	\$19.33	\$11.10	\$8.44	\$7.16	\$6.44	\$6.00
6.5	\$19.57	\$11.35	\$8.71	\$7.46	\$6.75	\$6.32
7	\$19.80	\$11.61	\$8.99	\$7.75	\$7.07	\$6.65
7.5	\$20.04	\$11.87	\$9.27	\$8.06	\$7.39	\$6.99
8	\$20.28	\$12.13	\$9.56	\$8.36	\$7.72	\$7.34

ISBN 9780170413220

EXAMPLE 21

a Piri wants to borrow $350 000 over 30 years to buy a unit, but she is not sure she can afford to pay the monthly repayments. If interest is 4.5% per month, calculate:

i the amount of each monthly repayment

ii the total amount Piri would pay

b Hamish borrows $25 000 over 5 years to buy a car. Interest is 2% per month. Find:

i the amount of each monthly repayment

ii the total amount Hamish pays

iii the flat rate of interest on the loan

Solution

a **i** From the table, 30 years at 4.5% p.a. gives $5.07.

This is on a loan of $1000, so for $350 000 we multiply the value by 350.

$\$5.07 \times 350 = \$1774.50$

So Piri would pay $1774.50 each month.

ii $30 \text{ years} = 30 \times 12 = 360 \text{ months}$

$\text{Total amount repaid} = \$1774.50 \times 360$

$= \$638\,820$

b **i** From the table, 5 years at 2% p.a. gives $17.53.

This is on a loan of $1000, so for $25 000 we multiply the value by 25.

$\$17.53 \times 25 = \$438.25$

So Hamish pays $438.25 each month.

ii $5 \text{ years} = 5 \times 12 = 60 \text{ months}$

$\text{Total amount repaid} = \$438.25 \times 60$

$= \$26\,295$

iii $\text{Interest} = \$26\,295 - \$25\,000$

$= \$1295$

$\frac{1295}{25000} \times 100\% = 5.18\%$

So the flat rate of interest is 5.18%.

ISBN 9780170413220

You can use the table to do other calculations.

EXAMPLE 22

a The monthly repayments on a loan of \$70 000 at 5% p.a. are \$553.70. Find the term of the loan.

b A \$150 000 loan with a term of 20 years has monthly instalments of \$1119. Find the interest rate.

Solution

a Let the value in the table be x.

The table is for loans of \$1000.

$\$70\,000 \div \$1000 = 70$

$70 \times x = \$553.70$

$$x = \frac{\$553.70}{70}$$

$\approx \$7.91$

Looking at the table in the 5% row, \$7.91 is in the 15 year column.

So the term of the loan is 15 years.

b Let the value in the table be x.

$\$150\,000 \div \$1000 = 150$

$150 \times x = \$1119$

$$x = \frac{\$1119}{150}$$

$\approx \$7.46$

Looking at the table in the 20 year column, \$7.46 is in the 6.5% row.

So the interest rate is 6.5% p.a.

INVESTIGATION

FINANCIAL CALCULATORS

Most bank and other financial websites have calculators rather than tables for loan repayments, values of investments and annuities. Search the websites of banks or general websites that have these and try using these calculators.

Exercise 9.07 Reducing balance loans

1 Calculate the amount owing after 3 months on a loan of:

a \$20 000 at 0.9% per month with repayments of \$432.87 per month

b \$3500 at 1.3% per month with repayments of \$151.57 per month

c \$100 000 at 2.2% per month with repayments of \$2203.22 per month

d \$2000 at 2% per month with repayments of \$105.74 per month

e \$45 800 at 12% p.a. with repayments of \$504.30 per month

ISBN 9780170413220

2 For each loan below, find:

- **i** the total amount repaid
- **ii** total amount of interest paid
- **iii** the flat interest rate of interest p.a.

a $5000 over 3 years with a monthly payment of $166.07

b $15 900 over 5 years with a monthly payment of $403.76

c $80 000 over 12 years with a monthly payment of $1109.62

d $235 000 over 25 years with a monthly payment of $907.09

e $1348 over 2 years with a monthly payment of $71.27

Use the table of loan repayments on page 401 to answer the rest of the questions.

