BOARD OF STUDIES
NEW SOUTH WALES

2012

HIGHER SCHOOL CERTIFICATE EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
  Black pen is preferred
- Calculators may be used
- A formulae sheet is provided at the back of this paper
- In Questions 26–30, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number on the Question 28 Writing Booklet

Total marks – 100

Section I Pages 2–13

25 marks

- Attempt Questions 1–25
- Allow about 35 minutes for this section

Section II Pages 14–27

75 marks

- Attempt Questions 26–30
- Allow about 1 hour and 55 minutes for this section

Section I

25 marks
Attempt Questions 1–25
Allow about 35 minutes for this section

Use the multiple-choice answer sheet for Questions 1–25.

---

1 A set of 15 scores is displayed in a stem-and-leaf plot.

5	3	4		
6	2	6	7	
7	7	7	8	9
8	2	4		
9	1	3	5	7

What is the median of these scores?

(A) 7

(B) 8

(C) 77

(D) 78

2 Handmade chocolates are checked for size and shape. Every 30th chocolate is sampled.

Which term best describes this type of sampling?

(A) Census

(B) Random

(C) Stratified

(D) Systematic

3 A pair of players is to be selected from 6 people.

How many different pairs of players can be selected?

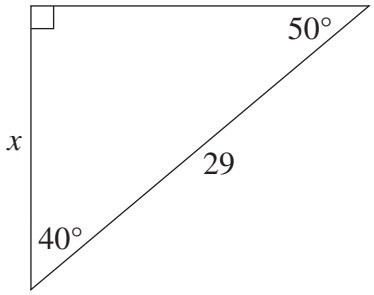
(A) 6

(B) 12

(C) 15

(D) 30

4



NOT TO SCALE

Which expression could be used to calculate the value of x in this triangle?

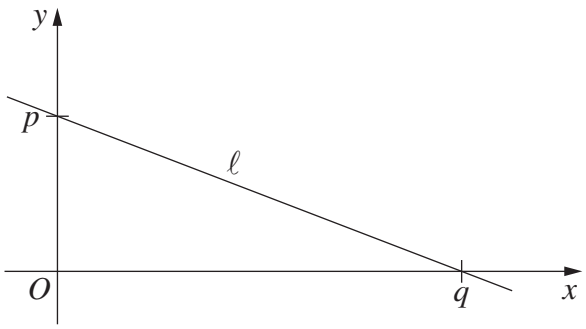
(A) $29 \times \cos 40°$

(B) $29 \times \cos 50°$

(C) $\dfrac{\cos 40°}{29}$

(D) $\dfrac{\cos 50°}{29}$

5 The line ℓ has intercepts p and q, where p and q are positive integers.



What is the gradient of line ℓ?

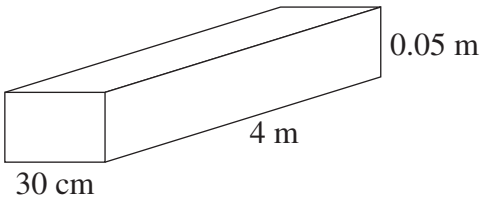
(A) $-\dfrac{p}{q}$

(B) $-\dfrac{q}{p}$

(C) $\dfrac{p}{q}$

(D) $\dfrac{q}{p}$

6



NOT TO SCALE

What is the volume of this rectangular prism in cubic centimetres?

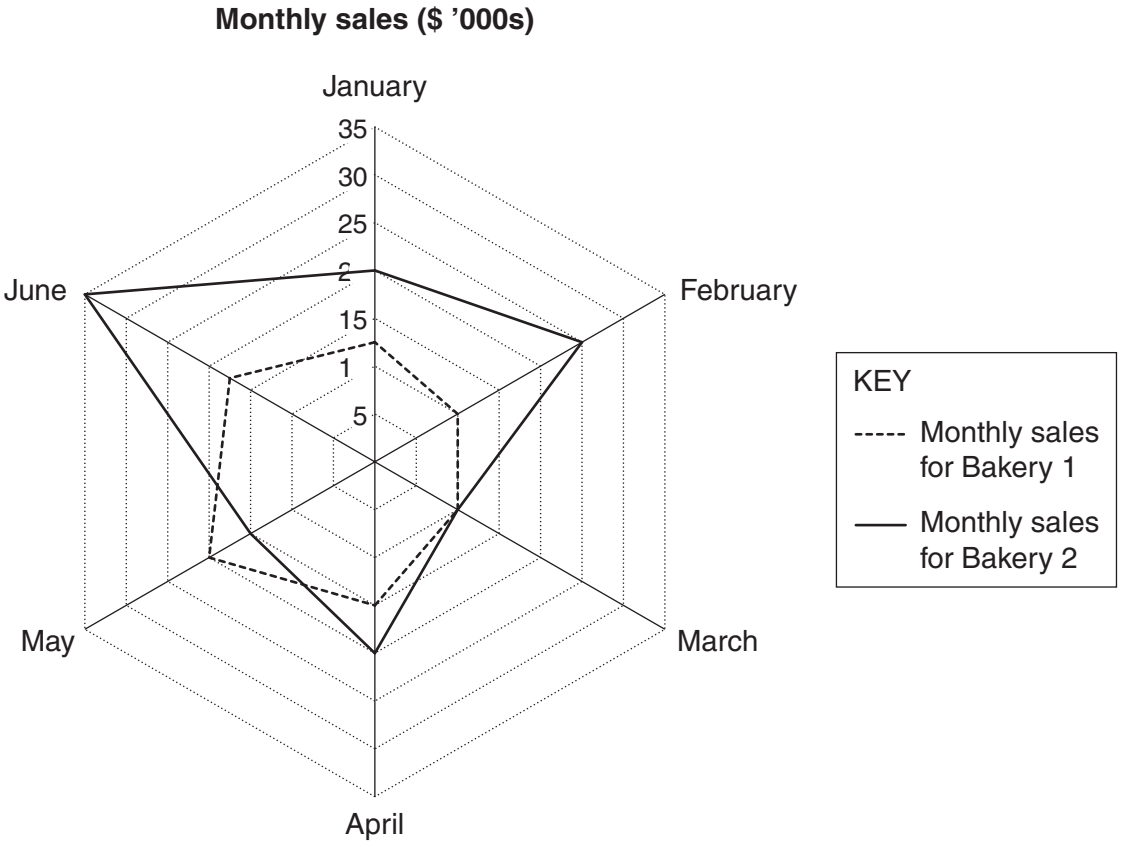
(A) 6 cm^3

(B) 600 cm^3

(C) 60 000 cm^3

(D) 6 000 000 cm^3

7 The Pi Company has two bakeries. The radar chart displays the monthly sales for the bakeries.



What was the difference in sales in June between the two bakeries?

(A) $7.50

(B) $17.50

(C) $7500

(D) $17 500

8 Dots were used to create a pattern. The first three shapes in the pattern are shown.



Shape 1 Shape 2 Shape 3

The number of dots used in each shape is recorded in the table.

Shape (S)	1	2	3
Number of dots (N)	6	8	10

How many dots would be required for Shape 156?

