Question 1

(a) $\int x\ln x dx.$

Let $u=\ln x, du=\frac{1}{x}dx$ and $dv=xdx, v=\frac{x^2}{2}.$

$\therefore I=\frac{x^2}{2}\ln x-\int\frac{x}{2}dx=\frac{x^2}{2}\ln x-\frac{x^2}{4}+C.$

(b) $\int_0^3 x\sqrt{x+1}\,dx=\int_0^3(x+1-1)\sqrt{x+1}\,dx$

$=\int_0^3\sqrt{(x+1)^3}\,dx-\int_0^3\sqrt{x+1}\,dx$

$=\left[\frac{2\sqrt{(x+1)^5}}{5}-\frac{2\sqrt{(x+1)^3}}{3}\right]_0^3$

$=\left(\frac{2}{5}\times 32-\frac{2}{3}\times 8\right)-\left(\frac{2}{5}-\frac{2}{3}\right)=\frac{116}{15}.$

(c) $\frac{1}{x^2(x-1)}=\frac{1}{x-1}+\frac{-x-1}{x^2}=\frac{1}{x-1}-\frac{1}{x}-\frac{1}{x^2}.$

$\therefore c=1, b=-1, a=-1.$

$\therefore\int\frac{1}{x^2(x-1)}dx=\ln(x-1)-\ln x+\frac{1}{x}+C.$

(d) $\int\cos^3\theta\,d\theta=\int(1-\sin^2\theta)\cos\theta\,d\theta$

$=\sin\theta-\frac{\sin^3\theta}{3}+C.$

(e) $\int_{-1}^1\frac{1}{5-2t+t^2}dt=\int_{-1}^1\frac{1}{(t-1)^2+4}dt$

$=\frac{1}{2}\left[\tan^{-1}\frac{t-1}{2}\right]_{-1}^1=\frac{1}{2}\left(0-\tan^{-1}(-1)\right)$

$=\frac{1}{2}\tan^{-1}1=\frac{\pi}{8}.$

Question 2

(a) (i) $\overline{w}+z=2+3i+3+4i=5+7i$

(ii) $|w|=\sqrt{2^2+3^2}=\sqrt{13}$

(iii) $\frac{w}{z}=\frac{2-3i}{3+4i}=\frac{2-3i}{3+4i}\times\frac{3-4i}{3-4i}$

$=\frac{6-12-9i-8i}{25}=\frac{-6-17i}{25}.$

(b) (i) $z=\left(\sqrt{3}+i\right)+\left(1+i\sqrt{3}\right)=\left(1+\sqrt{3}\right)+i\left(1+\sqrt{3}\right)$

(ii) Let $A=\sqrt{3}+i, B=1+i\sqrt{3},$

$\angle BOA=\tan^{-1}\sqrt{3}-\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}.$

$\therefore\theta=\pi-\frac{\pi}{6}$ (cointerior angles on parallel lines)

$\therefore\theta=\frac{5\pi}{6}.$

(c) $z^3=8=8\text{cis}(2k\pi).$

$\therefore z=2\text{cis}\frac{2k\pi}{3}, k=0,\pm1$

$=2\text{cis}\,0, 2\text{cis}\frac{2\pi}{3}, 2\text{cis}\left(-\frac{2\pi}{3}\right).$

(d) (i) $(\cos\theta+i\sin\theta)^3=c^3+3c^2(is)+3c(is)^2+(is)^3,$

where $c=\cos\theta, s=\sin\theta,$

$=\cos^3\theta+i3\cos^2\theta\sin\theta-3\cos\theta\sin^2\theta-i\sin^3\theta$

(ii) $(\cos\theta+i\sin\theta)^3=\cos3\theta+i\sin3\theta.$

$\therefore\cos3\theta=\cos^3\theta-3\cos\theta\sin^2\theta,$ equating the real parts