3 Find the amount of the monthly repayment on a loan of:

a $8 000 over 5 years at 6% p.a.

b $15 000 over 5 years at 8% p.a.

c $72 000 over 10 years at 7.5% p.a.

d $430 000 over 20 years at 4% p.a.

e $312 000 over 15 years at 5.5% p.a.

f $137 000 over 25 years at 3.5% p.a.

g $49 000 over 10 years at 7% p.a.

h $765 000 over 30 years at 2.5% p.a.

i $925 000 over 25 years at 2% p.a.

j $1 000 000 over 30 years at 5.5% p.a.

4 Markus takes out a mortgage of $680 500 over 20 years at 3.5% interest.

a Find his monthly repayment.

b Find the total amount he will pay.

c How much interest does he pay?

d Calculate the flat rate of interest over the whole loan.

5 Find the term of each loan given the monthly payments of:

a $81.12 for a $4 000 loan at 8% p.a.

b $777 for a $75 000 loan at 4.5% p.a.

c $937.95 for a $169 000 loan at 3% p.a.

d $1560.25 for a $395 000 loan at 2.5% p.a.

e $232.20 for a $20 000 loan at 7% p.a.

f $3131.25 for a $625 000 loan at 3.5% p.a.

g $1302 for a $120 000 loan at 5.5% p.a.

h $1809.64 for a $281 000 loan at 2% p.a.

i $72.15 for a $6 500 loan at 6% p.a.

j $474.15 for a $81 750 loan at 3.5% p.a.

ISBN 9780170413220

6 Find the interest rate of each loan if the monthly instalment is:

- **a** $57.99 for a $3000 loan for 5 years
- **b** $619.65 for an $81 000 loan for 15 years
- **c** $2307.36 for a $456 000 loan for 20 years
- **d** $2571.56 for a $212 000 loan for 10 years
- **e** $6515.36 for a $947 000 loan for 20 years
- **f** $178.20 for a $9000 loan for 5 years
- **g** $2709.70 for a $686 000 loan for 30 years
- **h** $1422 for a $300 000 loan for 25 years
- **i** $6814.73 for an $845 500 loan for 20 years
- **j** $3127.24 for a $422 600 loan for 15 years

9.08 Loans and geometric series

We can apply the formulas for compound interest and geometric series to work out the amount of the regular repayments of a reducing balance loan.

WS Loans and geometric series

WS Applications of series

EXAMPLE 23

Find the amount of each monthly repayment on a loan of $20 000 at 12% p.a. over 4 years.

Solution

Let M stand for the monthly repayment.

Number of payments is $4 \times 12 = 48$.

Monthly interest is $12\% \div 12 = 1\% = 0.01$

Each month, we add interest and subtract the repayment.

Amount owing after 1 month:

$A_1 = 20\,000(1 + 0.01)^1 - M$

$= 20\,000(1.01)^1 - M$

Amount owing after 2 months:

$A_2 = A_1(1.01)^1 - M$

$= [20\,000(1.01)^1 - M](1.01)^1 - M$

$= 20\,000(1.01)^2 - M(1.01)^1 - M$ (expanding brackets)

$= 20\,000(1.01)^2 - M(1.01^1 + 1)$ (factorising)

ISBN 9780170413220

Amount owing after 3 months:

$A_3 = A_2(1.01)^1 - M$

$= [20\,000(1.01)^2 - M(1.01^1 + 1)](1.01)^1 - M$

$= 20\,000(1.01)^3 - M(1.01^1 + 1)(1.01)^1 - M$ (expanding brackets)

$= 20\,000(1.01)^3 - M(1.01^2 + 1.01^1)) - M$

$= 20\,000(1.01)^3 - M(1.01^2 + 1.01^1 + 1)$ (factorising)

Continuing this pattern, after 48 months the amount owing is:

$A_{48} = 20\,000(1.01)^{48} - M(1.01^{47} + 1.01^{46} + 1.01^{45} + \ldots + 1.01^2 + 1.01^1 + 1)$

But the loan is paid out after 48 months.