(A) 316

(B) 520

(C) 624

(D) 936

9 Tracy invests some money for 2 years at 4% per annum, compounded quarterly.

Compounded values of $1

Period	Interest rate per period				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.010	1.020	1.030	1.040	1.050
2	1.020	1.040	1.061	1.082	1.103
3	1.030	1.061	1.093	1.125	1.158
4	1.041	1.082	1.126	1.170	1.216
5	1.051	1.104	1.159	1.217	1.276
6	1.062	1.126	1.194	1.265	1.340
7	1.072	1.149	1.230	1.316	1.407
8	1.083	1.172	1.267	1.369	1.477

Which figure from the table should Tracy use to calculate the value of her investment at the end of 2 years?

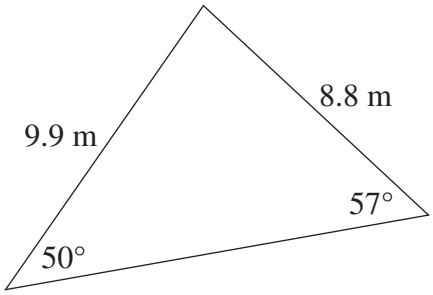
(A) 1.020

(B) 1.082

(C) 1.083

(D) 1.369

10



NOT TO SCALE

What is the area of this triangle, to the nearest square metre?

(A) 33 m^2

(B) 37 m^2

(C) 42 m^2

(D) 44 m^2

11 Which of the following relationships would most likely show a negative correlation?

(A) The population of a town and the number of hospitals in that town.

(B) The hours spent training for a race and the time taken to complete the race.

(C) The price per litre of petrol and the number of people riding bicycles to work.

(D) The number of pets per household and the number of computers per household.

12 Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled.

What is the probability of a 6 appearing on at least one of the dice?

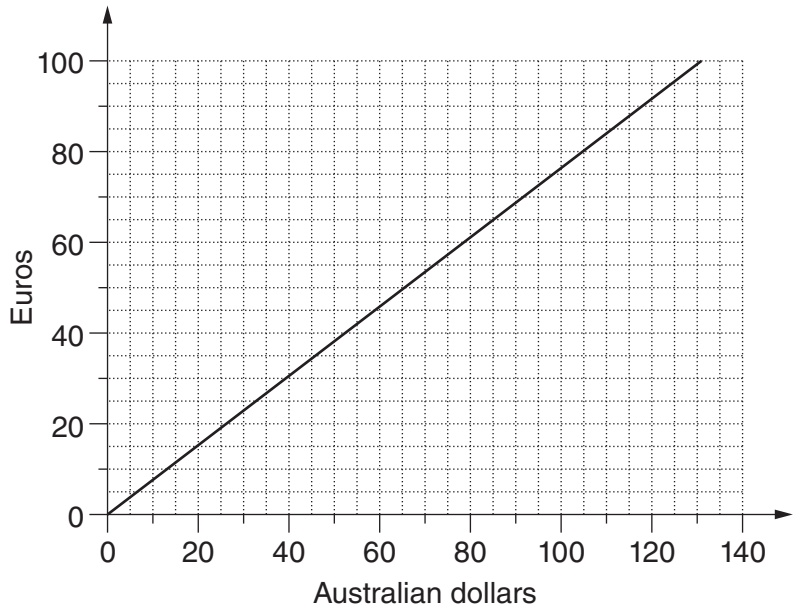
(A) $\frac{1}{6}$

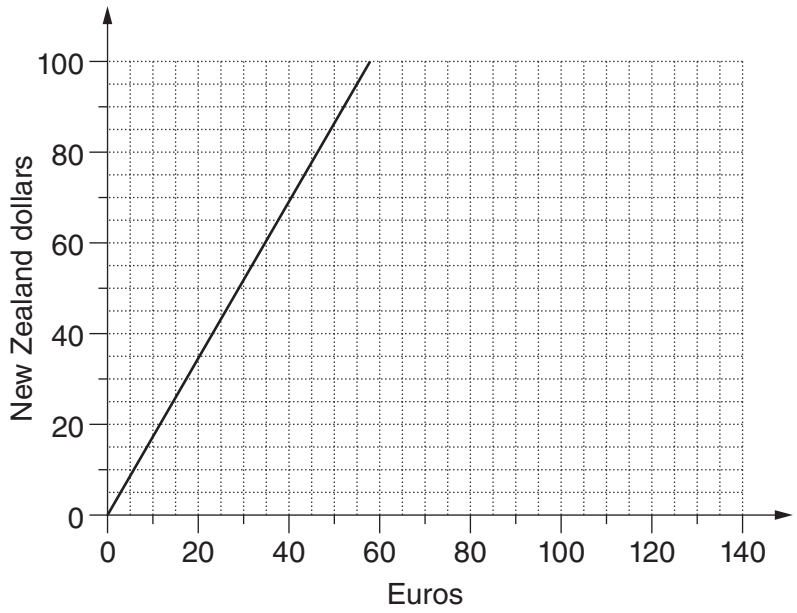
(B) $\frac{11}{36}$

(C) $\frac{25}{36}$

(D) $\frac{5}{6}$

13 Conversion graphs can be used to convert from one currency to another.





Sarah converted 60 Australian dollars into Euros. She then converted all of these Euros into New Zealand dollars.

How much money, in New Zealand dollars, should Sarah have?

(A) $26

(B) $45

(C) $78

(D) $135

14 Which of the following expresses $2x^2(5-x)-x(x-2)$ in its simplest form?

(A) $-2x^3+9x^2+2x$

(B) $-2x^3-9x^2-2x$

(C) $9x^2-x+2$

(D) $9x^2-x-2$

15 The time taken to complete a journey varies inversely with the speed of a car. A car takes 6 hours to complete a journey when travelling at 60 km/h.

How long would the same journey take if the car were travelling at 100 km/h?

(A) 36 minutes

(B) 1 hour and 40 minutes

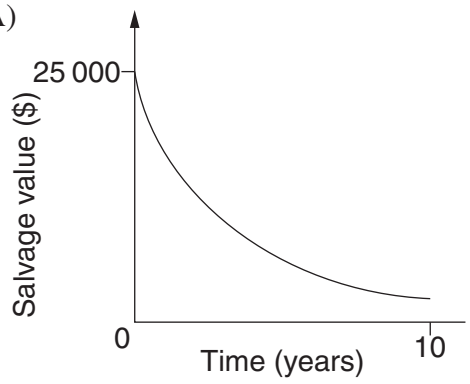
(C) 3 hours and 6 minutes

(D) 3 hours and 36 minutes

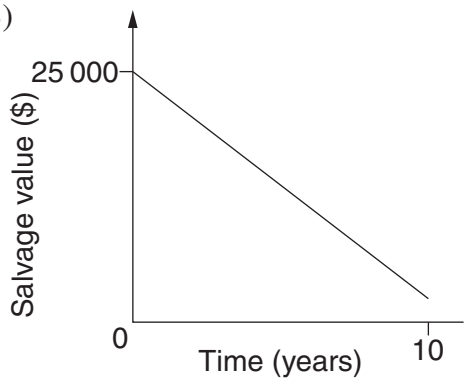
16 A machine was bought for \$25 000.

Which graph best represents the salvage value of the machine over 10 years using the declining balance method of depreciation?