$=\cos^3\theta-3\cos\theta(1-\cos^2\theta)$

$=4\cos^3\theta-3\cos\theta.$

$\therefore 4\cos^3\theta=\cos3\theta+3\cos\theta.$

$\therefore\cos^3\theta=\frac{1}{4}\cos3\theta+\frac{3}{4}\cos\theta.$

(iii) $4\cos^3\theta-3\cos\theta=\cos3\theta=1.$

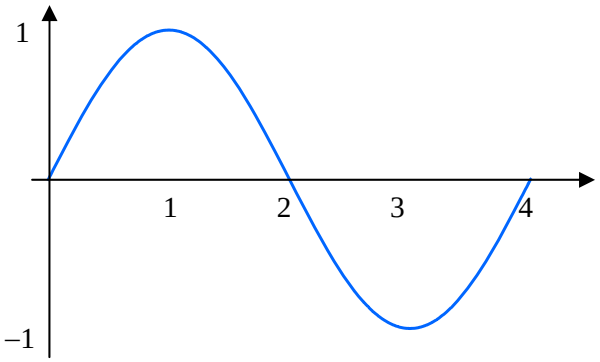
$\therefore 3\theta=2k\pi, k\in Z.$

$\therefore\theta=\frac{2k\pi}{3}.$

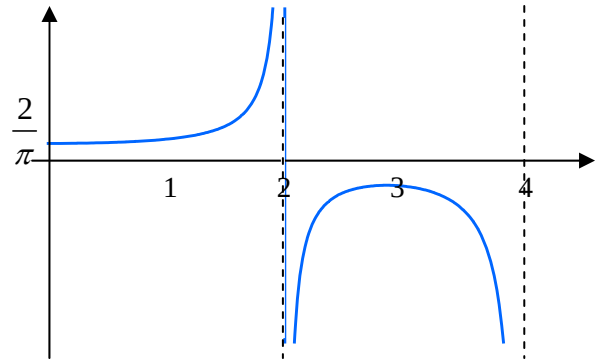
$\therefore$ The smallest positive solution is $\theta=\frac{2\pi}{3}.$

Question 3

(a) (i) Amplitude $=1$, Period $=4$



(ii) $\lim\limits_{x\to 0}\dfrac{x}{\sin\dfrac{\pi x}{2}}=\lim\limits_{x\to 0}\dfrac{\dfrac{\pi x}{2}}{\sin\dfrac{\pi x}{2}}\times\dfrac{2}{\pi}=\dfrac{2}{\pi}.$



(b) Cross-section has base $2y$, height $\sqrt{1-y^2}$

$\therefore A=\dfrac{1}{2}\times 2y\times\sqrt{1-y^2}=y\sqrt{1-y^2}.$

$\partial V=y\sqrt{1-y^2}\times\partial x=\cos x\sqrt{1-\cos^2 x}\partial x$

$=\cos x\sin x\partial x.$

$\therefore V=\int_0^{\frac{\pi}{2}}\cos x\sin x dx=\dfrac{1}{2}\left[\sin^2 x\right]_0^{\frac{\pi}{2}}=\dfrac{1}{2}\ \text{u}^3.$

(c) Let $n=1$, LHS $=2$, RHS $=2\times 1^2=2.$

$\therefore$ True for $n=1.$

Assume $(2n)!\geq 2^n(n!)^2$,

RTP $(2(n+1))!\geq 2^{n+1}((n+1)!)^2.$

LHS $=(2n)!(2n+1)(2n+2)\geq 2^n(n!)^2(2n+1)(2n+2)$

$>2^n(n!)^2(n+1)2(n+1),$ since $2n+1>n+1,$

$\geq 2^{n+1}(n!)^2(n+1)^2$

$=2^{n+1}((n+1)!)^2.$

$\therefore$ True for $n+1.$

$\therefore$ True for all $n\geq 1$ by the principle of Induction.

(d) (i) $9=16(e^2-1),$ using $b^2=a^2(e^2-1)$

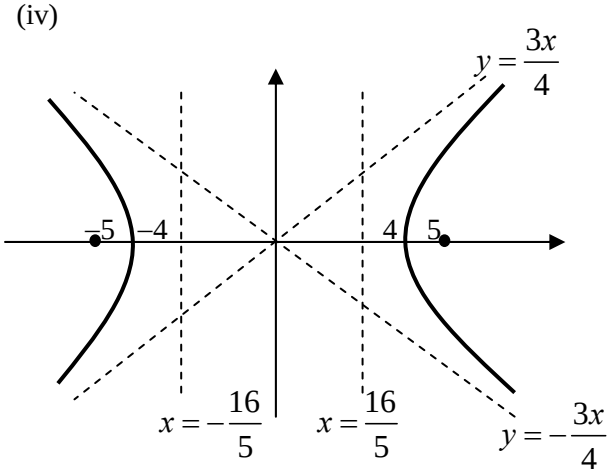
$\therefore e^2=\dfrac{9}{16}+1=\dfrac{25}{16}.$