So $A_{48} = 0$

$0 = 20\,000(1.01)^{48} - M(1.01^{47} + 1.01^{46} + 1.01^{45} + \ldots + 1.01^2 + 1.01^1 + 1)$

$M(1.01^{47} + 1.01^{46} + 1.01^{45} + \ldots + 1.01^2 + 1.01^1 + 1) = 20\,000(1.01)^{48}$

$$M = \frac{20000(1.01)^{48}}{1.01^{47} + 1.01^{46} + 1.01^{45} + \ldots + 1.01^2 + 1.01^1 + 1}$$

$$= \frac{20000(1.01)^{48}}{1 + 1.01^1 + 1.01^2 + 1.01^3 + \ldots + 1.01^{46} + 1.01^{47}}$$

The denominator is a geometric series with $a = 1$, $r = 1.01$ and $n = 48$.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{48} = \frac{1(1.01^{48} - 1)}{1.01 - 1}$$

$$= \frac{1.01^{48} - 1}{0.01}$$

≈ 61.223

$$M \approx \frac{20000(1.01)^{48}}{61.223}$$

$= 526.68$

So the monthly repayment is \$526.68.

We can use this method to find loan repayments for more complex questions.

ISBN 9780170413220

EXAMPLE 24

A store charges 9% p.a. for loans, and repayments do not have to be made until the 4th month. Ivan buys $8000 worth of furniture and pays it off over 3 years.

a How much does Ivan owe after 3 months?

b What are his monthly repayments?

c How much does Ivan pay altogether?

Solution

a Number of payments $= 3 \times 12 - 3 = 33$ (3 months of no repayments)

Monthly interest rate $= 0.09 \div 12 = 0.0075$

Let M stand for the monthly repayment.

The first repayment is made in the 4th month.

After 3 months, the amount owing is

$A = P(1 + r)^n$

$A_3 = 8000(1 + 0.0075)^3$

$= 8000(1.0075)^3$

$= 8181.35$

So the amount owing after 3 months is $8181.35.

b Amount owing after 4 months:

$A_4 = A_3(1.0075)^1 - M$

$= [8000(1.0075)^3](1.0075)^1 - M$

$= 8000(1.0075)^4 - M$

Amount owing after 5 months:

$A_5 = A_4(1.0075)^1 - M$

$= [8000(1.0075)^4 - M](1.0075)^1 - M$

$= 8000(1.0075)^5 - M(1.0075)^1 - M$

$= 8000(1.0075)^5 - M(1.0075 + 1)$ (factorising)

Continuing this pattern, after 36 months the amount owing will be:

$A_{36} = 8000(1.0075)^{36} - M(1.0075^{32} + 1.0075^{31} + 1.0075^{30} + \ldots + 1.0075^1 + 1)$

But the loan is paid out after 36 months.

So $A_{36} = 0$

$0 = 8000(1.00075)^{36} - M(1.0075^{32} + 1.0075^{31} + \ldots + 1.0075^1 + 1)$

$$M(1.0075^{32} + 1.0075^{31} + \ldots + 1.0075^{1} + 1) = 8000(1.0075)^{36}$$

$$M = \frac{8000(1.0075)^{36}}{1.0075^{32} + 1.0075^{31} + \ldots + 1.0075^{1} + 1}$$

$$= \frac{8000(1.0075)^{36}}{1 + 1.0075^{1} + 1.0075^{2} + \ldots + 1.0075^{32}}$$

The denominator is a geometric series with $a = 1$, $r = 1.0075$ and $n = 33$.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{33} = \frac{1(1.0075^{33} - 1)}{1.0075 - 1}$$

$$= \frac{1.0075^{33} - 1}{0.0075}$$

$$\approx 37.2849$$

$$M = \frac{8000(1.0075)^{36}}{37.2849\ldots}$$

$$\approx 280.79$$

So the monthly repayment is \$280.79.

c $\$280.79 \times 33 = \$9266.07$

So Ivan pays \$9266.07 altogether.

