1.

SEQUENCES AND SERIES

A sequence is a set of numbers that form a pattern. Many sequences occur in real life – the growth of plants, savings in the bank, populations, clearing of forests and so on. In the Year 11 course you looked at how things can grow or decay (decrease) exponentially. In this chapter you will look at two other types of patterns that apply to real-life applications.

CHAPTER OUTLINE

1.01 General sequences and series
1.02 Arithmetic sequences
1.03 Arithmetic series
1.04 Geometric sequences
1.05 Geometric series
1.06 Limiting sum of an infinite geometric series



IN THIS CHAPTER YOU WILL:

- identify the difference between a sequence and a series
- identify the difference between arithmetic and geometric sequences and series
- find the *n*th term of arithmetic and geometric sequences
- find the sum to *n* terms of arithmetic and geometric series
- understand and apply the limiting sum formula for infinite geometric series

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TERMINOLOGY

arithmetic sequence: A list of numbers where the difference between successive terms is a constant (called the common difference)
arithmetic series: A sum of the terms forming an arithmetic sequence
common difference: The constant difference between successive terms of an arithmetic sequence
common ratio: The constant multiplier of successive terms in a geometric sequence
geometric sequence: A list of numbers where the ratio of successive terms is a constant (called the common ratio)
geometric series: A sum of the terms forming a geometric sequence
limiting sum: The limit, where it exists, of a geometric series as $n \to \infty$
recurrence relation: An equation that defines a term of a sequence or series by referring to its previous term(s)
sequence: A list of numbers where each term of the sequence is related to the previous term by a particular pattern
series: The sum of terms of a sequence of numbers
term: A value of a sequence

WS Sequences and series

WS Classifying sequences

1.01 General sequences and series

Sequences

A **sequence** is an ordered list of numbers, called **terms** of the sequence, which follow a pattern. Some patterns are easy to see and some are more difficult to find.

EXAMPLE 1

Find the next 3 terms in the sequence:

a 14, 17, 20, … **b** 5, 10, 20, 40, … **c** 5, 1, –3, …

Solution

a For the sequence 14, 17, 20, … we add 3 to each term for the next term.

$14 + 3 = 17$ and $17 + 3 = 20$.

Following this pattern, the next 3 terms are 23, 26 and 29.

b For 5, 10, 20, 40, … we multiply each term by 2 for the next term.

$5 \times 2 = 10$, $10 \times 2 = 20$, $20 \times 2 = 40$.

So the next 3 terms are 80, 160 and 320.

c For the sequence 5, 1, –3, … we subtract 4 (or add –4) to each term for the next term.

$5 - 4 = 1$ and $1 - 4 = -3$.

So the next 3 terms are –7, –11 and –15.

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Series

A **series** is a sum of terms that form a sequence.

EXAMPLE 2

Find the sum of the series with 5 terms:

a $8 + 15 + 22 + \ldots$ **b** $4 + 8 + 16 + \ldots$

Solution

a We add 7 to each term in the series $8 + 15 + 22 + \ldots$ to find the next term.

So the series with 5 terms is $8 + 15 + 22 + 29 + 36$.

$\text{Sum} = 8 + 15 + 22 + 29 + 36$

$= 110$

b We multiply each term in the series $4 + 8 + 16 + \ldots$ by 2 to find the next term.

So the series with 5 terms is $4 + 8 + 16 + 32 + 64$.

$\text{Sum} = 4 + 8 + 16 + 32 + 64$

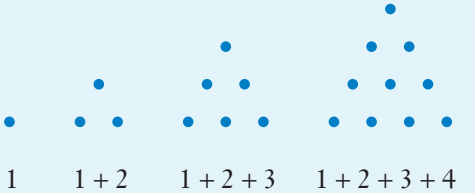
$= 124$

DID YOU KNOW?

Polygonal numbers

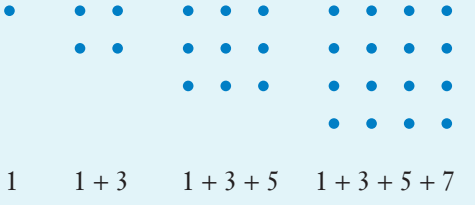
Around 500 BC the Pythagoreans explored different polygonal numbers:

Triangular numbers: $1 + 2 + 3 + 4 + \ldots$



$1 \quad 1 + 2 \quad 1 + 2 + 3 \quad 1 + 2 + 3 + 4$

Square numbers: $1 + 3 + 5 + 7 + \ldots$



$1 \quad 1 + 3 \quad 1 + 3 + 5 \quad 1 + 3 + 5 + 7$

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Exercise 1.01 General sequences and series

1 Find the next 3 terms in each sequence.

a 5, 8, 11, ...

b 8, 13, 18, ...

c 11, 22, 33, …

d 100, 95, 90, ...

e 7, 5, 3, …

f 12, 3, –6, ...

g $\frac{1}{2}, 1, 1\frac{1}{2}, \ldots$

h 1.3, 1.9, 2.5, …

i 2, – 4, 8, –16, ...

j $\frac{1}{5}, \frac{3}{20}, \frac{9}{80}, \ldots$

2 Find the sum of each series if it has 6 terms.

a 4 + 12 + 36 + …

b 1 + 2 + 4 + …

c 3 + 7 + 11 + …

d –6 + 12 – 24 + …

e 1 + 4 + 9 + 16 + …

f 1 + 8 + 27 + 64 + ...