$\therefore e=\dfrac{5}{4}.$

(ii) Foci $(\pm 5,0)$

(iii) Asymtptotes $y=\pm\dfrac{3x}{4}.$

(iv)



(v) $e^2=1+\dfrac{b^2}{a^2}$, $\therefore$ when $e\to\infty$, either $a\to 0$ or $b\to\infty$: the hyperbola flats out and becomes the y-axis or parallel to the y-axis.

Question4

(a) (i) $|z-a|^2-|z-b|^2=1$

$(x-a)^2+y^2-\left((x-b)^2+y^2\right)=1$

$x^2-2ax+a^2+y^2-\left(x^2-2bx+b^2+y^2\right)=1$

$2(b-a)x+a^2-b^2=1$

$2(b-a)x=1-a^2+b^2.$

$x=\frac{1}{2(b-a)}+\frac{b^2-a^2}{2(b-a)}$

$=\frac{1}{2(b-a)}+\frac{a+b}{2}.$

(ii) The locus of z is the vertical line of equation

$x=\frac{1}{2(b-a)}+\frac{a+b}{2}.$

(b) (i) $\angle ADG=\angle ABC$ (in a cyclic quad, the interior angle = the opposite exterior angle)

$\angle AFG=\pi-\angle ABC$ (cointerior angles on parallel lines)

$\therefore \angle ADG=\pi-\angle AFG.$

$\therefore AFGD$ is a cyclic quad (opposite angles are supplementary)

(ii) alternate angles on parallel lines

(iii) $\angle GAD=\angle GFD$ (angles subtending same arc)

$\therefore \angle GAD=\angle AED$ (because both $=\angle GFD$)

$\therefore GA$ is tangent to the circle $ABCD$ (angle between a tangent and a chord = angle in alternate segment)

(c) (i) Let $y=Af+Bg,$ where f and g satisfy the given differential equation,

$\frac{dy}{dt}=A\frac{df}{dt}+B\frac{dg}{dt}.$

$\frac{d^2y}{dt^2}=A\frac{d^2f}{dt^2}+B\frac{d^2g}{dt^2}.$

Substituting to $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y$ gives

$A\frac{d^2f}{dt^2}+B\frac{d^2g}{dt^2}+3A\frac{df}{dt}+3B\frac{dg}{dt}+2Af+2Bg$

$=A\left(\frac{d^2f}{dt^2}+3\frac{df}{dt}+2f\right)+B\left(\frac{d^2g}{dt^2}+3\frac{dg}{dt}+2g\right)$

$=A\times 0+B\times 0=0.$

$\therefore y=Af+Bg$ is also a solution.

(ii) If $y=e^{kt}$ is a solution,

$\frac{dy}{dx}=ke^{kt},\frac{d^2y}{dx^2}=k^2e^{kt},$

$\therefore k^2e^{kt}+3ke^{kt}+2e^{kt}=0.$

$e^{kt}\left(k^2+3k+2\right)=0.$

$k^2+3k+2=0,\text{since } e^{kt}\neq 0.$

$(k+2)(k+1)=0.$

$\therefore k=-2,-1.$

(iii) $y=Ae^{-2t}+Be^{-t},\frac{dy}{dx}=-2Ae^{-2t}-Be^{-t}.$

When $t=0, y=0, \therefore 0=A+B.$

When $t=0,\frac{dy}{dx}=1,\therefore 1=-2A-B.$

Adding the two equations, $1=-A.$

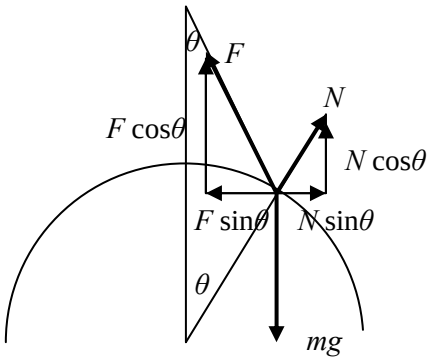
$\therefore A=-1, B=1.$

Question 5

(a) (i) Resolving the forces,

horizontally, $\sum \text{force} = F\sin\theta - N\sin\theta = \text{centripetal force } mr\omega^2$.

vertically, $\sum \text{force} = F\cos\theta + N\cos\theta - mg = 0$, as there is no vertical motion.