Exercise 9.08 Loans and geometric series

1 An amount of \$3000 is borrowed at 22% p.a. and paid off over 5 years with yearly repayments. How much is each repayment?

2 The sum of \$20 000 is borrowed at 18% p.a. interest calculated monthly over 8 years. How much are the monthly repayments?

3 David borrows \$5000 from the bank and pays back the loan in monthly instalments over 4 years. If the loan incurs interest of 15% p.a. calculated monthly, find the amount of each instalment.

4 Tri and Mai mortgage their house for \$150 000.

a Find the amount of the monthly repayments they will have to make if the mortgage is over 25 years with interest at 6% p.a. compounded monthly.

b If they want to pay their mortgage out after 15 years, what monthly repayments would they need to make?

5 A loan of \$6000 is paid back in equal annual instalments over 3 years. If the interest is 12.5% p.a., find the amount of each annual instalment.

6 Santi buys a car for \$38 000, paying a 10% deposit and taking out a loan for the balance. If the loan is over 5 years with interest of 1.5% monthly, find:

a the amount of each monthly loan repayment

b the total amount that Santi paid for the car.

ISBN 9780170413220

7 A $2000 loan is offered at 18% p.a. with interest charged monthly, over 3 years.

a If no repayment need be paid for the first 2 months, find the amount of each repayment.

b How much will be paid back altogether?

8 Breanna thinks she can afford a mortgage payment of $800 each month. How much can she borrow, to the nearest $100, over 25 years at 11.5% p.a.?

9 Get Rich Bank offers a mortgage at $7\frac{1}{2}$% p.a. over 10 years and Capital Bank offers a mortgage at $5\frac{1}{2}$% p.a. over 25 years, both with interest calculated monthly.

a Find the amount of the monthly repayments for each bank on a loan of $80 000.

b Find the difference in the total amount paid on each mortgage.

10 Majed buys a $35 000 car. He puts down a 5% deposit and pays the balance back in monthly instalments over 4 years at 12% p.a.

a Find the amount of the monthly payments.

b Find the total amount that Majed pays for the car.

11 Amy borrowed money over 7 years at 15.5% p.a. and she pays $1200 a month. How much did she borrow?

12 NSW Bank offers loans at 9% p.a. with no repayments for the first 3 months, while Sydney Bank offers loans at 7% p.a. Compare these loans on an amount of $5000 over 3 years and state which bank offers the better loan and why.

13 Danny buys a home cinema system for $10 000. He pays a $1500 deposit and borrows the balance at 18% p.a. over 4 years.

a Find the amount of each monthly repayment.

b How much did Danny pay altogether?

14 A store offers furniture on hire purchase at 20% p.a. over 5 years, with no repayments for 6 months. Ali buys furniture worth $12 000.

a How much does Ali owe after 6 months?

b What are the monthly repayments?

c How much does Ali pay for the furniture altogether?

15 A loan of $6000 over 5 years at 15% p.a. interest, charged monthly, is paid back in 5 annual instalments.

a What is the amount of each instalment?

b How much is paid back altogether?

16 **a** Using the table of loan repayments on page 401, find the amount of the monthly payments on a $1000 loan over 10 years at 4.5% p.a.

b Show that this table value is correct.

9. TEST YOURSELF

WS
Formula sheet: Measurement, Sequences and series

Qz
Practice quiz

In Questions **1** to **3**, select the correct answer **A**, **B**, **C** or **D**.

1 An amount of $2500 invested at 6% p.a. for 5 years with interest paid twice a year has a future value of:

A $\$2500(1.06)^5$ **B** $\$2500(1.03)^5$

C $\$2500(1.03)^{10}$ **D** $\$2500(1.06)^{10}$

2 An investment has a present value of $68 000 and a future value of $79 500. Find the flat interest rate on the investment.