3 Find the next 3 terms of the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

4 Find the next 4 terms in the series 3 + 6 + 11 + 18 + 27 + …

5 What are the next 5 terms in the sequence 1, 1, 2, 3, 5, 8, 13, ...?

6 Complete the next 3 rows in Pascal's triangle:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

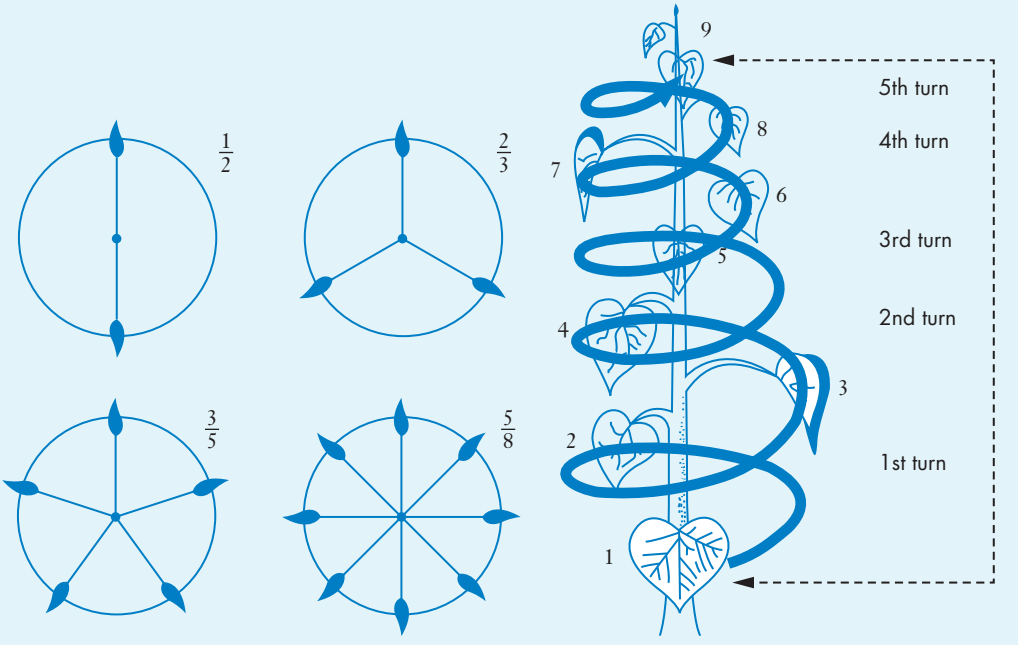
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DID YOU KNOW?

Fibonacci numbers

The numbers 1, 1, 2, 3, 5, 8, … are called **Fibonacci** numbers after **Leonardo Fibonacci** (1170–1250). These numbers occur in many natural situations.

For example, when new leaves grow on a plant's stem, they spiral around the stem. The ratio of the number of turns to the number of spaces between successive leaves gives the sequence of fractions $\frac{1}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \frac{13}{21}, \frac{21}{34}, \frac{34}{55}, \ldots$



The Fibonacci ratio is the number of turns divided by the number of spaces.

Research Fibonacci numbers and find out where else they appear in nature.

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WS Arithmetic sequences

WS Arithmetic progressions

1.02 Arithmetic sequences

In an **arithmetic sequence**, each term is a constant amount more than the previous term. The constant is called the **common difference**, d.

EXAMPLE 3

Find the common difference of the arithmetic sequence:

a 5, 9, 13, 17, … **b** 85, 80, 75, …

Solution

a For this sequence, $9 - 5 = 4$, $13 - 9 = 4$ and $17 - 13 = 4$.

So common difference $d = 4$.

b For this sequence, $80 - 85 = -5$ and $75 - 80 = -5$.

So common difference $d = -5$.

A **recurrence relation** is an equation that defines a term of a sequence by referring to its previous term. In any arithmetic sequence, a term is d more than the previous term. We can write this as a recurrence relation:

$$T_n = T_{n-1} + d \text{ where } T_n \text{ is the } n\text{th term of the sequence}$$

or $T_n - T_{n-1} = d$

EXAMPLE 4

a If 5, x, 31, … is an arithmetic sequence, find x.

b **i** Evaluate k if $k + 2$, $3k + 2$, $6k - 1$, … is an arithmetic sequence.

ii Write down the first 3 terms of the sequence.

iii Find the common difference d.

Solution

a For an arithmetic sequence,

$T_2 - T_1 = d$ and $T_3 - T_2 = d$

So $T_2 - T_1 = T_3 - T_2$

$$x - 5 = 31 - x$$
$$2x - 5 = 31$$
$$2x = 36$$
$$x = 18$$

Note: x is called the **arithmetic mean** because $x = \frac{5 + 31}{2}$.

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b **i** For an arithmetic sequence,

$$T_2 - T_1 = T_3 - T_2$$
$$(3k + 2) - (k + 2) = (6k - 1) - (3k + 2)$$
$$3k + 2 - k - 2 = 6k - 1 - 3k - 2$$
$$2k = 3k - 3$$
$$2k + 3 = 3k$$
$$3 = k$$

ii Substituting $k = 3$ into the terms of the sequence:

$T_1 = k + 2$ $\quad$ $T_2 = 3k + 2$ $\quad$ $T_3 = 6k - 1$

$= 3 + 2$ $\quad$ $= 3(3) + 2$ $\quad$ $= 6(3) - 1$

$= 5$ $\quad$ $= 11$ $\quad$ $= 17$

iii The sequence is 5, 11, 17, …

$d = 11 - 5$ or $17 - 11$

$= 6$

So $d = 6$.

The general term of an arithmetic sequence

Given an arithmetic sequence with 1st term $T_1 = a$ and common difference d:

$T_1 = a$ $\quad$ $T_2 = T_1 + d$ $\quad$ $T_3 = T_2 + d$ $\quad$ $T_4 = T_3 + d$

$= a + d$ $\quad$ $= (a + d) + d$ $\quad$ $= (a + 2d) + d$

$= a + 2d$ $\quad$ $= a + 3d$

Notice that the multiple of d is one less than the number of the term. So the multiple of d for the nth term T_n is $n - 1$.

nth term of an arithmetic sequence

$$T_n = a + (n - 1)d$$

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EXAMPLE 5

a Find the 20th term of the sequence 3, 10, 17, …

b Find a formula for the nth term of the sequence 2, 8, 14, …

c Find the first positive term of the sequence –50, –47, –44, …

Solution

a $a = 3, d = 7, n = 20$

$T_n = a + (n-1)d$

$T_{20} = 3 + (20-1) \times 7$

$= 3 + 19 \times 7$

$= 136$

b $a = 2, d = 6$

$T_n = a + (n-1)d$

$= 2 + (n-1) \times 6$

$= 2 + 6n - 6$

$= 6n - 4$

c $a = -50, d = 3$

For the first positive term,

$T_n > 0$

$a + (n-1)d > 0$

$-50 + (n-1) \times 3 > 0$

$-50 + 3n - 3 > 0$

$3n - 53 > 0$

$3n > 53$

$n > 17.66...$

So $n = 18$ gives the first positive term.

$T_{18} = -50 + (18-1) \times 3$

$= 1$

So the first positive term is 1.

EXAMPLE 6

The 5th term of an arithmetic sequence is 37 and the 8th term is 55. Find the common difference and the first term of the sequence.

Solution

$T_n = a + (n-1)d$

Given $T_5 = 37$:

$a + (5-1)d = 37$

$a + 4d = 37$ [1]

Given $T_8 = 55$:

$a + (8-1)d = 55$

$a + 7d = 55$ [2]

Solve [1] and [2] simultaneously:

$3d = 18$ [2] – [1]

$d = 6$

Substitute $d = 6$ into [1]:

$a + 4(6) = 37$

$a + 24 = 37$

$a = 13$

So the common difference is 6 and the first term is 13.