(ii) $(F-N)\sin\theta = mr\omega^2, \therefore F - N = \dfrac{mr\omega^2}{\sin\theta}$. (1)

$(F+N)\cos\theta = mg, \therefore F + N = \dfrac{mg}{\cos\theta}$. (2)

(2) − (1) gives $2N = \dfrac{mg}{\cos\theta} - \dfrac{mr\omega^2}{\sin\theta}$.

$\therefore N = \dfrac{1}{2}mg\sec\theta - \dfrac{1}{2}mr\omega^2\operatorname{cosec}\theta$.

(iii) The bead remains in contact with the sphere when $N \geq 0$.

$g\sec\theta - r\omega^2\operatorname{cosec}\theta \geq 0$.

$\omega^2 \leq \dfrac{g\sec\theta}{r\operatorname{cosec}\theta} = \dfrac{g}{r}\tan\theta$.

But $\tan\theta = \dfrac{r}{h}, \therefore \omega^2 \leq \dfrac{g}{h}, \therefore \omega \leq \sqrt{\dfrac{g}{h}}$.

(b) $\dfrac{p}{1+p} + \dfrac{q}{1+q} - \dfrac{r}{1+r} = \dfrac{2pq+p+q}{1+p+q+pq} - \dfrac{r}{1+r}$

$= \dfrac{2pq+2pqr+p+rp+q+rq-r-rp-rq-pqr}{(1+p)(1+q)(1+r)}$

$= \dfrac{2pq+pqr+p+q-r}{(1+p)(1+q)(1+r)}$

$\geq \dfrac{2pq+pqr}{(1+p)(1+q)(1+r)}$, since $p+q \geq r$

≥ 0.

(c) (i) The reflection property of the ellipse:

When a light source is placed at a focus, the light ray from a focus, when hitting an elliptical mirror, will reflect through the other focus.

When light reflects at P, it reflects as if it moves from a light source at R, so that the distance travelled from S to P is the same as the distance travelled from R to P.

$\therefore PS = PR$.

$\therefore \Delta PQS \equiv \Delta PQR$ (RHS).

(Alternatively, let QPN be the tangent,

$\angle RPQ = \angle S'PN$ (vertically opposite angles)

$\angle S'PN = \angle SPQ$ (reflection angles)

$\therefore \angle RPQ = \angle SPQ$.

$\therefore \Delta PRQ \equiv \Delta PQS$ (AAS))

$\therefore SQ = RQ$ (corresponding sides in congruent triangles).

(ii) $SP + S'P = 2a$, by definition of the ellipse.

But $SP = PR$, $\therefore S'P + PR = S'R = 2a$.

(iii) O is the midpoint of $S'S$ and Q is the midpoint of RS, $\therefore OQ = \dfrac{1}{2}S'S$ (the join of the midpoints of two sides of a triangle is parallel to and equals half the third side).

$\therefore OQ = a$.

As O is fixed, the locus of Q is a circle of centre O, radius $a, \therefore Q \in x^2 + y^2 = a^2$.

Question 6

(a) (i) Terminal velocity occurs when acceleration $= 0$

$\therefore mg = kv^2.$

$\therefore v_T = \sqrt{\dfrac{mg}{k}}.$

(ii) $m\dfrac{dv}{dt} = mg - kv^2.$

$\therefore m\displaystyle\int \frac{dv}{mg - kv^2} = \int dt.$

$\therefore t = m\displaystyle\int \frac{dv}{\left(\sqrt{mg} - \sqrt{k}v\right)\left(\sqrt{mg} + \sqrt{k}v\right)}$

$= m \times \dfrac{1}{2\sqrt{mg}} \displaystyle\int \left(\frac{1}{\sqrt{mg} - \sqrt{k}v} + \frac{1}{\sqrt{mg} + \sqrt{k}v} \right) dv$