A 11.5% **B** 16.9%

C 85.5% **D** 14.5%

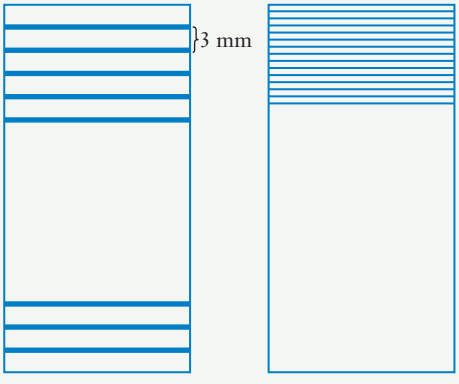
3 A tree is planted when it is 1.2 m tall. Every year its growth is $\frac{3}{8}$ of its previous year's height. Find how tall the tree will grow.

A 1.92 m **B** 2 m **C** 2.5 m **D** 3.2 m

4 A loan of $22 000 at 12% p. a. is paid off in monthly payments of $226.29. Find the amount owing after 3 months.

5 Zac puts $1500 into a savings account that earns 3.7% p.a. How much will Zac have in the account after 3 years?

6 A bamboo blind has 30 slats. It is attached to the window at the top and when the blind is down, the gap between each slat and the next, and between the top slat and the top of the window, is 3 mm. When the blind is up, the slats have no gaps between them.



a Show that when the blind is up, the bottom slat rises 90 mm.

b How far does the next slat rise?

c Explain briefly why the distances the slats rise when the blind is up form an arithmetic sequence.

d Find the distance the 17th slat from the bottom rises.

e What is the sum of the distances that all slats rise?

7 Cristina borrows $62 500 over 5 years with monthly repayments of $2022. Find:

a the total amount Cristina pays

b the amount of interest she pays

c the flat interest rate on the total loan

ISBN 9780170413220

8 Use the table of future values of an investment on page 380 to find the future value of:

a $595 over 4 years at 5% p.a.
b $5 000 over 9 years at 8% p.a.
c $1651.20 over 6 years at 10% p.a.
d $13 500 over 11 years at 2% p.a.
e $9 485 over 18 years at 1% p.a.

9 Murat earned $20 000 in one year. At the beginning of the 2nd year he received a salary increase of $450. He now receives the same increase each year.

a What will his salary be after 10 years?
b How much will Murat earn altogether over the 10 years?

10 Ana puts her superannuation payout of $186 900 into an account that earns 3% p.a. paid monthly. She withdraws $2500 at the end of each month as a pension.
Find the amount in the account after:

a 1 month
b 2 months
c 3 months

11 Gerri wants to contribute a certain amount of money at the end of each year into a superannuation fund so that she will have $200 000 at the end of 25 years. If the fund averages 13% p.a., find the amount of the money Gerri would contribute each year.

12 A supermarket stacks boxes with 20 boxes in the bottom stack, 18 boxes in the next stack, 16 in the next and so on.

a How many stacks are there?
b How many boxes are there?

13 Convert each recurring decimal to a fraction.

a $0.\dot{4}$
b $0.7\dot{2}$
c $1.\dot{5}\dot{7}$

14 Find the amount invested in a bank account at 9.5% p.a. if the balance in the account is $5860.91 after 6 years.

15 Haylee borrows $50 000 for farm machinery at 18% p.a. over 5 years and makes equal yearly repayments on the loan at the end of each year.

a How much does she owe at the end of the first year, just before she makes the first repayment?
b How much is each yearly repayment?

16 **a** If $2000 is invested at 4.5% p.a., how much will it be worth after 4 years?
b If interest is paid quarterly, how much would the investment be worth after 4 years?

17 Pedro borrows $200 000 to buy a house. If the interest is 6% p.a. compounded monthly and the loan is over 20 years:

a how much is each monthly repayment?
b how much does Pedro pay altogether?

18 **a** Find the annual contribution needed for an annuity to have a future value of $12 000 after 4 years at 5% p.a. if the contribution is made at the *beginning* of the year.

b Find the single investment that would need to be invested at the same interest rate now to have this future value.