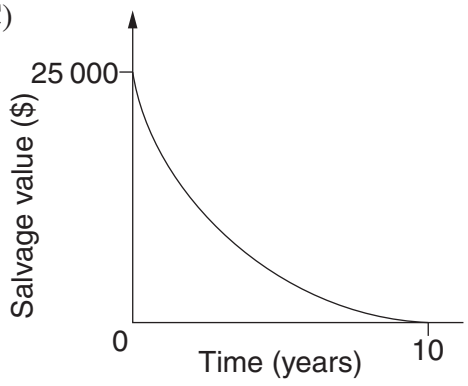
(A)



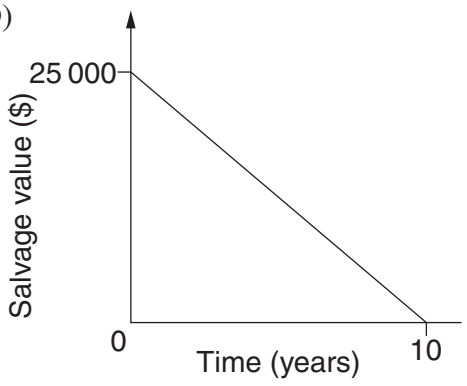
(B)



(C)



(D)



17 A spinner with different coloured sectors is spun 40 times. The results are recorded in the table.

Colour obtained	*Frequency*
Red	2
Yellow	4
Blue	6
Orange	
Green	10
Purple	12

What is the relative frequency of obtaining the colour orange?

(A) $\frac{3}{20}$

(B) $\frac{1}{5}$

(C) 6

(D) 8

18 Jo qualifies for both Rent Assistance and Youth Allowance and receives a fortnightly payment from the government.

Rent Assistance is $119.40 per fortnight.

The maximum Youth Allowance is $402.70 per fortnight. It is reduced by 50 cents in the dollar for any income earned over $236 per fortnight.

Jo earns $300 per fortnight from a part-time job.

What is the total payment Jo receives each fortnight from the government?

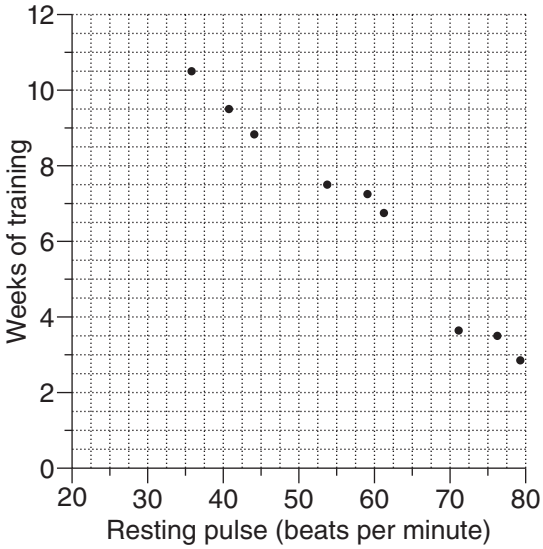
(A) $370.70

(B) $372.10

(C) $458.60

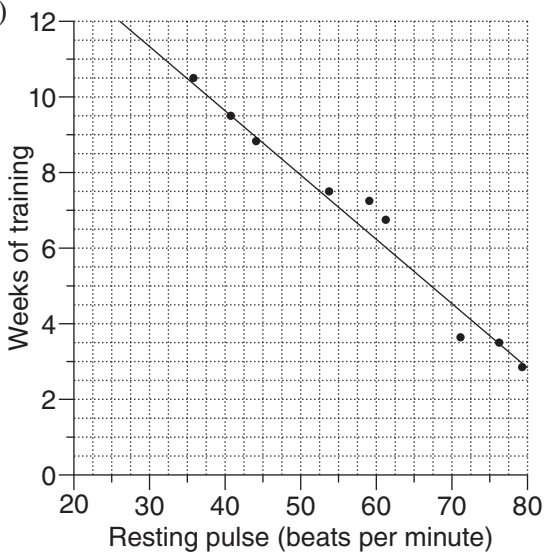
(D) $490.10

19 A coach compares the resting pulse and the number of weeks of training for nine cyclists. The information is graphed in order to draw a median regression line.

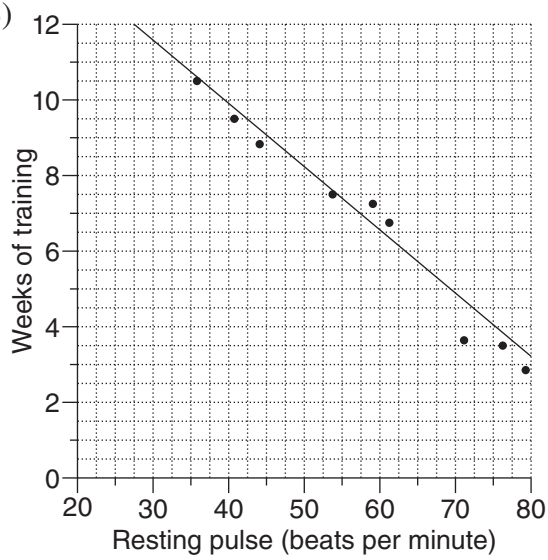


Which of the following graphs best shows the median regression line for the data?

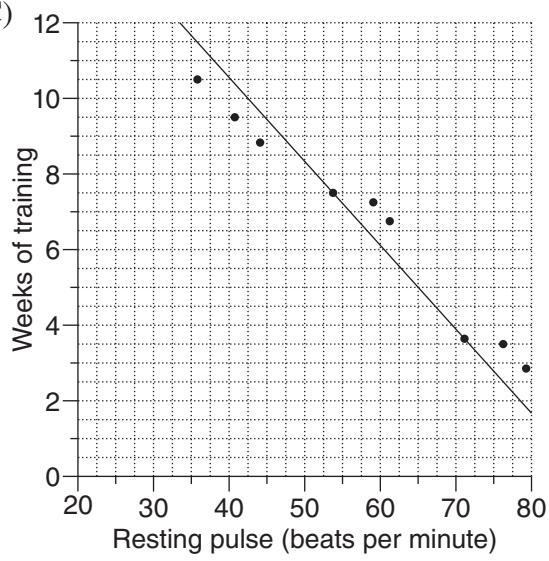
(A)



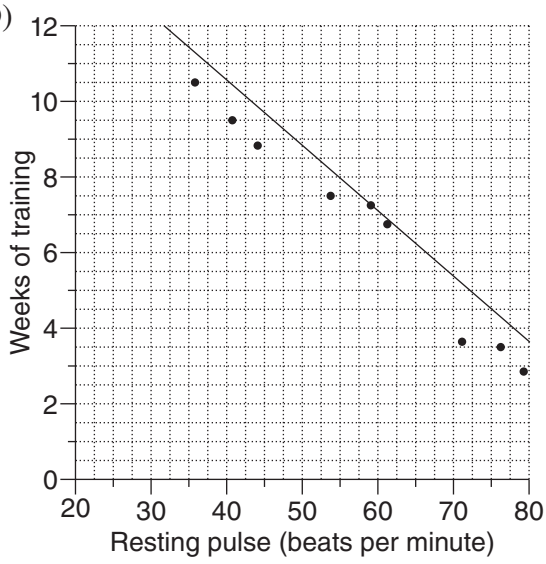
(B)



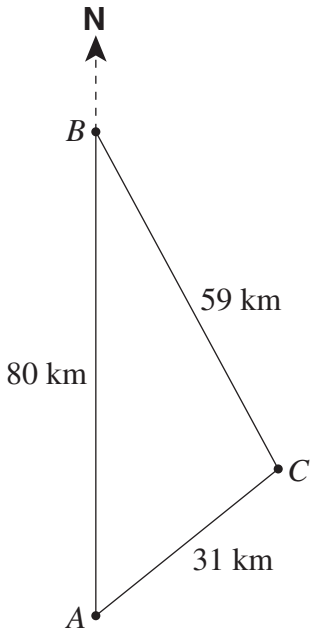
(C)



(D)



20 Town B is 80 km due north of Town A and 59 km from Town C.
Town A is 31 km from Town C.



NOT TO SCALE

What is the bearing of Town C from Town B?