$= \dfrac{1}{2}\sqrt{\dfrac{m}{g}} \times \dfrac{1}{\sqrt{k}} \ln \dfrac{\sqrt{mg} + \sqrt{k}v}{\sqrt{mg} - \sqrt{k}v} + C$

$= \dfrac{1}{2g}\sqrt{\dfrac{mg}{k}} \ln \dfrac{\sqrt{mg} + \sqrt{k}v}{\sqrt{mg} - \sqrt{k}v} + C$

$= \dfrac{1}{2g}\sqrt{\dfrac{mg}{k}} \ln \dfrac{\sqrt{\dfrac{mg}{k}} + v}{\sqrt{\dfrac{mg}{k}} - v} + C$

$= \dfrac{v_T}{2g} \ln \dfrac{v_T + v}{v_T - v} + C$

When $t = 0, v = v_0, \therefore C = -\dfrac{v_T}{2g} \ln \dfrac{v_T + v_0}{v_T - v_0}.$

$\therefore t = \dfrac{v_T}{2g} \ln \dfrac{(v_T + v)(v_T - v_0)}{(v_T - v)(v_T + v_0)}.$

(iii) For Jac, $v_0 = \dfrac{1}{3}v_T, \therefore$ when his speed $= \dfrac{2}{3}v_T,$

the time taken is $\dfrac{v_T}{2g} \ln \dfrac{\left(v_T + \dfrac{2}{3}v_T\right)\left(v_T - \dfrac{1}{3}v_T\right)}{\left(v_T - \dfrac{2}{3}v_T\right)\left(v_T + \dfrac{1}{3}v_T\right)}$

$= \dfrac{v_T}{2g} \ln \dfrac{\dfrac{5}{3} \times \dfrac{2}{3}}{\dfrac{1}{3} \times \dfrac{4}{3}} = \dfrac{v_T}{2g} \ln \dfrac{5}{2}.$

For Gil, $v_0 = 3v_T, \therefore$ when her speed $= \dfrac{3}{2}v_T,$

the time taken is $\dfrac{v_T}{2g} \ln \dfrac{\left(v_T + \dfrac{3}{2}v_T\right)\left(v_T - 3v_T\right)}{\left(v_T - \dfrac{3}{2}v_T\right)\left(v_T + 3v_T\right)}$

$= \dfrac{v_T}{2g} \ln \dfrac{\dfrac{5}{2} \times -2}{-\dfrac{1}{2} \times 4} = \dfrac{v_T}{2g} \ln \dfrac{5}{2}.$

$\therefore$ The time taken for Jac's speed to double is equal to the time taken for Gil's speed to halve.

(b) (i) $y = \left(f(x)\right)^3, y' = 3\left(f(x)\right)^2 f'(x).$

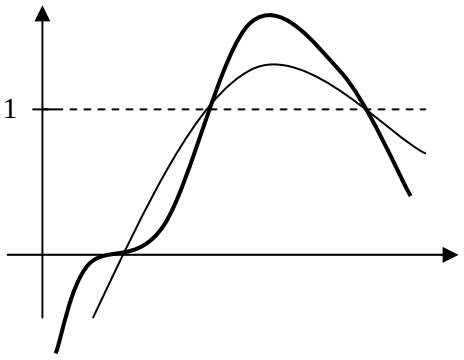
$\therefore$ If $f(a) = 0$ or $f'(a) = 0, y' = 0.$

$\therefore$ Stationary point at $x = a$ if $f(a) = 0$ or $f'(a) = 0.$

(ii) If $f(a) = 0$ and $f'(a) \neq 0,$ then $y = 0, \therefore$ the stationary point occurs on the x-axis, $\therefore$ this is the triple root.

$\therefore$ It has a horizontal point of inflexion at $x = a$ if $f(a) = 0$ and $f'(a) \neq 0.$

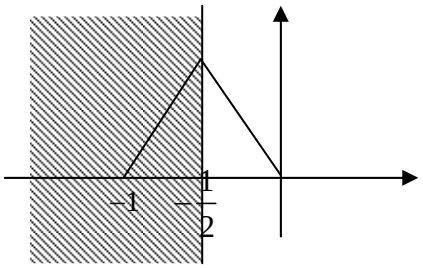
(iii)



(c) $\left|1 + \dfrac{1}{z}\right| = \left|\dfrac{z+1}{z}\right| = \dfrac{|z+1|}{|z|}.$