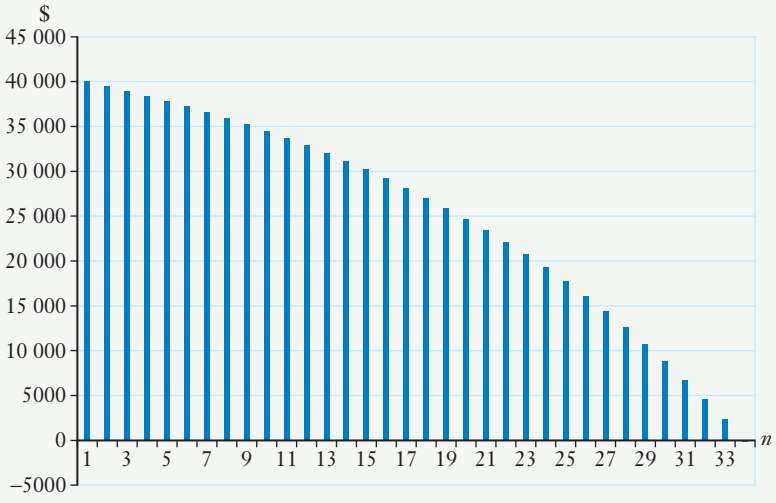
19 Every week during a typing course, Jamal improves his typing speed by 3 words per minute until he reaches 60 words per minute by the end of the course.

a If he can type 18 words per minute in the first week of the course, how many words per minute can he type by week 8?

b How many weeks does the course run for?

20 A ball drops from a height of 1.2 metres then bounces back to $\frac{3}{5}$ of this height. On the next bounce, it bounces up to $\frac{3}{5}$ of this height and so on. Through what distance will the ball travel?

21 This graph shows an annuity of $40 000 earning 5% p.a. with withdrawals of $2500 at the end of each year.



a What is the annuity worth after:

i 10 years? **ii** 20 years?

b When is the annuity worth:

i $30 000? **ii** $10 000?

c How long will it take for the annuity to run out?

ISBN 9780170413220

9. CHALLENGE EXERCISE

1 Jane puts $300 into an account at the beginning of each year to pay for her daughter's education in 5 years' time. If 6% p.a. interest is paid quarterly, how much money will Jane have at the end of the 5 years?

2 A factory sells shoes at $60 a pair. For 10 pairs of shoes there is a discount, whereby each pair costs $58. For 20 pairs, the cost is $56 a pair and so on. Find:

- **a** the price of each pair of shoes on an order of 100 pairs of shoes
- **b** the total price on an order of 60 pairs of shoes

3 Find the amount of money in a bank account if $5000 earns 8.5% p.a. for 4 years, then 6.5% p.a. for 3 years, with interest paid monthly for all 7 years.

4 A metal is heated to 500°C. A minute later it cools to 425°C, then a minute later it cools down to 361.25°C. If the metal continues to cool in the same way, find:

- **a** its temperature after:
  - **i** 10 minutes
  - **ii** 15 minutes
- **b** how long it will take to cool down to:
  - **i** 200°
  - **ii** 100°

5 Lukas puts $1000 into a superannuation account at the beginning of each year where it earns 6% p.a. He retires and collects the superannuation at the end of 25 years.

- **a** How much will the first $1000 be worth at the end of 25 years, in index form?
- **b** When Lukas deposits the second $1000 at the end of the 2nd year, how much will it be worth after 25 years?
- **c** How much will the third $1000 be worth after 25 years?
- **d** How much will the final $1000 be worth that Lukas deposits at the beginning of the 25th year?
- **e** Show that the total amount in the account after 25 years is $1000 \times \frac{1.06(1.06^{25} - 1)}{0.06}$.
- **f** Find the amount that Lukas will have at the end of the 25 years.

6 Kim borrows $10 000 over 3 years at a rate of 1% interest compounded each month. If she pays off the loan in three equal annual instalments, find:

- **a** the amount Kim owes after one month
- **b** the amount she owes after the first year, just before she pays the first instalment
- **c** the amount of each instalment
- **d** the total amount of interest Kim pays.

7 A superannuation fund paid 6% p.a. for the first 10 years and then 10% p.a. after that. If Thanh put $5000 into this fund at the beginning of each year, how much would she have at the end of 25 years?

ISBN 9780170413220