(A) 019°

(B) 122°

(C) 161°

(D) 341°

21 Which of the following correctly expresses c as the subject of $E = mc^2 + p$?

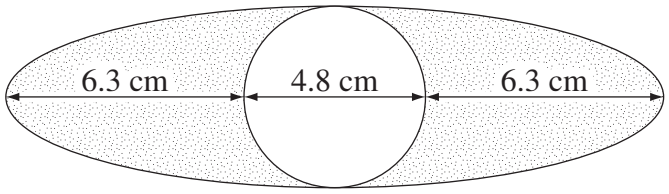
(A) $c = \pm\sqrt{\dfrac{E - p}{m}}$

(B) $c = \pm\dfrac{\sqrt{E - p}}{m}$

(C) $c = \pm\sqrt{\dfrac{E}{m}} - p$

(D) $c = \pm\sqrt{\dfrac{E}{m} - p}$

22 A label is designed with a circle inside an ellipse as shown.



NOT TO SCALE

What is the area of the shaded part of the label, to the nearest square centimetre?

(A) 29 cm^2

(B) 48 cm^2

(C) 113 cm^2

(D) 244 cm^2

23 A football club knows that at the end of 10 years it will need to replace goal posts and other equipment. It is estimated that the replacement cost will be \$12 000. For 10 years, the club will invest an amount at the end of each month at 6% per annum, compounded monthly.

Which equation should the club use to calculate the amount, M, it will need to deposit each month in order to have \$12 000 at the end of 10 years?

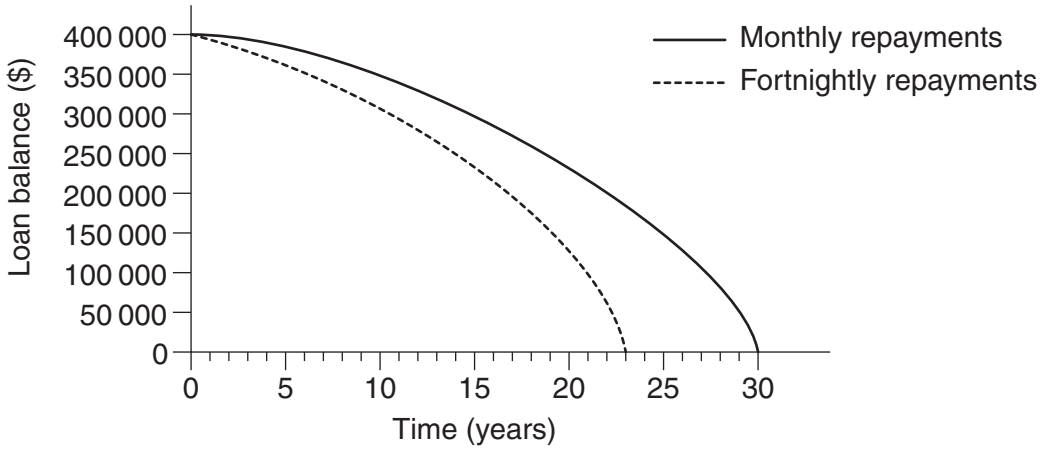
(A) $12\,000 = M\left\{\dfrac{(1+0.06)^{10}-1}{0.06}\right\}$

(B) $12\,000 = M\left\{\dfrac{(1+0.06)^{10}-1}{0.06(1+0.06)^{10}}\right\}$

(C) $12\,000 = M\left\{\dfrac{(1+0.005)^{120}-1}{0.005(1+0.005)^{120}}\right\}$

(D) $12\,000 = M\left\{\dfrac{(1+0.005)^{120}-1}{0.005}\right\}$

24 A $400 000 loan can be repaid by making either monthly or fortnightly repayments. The graph shows the loan balances over time using these two different methods of repayment.



The monthly repayment is $2796.86 and the fortnightly repayment is $1404.76.

What is the difference in the total interest paid using the two different methods of repayment, to the nearest dollar?

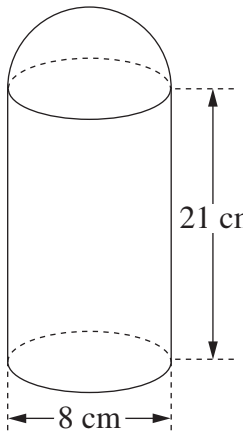
(A) $51 596

(B) $166 823

(C) $210 000

(D) $234 936

25 The solid shown is made of a cylinder with a hemisphere (half a sphere) on top.



NOT TO SCALE

What is the total surface area of the solid, to the nearest square centimetre?

(A) 628 cm^2

(B) 679 cm^2

(C) 729 cm^2

(D) 829 cm^2

Section II

75 marks
Attempt Questions 26–30
Allow about 1 hour and 55 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 26–30, your responses should include relevant mathematical reasoning and/or calculations.

Question 26 (15 marks) Use the Question 26 Writing Booklet.

(a) Postcodes in Australia are made up of four digits eg 2040.

(i) How many different postcodes beginning with a 2 are possible? **1**

(ii) Peta remembers that the first two digits of a town's postcode are 2 and then 4. She is unable to remember the rest of the postcode. **1**

2	4	?	?

What is the probability that Peta guesses the correct postcode?

(b) Jim buys a photocopier for $22 000. Its value is depreciated using the declining balance method at the rate of 15% per annum. **2**

What is its value at the end of 3 years?

(c) Heather used her credit card to purchase a plane ticket valued at $1990 on 28 January 2011. She made no other purchases on her credit card account in January. She paid the January account in full on 19 February 2011. **2**

The credit card account has no interest free period. Simple interest is charged daily at the rate of 20% per annum, including the date of purchase and the date the account is paid.

How much interest did she pay, to the nearest cent?

Question 26 continues on page 15

(d) Greg needs to conduct a statistical inquiry into how much time people aged 18–25 years have spent accessing social media websites in the last two weeks. He has decided to survey a sample of students from his university.

The process of statistical inquiry includes the following steps, which are NOT in order.