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Exercise 1.02 Arithmetic sequences

1 Find the value of the pronumeral in each arithmetic sequence.

a $5, 9, y, \ldots$ **b** $8, 2, x \ldots$ **c** $45, x, 99, \ldots$

d $16, b, 6, \ldots$ **e** $x, 14, 21, \ldots$ **f** $32, x - 1, 50, \ldots$

g $3, 5k + 2, 21, \ldots$ **h** $x, x + 3, 2x + 5, \ldots$ **i** $t - 5, 3t, 3t + 1, \ldots$

j $2t - 3, 3t + 1, 5t + 2, \ldots$

2 Find the 15th term of each sequence.

a $4, 7, 10, \ldots$ **b** $8, 13, 18, \ldots$ **c** $10, 16, 22, \ldots$

d $120, 111, 102, \ldots$ **e** $-3, 2, 7, \ldots$

3 Find the 100th term of each sequence.

a $-4, 2, 8, \ldots$ **b** $41, 32, 23, \ldots$ **c** $18, 22, 26, \ldots$

d $125, 140, 155, \ldots$ **e** $-1, -5, -9, \ldots$

4 What is the 25th term of each sequence?

a $-14, -18, -22, \ldots$ **b** $0.4, 0.9, 1.4, \ldots$ **c** $1.3, 0.9, 0.5, \ldots$

d $1, 2\frac{1}{2}, 4, \ldots$ **e** $1\frac{2}{5}, 2, 2\frac{3}{5}, \ldots$

5 Find the formula for the nth term of the sequence $3, 5, 7, \ldots$

6 Find the formula for the nth term of each sequence.

a $9, 17, 25, \ldots$ **b** $100, 102, 104, \ldots$ **c** $6, 9, 12, \ldots$

d $80, 86, 92, \ldots$ **e** $-21, -17, -13, \ldots$ **f** $15, 10, 5, \ldots$

g $\frac{7}{8}, 1, 1\frac{1}{8}, \ldots$ **h** $-30, -32, -34, \ldots$ **i** $3.2, 4.4, 5.6, \ldots$

j $\frac{1}{2}, 1\frac{1}{4}, 2, \ldots$

7 Find which term of $3, 7, 11, \ldots$ is equal to 111.

8 Which term of the sequence $1, 5, 9, \ldots$ is 213?

9 Which term of the sequence $15, 24, 33, \ldots$ is 276?

10 Which term of the sequence $25, 18, 11, \ldots$ is equal to -73?

11 Is 0 a term of the sequence $48, 45, 42, \ldots$?

12 Is 270 a term of the sequence $3, 11, 19, \ldots$?

13 Is 405 a term of the sequence $0, 3, 6, \ldots$?

14 Find the first value of n for which the terms of the sequence $100, 93, 86, \ldots$ is less than 20.

15 Find the values of n for which the terms of the sequence $-86, -83, -80, \ldots$ are positive.

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16 Find the first negative term of the sequence 54, 50, 46, …

17 Find the first term that is greater than 100 in the sequence 3, 7, 11, …

18 The first term of an arithmetic sequence is –7 and the common difference is 8. Find the 100th term.

19 The first term of an arithmetic sequence is 15 and the 3rd term is 31.

a Find the common difference.

b Find the 10th term of the sequence.

20 The first term of an arithmetic sequence is 3 and the 5th term is 39. Find the common difference.

21 The 2nd term of an arithmetic sequence is 19 and the 7th term is 54. Find the first term and common difference.

22 Find the 20th term in an arithmetic sequence with 4th term 29 and 10th term 83.

23 The common difference of an arithmetic sequence is 6 and the 5th term is 29. Find the first term of the sequence.

24 If the 3rd term of an arithmetic sequence is 45 and the 9th term is 75, find the 50th term of the sequence.

25 The 7th term of an arithmetic sequence is 17 and the 10th term is 53. Find the 100th term of the sequence.

26 **a** Show that $\log_5 x, \log_5 x^2, \log_5 x^3, \ldots$ is an arithmetic sequence.

b Find the 80th term.

c If $x = 4$, evaluate the 10th term correct to 1 decimal place.

27 **a** Show that $\sqrt{3}, \sqrt{12}, \sqrt{27}, \ldots$ is an arithmetic sequence.

b Find the 50th term in simplest form.

28 Find the 25th term of $\log_2 4, \log_2 8, \log_2 16, \ldots$

29 Find the 40th term of $5b, 8b, 11b, \ldots$

30 Which term is $213y$ of the sequence $28y, 33y, 38y, \ldots$?

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1.03 Arithmetic series

The sum of an **arithmetic series** with n terms is given by the formula:

Sum of an arithmetic series with n terms

$S_n = \frac{n}{2}(a + l)$ where a = 1st term and l = last (nth) term

Proof

Let the last or nth term be l.

$$S_n = a + (a + d) + (a + 2d) + \ldots + l \qquad [1]$$

Writing this around the other way:

$$S_n = l + (l - d) + (l - 2d) + \ldots. + a \qquad [2]$$

[1] + [2]

$2S_n = (a + l) + (a + l) + (a + l) + \ldots. + (a + l)$ n times

$= n(a + l)$

$S_n = \frac{n}{2}(a + l)$

We can find a more general formula if we substitute $T_n = a + (n - 1)d$ for l:

Sum of an arithmetic series with n terms

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Proof

$S_n = \frac{n}{2}(a + l)$

$= \frac{n}{2}[a + a + (n - 1)d]$

$= \frac{n}{2}[2a + (n - 1)d]$

We can also use these formulas to find the sum of the first n terms of an arithmetic sequence (also called the **nth partial sum**).

EXAMPLE 7

a Evaluate $9 + 14 + 19 + \ldots + 224$.

b For what value of n is the sum of the series $2 + 11 + 20 + \ldots$ equal to 618?

c The 6th term of an arithmetic sequence is 23 and the sum of the first 10 terms is 210. Find the sum of the first 20 terms of the sequence.

Solution

a $a = 9, d = 5, T_n = 224$

$T_n = a + (n-1)d$

$224 = 9 + (n-1) \times 5$

$= 9 + 5n - 5$

$= 5n + 4$

$220 = 5n$

$44 = n$

$S_n = \frac{n}{2}(a + l)$

$= \frac{44}{2}(9 + 224)$

$= 5126$

b $a = 2, d = 9, S_n = 618$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$618 = \frac{n}{2}[2 \times 2 + (n-1) \times 9]$

$1236 = n(4 + 9n - 9)$

$= n(9n - 5)$

$= 9n^2 - 5n$

$0 = 9n^2 - 5n - 1236$

$= (n - 12)(9n + 103)$

(or use the quadratic formula)

$n - 12 = 0, 9n + 103 = 0$

$n = 12$

($9n + 103 = 0$ gives a negative value of n)

c $T_n = a + (n-1)d$

$T_6 = a + (6-1)d = 23$

$a + 5d = 23$ [1]

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + (10-1)d] = 210$

$5(2a + 9d) = 210$

$2a + 9d = 42$ [2]

[1] × 2: $2a + 10d = 46$ [3]

[3] − [2]: $d = 4$

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Substitute $d = 4$ in [1]

$a + 5(4) = 23$

$a + 20 = 23$

$a = 3$

Substitute $a = 3, d = 2, n = 20$ into the formula for S_n.