$\therefore \left|1 + \dfrac{1}{z}\right| \leq 1$ is equivalent to $|z+1| \leq |z|.$

$\therefore$ The locus of z is the region in the left of the perpendicular bisector of the join of 0 and $-1.$



Question 7

(a) $\partial V = 2\pi Rh\partial x = 2\pi(1-x)y\partial x$

$= 2\pi\dfrac{(1-x)x}{1+x^2}\partial x = 2\pi\dfrac{x-x^2}{1+x^2}\partial x$

$= 2\pi\left(-1+\dfrac{x+1}{1+x^2}\right)\partial x$

$\therefore V = 2\pi\displaystyle\int_0^1\left(-1+\frac{x+1}{1+x^2}\right)dx$

$= 2\pi\left[-x+\dfrac{1}{2}\ln(1+x^2)+\tan^{-1}x\right]_0^1$

$= 2\pi\left(-1+\dfrac{1}{2}\ln 2+\dfrac{\pi}{4}\right)$ u^3.

(b) (i) Let $u = 4-x, du = -dx$.

When $x=1, u=3$; when $x=3, u=1$.

$\displaystyle\int_1^3\frac{\cos^2\left(\frac{\pi}{8}x\right)}{x(4-x)}dx = \int_3^1\frac{\cos^2\left(\frac{\pi}{8}(4-u)\right)}{u(4-u)}(-du)$

$= \displaystyle\int_1^3\frac{\cos^2\left(\frac{\pi}{2}-\frac{\pi}{8}u\right)}{u(4-u)}du = \int_1^3\frac{\sin^2\left(\frac{\pi}{8}u\right)}{u(4-u)}du$

(ii) $\therefore I = \displaystyle\int_1^3\frac{\cos^2\left(\frac{\pi}{8}x\right)}{x(4-x)}dx = \int_1^3\frac{\sin^2\left(\frac{\pi}{8}x\right)}{x(4-x)}dx.$

$\therefore I = \dfrac{1}{2}\displaystyle\int_1^3\frac{\cos^2\left(\frac{\pi}{8}x\right)+\sin^2\left(\frac{\pi}{8}x\right)}{x(4-x)}dx$

$= \dfrac{1}{2}\displaystyle\int_1^3\frac{1}{x(4-x)}dx = \frac{1}{8}\int_1^3\left(\frac{1}{x}+\frac{1}{4-x}\right)dx$

$= \dfrac{1}{8}\left[\ln\dfrac{x}{4-x}\right]_1^3 = \dfrac{1}{8}\ln\dfrac{3}{\frac{1}{3}} = \dfrac{1}{8}\ln 9.$

(c) (i) $\dfrac{x^2}{a^2}+\dfrac{(mx+c)^2}{b^2}=1.$

$\left(b^2+a^2m^2\right)x^2+2a^2mcx+\left(a^2c^2-a^2b^2\right)=0.$

The line is a tangent, $\therefore \Delta = 0.$

$4a^4m^2c^2-4\left(b^2+a^2m^2\right)\left(a^2c^2-a^2b^2\right)=0.$

$a^2m^2c^2-\left(b^2+a^2m^2\right)\left(c^2-b^2\right)=0$

$a^2m^2c^2-b^2c^2+b^4-a^2m^2c^2+a^2m^2b^2=0$

$-c^2+b^2+a^2m^2=0$

$\therefore b^2+a^2m^2=c^2.$

(ii) The equation of ℓ is $mx-y+c=0$,

$\therefore QS = \left|\dfrac{mae-0+c}{\sqrt{m^2+1}}\right| = \left|\dfrac{mae+c}{\sqrt{m^2+1}}\right|$

(iii) $QS\times Q'S' = \left|\dfrac{mae+c}{\sqrt{m^2+1}}\right|\left|\dfrac{mae-c}{\sqrt{m^2+1}}\right|$

$= \left|\dfrac{m^2a^2e^2-c^2}{m^2+1}\right| = \left|\dfrac{m^2a^2e^2-(b^2+a^2m^2)}{m^2+1}\right|$

$= \left|\dfrac{m^2a^2\left(e^2-1\right)-b^2}{m^2+1}\right| = \left|\dfrac{-m^2b^2-b^2}{m^2+1}\right|$

$= b^2\left|\dfrac{m^2+1}{m^2+1}\right| = b^2.$

Question 8

(a) $I_m = \int_0^1 x^m (x^2-1)^5 dx$

Let $u = x^{m-1}, du = (m-1)x^{m-2}$

and $dv = x(x^2-1)^5 dx, v = \dfrac{(x^2-1)^6}{12}$.