A Writing a report

B Posing questions

C Organising data

D Analysing data and drawing conclusions

E Collecting data

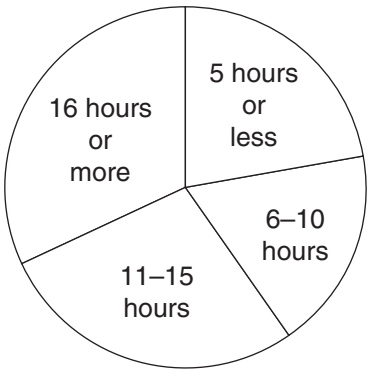
F Summarising and displaying data

(i) Using the letters A, B, C, D, E and F, list the steps in the most appropriate order for Greg to conduct his statistical inquiry. **2**

(ii) Greg conducts his statistical inquiry. **1**

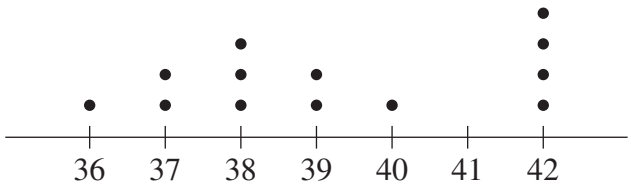
At which step in the process would he have drawn this graph?

Time spent accessing social media websites (in hours)



Question 26 continues on page 16

(e) The dot plot shows the number of push-ups that 13 members of a fitness class can do in one minute.



(i) What is the probability that a member selected at random from the class can do more than 38 push-ups in one minute? **1**

(ii) A new member who can do 32 push-ups in one minute joins the class. Does the addition of this new member to the class change the probability calculated in part (e) (i)? Justify your answer. **1**

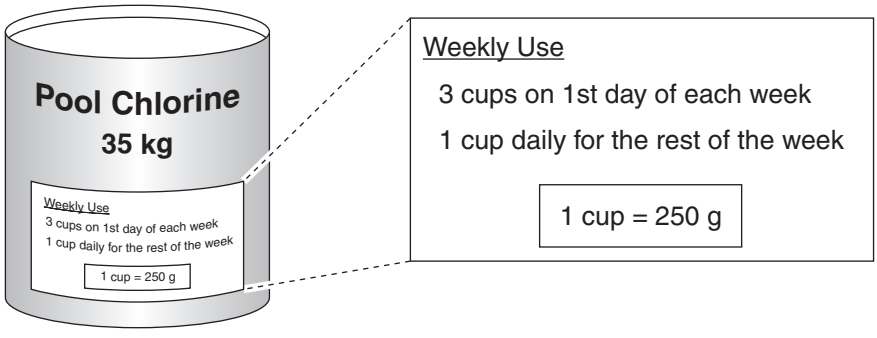
(f) The capture-recapture technique was used to estimate a population of seals in 2012. **2**

- 60 seals were caught, tagged and released.
- Later, 120 seals were caught at random.
- 30 of these 120 seals had been tagged.

The estimated population of seals in 2012 was 11% less than the estimated population for 2008.

What was the estimated population for 2008?

(g) Bhawana purchases pool chlorine in a new container which holds 35 kg. **2**



She begins using this new container on the first day of a week.

How many full weeks should this container last?

End of Question 26

Question 27 (15 marks) Use the Question 27 Writing Booklet.

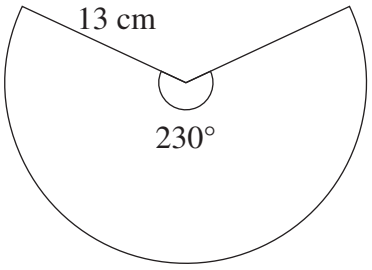
(a) Tai earns a gross weekly wage of \$1024. Each week her deductions are: **3**

- tax instalment of \$296.40
- health fund contribution of \$24.50
- union fees of \$15.80.

She also pays \$3640 over the year as her share of the household expenses.

What percentage of her net wage does Tai pay for household expenses?

(b) The sector shown has a radius of 13 cm and an angle of 230°. **2**



NOT TO SCALE

What is the perimeter of the sector to the nearest centimetre?

(c) A map has a scale of 1 : 500 000.

(i) Two mountain peaks are 2 cm apart on the map. **1**

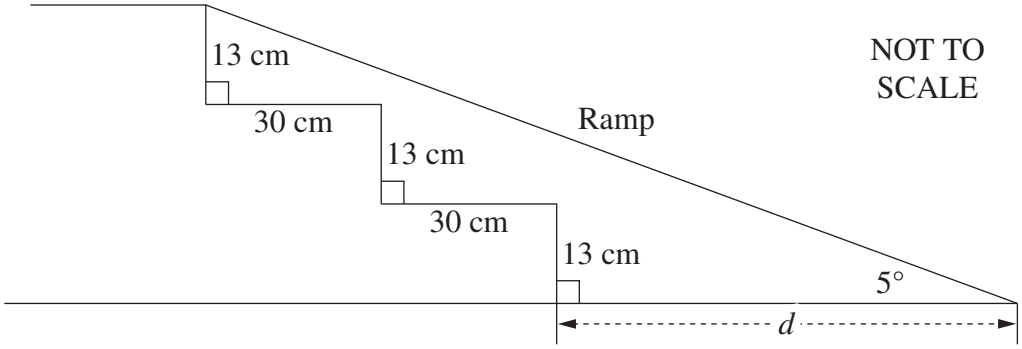
What is the actual distance between the two mountain peaks, in kilometres?

(ii) Two cities are 75 km apart. **1**

How far apart are the two cities on the map, in centimetres?

Question 27 continues on page 18

(d) A disability ramp is to be constructed to replace steps, as shown in the diagram. The angle of inclination for the ramp is to be 5°. **3**

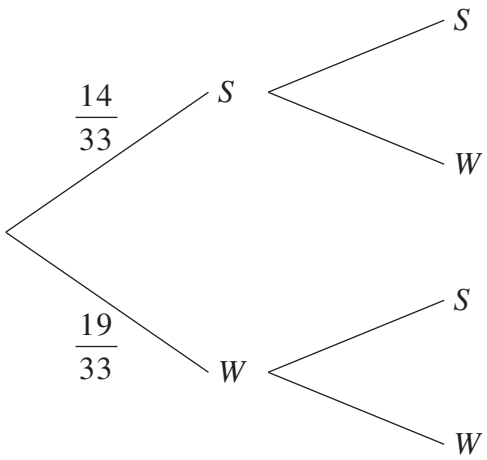


Calculate the extra distance, d, that the ramp will extend beyond the bottom step. Give your answer to the nearest centimetre.

(e) A box contains 33 scarves made from two different fabrics. There are 14 scarves made from silk (S) and 19 made from wool (W).

Two girls each select, at random, a scarf to wear from the box.

(i) Copy and complete the probability tree diagram in your answer booklet. **2**



(ii) Calculate the probability that the two scarves selected are made from silk. **1**

(iii) Calculate the probability that the two scarves selected are made from different fabrics. **2**

End of Question 27

Question 28 (15 marks) Use the Question 28 Writing Booklet.