$S_{20} = \frac{20}{2}[2(3) + (20 - 1)4]$

$= 10(6 + 19 \times 4)$

$= 820$

Exercise 1.03 Arithmetic series

1 Find the sum of 15 terms of each series.

a $4 + 7 + 10 + \ldots$ **b** $2 + 7 + 12 + \ldots$ **c** $60 + 56 + 52 + \ldots$

2 Find the sum of 30 terms of each series.

a $1 + 7 + 13 + \ldots$ **b** $15 + 24 + 33 + \ldots$ **c** $95 + 89 + 83 + \ldots$

3 Find the sum of 25 terms of each series.

a $-2 + 5 + 12 + \ldots$ **b** $5 - 4 - 13 - \ldots$

4 Find the sum of 50 terms of each series.

a $50 + 44 + 38 + \ldots$ **b** $11 + 14 + 17 + \ldots$

5 Evaluate each arithmetic series.

a $15 + 20 + 25 + \ldots + 535$ **b** $9 + 17 + 25 + \ldots + 225$

c $5 + 2 - 1 - \ldots - 91$ **d** $81 + 92 + 103 + \ldots + 378$

e $229 + 225 + 221 + \ldots + 25$ **f** $-2 + 6 + 14 + \ldots + 94$

g $0 - 9 - 18 - \ldots - 216$ **h** $79 + 81 + 83 + \ldots + 229$

i $14 + 11 + 8 + \ldots - 43$ **j** $1\frac{1}{2} + 1\frac{3}{4} + 2 + \ldots + 25\frac{1}{4}$

6 How many terms of the series $45 + 47 + 49 + \ldots$ are needed to give a sum of 1365?

7 For what value of n is the sum of the arithmetic series $5 + 9 + 13 + \ldots$ equal to 152?

8 How many terms of the series $80 + 73 + 66 + \ldots$ are needed to give a sum of 495?

9 How many terms of the series $20 + 18 + 16 + \ldots$ are needed to give a sum of 104?

10 The sum of the first 5 terms of an arithmetic sequence is 110 and the sum of the first 10 terms is 320. Find the first term and the common difference.

11 The sum of the first 5 terms of an arithmetic sequence is 35 and the sum of the next 5 terms is 160. Find the first term and the common difference.

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12 Find S_{25}, given an arithmetic series with 8th term 16 and 13th term 81.

13 The sum of 12 terms of an arithmetic series is 186 and the 20th term is 83. Find the sum of 40 terms of the series.

14 The sum of the first 4 terms of an arithmetic series is 42 and the sum of the 3rd and 7th term is 46. Find the sum of the first 20 terms.

15 **a** Show that $x + 1, 2x + 4, 3x + 7, \ldots$ are the first 3 terms in an arithmetic sequence.

b Find the sum of the first 50 terms of the sequence.

16 The 20th term of an arithmetic series is 131 and the sum of the 6th to 10th terms inclusive is 235. Find the sum of the first 20 terms.

17 The sum of 50 terms of an arithmetic series is 249 and the sum of 49 terms of the series is 233. Find the 50th term of the series.

18 Prove that $T_n = S_n - S_{n-1}$ for any arithmetic sequence.

19 **a** Find the sum of all integers from 1 to 100 that are multiples of 6.

b Find the sum of all integers from 1 to 100 that are not multiples of 6.

WS Geometric sequences

WS Geometric progressions 1

1.04 Geometric sequences

In a **geometric sequence**, each term is formed by multiplying the previous term by a constant. The constant is called the **common ratio** r.

EXAMPLE 8

Find the common ratio of the geometric sequence:

a $3, 6, 12, \ldots$ **b** $-2, 10, -50, \ldots$ **c** $\frac{1}{2}, \frac{1}{5}, \frac{2}{25}, \ldots$

Solution

a For this sequence, $6 \div 3 = 2, 12 \div 6 = 2$

So common ratio $r = 2$.

b For this sequence, $10 \div -2 = -5, -50 \div 10 = -5$

So common ratio $r = -5$.

c $\frac{1}{5} \div \frac{1}{2} = \frac{2}{5}, \frac{2}{25} \div \frac{1}{5} = \frac{2}{5}$

So common ratio $r = \frac{2}{5}$.

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In any geometric sequence, a term is r times more than the previous term. We can write this as a recurrence relation:

$$T_n = rT_{n-1}$$

or $\dfrac{T_n}{T_{n-1}} = r$

EXAMPLE 9

a **i** Find x if $5, x, 45, \ldots$ is a geometric sequence.

ii Find the sequence.

b Is $\dfrac{1}{4}, \dfrac{1}{6}, \dfrac{1}{18}, \ldots$ a geometric sequence?

Solution

a **i** For a geometric sequence:

$$\frac{T_2}{T_1} = r \text{ and } \frac{T_3}{T_2} = r$$

So $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_2}$

$$\frac{x}{5} = \frac{45}{x}$$

$$x^2 = 225$$

$$x = \pm\sqrt{225}$$

$$= \pm 15$$

Note: x is called the **geometric mean** because $x = \sqrt{5 \times 45}$.

ii If $x = 15$ the sequence is $5, 15, 45, \ldots$ $(r = 3)$

If $x = -15$ the sequence is $5, -15, 45, \ldots$ $(r = -3)$

b $\dfrac{T_2}{T_1} = \dfrac{1}{6} \div \dfrac{1}{4}$ $\qquad$ $\dfrac{T_3}{T_2} = \dfrac{1}{18} \div \dfrac{1}{6}$

$= \dfrac{2}{3}$ $\qquad$ $= \dfrac{1}{3}$

$\dfrac{T_2}{T_1} \neq \dfrac{T_3}{T_2}$ so the series is not geometric.

General term of a geometric sequence

Given a geometric sequence with 1st term $T_1 = a$ and common ratio r:

$T_1 = a$

$T_2 = T_1 \times r$
$= ar$

$T_3 = T_2 \times r$
$= (ar) \times r$
$= ar^2$

$T_4 = T_3 \times r$
$= (ar^2) \times r$
$= ar^3$

Notice that the power of r is one less than the number of the term. So the power of r for T_n is $n - 1$.

*n*th term of a geometric sequence

$$T_n = ar^{n-1}$$

EXAMPLE 10

a **i** Find the 10th term of the sequence 3, 6, 12, …

ii Find the formula for the *n*th term of the sequence.

b Find the 10th term of the sequence –5, 10, –20, …

c Which term of the sequence 4, 12, 36, … is equal to 78 732?

d The 3rd term of a geometric sequence is 18 and the 7th term is 1458. Find the first term and the common ratio.