$\therefore I_m = \left[x^{m-1} \dfrac{(x^2-1)^6}{12} \right]_0^1 - \dfrac{m-1}{12} \int_0^1 x^{m-2} (x^2-1)^6 dx$

$= -\dfrac{m-1}{12} \int_0^1 x^{m-2} (x^2-1)(x^2-1)^5 dx$

$= -\dfrac{m-1}{12} \int_0^1 (x^m - x^{m-2})(x^2-1)^5 dx$

$= -\dfrac{m-1}{12} (I_m - I_{m-2})$

$\therefore (12+m-1) I_m = (m-1) I_{m-2}$.

$\therefore I_m = \dfrac{m-1}{m+11} I_{m-2}$.

(b) (i) $\dfrac{7}{7} \times \dfrac{6}{7} \times \ldots \times \dfrac{1}{7} = \dfrac{7!}{7^7}$.

(ii) $1 - \Pr(\text{all balls are selected}) = 1 - \dfrac{7!}{7^7}$.

(iii) Choosing a ball not to be selected in ${}^7C_1$ ways, choosing a ball to repeat in ${}^6C_1$ ways, arranging the 7 balls, of which 2 are the same in $\dfrac{7!}{2!}$ ways, and each ball has $\dfrac{1}{7}$ chance of being selected, $\therefore \Pr(\text{exactly one ball is not selected})$

$= {}^7C_1 \times {}^6C_1 \times \dfrac{7!}{2!} \times \left(\dfrac{1}{7} \right)^7$.

(c) (i) Since β is a root,

$\beta^n + a_{n-1}\beta^{n-1} + \ldots + a_1\beta + a_0 = 0$.

$\therefore \beta^n = -a_{n-1}\beta^{n-1} - \ldots - a_1\beta - a_0$.

$\therefore |\beta^n| = |a_{n-1}\beta^{n-1} + \ldots + a_1\beta + a_0|$

$\le |a_{n-1}\beta^{n-1}| + \ldots + |a_1\beta| + |a_0|$, since the modulus of the sum of n complex numbers is always less than the sum of the moduli of these n complex numbers (triangular inequality is a special case of 2 numbers)

$\le M\left(|\beta^{n-1}| + \ldots + |\beta| + 1 \right)$, since $a_{n-1}, \ldots, a_1, a_0$ all are less than or equal M.

(ii) Inside the brackets is a GP, its sum = $\dfrac{|\beta|^n - 1}{|\beta| - 1}$.

$|\beta|^n \le M \left(\dfrac{|\beta|^n - 1}{|\beta| - 1} \right) = M \left| \dfrac{|\beta|^n - 1}{|\beta| - 1} \right|$, noting that $|\beta^{n-1}| + \ldots + |\beta| + 1 > 0$ always.

$1 \le M \left| \dfrac{1 - \dfrac{1}{|\beta|^n}}{|\beta| - 1} \right|$, on dividing by $|\beta|^n$

$1 < M \left| \dfrac{1}{|\beta| - 1} \right|$.

$\therefore |\beta| - 1 < M$, or $|\beta| < 1 + M$.

(d) Let β be the root of the equation $S(z)$

$= \sum_{k=0}^{n} c_k z^k$, where $z = x + \dfrac{1}{x}, \therefore \beta$ is also the root of $\sum_{k=0}^{n} \dfrac{c_k}{c_n} z^k$ (so that the coefficient of z^n is 1)

From part (c), $|\beta| < 1 + M$, where M is the maximum of $\left| \dfrac{c_k}{c_n} \right|$, but $|c_k| \le |c_n|, \therefore M \le 1$.

$\therefore |\beta| < 2, \therefore \left| x + \dfrac{1}{x} \right| < 2$.

$x^2 - 2x + 1 < 0$.

$(x-1)^2 < 0$.

$\therefore x$ is not real.