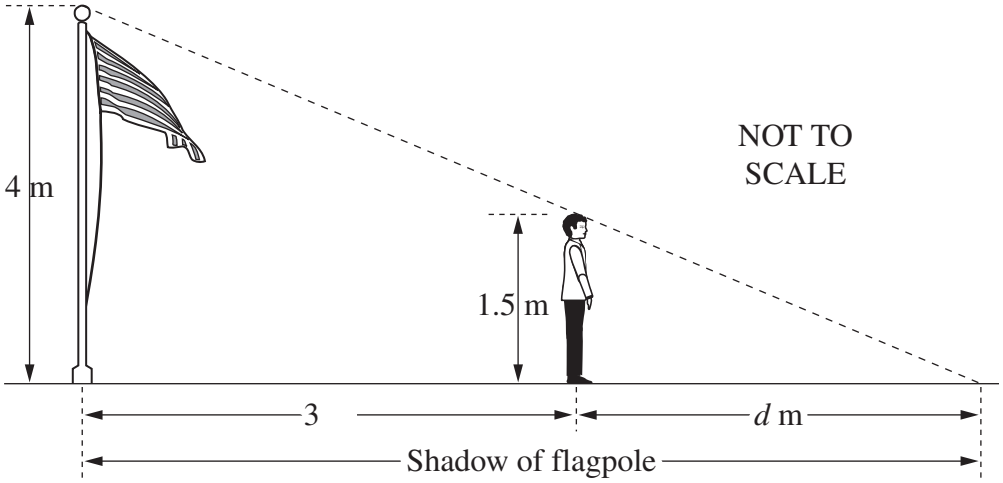
(a) The solid shown in the diagram on page 1 of the Question 28 Writing Booklet is a cube. **2**

Complete a sketch of the cube using the vanishing points V_1 and V_2.

Leave all construction lines on your diagram and label the vertices.

(b) Simplify fully $\dfrac{18ab}{3a^2} \times \dfrac{c}{b}$. **2**

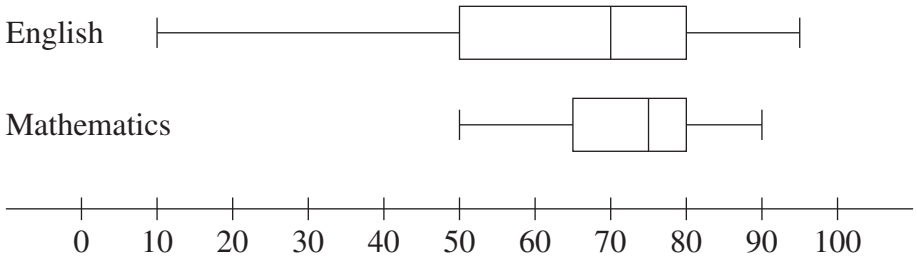
(c) Jacques and a flagpole both cast shadows on the ground. The difference between the lengths of their shadows is 3 metres. **3**



What is the value of d, the length of Jacques' shadow?

Question 28 continues on page 20

(d) The test results in English and Mathematics for a class were recorded and displayed in the box-and-whisker plots.



(i) What is the interquartile range for English? **1**

(ii) Compare and contrast the two data sets by referring to the skewness of the distributions and the measures of location and spread. **3**

(e) Matthew bought a laptop priced at $2800. He paid a 10% deposit and made monthly repayments of $95.20 for 3 years. **4**

What annual flat rate of interest was Matthew charged? Justify your answer with suitable calculations.

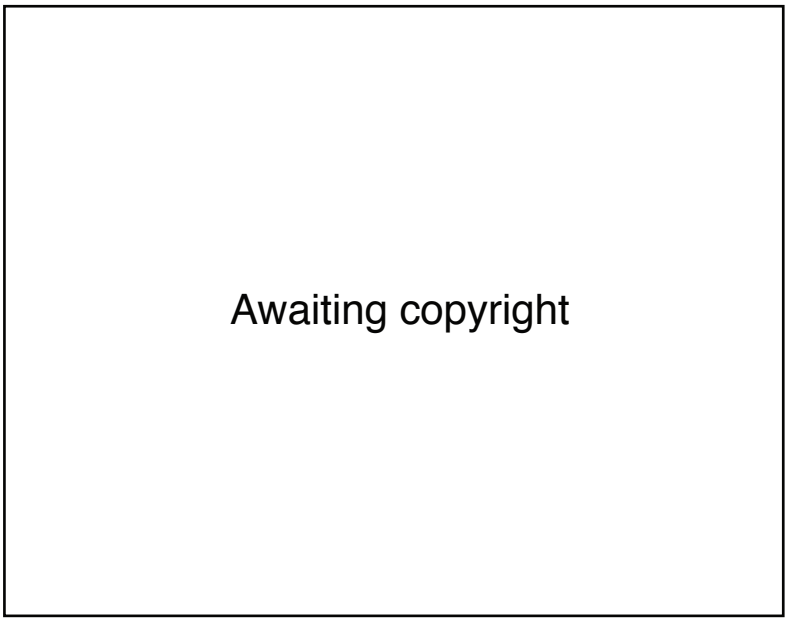
End of Question 28

Question 29 (15 marks) Use the Question 29 Writing Booklet.

(a) Tourists visit a park where steam erupts from a particular geyser.

The brochure for the park has a graph of the data collected for this geyser over a period of time.

The graph shows the duration of an eruption and the time until the next eruption, timed from the end of one eruption to the beginning of the next.



(i) Tony sees an eruption that lasts 4 minutes. Based on the data in the graph, what is the minimum time that he can expect to wait for the next eruption? **1**

(ii) Julia saw two consecutive eruptions, one hour apart. Based on the data in the graph, what was the longest possible duration of the first eruption that she saw? **1**

(iii) What does the graph suggest about the relationship between the duration of an eruption and the time to the next eruption? **1**

Question 29 continues on page 22

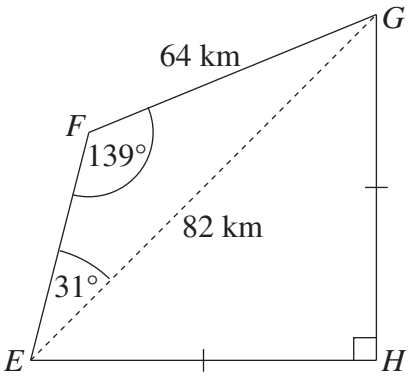
(b) A machine produces nails. When the machine is set correctly, the lengths of the nails are normally distributed with a mean of 6.000 cm and a standard deviation of 0.040 cm. **2**

To confirm the setting of the machine, three nails are randomly selected. In one sample the lengths are 5.950, 5.983 and 6.140.

The setting of the machine needs to be checked when the lengths of two or more nails in a sample lie more than 1 standard deviation from the mean.

Does the setting on the machine need to be checked? Justify your answer with suitable calculations.

(c) Raj cycles around a course. The course starts at E, passes through F, G and H and finishes at E. The distances EH and GH are equal.



NOT TO SCALE

(i) What is the length of EF, to the nearest kilometre? **2**

(ii) What is the total distance that Raj cycles, to the nearest kilometre? **3**

(d) Su-Lin pays a monthly contribution of 5% of her salary into a superannuation fund. Her salary is $81 600 per annum. The fund pays interest of 6.6% per annum, compounded monthly.

(i) What is the amount of Su-Lin's monthly contribution? **2**

(ii) Su-Lin made a contribution at the end of each month, starting at the end of January 2000. **3**

What will be the accumulated value of her superannuation at the end of December 2012 after making the December contribution?

End of Question 29

Question 30 (15 marks) Use the Question 30 Writing Booklet.