Geometric sequences

Solution

a **i** This is a geometric sequence with $a = 3$, $r = 2$ and $n = 10$.

$T_n = ar^{n-1}$
$T_{10} = 3(2)^{10-1}$
$= 3 \times 2^9$
$= 1536$

ii $T_n = ar^{n-1}$
$= 3(2)^{n-1}$

b $a = -5, r = -2, n = 10$

$T_n = ar^{n-1}$
$T_{10} = -5(-2)^{10-1}$
$= -5 \times (-2)^9$
$= 2560$

c This is a geometric sequence with $a = 4$, $r = 3$ and $T_n = 78\,732$.

$T_n = ar^{n-1}$
$78\,732 = 4(3)^{n-1}$
$19\,683 = 3^{n-1}$
$\log 19\,683 = \log 3^{n-1}$
$= (n-1)\log 3$

$\frac{\log 19\,683}{\log 3} = n - 1$

$\frac{\log 19\,683}{\log 3} + 1 = n$

$10 = n$

So the 10th term is 78 732.

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d Given $T_3 = 18$

$ar^{3-1} = 18$

$ar^2 = 18$ [1]

Given $T_7 = 1458$

$ar^{7-1} = 1458$

$ar^6 = 1458$ [2]

[2] ÷ [1]:

$$\frac{ar^6}{ar^2} = \frac{1458}{18}$$

$r^4 = 81$

$r = \pm\sqrt[4]{81}$

$= \pm 3$

Substitute $r = 3$ into [1]

$a(3)^2 = 18$

$9a = 18$

$a = 2$

Substitute $r = -3$ into [1]

$a(-3)^2 = 18$

$9a = 18$

$a = 2$

The first term is 2 and the common ratio is ± 3.

Here is an example of a geometric sequence involving fractions.

EXAMPLE 11

a Find the 8th term of $\frac{2}{3}, \frac{4}{15}, \frac{8}{75}, \ldots$ in index form.

b Find the first value of n for which the terms of the sequence $\frac{1}{5}, 1, 5, \ldots$ exceed 3000.

Solution

a
$$\frac{T_2}{T_1} = \frac{4}{15} \div \frac{2}{3}$$
$$= \frac{2}{5}$$
$$\frac{T_3}{T_2} = \frac{8}{75} \div \frac{4}{15}$$
$$= \frac{2}{5}$$

So $r = \frac{2}{5}$

$$T_n = ar^{n-1}$$
$$= \frac{2}{3}\left(\frac{2}{5}\right)^{8-1}$$
$$= \frac{2}{3}\left(\frac{2}{5}\right)^{7}$$
$$= \frac{2^8}{3(5^7)}$$

b $a = \frac{1}{5}, r = 5$

$T_n > 3000$

$ar^{n-1} > 3000$

$\frac{1}{5}(5)^{n-1} > 3000$

$5^{n-1} > 15\,000$

$\log 5^{n-1} > \log 15\,000$

$(n-1)\log 5 > \log 15\,000$

$$n - 1 > \frac{\log 15000}{\log 5}$$
$$n > \frac{\log 15000}{\log 5} + 1$$
$$> 6.974...$$

So $n = 7$

The 7th term is the first term to exceed 3000.

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Exercise 1.04 Geometric sequences

1 Is each sequence geometric? If so, find the common ratio.

a $5, 20, 60, \ldots$

b $-4, 3, -2\frac{1}{4}, \ldots$

c $\frac{3}{4}, \frac{3}{14}, \frac{3}{49}, \ldots$

d $7, 5\frac{5}{6}, 3\frac{1}{3}, \ldots$

e $-14, 42, -168, \ldots$

f $1\frac{1}{3}, \frac{8}{9}, \frac{8}{27}, \ldots$

g $5.7, 1.71, 0.513, \ldots$

h $2\frac{1}{4}, -1\frac{7}{20}, \frac{81}{100}, \ldots$

i $63, 9, 1\frac{7}{8}, \ldots$

j $-1\frac{7}{8}, 15, -120, \ldots$

2 Find the pronumeral in each geometric sequence.

a $4, 28, x, \ldots$

b $-3, 12, y, \ldots$

c $2, a, 72, \ldots$

d $y, 2, 6, \ldots$

e $x, 8, 32, \ldots$

f $5, p, 20, \ldots$

g $7, y, 63, \ldots$

h $-3, m, -12, \ldots$

i $3, x - 4, 15, \ldots$

j $3, k - 1, 21, \ldots$

k $\frac{1}{4}, t, \frac{1}{9}, \ldots$

l $\frac{1}{3}, t, \frac{4}{3}, \ldots$

3 Find the formula for the nth term of each sequence.

a $1, 5, 25, \ldots$

b $1, 1.02, 1.0404, \ldots$

c $1, 9, 81, \ldots$

d $2, 10, 50, \ldots$

e $6, 18, 54, \ldots$

f $8, 16, 32, \ldots$

g $\frac{1}{4}, 1, 4, \ldots$

h $1000, -100, 10, \ldots$

i $-3, 9, -27, \ldots$

j $\frac{1}{3}, \frac{2}{15}, \frac{4}{75}, \ldots$

4 Find the 6th term of each sequence.

a $8, 24, 72, \ldots$

b $9, 36, 144, \ldots$

c $8, -32, 128, \ldots$

d $-1, 5, -25, \ldots$

e $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \ldots$

5 What is the 9th term of each sequence?

a $1, 2, 4, \ldots$

b $4, 12, 36, \ldots$

c $1, 1.04, 1.0816, \ldots$

d $-3, 6, -12, \ldots$

e $\frac{3}{4}, -\frac{3}{8}, \frac{3}{16}, \ldots$

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6 Find the 8th term of each sequence.

a 3, 15, 75, …

b 2.1, 4.2, 8.4, …

c 5, –20, 80, …

d $-\frac{1}{2}, \frac{3}{10}, -\frac{9}{50}, \ldots$

e $1\frac{47}{81}, 2\frac{10}{27}, 3\frac{5}{9}, \ldots$

7 Find the 20th term of each sequence, leaving the answer in index form.

a 3, 6, 12, …

b 1, 7, 49, …

c $1.04, 1.04^2, 1.04^3, \ldots$

d $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

e $\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \ldots$

8 Find the 50th term of 1, 11, 121, … in index form.

9 Which term of the sequence 4, 20, 100, … is equal to 12 500?

10 Which term of 6, 36, 216, … is equal to 7776?

11 Is 1200 a term of the sequence 2, 16, 128, …?

12 Which term of 3, 21, 147, … is equal to 352 947?

13 Which term of the sequence 8, –4, 2, … is $\frac{1}{128}$?

14 Which term of 54, 18, 6, … is $\frac{2}{243}$?

15 Find the value of n if the nth term of the sequence $-2, 1\frac{1}{2}, -1\frac{1}{8}, \ldots$ is $-\frac{81}{128}$.

16 The first term of a geometric sequence is 7 and the 6th term is 1701.
Find the common ratio.

17 The 4th term of a geometric sequence is –648 and the 5th term is 3888.

a Find the common ratio.

b Find the 2nd term.

18 The 3rd term of a geometric sequence is $\frac{2}{5}$ and the 5th term is $1\frac{3}{5}$.
Find the first term and common ratio.

19 Find the value of n for the first term of the sequence 5000, 1000, 200, … that is less than 1.

20 Find the first term of the sequence $\frac{2}{7}, \frac{6}{7}, 2\frac{4}{7}, \ldots$ that is greater than 100.

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WS Geometric series

WS Geometric series

WS Geometric series assignment

WS Geometric series problems

1.05 Geometric series

The sum of a **geometric series** with n terms is given by the formulas:

Sum of a geometric series with n terms

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

This formula can also be written as:

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

to be used if r is a fraction, that is, $-1 < r < 1$, also written as $|r| < 1$.

Proof

The sum of a geometric series can be written

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1} \qquad [1]$$

Multiplying both sides by r:

$$rS_n = r(a + ar + ar^2 + \ldots + ar^{n-1})$$
$$= ar + ar^2 + ar^3 + \ldots + ar^n \qquad [2]$$

[2] – [1]:

$$rS_n - S_n = ar^n - a$$

$$S_n(r - 1) = a(r^n - 1)$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

[1] – [2] gives the formula

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

We can also use these formulas to find the sum of the first n terms of a geometric sequence (also called the **nth partial sum**).

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EXAMPLE 12

a Find the sum of the first 10 terms of the series $3 + 12 + 48 + \ldots$

b Evaluate $60 + 20 + 6\frac{2}{3} + \ldots + \frac{20}{81}$.

c The sum of n terms of $1 + 4 + 16 + \ldots$ is 21 845. Find the value of n.

Solution

a This is a geometric series with $a = 3, r = 4, n = 10$.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{3(4^{10} - 1)}{4 - 1}$$

$$= \frac{3(4^{10} - 1)}{3}$$

$$= 4^{10} - 1$$

$$= 1\,048\,575$$

b $a = 60, r = \frac{1}{3}, T_n = \frac{20}{81}$

$$T_n = ar^{n-1}$$

$$ar^{n-1} = \frac{20}{81}$$

$$\text{So } 3^{n-1} = 243$$

$$= 3^5$$

$$n - 1 = 5$$

$$n = 6$$

Since $|r| < 1$, we use the second formula.

$$S_n = \frac{a(1 - r^n)}{r - 1}$$

$$60\left(\frac{1}{3}\right)^{n-1} = \frac{20}{81}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{243}$$

$$\frac{1}{3^{n-1}} = \frac{1}{243}$$

$$S_6 = \frac{60\left(1 - \left(\frac{1}{3}\right)^6\right)}{1 - \frac{1}{3}}$$

$$= \frac{60\left(1 - \frac{1}{729}\right)}{\frac{2}{3}}$$

$$= 60 \times \frac{728}{729} \times \frac{3}{2}$$

$$= 89\frac{71}{81}$$

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c $a = 1, r = 4, S_n = 21\,845$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$21\,845 = \frac{1(4^n - 1)}{4 - 1}$$

$$= \frac{4^n - 1}{3}$$

$$65\,535 = 4^n - 1$$

$$65\,536 = 4^n$$

$$\log 65\,536 = \log 4^n$$

$$= n \log 4$$

$$\frac{\log 65\,536}{\log 4} = n$$

$$8 = n$$

So 8 terms give a sum of 21 845.

Exercise 1.05 Geometric series

1 Find the sum of 10 terms of each geometric series.

a $6 + 24 + 96 + \ldots$ **b** $3 + 15 + 75 + \ldots$

2 Find the sum of 8 terms of each series.

a $-1 + 7 - 49 + \ldots$ **b** $8 + 24 + 72 + \ldots$

3 Find the sum of 15 terms of each series.

a $4 + 8 + 16 + \ldots$ **b** $\frac{3}{4} - \frac{3}{8} + \frac{3}{16} - \ldots$(to 1 decimal place)

4 Evaluate:

a $2 + 10 + 50 + \ldots + 6250$ **b** $18 + 9 + 4\frac{1}{2} + \ldots + \frac{9}{64}$

c $3 + 21 + 147 + \ldots + 7203$ **d** $\frac{3}{4} + 2\frac{1}{4} + 6\frac{3}{4} + \ldots + 182\frac{1}{4}$

e $-3 + 6 - 12 + \ldots + 384$

5 For the series $7 + 14 + 28 + \ldots$ find:

a the 9th term

b the sum of the first 9 terms.

6 Find the sum of 30 terms of the series $1.09 + 1.09^2 + 1.09^3 + \ldots$ correct to 2 decimal places.

7 Find the sum of 25 terms of the series $1 + 1.12 + 1.12^2 + \ldots$ correct to 2 decimal places.

8 Find the value of n if the sum of n terms of the series $11 + 33 + 99 + \ldots$ is equal to 108 251.

9 How many terms of the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ give a sum of $\frac{1023}{1024}$?

10 The common ratio of a geometric series is 4 and the sum of the first 5 terms is 3069. Find the first term.

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11 Find the number of terms needed to be added for the sum to exceed 1 000 000 in the series $4 + 16 + 64 + \ldots$

12 **a** Find the sum of 10 terms of the series $2 + 4 + 8 + \ldots$

b Find the sum of 10 terms of the series $1 + 3 + 5 + \ldots$

c Find the sum of the first 10 terms of the series $3 + 7 + 13 + \ldots$

PUZZLES

1 A poor girl saved a rich king from drowning one day. The king offered the girl a reward of a sum of money in 30 daily payments. He gave her a choice of payments:

Choice 1: \$1 the first day, \$2 the second day, \$3 the third day and so on.

Choice 2: 1 cent the first day, 2 cents the second day, 4 cents the third day and so on, the payment doubling each day.

How much money would the girl receive for each choice? Which plan would give the girl more money?

2 Can you solve Fibonacci's problem?

A man entered an orchard through 7 guarded gates and gathered a certain number of apples. As he left the orchard he gave the guard at the first gate half the apples he had and 1 apple more. He repeated this process for each of the remaining 6 guards and eventually left the orchard with 1 apple. How many apples did he gather? (He did not give away any half-apples.)