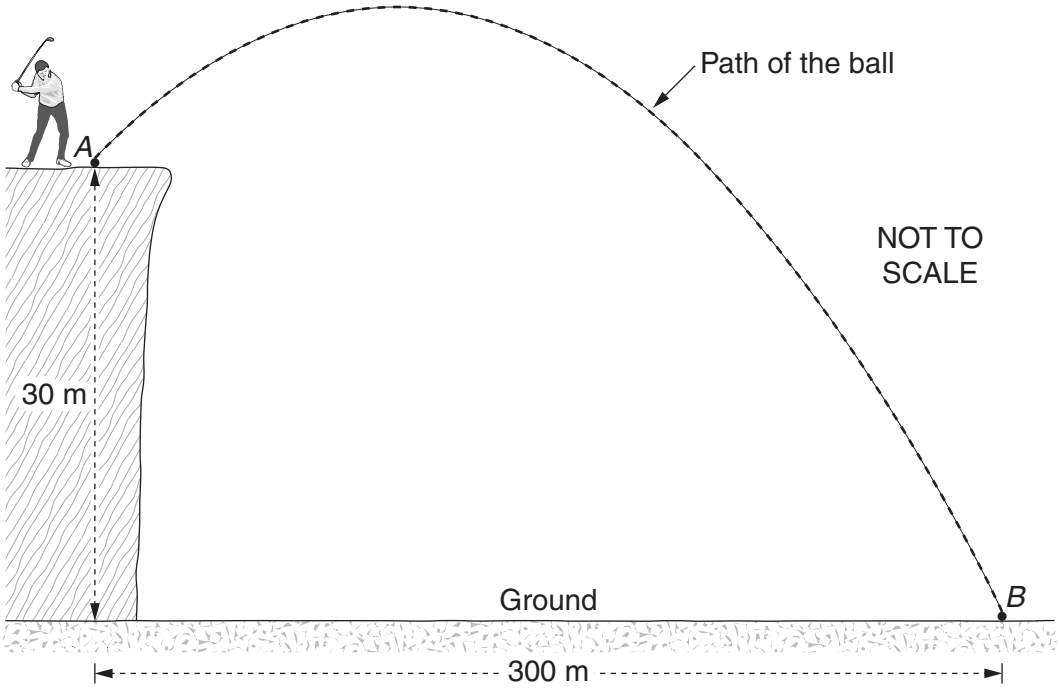
(a) A ship located at 4°N 160°E needs assistance. **3**

A rescue boat leaves Honiara 9°S 160°E and travels due north at a speed of 30 knots to reach the ship.

How long will it take the rescue boat to reach the ship? (You may assume that 1° on a great circle equals 60 nautical miles.)

Question 30 continues on page 24

(b) A golf ball is hit from point *A* to point *B*, which is on the ground as shown. Point *A* is 30 metres above the ground and the horizontal distance from point *A* to point *B* is 300 m.



The path of the golf ball is modelled using the equation

$$h = 30 + 0.2d - 0.001d^2$$

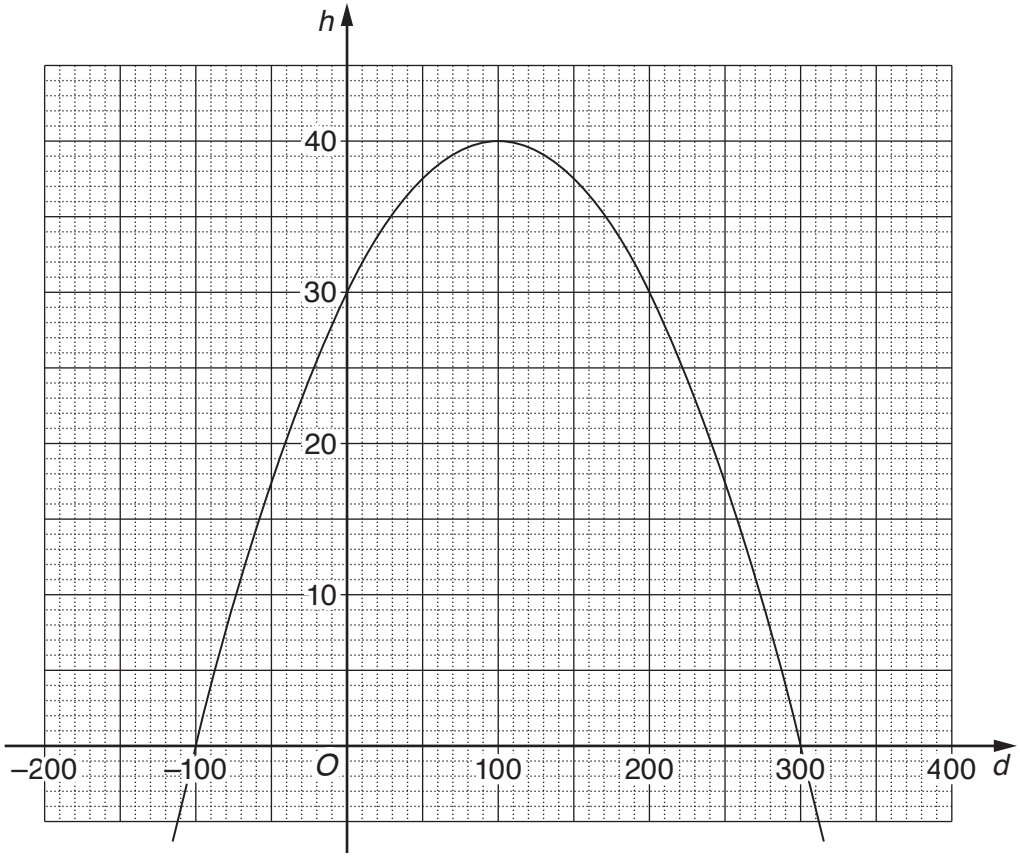
where

h is the height of the golf ball above the ground in metres, and

d is the horizontal distance of the golf ball from point *A* in metres.

Question 30 continues on page 25

The graph of this equation is drawn below.



(i) What is the maximum height the ball reaches above the ground? **1**

(ii) There are two occasions when the golf ball is at a height of 35 metres. **1**

What horizontal distance does the ball travel in the period between these two occasions?

(iii) What is the height of the ball above the ground when it still has to travel a horizontal distance of 50 metres to hit the ground at point B? **1**

(iv) Only part of the graph applies to this model. **2**

Find all values of d that are not suitable to use with this model, and explain why these values are not suitable.

Question 30 continues on page 26

Question 30 (continued)

(c) In 2010, the city of Thagoras modelled the predicted population of the city using the equation