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Limiting sum of an infinite geometric series

Geometric series crossword

1.06 Limiting sum of an infinite geometric series

In some geometric sequences the sum becomes very large as n increases, for example, the series $2 + 4 + 8 + 16 + 32 + \ldots$ We say these series **diverge** (their sum is infinite).

In other geometric sequences, however, such as $8 + 4 + 2 + 1 + \ldots$, the sum does not increase greatly after a few terms, but approaches some constant value. We say these series **converge** (they have a **limiting sum** that is a specific value, sometimes called the **sum to infinity**).

EXAMPLE 13

a Find the sum of 15 terms of $2 + 6 + 18 + \ldots$

b By evaluating the sum of 10 terms and 20 terms correct to 4 decimal places for the series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$, estimate its limiting sum.

Solution

a $a = 2, r = 3, n = 15$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{15} = \frac{2(3^{15} - 1)}{3 - 1}$$

$$= \frac{2(3^{15} - 1)}{2}$$

$$= 3^{15} - 1$$

$$= 14\,348\,906$$

b $a = 2, r = \frac{1}{2}$: $S_n = \frac{a(1 - r^n)}{1 - r}$

Sum to 10 terms:

$$S_{10} = \frac{2\left[1 - \left(\frac{1}{2}\right)^{10}\right]}{1 - \frac{1}{2}}$$

$$= \frac{2\left[1 - \frac{1}{2^{10}}\right]}{\frac{1}{2}}$$

$$= 3.9961$$

Sum to 20 terms:

$$S_{20} = \frac{2\left[1 - \left(\frac{1}{2}\right)^{20}\right]}{1 - \frac{1}{2}}$$

$$= \frac{2\left[1 - \frac{1}{2^{20}}\right]}{\frac{1}{2}}$$

$$= 4.0000$$

The limiting sum is 4.

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Can you see why the series $2 + 6 + 18 + \ldots$ does not have a limiting sum and the series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ has a limiting sum?

It's because of the common ratio. Only geometric series with common ratios that are fractions, $|r| < 1$, will have a limiting sum.

For $S_n = \frac{a(1-r^n)}{1-r}$:

As $n \to \infty$, $r^n \to 0$ when $-1 < r < 1$

We write $\lim_{n\to\infty} r^n = 0$

Limiting sum of a geometric series

$$S = \frac{a}{1-r} \text{ when } |r| < 1.$$

Proof

$$S_n = \frac{a(1-r^n)}{1-r}$$

For $|r| < 1$, $\lim_{n\to\infty} r^n = 0$

$$S_\infty = \frac{a(1-0)}{1-r}$$

$$= \frac{a}{1-r}$$

EXAMPLE 14

a Find the limiting sum of the series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

b Find the sum to infinity of the series $6 + 2 + \frac{2}{3} + \ldots$

c Does the series $\frac{3}{4} + \frac{15}{16} + 1\frac{11}{64} + \ldots$ have a limiting sum?

Solution

a $a = 2, r = \frac{1}{2}$

Since $|r| < 1$, the series has a limiting sum.

$$S = \frac{a}{1-r}$$

$$= \frac{2}{1-\frac{1}{2}}$$

$$= \frac{2}{\frac{1}{2}}$$

$$= 2 \times \frac{2}{1}$$

$$= 4$$

So the limiting sum is 4.

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b $a = 6$

$2 \div 6 = \frac{1}{3}$ and $\frac{2}{3} \div 2 = \frac{1}{3}$ so $r = \frac{1}{3}$

Since $|r| < 1$, the series has a limiting sum.

$$S = \frac{a}{1-r}$$

$$= \frac{6}{1-\frac{1}{3}}$$

$$= \frac{6}{\frac{2}{3}}$$

$$= 6 \times \frac{3}{2}$$

$$= 9$$

So the limiting sum is 9.

c For $\frac{3}{4} + \frac{15}{16} + 1\frac{11}{64} + \ldots$

$r = \frac{15}{16} \div \frac{3}{4}$ or $1\frac{11}{64} \div \frac{15}{16}$

$= 1\frac{1}{4}$

Since $|r| > 1$, this series does not have a limiting sum.

DID YOU KNOW?

A series involving π and *e*

Here is an interesting series involving π:

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$$

Gottfried Wilhelm Leibniz (1646–1716) discovered this result. It is interesting that while π is an irrational number, it can be written as the sum of rational numbers.

Here is another interesting series involving *e*:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

Remember: $2! = 2 \times 1$, $3! = 3 \times 2 \times 1$, $4! = 4 \times 3 \times 2 \times 1$ and so on.

Research these and other series.

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Exercise 1.06 Limiting sum of an infinite geometric series

1 Which series has a limiting sum? Find the limiting sum where it exists.

a $9 + 3 + 1 + \ldots$ **b** $\frac{1}{4} + \frac{1}{2} + 1 + \ldots$ **c** $16 - 4 + 1 - \ldots$

d $\frac{2}{3} + \frac{7}{9} + \frac{49}{54} + \ldots$ **e** $1 + \frac{2}{3} + \frac{4}{9} + \ldots$ **f** $\frac{5}{8} + \frac{1}{8} + \frac{1}{40} + \ldots$

g $-6 + 36 - 216 + \ldots$ **h** $-2\frac{1}{4} + 1\frac{7}{8} - 1\frac{27}{48} + \ldots$ **i** $\frac{1}{9} + \frac{1}{6} + \frac{1}{4} + \ldots$

j $2 - \frac{4}{5} + \frac{8}{25} - \ldots$

2 Find the limiting sum of each series.

a $40 + 20 + 10 + \ldots$ **b** $320 + 80 + 20 + \ldots$ **c** $100 - 50 + 25 - \ldots$

d $6 + 3 + 1\frac{1}{2} + \ldots$ **e** $\frac{2}{5} + \frac{6}{35} + \frac{18}{245} + \ldots$ **f** $72 - 24 + 8 - \ldots$

g $-12 + 2 - \frac{1}{3} + \ldots$ **h** $\frac{3}{4} - \frac{1}{2} + \frac{1}{3} - \ldots$ **i** $12 + 9 + 6\frac{3}{4} + \ldots$

j $-\frac{2}{3} + \frac{5}{12} - \frac{25}{96} + \ldots$

3 Find the difference between the limiting sum and the sum of 6 terms of each series, correct to 2 significant figures.

a $56 - 28 + 14 - \ldots$ **b** $72 + 24 + 8 + \ldots$ **c** $1 + \frac{1}{5} + \frac{1}{25} + \ldots$

d $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ **e** $1\frac{1}{4} + \frac{15}{16} + \frac{45}{64} + \ldots$

4 A geometric series has limiting sum 6 and common ratio $\frac{1}{3}$. Evaluate the first term of the series.

5 A geometric series has a limiting sum of 5 and first term 3. Find the common ratio.

6 The limiting sum of a geometric series is $9\frac{1}{3}$ and the common ratio is $\frac{2}{5}$.
Find the first term of the series.

7 A geometric series has limiting sum 40 and its first term is 5. Find the common ratio of the series.

8 A geometric series has limiting sum $-6\frac{2}{5}$ and first term -8. Find its common ratio.

9 The limiting sum of a geometric series is $-\frac{3}{10}$ and its first term is $-\frac{1}{2}$.
Find the common ratio of the series.

10 The second term of a geometric series is 2 and its limiting sum is 9.
Find the values of first term a and common ratio r.

11 A geometric series has 3rd term 12 and 4th term -3. Find a, r and the limiting sum.

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12 A geometric series has 2nd term $\frac{2}{3}$ and 4th term $\frac{8}{27}$. Find a, r and its limiting sum.

13 The 3rd term of a geometric series is 54 and the 6th term is $11\frac{83}{125}$. Evaluate a, r and the limiting sum.

14 The 2nd term of a geometric series is $\frac{4}{15}$ and the 5th term is $\frac{32}{405}$. Find the values of a and r and its limiting sum.

15 The limiting sum of a geometric series is 5 and the 2nd term is $1\frac{1}{5}$. Find the first term and the common ratio.

16 The series $x + \frac{x}{4} + \frac{x}{16} + \ldots$ has a limiting sum of $\frac{7}{8}$. Evaluate x.

17 **a** For what values of k does the limiting sum exist for the series $k + k^2 + k^3 + \ldots$?

b Find the limiting sum of the series when $k = -\frac{2}{3}$.

c Evaluate k if the limiting sum of the series is 3.

18 Show that in any geometric series the difference between the limiting sum and the sum of n terms is $\frac{ar^n}{1-r}$.



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1. TEST YOURSELF



Formula sheet: Measurement, Sequences and series



Practice quiz

For Questions **1** to **3**, select the correct answer **A**, **B**, **C** or **D**.

1 The sum of n terms of a geometric sequence is:

A $S_n = \frac{n}{2}[2a + (n-1)d]$

B $S_n = \frac{a}{1-r}$

C $S_n = \frac{a(1-r^n)}{1-r}$

D $S_n = \frac{n}{2}(a + l)$

2 The limiting sum of an infinite geometric series exists when:

A $r > 1$ **B** $|r| > 1$ **C** $|r| < 1$ **D** $r < 1$

3 The nth term of the statement that can be proved 12, 9, 6, … is:

A $9 + 3n$ **B** $15 - 3n$ **C** $9 - 3n$ **D** $12n - 15$

4 Find a formula for the nth term of each sequence.

a 9, 13, 17, … **b** 7, 0, –7, … **c** 2, 6, 18, …

d $200, 50, 12\frac{1}{2}, \ldots$ **e** –2, 4, –8, …

5 For the series 156 + 145 + 134 + …:

a Find the 15th term.

b Find the sum of 15 terms.

c Find the sum of 14 terms.

d Write a relationship between T_{15}, S_{15} and S_{14}.

e Find the value of n for the first negative term of the series.

6 Find whether each sequence is:

i arithmetic **ii** geometric **iii** neither.

a 97, 93, 89, …

b $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \ldots$

c $\sqrt{5}, \sqrt{20}, \sqrt{45}, \ldots$

d –1.6, –0.4, 0.6, …

e 3.4, 7.5, 11.6, …

f 48, 24, 12, …

g $-\frac{1}{5}, 1, -5, \ldots$

h 105, 100, 95, …

i $1\frac{1}{2}, 1\frac{1}{4}, 1, \ldots$

j $\log x, \log x^2, \log x^3, \ldots$

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7 The nth term of the sequence 8, 13, 18, … is 543. Evaluate n.

8 The 11th term of an arithmetic sequence is 97 and the 6th term is 32.
Find the first term and common difference.

9 A sequence has nth term given by $T_n = n^3 - 5$. Find:

a the 4th term **b** the sum of 4 terms **c** which term is 5827.

10 A sequence has terms $5, x, 45, \ldots$ Evaluate x if the sequence is:

a arithmetic **b** geometric.

11 If x, $2x + 3$ and $5x$ are the first 3 terms of an arithmetic series, calculate the value of x.

12 Find the 20th term of:

a 3, 10, 17, … **b** 101, 98, 95, … **c** 0.3, 0.6, 0.9, …

13 Find the limiting sum of the series $81 + 27 + 9 + \ldots$

14 For each series, find the formula for the sum of n terms.

a $5 + 9 + 13 + \ldots$ **b** $1 + 1.07 + 1.07^2 + \ldots$

15 **a** For what values of x does the geometric series $1 + x + x^2 + \ldots$ have a limiting sum?

b Find the limiting sum when $x = \frac{3}{5}$.

c Evaluate x when the limiting sum is $1\frac{1}{2}$.

16 The first term of an arithmetic series is 4 and the sum of 10 terms is 265.
Find the common difference.

17 If $x + 2$, $7x - 2$ and $15x + 6$ are consecutive terms in a geometric sequence, evaluate x.

18 Evaluate $8 + 14 + 20 + \ldots + 122$.

19 **a** Calculate the sum of all the multiples of 7 from 1 to 100.

b Calculate the sum of all numbers from 1 to 100 that are not multiples of 7.

20 The sum of n terms of the series $214 + 206 + 198 + \ldots$ is 2760. Evaluate n.

21 Evaluate n if the nth term of the sequence 4, 12, 36, … is 236 196.

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1. CHALLENGE EXERCISE

1 The nth term of a sequence is given by $T_n = \dfrac{n^2}{n+1}$.

a What is the 9th term of the sequence?

b Which term is equal to $18\frac{1}{20}$?

2 For the series $\frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \ldots$ find the exact value of:

a the common difference

b the 7th term

c the sum of 6 terms.

3 Evaluate the sum of the first 20 terms of the series:

a $3 + 5 + 9 + 17 + 33 + 65 + \ldots$

b $5 - 2 + 10 - 8 + 15 - 32 + \ldots$

4 Which term of the sequence $\frac{7}{9}, \frac{14}{45}, \frac{28}{225}, \ldots$ is equal to $\frac{224}{28\,125}$?

5 Find the sum of all integers between 1 and 200 that are not multiples of 9.

6 Find the values of n for which $S_n > 24.99$ for the series $20 + 4 + \frac{4}{5} + \ldots$

7 The sum of the first 5 terms of a geometric series is 77 and the sum of the next 5 terms is –2464.

a Find the first term and common ratio of the series.

b Find the 4th term of the series.

8 **a** Find the limiting sum of the series $1 + \cos^2 x + \cos^4 x + \ldots$ where $\cos^2 x \neq 0, 1$.

b Why does this series have a limiting sum?

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