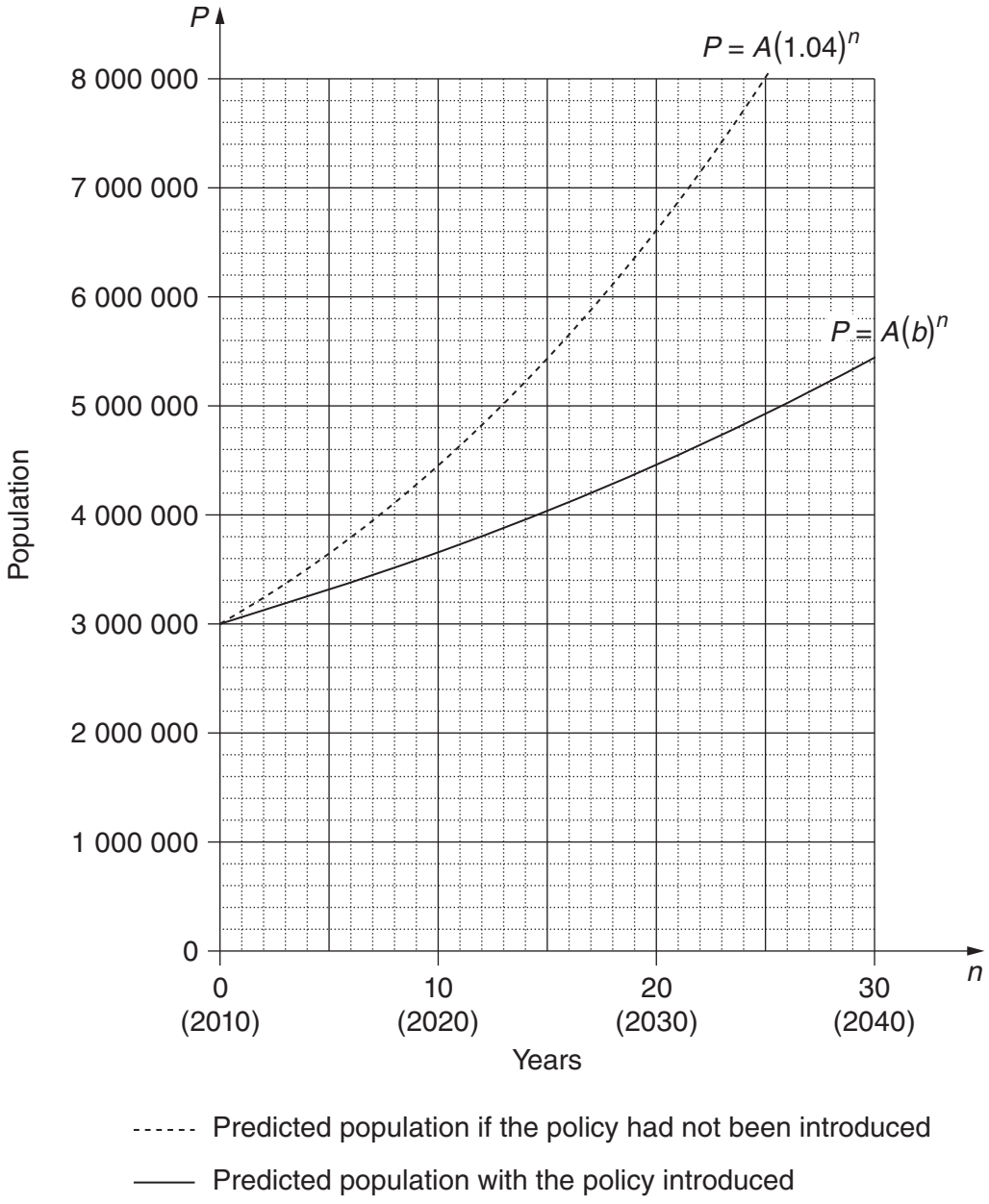
$$P = A(1.04)^n.$$

That year, the city introduced a policy to slow its population growth. The new predicted population was modelled using the equation

$$P = A(b)^n.$$

In both equations, P is the predicted population and n is the number of years after 2010.

The graph shows the two predicted populations.



Question 30 continues on page 27

(i) Use the graph to find the predicted population of Thagoras in 2030 if the population policy had NOT been introduced. **1**

(ii) In each of the two equations given, the value of A is $3\,000\,000$. **1**

What does A represent?

(iii) The guess-and-check method is to be used to find the value of b, in $P = A(b)^n$.

(1) Explain, with or without calculations, why 1.05 is not a suitable first estimate for b. **1**

(2) With $n = 20$ and $P = 4\,460\,000$, use the guess-and-check method and the equation $P = A(b)^n$ to estimate the value of b to two decimal places. Show at least TWO estimate values for b, including calculations and conclusions. **2**

(iv) The city of Thagoras was aiming to have a population under 7 000 000 in 2050. Does the model indicate that the city will achieve this aim? Justify your answer with suitable calculations. **2**

End of paper

BLANK PAGE

© Board of Studies NSW 2012

FORMULAE SHEET

Area of an annulus

$A = \pi\left(R^2 - r^2\right)$

R = radius of outer circle
r = radius of inner circle

Area of an ellipse

$A = \pi ab$

a = length of semi-major axis
b = length of semi-minor axis

Area of a sector

$A = \dfrac{\theta}{360}\pi r^2$

θ = number of degrees in central angle

Arc length of a circle

$l = \dfrac{\theta}{360}2\pi r$

θ = number of degrees in central angle

Simpson's rule for area approximation

$A \approx \dfrac{h}{3}\left(d_f + 4d_m + d_l\right)$

h = distance between successive measurements
d_f = first measurement
d_m = middle measurement
d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius
h = perpendicular height

Volume

Cone $V = \dfrac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \dfrac{1}{3}Ah$

Sphere $V = \dfrac{4}{3}\pi r^3$

r = radius
h = perpendicular height
A = area of base

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area of a triangle

$A = \dfrac{1}{2}ab\sin C$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

or

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

FORMULAE SHEET

Simple interest

$$I = Prn$$

P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

Compound interest

$$A = P(1+r)^n$$

A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods
V_0 = purchase price of the asset
D = amount of depreciation apportioned per period
n = number of periods

Declining balance formula for depreciation

$$S = V_0(1-r)^n$$

S = salvage value of asset after n periods
r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean
x = individual score
n = number of scores
f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$$y = mx + b$$

m = gradient
b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

BOARD OF STUDIES
NEW SOUTH WALES

Centre Number

Student Number

2012

HIGHER SCHOOL CERTIFICATE EXAMINATION

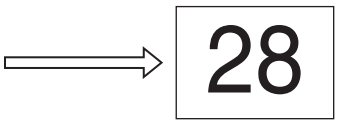
General Mathematics

Writing Booklet

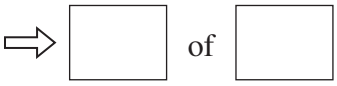
Question 28

Instructions

- Use this Writing Booklet to answer Question 28.



- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3).



this booklet

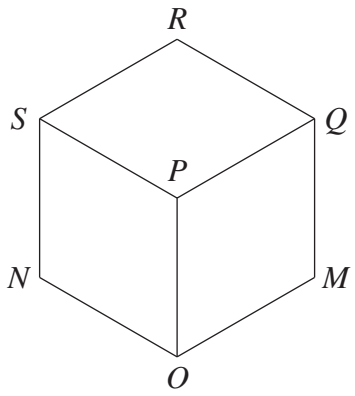
number of booklets for this question

- Write your Centre Number and Student Number at the top of this page.
- Write in black or blue pen (black is recommended).
- You may ask for an extra writing booklet if you need more space.
- If you have not attempted the question(s), you must still hand in the writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.
- You may NOT take any writing booklets, used or unused, from the examination room.

Start here for Question Number: 28

Question 28 (a)

The solid shown is a cube.



Complete a sketch of this cube using the vanishing points V_1 and V_2.

The edge OP has been drawn for you.

Leave all construction lines on your diagram and label the vertices.

V_1 •

• V_2



Additional writing space on back page.

Tick this box if you have continued this answer in another writing booklet.

Office Use Only – Do NOT write anything, or make any marks below this line.